

# Classification and Regression Trees and Forests for Incomplete Data from Sample Surveys

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and do not necessarily reflect the policies  
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## Motivation

Analysis of sample survey data often requires adjustments for **missing** values.

Standard adjustments rely on auxiliary variables for **both** responding and non-responding units.

Their application can be challenging when the **auxiliary** variables are numerous and are themselves subject to **incomplete-data** problems.

Performance depends on the **number** of X variables and their **incomplete-data patterns**.

This paper shows how classification and regression trees and forests can be applied to these cases.

# *The U.S. Consumer Expenditure (CE) Quarterly Interview Survey*

A nationwide household survey conducted  
by the U.S. Bureau of Labor Statistics.

It collects information  
on consumers' **expenditures** and **incomes**  
as well as **characteristics** of the consumers.

## Goal

Estimate the population mean of **INTRDVX**,  
the amount of interest and dividend income  
received during the past 12 months.

*High rate of item missingness*

## *CE Public-Use Microdata (2013)*

- ▶ **Consumer Units** (CUs) are roughly equivalent to households.
- ▶ Excluded CUs for which INTRDVX codes were “**validly missing**” or “**topcoded.**”
- ▶ Remaining 4609 CUs:  
1771 **missing** and 2838 **non-missing** INTRDVX

## *Missingness in Predictor Variables*

Potential **predictor variables** were themselves subject to relatively high item-missingness rates.

- ▶ **630** predictor variables available
- ▶ **124** variables have missing values
- ▶ **67** variables have more than 95% values missing



## Adjusting for Missingness

Form cells to have

**common response propensity**  $\pi$  or  
**common mean** of  $Y$

Bias under stochastic response model  
(Kalton & Maligalig 1991)

$$B(\hat{y}_{\pi}) \doteq \frac{1}{N\bar{\pi}} \sum (\pi_i - \bar{\pi}) (y_i - \bar{Y}_U)$$

## *Classification and Regression Trees and Forests*

**Classification** trees and forests  
to estimate the unit-level **propensity** for item  
missingness and obtain inverse probability weighted  
(**IPW**) estimates.

**Regression** trees and forests  
to estimate conditional **means** in **adjustment cells**  
defined by the nodes of the trees.

# Features of GUIDE

*Generalized, Unbiased, Interaction Detection and Estimation by W-Y Loh*

For the best split variable, **first selects** an  $X$  variable, then finds the best split on the **selected  $X$** .

For **missing** values in the  $X$  variables, it creates a **missing level** to use in the chi-square tests for variable selection.

## Classification Trees and Forests

**INTRDVX**\_, a flag variable for INTRDVX, is a dependent variable.

Traditional methods of obtaining the estimated **probability** that  $y_i$  is responding, are difficult to apply due to the **many**  $X$  variables and the large numbers of **missing** values in  $X$ .

**Classification** trees and forests  
to estimate the unit-level **propensity**  
for item missingness  
and obtain inverse probability weighted (**IPW**)  
estimates.

## *Inverse Probability Weighted (IPW) Estimate (Little, 1986)*

$$\left( \sum_{i \in S_R} \hat{\pi}_i^{-1} w_i \right)^{-1} \sum_{i \in S_R} \hat{\pi}_i^{-1} w_i y_i$$

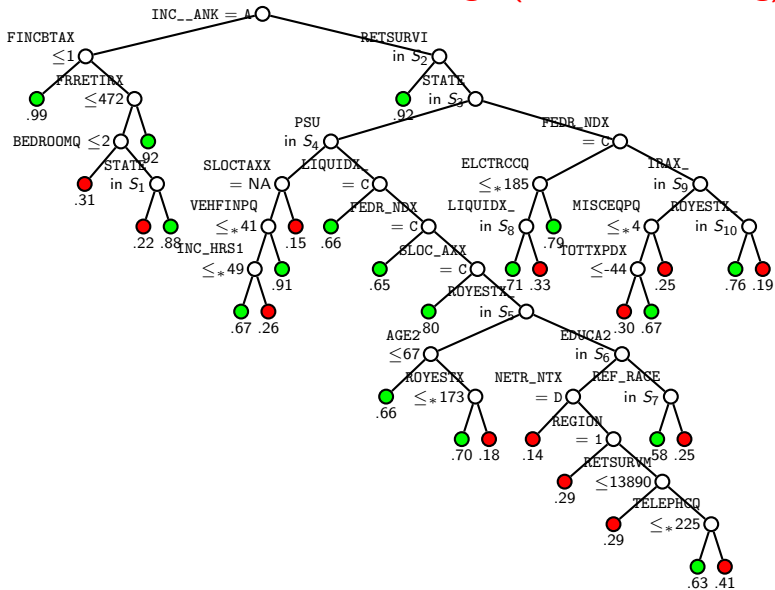
where

$S_R$  is the sample subset of responding  $y_i$ ,

$\hat{\pi}_i$  the estimated probability that  $y_i$  is responding,

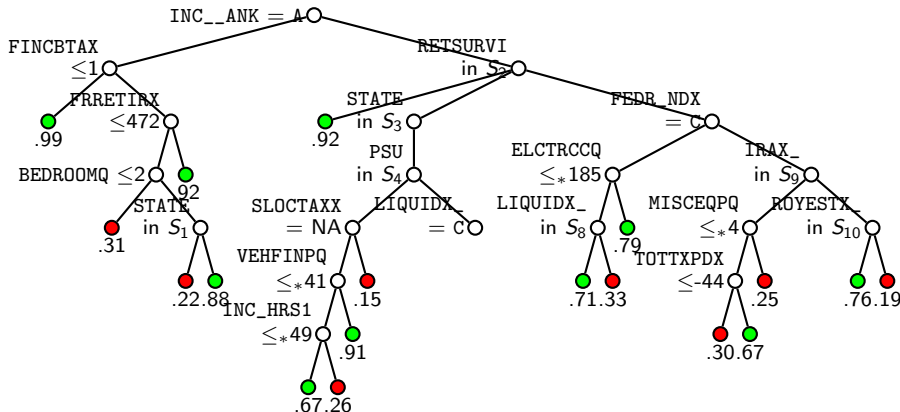
$w_i$  sampling weight.

# GUIDE classification tree estimating P(INTRDVX missing)



# Enlarged View

## GUIDE classification tree estimating $P(\text{INTRDVX missing})$





## Missing-variable flags

are important predictors of missingness propensity of INTRDVX.

Tree methods can explore the use of **both** observed values and related missing-variable flags.

## *Regression Trees and Forests*

**Regression** trees and forests are used to model the conditional mean of INTRDVX.

Unlike classification models, the regression tree uses only 2838 CUs with **non-missing** INTRDVX.

## Mean Imputation Estimate

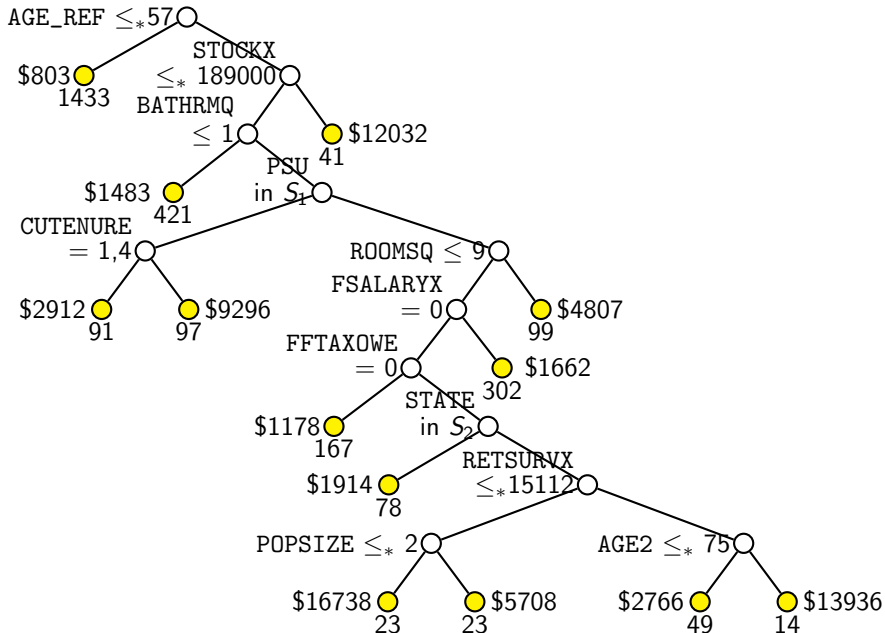
$$\left( \sum_{k \in S} w_k \right)^{-1} \left( \sum_{k \in S_R} w_i y_i + \sum_{j \in S_{NR}} w_j \hat{y}_j \right)$$

where

$$S = S_R \cup S_{NR},$$

$\hat{y}_j$  the predicted from  $X$  values in  $S_{NR}$ .

## GUIDE regression tree estimating $y_i = \text{INTRDVX}$



## Comparison of Methods

<b>AME</b>	AMELIA imputation (multivariate normal likelihood and EM)
<b>AIPW</b>	IPW using logistic regression with AMELIA for X imputation
<b>MICE</b>	MICE imputation
<b>GMICE</b>	MICE using GUIDE instead of linear and logistic regression
<b>GCT</b>	IPW using GUIDE classification tree
<b>GRT</b>	GUIDE regression tree imputation
<b>GCF</b>	IPW using GUIDE classification forest
<b>GRF</b>	GUIDE regression forest imputation
<b>SIM</b>	Simple estimate ignoring missing responses

## Comparison of Methods

Applied methods to 3 nested sets of  $X$  variables

- ▶ The set of **19** variables for which MICE does not fail
- ▶ The set of **52** variables by combining 19 above and the top 20  $X$  variables for predicting INTRDVX\_ and INTRDVX
- ▶ The full set of **587** variables

## *Estimates of Mean INTRDVX (SIM = 1900)*

	19 variables		52 variables		587 variables	
	Est.	Sec.	Est.	Sec.	Est.	Sec.
AME	2088	139	2184	111068	-	-
AIPW	2055	122	1900	72029	-	-
GCT	1925	8	1946	13	1969	197
GCF	1983	113	1926	173	1914	2028
GRT	2055	8	2010	14	2009	190
GRF	2007	248	1993	360	1944	2030
GMICE	2094	57	2005	434	2002	76874
MICE	2031	430	Fail	-	Fail	-

## Computation Time

- ▶ Every method works on the set of 19 variables.
- ▶ **MICE** is the slowest for 19 predictors and fails for other two sets.
- ▶ **AME** is the second slowest for 19 variables. Computation was terminated for 587 variables.
- ▶ Single tree is much faster than forest.



## *Estimates of Mean INTRDVX*

- ▶ **SIM** estimates as \$1900 for all three sets.
- ▶ Every method works on the set of 19 variables.  
**MICE** fails for the other two sets
- ▶ The estimates range from a low of \$1900 (**SIM**) to a high of \$2184 (**AME**, 52 variables)
- ▶ Majority of the estimates lie with one **s.e.** (\$146) of balanced repeated replicate variance estimate and all within two s.e..

## Findings

Classification and Regression Trees and Forests methods are

- ▶ often **competitive** with traditional methods in terms of bias and mean squared error for mean estimation.
- ▶ **not** limited by sample size.
- ▶ **not** hindered or crippled by multicollinearity or quasi-complete separation.
- ▶ orders of magnitude **faster** compared to traditional methods.

## Summary

Potential predictor variables were many and were themselves subject to relatively high item-missingness rates.

- ▶ Applied **classification** trees to estimate the **propensity** for item missingness, to be used in inverse probability weighting.
- ▶ Applied **regression** trees to estimate **conditional means** in adjustment cells defined by the nodes of the trees.

# References

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