On Combining Data with Prior Parameter Information

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Question: How much prior information do we need to recover information given an incomplete data set?

Purpose:

- Predictive models contain unknown parameters that can be estimated with observational data.
- The process of combining models with data to obtain unknowns is called inversion.
- Inversion requires prior information for unknowns.
- Prior information adds bias which can be alleviated with uncertainty estimates for them.

Accurate uncertainty estimates are difficult to obtain.

Methods:

u=G(z)

G- predictive model

u- data

z - unknows

$z= G^{-1}(u)$ is typically not possible.

Singular values indicate if we have enough prior information to recover z.

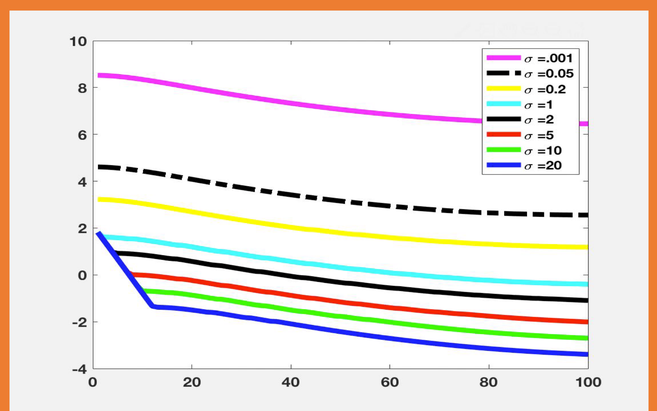


Figure 1: Log of singular values with varying prior uncertainty.

- Adding prior information to the predictive model produces a mathematical model with singular values that do not go to zero, indicating the inverse problem is stable.
- Less prior uncertainty produces a more stable problem.
- As prior uncertainty increases the largest singular values do not change while the smaller ones to go zero.

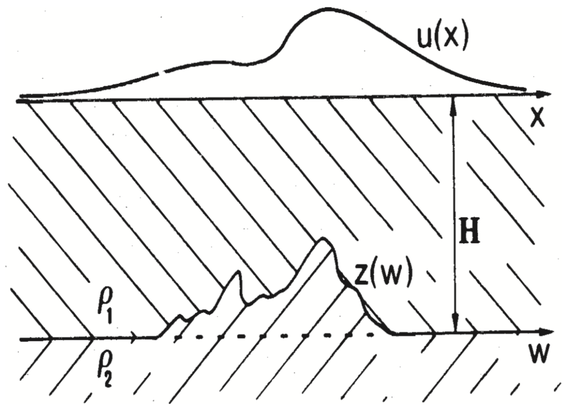


Figure 2: The gravitational inverse problem. [1]

Predictive Model: Given anomalies u(x) recover frontier z(w)

$$u(x) = \int_a^b log \frac{(x-w)^2+H^2}{(x-w)^2+[H-z(w)]^2} dw$$

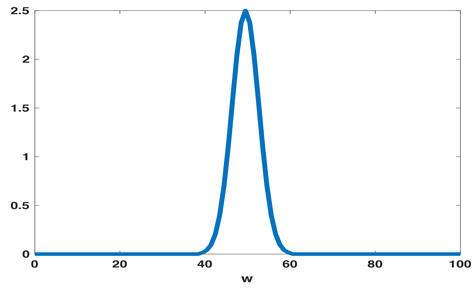


Figure 3: True z(w)

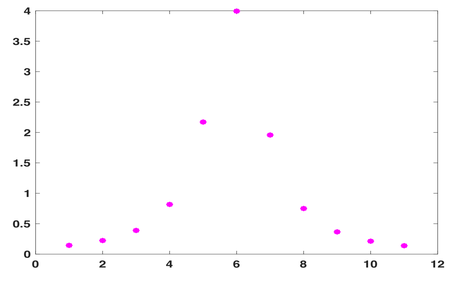
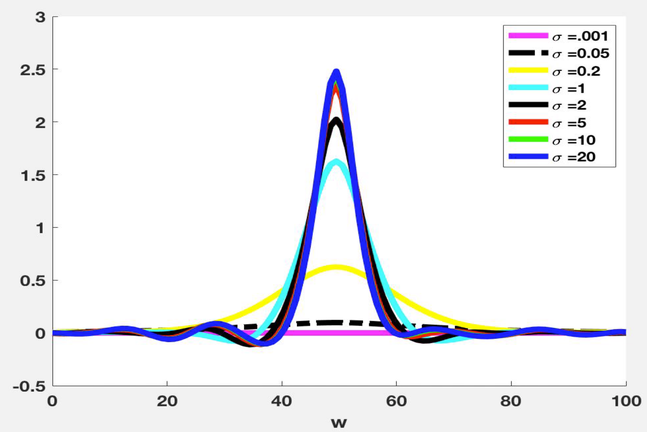


Figure 4: Observed u(x)

Recovered Parameters z(w) :



Prior Information: $z_0 = 0$ and

$$C_{p_0p_0}(w,w') = \sigma^2 \exp\left\{-\frac{1}{2}\frac{(w-w')^2}{\Delta^2}\right\}$$

- Small prior uncertainty ensures problem is stable but recovered z(w) *(magenta)* remains at z=0 which is far from true z(w).
- Accurate prior uncertainty $(\sigma = 5)$ recovers z(w) well.
- Large prior uncertainty $(\sigma = 10, 20)$ *also* recovers z(w) well.

References:

[1] Albert Tarantola, Bernard Valette. Generalized Nonlinear Inverse Problems Solved Using the Least Squares Criterion. Institut de Physique du Globe de Paris, 75005 Paris, France. Reviews of Geophysics and Space Physics, Vol. 20, No. 2, pages 219–232; May 1982.

Conclusions:

- Large prior uncertainty estimates allow the mathematical model to match the data more quickly in nonlinear iterations.
- Effective prior uncertainty estimates can be chosen by analyzing singular values with varying uncertainties.

Future Work:

Use singular value curves (Figure 1) to identify appropriate uncertainty necessary to recover unknowns parameters. This requires significantly less computation than actually recovering the parameters with different uncertainty estimates.