The 2020 U.S. Stock Market Crash

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Abstract
We used the log-periodic power law singularity (LPPLS) methodology to systematically investigate the 2020 stock market crash in the U.S. equities sectors using the Wilshire 5000 Total Market index, the S&P 500 index, the S&P MidCap 400 index, and the Russell 2000 index. During the crash, all four indexes lost more than a third of their values within five weeks, while both the middle capitalization stocks and the small capitalization stocks have suffered much greater losses than the large capitalization stocks and stocks overall. The results indicate that the price trajectories of these indexes prior to the 2020 stock market crash have clearly featured the obvious LPPLS bubble pattern and were indeed in a positive bubble regime. Contrary to the popular belief that the COVID-19 led to the 2020 stock market crash, the 2020 U.S. stock market crash was largely endogenous, stemming from the increasingly systemic instability of the stock market itself.

Key Words: 2020 U.S. stock market crash, COVID-19, Log-periodic power law singularity (LPPLS), Financial bubble and crash

1. Introduction

Starting on February 20, 2020, the Wilshire 5000 Total Market index, a benchmark for the market value of all stocks actively traded in the United States, dropped by 34.9% in the next five weeks, which is the worst percentage loss in approximately one month since the Great Recession in 2008, indicating the end of the 11-year boom in U.S. stock markets. During this crash, the S&P 500 index, the most commonly tracked stock index, lost 33.9% of its value and triggered the level-1 trading curbs for the consecutive four times within 10 days, resulting in major U.S. stock markets to suspend trading for 15 minutes on 3/9/2020, 3/12/2020, 3/16/2020, and 3/18/2020, respectively. The 2020 U.S. Stock market Crash had a great impact on the lives and livelihoods of many people throughout the country and caused permanent losses to the wealth of many investors, especially those who lacked risk management experience. To stabilize the financial system and prevent the economic recession from intensifying, the Federal Reserve had adopted a series of critical measures, including reducing the target range of the federal funds rate to near zero and reviving the Quantitative Easing (QE) program and expanding the QE purchases to an unlimited amount.

In this study, we employed the Log Periodic Power Law Singularity (LPPLS) model to systematically analyze the 2020 stock market crash in the United States. By syndicating the mathematical and statistical physics of bifurcations and phase transitions, the economic theory of rational expectations, and behavioral finance of herding of traders, the LPPLS model defines a positive (or negative) financial bubbles as a process of unsustainably faster-than-exponential growth (or drop) to reach an infinite return in finite time, leading to a short-term correction molded by the symmetry of discrete scale invariance (Sornette, 1998). Two types of agents are taken into account in the LPPLS model, namely the rational traders who trade based on rational expectations, and the noise traders who are prone to imitation and herding behavior. Assuming that the collective behavior of noise traders will destabilize asset prices through large-scale herding and imitation transactions, the LPPLS model diagnoses financial bubbles by capturing two distinct characteristics of price trajectories in bubble regimes: the faster-than-exponential growth resulting from positive feedbacks by imitation and herding behavior of noise traders, and the accelerating log-periodic volatility fluctuations of the price growth due to expectations of higher returns and an upcoming crash. Recently, the versatile LPPLS model has made waves diagnosing bubbles and crashes in various financial markets including the stock markets (Demirer et al., 2019; Shu, 2019; Shu & Zhu, 2019, 2020a; Song et al., 2021; Sornette et al., 2015) and the cryptocurrency market (Shu & Zhu, 2020b; Wheatley et al., 2019).

To study the performance of U.S. equities sectors with different levels of total market capitalization in the 2020 stock market crash, we adopted four major U.S. stock market indexes, including the Wilshire 5000 Total Market (W5000) index, the S&P 500 (SP500) index, the S&P MidCap 400 (SP400) index, and the Russell 2000 (R2000) index, representing the benchmarks for the market value of all stocks actively traded in the United States, the 500 large capitalization stocks, the middle capitalization stocks, and the small capitalization stocks listed on the U.S. stock exchanges, respectively. Figure 1 shows the evolution of their price trajectories from January 2019 to December 2020.

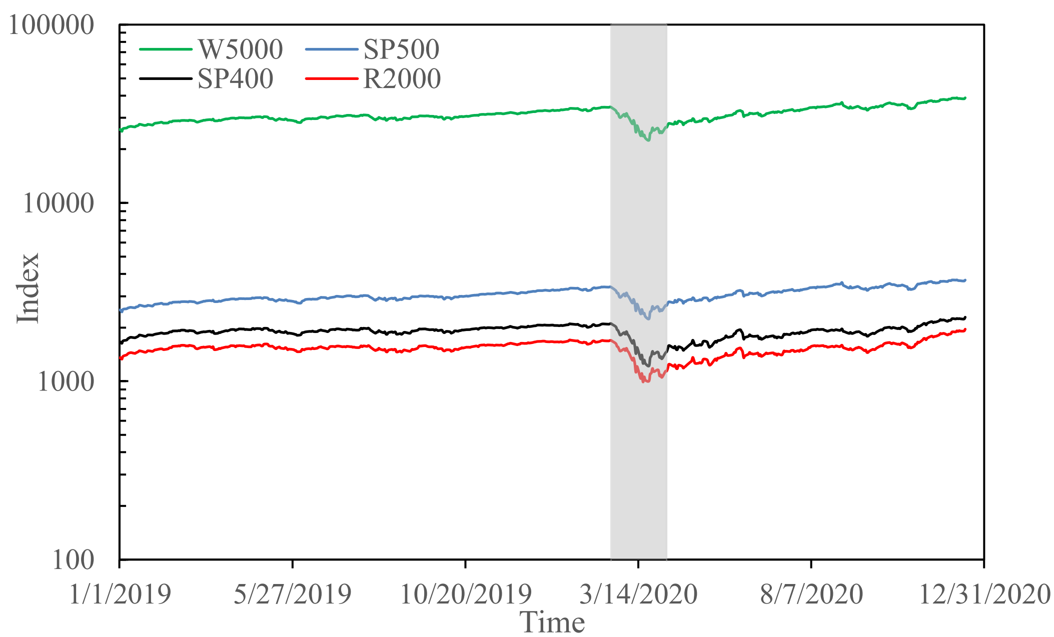


Figure 1: Evolution of price trajectories of the W5000, SP500, SP400, and R2000 indexes from January 2019 to December 2020. The shadowed band shows the period of the 2020 U.S. stock market crash.

2. Methodology

2.1 The Log-Periodic Power Law Singularity (LPPLS) Model

The LPPLS model is also called the Johansen - Ledoit - Sornette (JLS) model or Log-Periodic Power Law (LPPL) Model. The simple mathematical formula of the LPPLS can be described as (Filimonov & Sornette, 2013):

$$\mathrm{LPPLS}(t) \equiv E[\ln p(t)] = A + B(t_c - t)^m + C_1(t_c - t)^m \cos[\omega\, ln(t_c - t)] \\ +C_2(t_c - t)^m \sin[\omega\, ln(t_c - t)] \quad (1)$$

where $p(t)$ is the observed asset price and A is the expected value of the log-price at the critical time t_c. The critical time t_c is the most probable time for a regime change in a form of a major crash or a great change of growth rate. In a bubble regime, the power parameter m ranges between 0 and 1 to ensure that not only the price remains finite at the t_c, but also the expected logarithmic price diverges at the t_c.

The LPPLS model contains three nonlinear parameters (t_c, m, ω) and four linear parameters (A, B, C_1, C_2). Using the L^2 norm, the sum of squared residuals of the LPPLS formula can be written as:

$$F(t_c, m, \omega, A, B, C_1, C_2) = \sum_{i=1}^{N} [ln\, p\,(\tau_i) - A - B(t_c - \tau_i)^m - C_1(t_c - \tau_i)^m\, cos(\,\omega\, ln(\,t_c - \tau_i)) \\ - C_2(t_c - \tau_i)^m\, sin(\,\omega\, ln(\,t_c - \tau_i))]^2 \quad (2)$$

where $\tau_1 = t_1$ and $\tau_N = t_2$. The 4 linear parameters (A, B, C_1, C_2) can be solved analytically through the following matrix equations

$$\begin{pmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum f^2{}_i & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum g^2{}_i & \sum h_i g_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h^2{}_i \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C}_1 \\ \hat{C}_2 \end{pmatrix} = \begin{pmatrix} \sum ln\, p_i \\ \sum f_i\, ln\, p_i \\ \sum g_i\, ln\, p_i \\ \sum h_i\, ln\, p_i \end{pmatrix} \quad (3)$$

The three nonlinear parameters (t_c, m, ω) can be estimated by solving the resulting nonlinear optimization problem:

$$\{\hat{t}_c, \widehat{m}, \widehat{\omega}\} = arg \min_{\{t_c, m, \omega\}} F_1(t_c, m, \omega) \quad (4)$$

In this study, we use the covariance matrix adaptation evolution strategy (Hansen et al., 1995) to solve this optimization problem.

2.2 LPPLS Confidence Indicator

The LPPLS confidence indicator (Sornette et al., 2015) is defined as the fraction of fitting windows where the calibrated LPPLS models meet the specified filter constrains to quantity the sensitivity of the detected bubble pattern to the selection of the start time t_1 in the fitting windows. Larger LPPLS confidence indicator simply indicates more fitting windows having the signatures of the LPPLS model and hence the detected LPPLS bubble patterns are more reliable.

To ensure model robustness, the search space is set to (Shu & Zhu, 2020b):

$$m \in [0,1], \omega \in [1,50], t_c \in \left[t_2, t_2 + \frac{t_2 - t_1}{3}\right], \frac{m|B|}{\omega\sqrt{C_1^2 + C_2^2}} \geq 1 \quad (5)$$

To determine the valid LPPLS fits, the calibrated LPPLS model is filtered under the following constrains (Shu & Zhu, 2020b):

$$m \in [0.01,0.99], \omega \in [2,25], t_c \in \left[t_2, t_2 + \frac{t_2 - t_1}{5}\right], \frac{\omega}{2}\ln\left(\frac{t_c - t_1}{t_c - t_2}\right) \geq 2.5,$$
$$\max\left(\frac{|\widehat{p_t} - p_t|}{p_t}\right) \leq 0.20,\ p_{lomb} \leq \alpha_{sign}, \ln(\widehat{p_t}) - \ln(p_t) \sim \mathrm{AR}(1) \quad (6)$$

3. LPPLS Bubble Identification

Using the daily data of the W5000, SP500, SP400, and R2000 indexes from January 2, 2019, to June 30, 2020 (https://finance.yahoo.com/), we detected both positive and negative bubbles in the U.S. stock markets based on the LPPLS confidence indicator. A positive bubble is related to accelerating growth trend which is vulnerable to regime changes in the form of volatile sideway plateaus or large crashes. In contrast, a negative bubble is related to accelerating downward trend susceptible to regime changes in the form of rallies or volatile sideway plateaus. In this study, the LPPLS confidence indicator is calculated by shrinking the length of time windows $t_2 - t_1$ from 650 trading days to 30 trading days in steps of 5 trading days, thereby creating 125 fitting windows for each t_2, and moving the endpoint t_2 from January 2, 2019 to June 30, 2020.

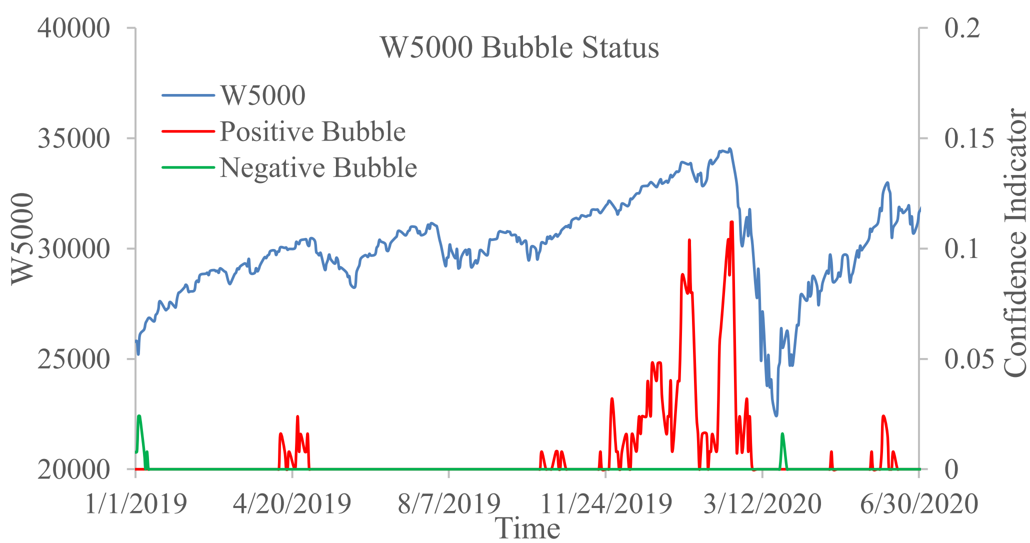
Figure 2 shows the results of the LPPLS confidence indicator for these four US stock market indexes. Positive bubbles are shown in red and negative bubbles in green (right scale) along with the index price in blue (left scale). From Figure 2, we can perceive visually the confidence level of the LPPLS bubbles detected in the price trajectories, as the confidence indicator measures the sensitivity of the fitting results with respect to the start time selection. Figure 2 (a) shows the detected bubble status of W5000 index, including two obvious clusters of positive bubbles between December 13, 2019 and January 28, 2020, and between February 11 and March 4, 2020, plus one subtle cluster of negative bubbles between March 26 and March 27, 2020. The positive LPPLS confidence indicator reached the peak value of 11.2% on February 20, 2020, indicating that 14 out of 125 fitting windows can satisfy the filter conditions, and the price trajectory of W5000 index can be confirmed in a positive bubble regime. The accelerated growth trend of the W5000 index is highly unsustainable, and the positive bubble regime of the W5000 index tends to change in the forms of volatile sideway plateaus or large crashes. The forecast of bubble regime change was consistent with the fact that the W5000 plunged dramatically from 34,533.9 on February 19, 2020 to 22,465.1 on March 23, 2020, losing 34.9% of its value within 24 trading days. During the 2020 U.S. stock market crash, similar positive bubble patterns detected as shown in Figure 2 (a) for the W5000 index can also be found in the other three indices including (b) SP500, (c) SP400, and (d) R2000, as shown in the remaining subfigures in Figure 2. Furthermore, The W5000 index reached the peak negative LPPLS confidence indicator value of 1.6% on March 26, 2020, the SP500 index peaked at 1.6% on March 26, 2020, the SP400 index peaked at 0.8% on March 19, 2020, and the R2000 index peaked at 1.6% on March 24, 2020, respectively.

Table 1 summarizes the statistics of the positive bubbles detected among these four stock indexes during the 2020 U.S. stock market crash and the related information about the peaks and valleys. During the 2020 stock market crash, all four indexes fell by more than a third of their values in five weeks. Among these four major U.S. indexes, the SP400 index fell the most by 42.1%, from 2,106.1 on February 20, 2020 to 1,218.6 on March 23, 2020, followed by the R2000 index which fell by 41.6% from 1,696.1 on February 20, 2020 to 991.2 on March 23, 2020, while the SP500 index dropped 33.9% from 3,386.1 on February 19, 2020 to 2,237.4 on March 23, 2020, the smallest amount relatively, indicating that both middle capitalization stocks and small capitalization stocks have suffered much greater losses than the large capitalization stocks.

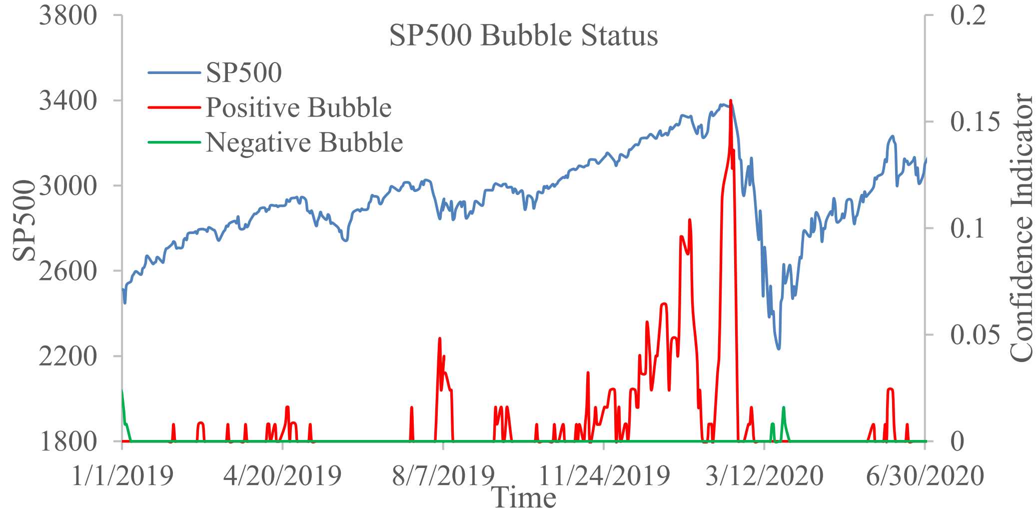
The peak values of the LPPLS confidence indicator (CI) during the 2020 U.S. stock market crash are also listed in Table 1. Among the four indexes, the SP500 index has the largest peak CI value of 16.0%, which means that 20 out of 125 fitting windows can successfully meet the filter constraints, signifying a strong LPPLS bubble appeared in the price trajectory of the SP500 index in the 2020 U.S. stock market crash. The peak confidence indicators of the four indexes all exceed 6.4%, indicating that the price trajectories of the four stock indexes clearly feature the LPPLS bubble pattern, and they are indeed in a positive bubble state. Given that the LPPLS model can only detect endogenous bubbles, the positive bubbles of the four indexes during the 2020 U.S. stock market crash and the subsequent crashes are endogenous, stemming from the increasing systemic instability of the stock market itself, and the well-known external shocks, such as the COVID-19 pandemic-induced market instability, the mass hysteria, and the corporate debt bubble, are not the root causes of the 2020 U.S. stock market crash, and they only acted as sparks during the 2020 stock market crash.

Table 1: Statistics of positive bubble detection based on daily data during the 2020 U.S. stock market crash

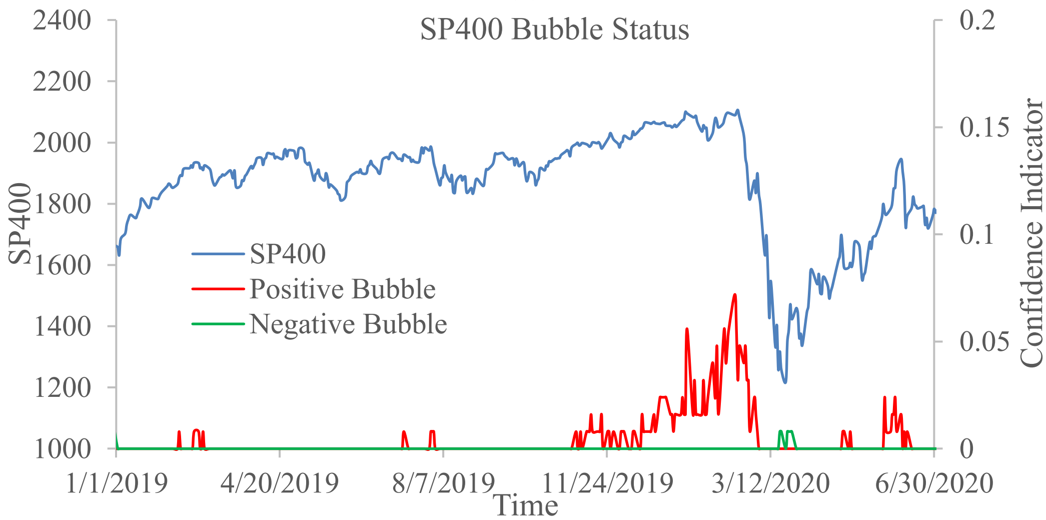
Index	Peak Day	Peak Price	Valley Date	Valley Price	Crash Size (%)	Peak CI (%)
W5000	2/19/2020	34533.9	3/23/2020	22465.1	34.9	11.2
SP500	2/19/2020	3386.1	3/23/2020	2237.4	33.9	16.0
SP400	2/20/2020	2106.1	3/23/2020	1218.6	42.1	7.2
R2000	2/20/2020	1696.1	3/18/2020	991.2	41.6	6.4



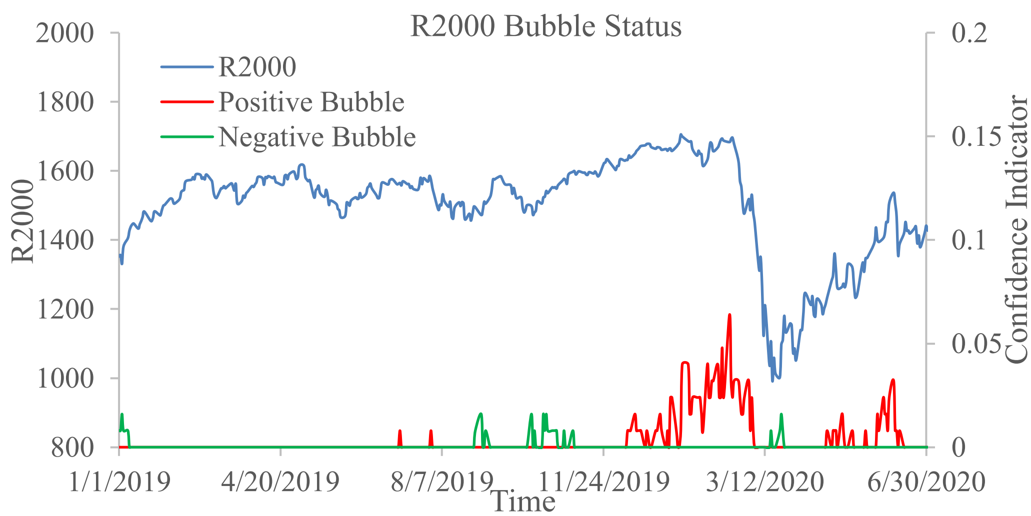
(a)


SP500 Bubble Status
SP500
Positive Bubble
Negative Bubble
SP500
3800
3400
3000
2600
2200
1800
0.2
0.15
0.1
0.05
0
Confidence Indicator
1/1/2019
4/20/2019
8/7/2019
11/24/2019
3/12/2020
6/30/2020
Time

(b)


SP400 Bubble Status
SP400
Positive Bubble
Negative Bubble
SP400
2400
2200
2000
1800
1600
1400
1200
1000
0.2
0.15
0.1
0.05
0
Confidence Indicator
1/1/2019
4/20/2019
8/7/2019
11/24/2019
3/12/2020
6/30/2020
Time

(c)


R2000 Bubble Status
R2000
Positive Bubble
Negative Bubble
R2000
2000
1800
1600
1400
1200
1000
800
0.2
0.15
0.1
0.05
0
Confidence Indicator
1/1/2019
4/20/2019
8/7/2019
11/24/2019
3/12/2020
6/30/2020
Time

(d)

Figure 2: LPPLS confidence indicator for positive bubbles is shown in red and negative bubbles in green (right scale) along with the index price in blue (left scale) for the four U.S. stock market indexes based on daily data from January 2019 to June 2020.

4. Conclusions

In this study, we applied the LPPLS methodology to systematically analyze the 2020 stock market crash in the U.S. equities sectors with different levels of total market capitalizations through four major U.S. stock market indexes, W5000, SP500, SP400 and R2000, representing the overall stocks, the large capitalization stocks, the middle capitalization stocks and the small capitalization stocks, respectively. During the 2020 U.S. stock market crash, all four indexes dropped over 33% within five weeks, with the middle capitalization stocks and small capitalization stocks suffering much greater losses at 42.1% and 41.6% each, than the large capitalization stocks at 33.9%.

Our results show that the price trajectories of these four stock market indexes prior to 2020 stock market crash have clearly contained the distinct LPPLS bubble pattern of the faster-than-exponential growth corrected by the accelerating logarithm-periodic oscillations and are indeed in a positive bubble regime. Contrary to the popular belief that the COVID-19 pandemic-induced market instability led to the 2020 U.S. stock market crash, the crashes in these four indexes during the 2020 U.S. stock market crash are predominantly endogenous, stemming from the increasingly systemic instability of the stock markets. The well-known external shocks, such as the COVID-19 pandemic-induced market instability, served as sparks but not the root causes of the 2020 U.S. stock market crash.

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