Automatically Extracting Differential Equations from Data

Sparse Regression Tools for System Identification

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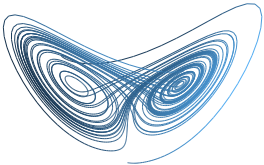
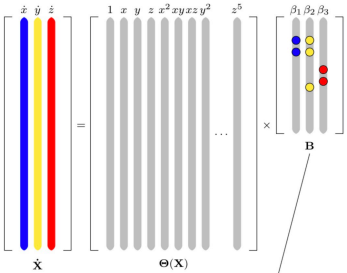
June 4, 2021

Lorenz System

Lorenz Equations:

$$\dot{x} = \sigma(y - x),$$
$$\dot{y} = x(\rho - z) - y,$$
$$\dot{z} = xy - \zeta z.$$

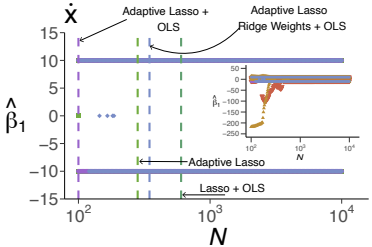


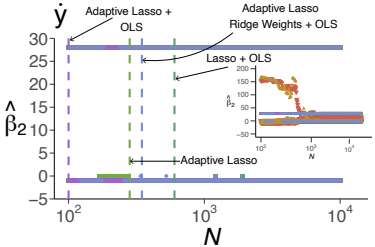


$$\dot{\mathbf{X}} = \mathbf{\Theta}(\mathbf{X})\mathbf{B} + \eta\mathbf{Z}$$

- For the constants, we use the standard parameter values $\sigma = 10$, $\rho = 28$, and $\zeta = 8/3$.

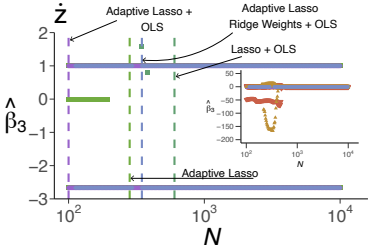
Lorenz System Identification as the Number of Time Steps Increases





$$\dot{x} = \sigma(y - x),$$
$$\dot{y} = x(\rho - z) - y,$$
$$\dot{z} = xy - \zeta z.$$

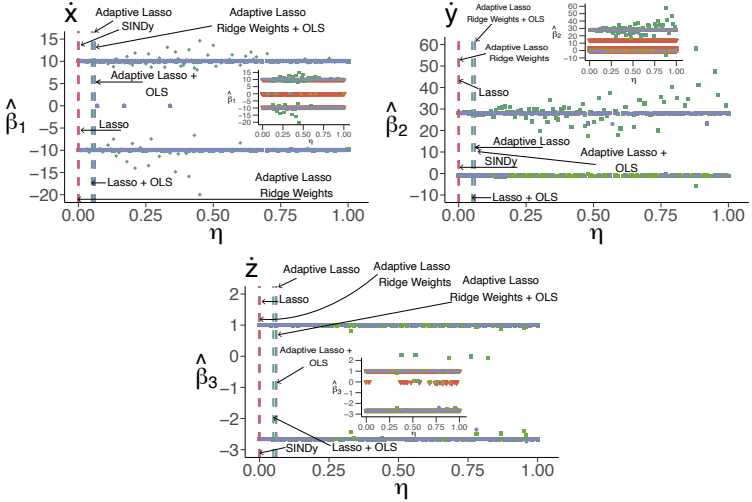
$$\begin{array}{c} \\ 1 \\ x \\ y \\ z \\ x^2 \\ xy \\ xz \\ \vdots \\ z^5 \end{array} \begin{array}{c} \begin{array}{ccc} \beta_1 & \beta_2 & \beta_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ -\sigma & \rho & 0 \\ \sigma & -1 & 0 \\ 0 & 0 & -\zeta \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$



Algorithm

- Adaptive Lasso
- Adaptive Lasso Ridge Weights
- Lasso
- SINDy
- Adaptive Lasso + OLS
- Adaptive Lasso Ridge Weights + OLS
- Lasso + OLS

Lorenz System Identification as Noise Magnitude Increases



Conclusion

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