# A Sequential Discrimination Procedure for Two Almost Identically Shaped Real Distributions

Silvey Shamsi[1], Mian Arif Shams Adnan[2]
[1]Ball State University, Muncie, IN 47304
[2]Bowling Green State University, Bowling Green, OH 43403

**Abstract**
The way of investigating a distribution knowing its interesting properties might be often inadequate when the shapes of two real distributions are almost similar. In each of these circumstances, the accurate decision about the genesis of a random sample from any of the two parent real distributions will be very much ambiguous even in the presence of the existing testing procedure of the real data. A sequential discrimination procedure has been suggested which is consisting of two tests. It is also invariant to the sample size. The pragmatic performance of the proposed discrimination procedure has been evaluated by checking its meticulous capacity of detecting the genesis of the known samples from the two identically shaped real distributions. Long run simulation studies also show that the proposed test is perfectly correct whereas the individual traditional tests were highly capricious in between the range of 3% to 75%. Further scopes have also been captivated by the proposed tests.

**Key Words:** Density Plot, Discrepancy, Likelihood Ratio Test, Maximum likelihood.

## 1. Introduction

The investigation of a distribution along with its available properties may be very ambiguous when several distributions have similar shapes. As for an example, the shapes of Gamma and Beta distribution are identical for individual specification of the parameters of the corresponding distributions (Adnan *et al*, 2011, 2010). An accurate decision of the genesis of a random sample from any of the two aforesaid parent distributions will be very much puzzling. Adnan *et al* (2012, 2016) demonstrated some new wrapped distributions as well as a sequential discriminating procedure for two almost identically shaped wrapped distributions. An appropriate step by step test procedure of discrimination has been suggested which is also invariant to the size of the sample. The performance of the newly suggested discrimination procedure has been evaluated by comparing the overall statistical distances between the observed and the fitted models to check whether the discrimination procedure can correctly identify the genesis of the known samples from the identically shaped distributions with different specification of the respective individual parameters.

The shape characteristics of these two distributions are discussed in section 2. The section 3 demonstrates the proposed discrimination procedure of the two distributions with two steps. In section 4, a simulation study shows the activities and performance of the proposed discrimination procedure. Section 5 focuses on the use and application of the proposed test. Conclusion has been drawn in section 6.

## 2. Densities and Basic Properties

The basic properties (Mood, Graybill and Boes and Devore, J. L., 2016) and the shape characteristics of both the Normal and Laplace Double Exponential distributions are discussed first. The Normal distribution, denoted by $\mathrm{N}(\mu, \sigma)$, has the density function of the form

$$\mathrm{f}(x, \mu, \sigma) \; = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < \mathrm{x} < \infty \tag{1}$$

and distribution function as

$$\mathrm{F}(x, \mu, \sigma) \; = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx, \tag{2}$$

where $\mu$ and $\sigma$ represent the location and the scale parameters respectively. The shape of the density depends on $\sigma$ but not on $\mu$. The density is bell-shaped. On the other hand, the density function of Laplace distribution, represented by $L(\theta, b)$

$$\mathrm{f}(x, \theta, b) \; = \frac{1}{2} e^{-\left|\frac{x-\theta}{b}\right|}, -\infty < \mathrm{x} < \infty \tag{3}$$

and the distribution function is

$$\mathrm{F}(x, \theta, b) \; = \frac{1}{2b} \int_{-\infty}^{x} e^{-\left|\frac{x-\theta}{b}\right|} dx \tag{4}$$

$\theta, b$ are the location as well as scale parameters respectively. $L(\theta, b)$ density function is symmetric. Now, for some specifications, Laplace Distribution $L(\theta, \lambda = \sqrt{2})$ and the Normal Distribution $\mathrm{N}(\mu, \sigma = 2)$, have similar shapes when $\theta = \mu$. As for example, in the following figure [Figure 1], the shapes of N(10, 2) and L(10, √2) the following graph shows that the two distributions have relatively similar shapes.

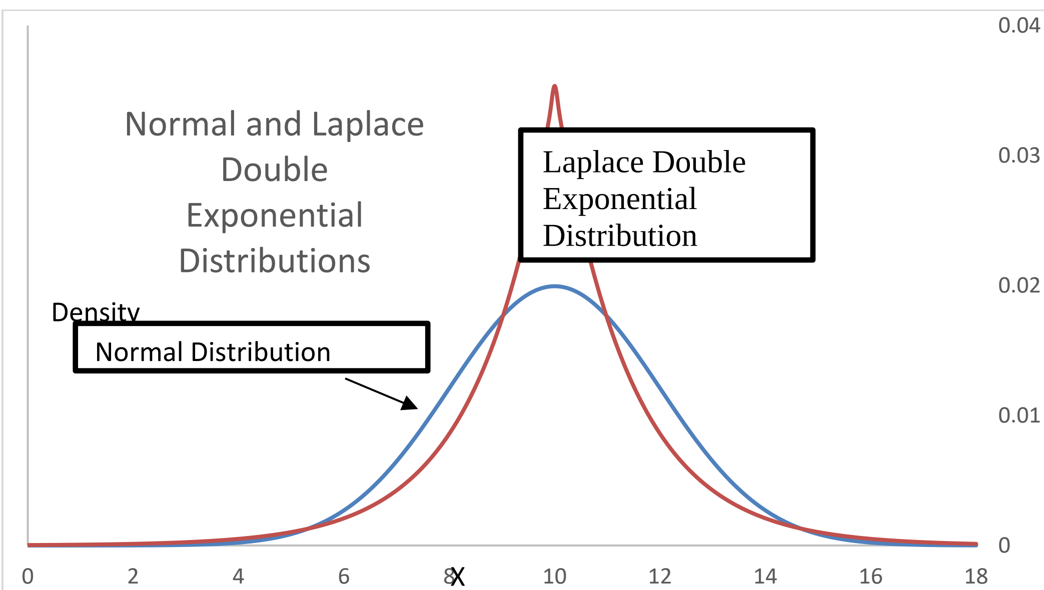


**Figure 1**: The density functions N(10, 2) and L(10, √2)

## 3. Discrimination Procedure

We want to propose a sequential discrimination procedure consisting of two tests to be conducted in two steps which are as below:

(i) Step 1: Maximum likelihood ratio test,
(ii) Step 2: Kolmogorov Smirnov (KS) two sample test.

Here, maximum likelihood ratio test (Casella and Berger, 2002) in step 1 is employed to discriminate two samples for two parent distributions and the KS statistics (Lehman, 2006) in step 2 are adopted to check whether the decision results obtained at step 1 are similar to those at step 2. At step 2 we not only measure the statistical distances between each of the resulted fitted distribution functions and each of the observed empirical distribution functions for each of the sample data sets but also want to confirm whether each statistical distance (minimum KS test statistic) between the resulted fitted distribution function and the observed empirical distribution function for each sample is less (minimum) than the statistical distance between the in-discriminated fitted distribution function and the observed empirical distribution function.

We intend to quantify to what extent the null hypothesized fitted distribution function, (by maximum likelihood estimation) for a discriminated sample data set at step 1, can fit the empirical distribution function of the observed data of that sample data set at step 2. At step 2, the KS test statistic represents the statistical distance between a fitted distribution function (decided at the end of step 1) and the empirically observed distribution function of a data set. So, step 2 shows the performance of the discrimination held in step 1 since the KS's two sample test shows whether a discriminated-fitted distribution function is statistically closer to the respective empirical distribution function of an observed data set compared to the discouraged (alternative hypothesized) fitted distribution function which was not supported by discrimination procedure at 1st step.

In step 1, during the Maximum Likelihood Ratio test, we calculate the ratio of the estimated maximum likelihood functions using the maximum likelihood estimates of the parameters of two distributions based on corresponding realized data sets.

The likelihood ratio test statistic will be

$$\boldsymbol{T = log\left[\frac{L_1}{L_2}\right]}. \quad (5)$$

For a given sample of size *n*, the realized value of *T*, *t*, is calculated and compared with some 'specified' value, $c_n = 0$. If *t* is greater than $c_n$, the sample is classified as having been drawn from the 1st distribution (referred to the numerator of the likelihood ratio test statistic). Otherwise, the sample is classified from the 2nd distribution (referred to the denominator of the likelihood ratio test statistic). Here the likelihood decision process for real data is different from that of the traditional likelihood decision process for the real data since the corresponding likelihood as well as the density functions are not inverse functions in real data.

In step 2, the $\boldsymbol{KS}$'s two sample test statistic for a data set is

$$D_{n_1,n_2} = Sup|F_1(x) - F_2(x)|$$

where $F_1(x)$ is the empirical distribution function of an observed data set and $F_2(x)$ is the fitted distribution function of a real distribution function.

Again, the $KS$'s two sample test statistic for that data set is

$$W_{n_1,n_2} = Sup|S_1(x) - S_2(x)|$$

where $S_1(x)$ is the empirical distribution function of the same observed data set and $S_2(x)$ is the fitted distribution function of another real distribution.

The minimum value of the two KS test statistic, $min\{D_{n_1,n_2}, W_{n_1,n_2}\}$, is chosen for the current data set. It is inferred that this data set fits better the real distribution having minimum statistical distance between the fitted distribution function and the empirical distribution function of the observed data. The similar process is carried for second data set to confirm that the second data set come from the other real distribution.

So, we should calculate Kolmogrov-Smirnov test statistic for each of the two distributions for both the data sets. If for a data set, KS test statistic is less and insignificant at $100(1-\alpha)\%$ level of significance for one of the two distributions, we will infer that the current sample data comes from that distribution. And another data set comes from the other distribution.

## 4. Simulation Study

Two data sets, each of size *n* = 10, will be simulated independently from Real Normal and Real Double Exponential distributions. After confirming statistically, the origins of the data sets, drawn from the original distributions, it will be examined how better or worse the same data set fits the other distribution through the traditional chi-square goodness of fit test for real data. If the first data set drawn from one distribution fits better another distribution and vice versa, it would be claimed that the two individual data sets, coming from Real Normal and Real Double Exponential distributions, are not distinguishable for detecting their genesis from the respective original populations due to the lack of proper testing facilities.

In these circumstances, if the proposed discrimination method can accurately detect the original population individually for each of the known drawn data sets, the proposed method will be an appropriate discrimination procedure for the two aforesaid distributions. Kolmogorov Smirnov two sample test statistics will measure the statistical distance between each of the fitted distribution functions and the empirically observed distribution functions of the data sets.

### 4.1. Random Sample Generation

At first, we generate two random samples, each of which is of size *n* = 10, from N(10,2) and L(10,$\sqrt{2}$).

Data set N(10,2): 6.945648, 11.148451, 13.028798, 6.910681, 12.707729, 11.374669, 10.277903, 10.943627, 9.613667, 9.030217.

Data set L(10, $\sqrt{2}$): 10.010082, 9.291294, 9.383203, 12.313524, 9.853225, 9.355582, 9.839410, 9.165526, 10.199624, 10.507477.

Figure 2 indicates the similar shape of the density functions of N and L.

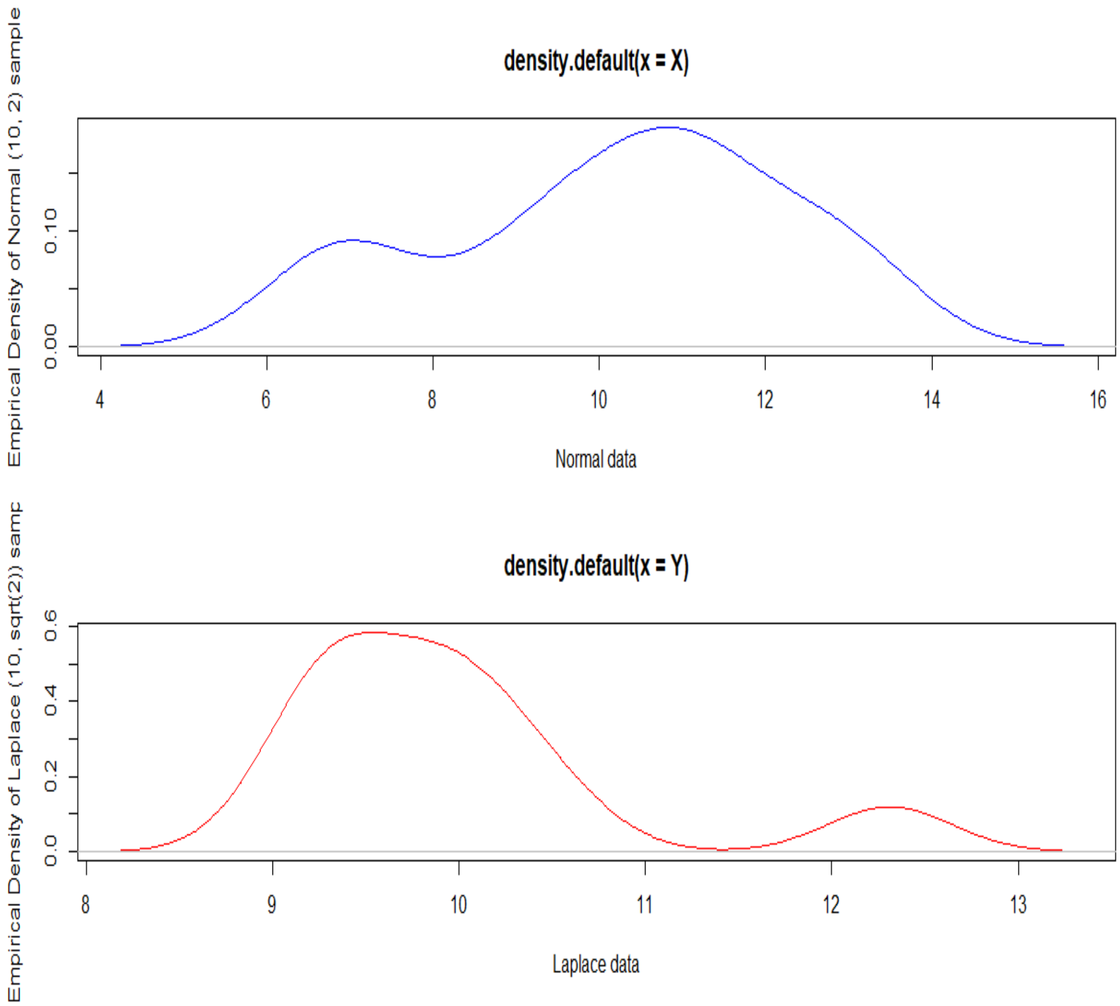


**Figure 2**: The Empirical density functions N, L for two data sets N(10,2) and L(10, $\sqrt{2}$)

After confirming statistically as the first data set coming from Real Laplace Double Exponential distribution, we assume that our null distribution is Real Normal. Again, after confirming the second data set coming from Real Normal distribution, we assume that our null distribution is Real Laplace Double Exponential. We test each of the null hypotheses by traditional goodness of fit test for two real data sets. If the available test fails to detect the genesis of the samples properly, we will conduct our proposed test by having step 1 check whether it can detect the genesis of the samples properly and then step 2 to check whether the statistical distance between each of the fitted distributions and the empirically observed distributions of the data sets are minimum for own individual cases.

To visualize the distance between the fitted distributions and the observed models, two graphical representations have been displayed in figure 3 and 4. The empirical distribution function, fitted Real Normal distribution function, and fitted Real Laplace Double Exponential distribution function for the data set L(10, $\sqrt{2}$) are plotted in figure 3. Similarly, the empirical distribution function, fitted Real Normal, and Real Laplace Double Exponential distribution functions for the data set N(0,2) are plotted in figure 4.

**4.2. Traditional Goodness of Fit Test for Real Data**

Now to check whether the data set L(10, $\sqrt{2}$) from Real Laplace Double Exponential distribution is drawn from Real Normal distribution, the traditional Kolmogorov Smirnov goodness of fit test has been conducted. As such, assuming $H_0$: the data set L(10, $\sqrt{2}$) is drawn from Real Normal distribution, we obtain

$$D_n = Sup(F_{n:L} - F_{H0:N})$$

Hence at 5 % level of significance, it is observed that the data set L(10, $\sqrt{2}$) comes from Real Normal distribution. Similarly, it is also noticed at the same level of significance that the data set N(10,2) comes from Real Laplace Double Exponential distribution. So, the Kolmogorov Smirnov test fails to accurately decide of the genesis of the two sample data sets.

**4.3. Maximum Likelihood Estimation for Real Laplace Double Exponential and Real Normal Distributions**

Kundu, D. (2005) used Maximum likelihood ratio statistic for discriminating Real Normal and Real Laplace distributions. Suppose that $x_1, x_2, \ldots x_n$ are independent and identically distributed random sample from N ($\mu$,σ). The likelihood function of the Real Normal N($\mu$, σ) is

$$L_N(\mu, \sigma) = \prod_{i=1}^{n} f_{n:N}(\mu, \sigma) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}\Sigma\left(\frac{x_i-\mu}{\sigma}\right)^2} \quad (6)$$

and hence the maximum likelihood estimators are

$$\hat{\mu} = \frac{\sum x_i}{n}, \widehat{\sigma^2} = \frac{\sum(x_i - \bar{x})^2}{n}.$$

Similarly, if the random sample $y_1, y_2, \ldots, y_n$ of the real data comes from the Real Laplace distribution, then the concerned likelihood function is

$$L_L(\theta, b) = \prod_{i=1}^{n} f_{n:L}(\theta, b) = \left(\frac{1}{2b}\right)^n e^{-\Sigma\left|\frac{y_i-\theta}{b}\right|} \quad (7)$$

and hence the maximum likelihood estimators are

$$\hat{\theta} = median(y_1, y_2, \ldots, y_n), \hat{b} = \frac{\Sigma|y_i - \hat{\theta}|}{n}.$$

**4.4. Proposed Step by Step Tests cum Discrimination Procedure**

Since the available test (Kolmogorov Smirnov test) fails to detect the genesis of the samples properly, we will conduct our proposed test by having step 1 check whether it can detect the genesis of the samples properly and then by having step 2 check the performance of step 1 by computing the statistical distance between each of the fitted distributions and the empirically observed distributions of the data sets.

*Step1: Maximum Likelihood Ratio (RML) Method*
Now according to the Maximum Likelihood Ratio (RML) method

$$\text{RML} = \frac{L_N(\hat{\mu}, \widehat{\sigma^2})}{L_L(\hat{\theta}, \hat{b})}$$

where $(\hat{\mu}, \widehat{\sigma^2})$ and $(\hat{\theta}, \hat{b})$ are the maximum likelihood estimators of $L_N$ and $L_L$ respectively.

Let us consider, the likelihood ratio test statistic is

$$T = \log(RML),$$

$$\text{or, } T = \log \frac{L_N(\hat{\mu}, \widehat{\sigma^2})}{L_L(\hat{\theta}, \hat{b})}$$

$$\text{or, } T = \log \frac{\left(\frac{1}{\hat{\sigma}\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}n}}{\left(\frac{1}{2\hat{b}}\right)^n e^{-n}}$$

Therefore, $$T = -n\log\hat{\sigma} - \frac{n}{2}log2 - \frac{n}{2}log\pi - \frac{n}{2} + nlog2 + nlog\hat{b} + n \qquad (8)$$

$$\text{or, } T = -n\log\hat{\sigma} + \frac{n}{2}log2 - \frac{n}{2}log\pi + nlog\hat{b} + \frac{n}{2}$$

For a given sample of size *n*, the realized value of *T*, *t*, is calculated and compared with some 'specified' value, $c_n = 0$. If *t* is greater than $c_n$, the sample is classified as having been drawn from the Real Normal distribution. Otherwise, the sample is classified from the Real Laplace Double Exponential distribution. Thus, in step 1, for a given real data, if $T > 0$, then we choose Normal distribution as the preferred model, otherwise we choose the Laplace distribution.

For the data set L(10, √2) the maximum likelihood test statistic has been found as T= -1.31 < 0 which leads us to infer that the data set L(10, √2) comes from Real Laplace Double Exponential distribution. Moreover, for the sample data N(0, 2) the maximum likelihood test statistic is, T= 0.74 > 0 that confers that the data set N(0,2) comes from the original Real Normal distribution. So, step 1 of the proposed test can easily detect the genesis of both samples.

*Step 2: Kolmogorov Smirnov Two Sample Test*
Now we want to check whether the results obtained, using maximum likelihood method, at step 1 are similar to the results to be carried by a distance-based KS test at step 2. We should calculate KS test for each of the two distributions Real Normal and Real Laplace Double Exponential for both the data set. If for a data set, KS is less and insignificant at $100(1-\alpha)\%$ level of significance for one of the two distributions, we will infer that the current sample data comes from that distribution. And another data set comes from the other distribution.

For the data set L(10, $\sqrt{2}$) the KS two sample test statistic between the empirical distribution function for the observed data and fitted Real Laplace distribution function due to discrimination, using the MLEs of the parameters $\hat{\theta}$ = 9.846317 and $\hat{b}$ = 0.5848916 , is 0.1734847 with the *P*-value >> 0.2. Similarly, for the same data set, the KS two sample test between the empirical distribution function for the observed data and the fitted Real Normal distribution function due to indiscrimination, having the MLEs of the parameters used as $\hat{\mu}$ = 9.991895 and $\hat{\sigma}$ = 0.8769763, is 0.9493495 with the *P*-value <<0.001. Henceforth, the smaller value (0.1734847) of the KS two sample test statistic (less statistical distance between the empirical distribution function of the observed data set L(10, $\sqrt{2}$) and the fitted Real Laplace Double Exponential distribution function due to discrimination) suggests that the data set L(10, $\sqrt{2}$) fits better (with greater *P*-value >> 0.2) the Real Laplace Double Exponential distribution (which is also clear from the figure 3).

Again for the sample data N(10, 2) the KS two sample test statistic between the empirical distribution function for the observed data and the fitted Real Normal distribution function due to discrimination, using the MLEs of the parameters as $\hat{\mu}$ = 10.19814 and $\hat{\sigma}$ = 2.006286, is 0.1475073 with the corresponding *P*-value >> 0.2. Similarly, for the same data set, the KS two sample test statistic between the empirical distribution function for the observed data and the fitted Real Laplace Double Exponential distribution function due to indiscrimination, having the MLEs of the parameters used as $\hat{\theta}$ = 10.61076 and $\hat{b}$ = 1.642516, is 0.8926376 with the *P*-value >> 0.2. Henceforth, the smaller value (0.1475073) of the KS two sample test statistic (less statistical distance between the empirical distribution function of the observed data set N(10, 2) and the fitted Real Normal distribution function due to discrimination) suggests that the data set N(10, 2) fits better (with greater *P*-value >> 0.2) the Real Normal distribution (which is also evident from the figure 4). Hence the proposed discrimination procedure along with steps 1 and 2 can accurately detect the origins of the two real data sets.

The graphical representation of the distances between the empirical cdf for each of the observed data sets and each of the fitted cdfs has been displayed to show the closeness between the empirical distributions and the fitted distributions. Figure 3 addresses that the empirical distribution for the data set L(10, $\sqrt{2}$) stays more close to the fitted Real Laplace Double Exponential distribution. Figure 4 shows that the empirical distribution for the data set N(10, 2)is more close to the fitted Real Normal distribution. The graphs demonstrate the same results as addressed by the proposed discrimination procedure.

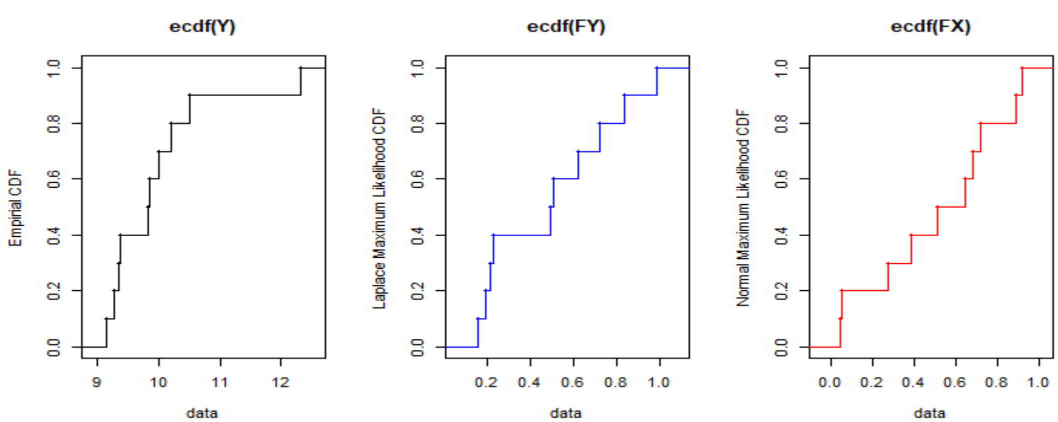


**Figure 3**: Empirical distribution function, fitted distribution functions of Laplace and Normal for data L(10, $\sqrt{2}$).

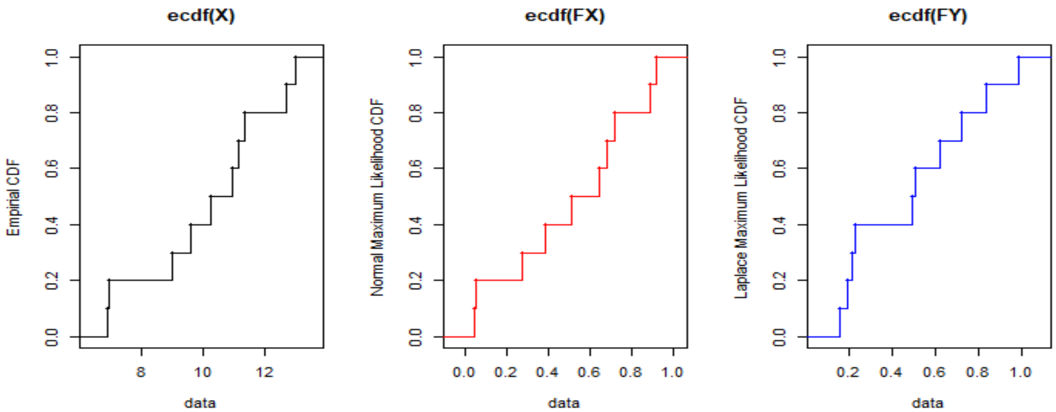


**Figure 4**: Empirical distribution function, fitted distribution functions of Laplace and Normal for data N(10, 2).

**4.5. Long Run Behavior of the Proposed Tests**

The tests have been carried out for 10,000 times. At least 25 % cases, the individual likelihood ratio test miscalculates that the samples come from the Laplace distribution L(10, $\sqrt{2}$) whereas the true distribution is Normal N(10, 2). The individual likelihood ratio test cannot properly conclude the genesis of the Normal N(10, 2) samples for more than 55% cases, rather concludes them come from Laplace distribution L(10, $\sqrt{2}$). The empirical ratio of concluding the genesis of the samples from distributions by likelihood ratio test is 40:60 whereas it should be 50:50. For at least 96% times, the individual traditional KS test demonstrates that the two types of samples come from the same distribution whereas they originated from two distributions.

Since the empirical ratio (40:60) of concluding the genesis of the samples from distributions by likelihood ratio test is close to 50:50, the likelihood ratio test can be carried in the first step of the sequential tests.

In step 2 for the sequential tests for 10,000 times, the proposed KS tests have been found to be perfectly correct in deciding the genesis of the samples. Therefore, the sequential test consisting of steps 1 and 2 can perfectly detect the genesis of the sample from the true population. The following table (table 1) shows the percentage of correct decisions by the individual tests for the genesis of samples.

**Table 1**: Percentage of Correct Decision by the tests for the Genesis of Samples

| *Samples* | *LRT* | *KS* | *Sequential Tests* | *Correct Decision* |
|---|---|---|---|---|
| N(10, 2) | 45% | 3% | 100% | Samples come from N(10, 2) Distribution. |
| L(10, $\sqrt{2}$) | 75% | 3% | 100% | Samples come from L(10, $\sqrt{2}$) Distribution. |

## 5. Justification of Using the Sequential Tests and its Applications

Adnan and Kiser (2010) developed a Generalized Double Exponential distribution from where most of the exponential distributions like Normal, Lognormal, Gamma, Rayleigh, Laplace, etc can be unfolded through proper specification of the parameters of the generalized form. The generalized forms of the Exponential, Gamma, Chi-square, t, F and a three parameter Double Exponential Distributions were developed via the parameters of the Generalized Double Exponential distribution. It can generate (Adnan and Kiser, 2011) a couple of other probability distributions such as Generalized Gamma, Generalized Chi-square, generalized $t$, Generalized $F$, etc. Generalized Beta 1st kind and 2nd kind distributions are also addressed from the generalized exponential distribution. The basic properties and the shape characteristics of the generalized double exponential distribution has been discussed here. The two-parameter generalized double exponential distribution was defined as a distribution of a random variable *X* having the probability density function

$$f(x) = \frac{d\sqrt[d]{a}}{2\Gamma(\frac{1}{d})}\, e^{-|ax^d|} \;;\; -\infty \le x \le \infty. \tag{9}$$

where $a$ is the scale-parameter and d is the shape-parameter such that $a, d > 0$. The authors demonstrated a classification of distributions based on various specification of the parameters of $a$ and d. The various specifications of $a$ and d along with the classification of distributions are addressed in the following table (table 2).

**Table 2**: A glance of the probability density functions as the special cases of the proposed Generalized Double Exponential distribution for various specifications.

| *Sl.* | *Name of the distribution* | $f(x)$ | *Support* | $a$ | $d$ | $x$ | *Mean* | *Variance* |
|---|---|---|---|---|---|---|---|---|
| 1 | Generalized Double Exponential | $\frac{d\sqrt[d]{a}}{2\Gamma(\frac{1}{b})}\, e^{-\lvert ax^b\rvert}$ | $-\infty \le x \le \infty$ | $a$ | d | $x$ | 0 | $\frac{\Gamma(\frac{3}{d})}{a^{2/d}\Gamma\left(\frac{1}{d}\right)}$ |
| 2 | Std. Laplace | $\frac{1}{2}e^{-\lvert x\rvert}$ | $-\infty \le x \le \infty$ | 1 | 1 | $x$ | 0 | 2 |
| 3 | Laplace | $\frac{1}{2\lambda}\, e^{-\left\lvert\frac{x-\theta}{\lambda}\right\rvert}$ | $-\infty \le x \le \infty$ | $\frac{1}{\lambda}$ | 1 | $x - \theta$ | $\theta$ | $2\lambda^2$ |
| 4 | Standard normal | $\frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2}$ | $-\infty \le x \le \infty$ | $\frac{1}{2}$ | 2 | $x$ | 0 | 1 |
| 5 | Normal | $\frac{1}{\sqrt{2\pi}\sigma}\, e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$ | $-\infty \le x \le \infty$ | $\frac{1}{2\sigma^2}$ | 2 | $x - \mu$ | $\mu$ | $\sigma^2$ |
| 6 | Log-normal | $\frac{1}{x\sqrt{2\pi}\sigma}\, e^{-\frac{1}{2}(\frac{\log x-\mu}{\sigma})^2}$ | $0 \le x \le \infty$ | $\frac{1}{2\sigma^2}$ | 2 | $\log x - \mu$ | $e^{\mu+\frac{1}{2}\sigma^2}$ | $e^{2\mu+\sigma^2}\{e^{\sigma^2} - 1\}$ |
| 7 | Exponential | $ae^{-ax}$ | $0 \le x \le \infty$ | $a$ | 1 | $x$ | $\frac{1}{a}$ | $\frac{1}{a^2}$ |
| 8 | Gamma | $\frac{x^{\frac{1}{b}-1}e^{-x}}{\Gamma(\frac{1}{b})}$ | $0 \le x \le \infty$ | 1 | d | $x^{1/b}$ | $1/d$ | $1/d$ |
| 9 | Rayleigh | $\frac{2x}{\lambda^2}e^{-\frac{x^2}{\lambda^2}}$ | $0 \le x \le \infty$ | $\frac{1}{\lambda^2}$ | 1 | $x^2$ | $0.886\lambda$ | $0.215\lambda^2$ |

The Generalized Laplace Double Exponential Distribution is the traditional Laplace Distribution L($\theta,\ \sqrt{2}$) for the specification $\alpha = \frac{1}{\sqrt{2}}$, d = 1 and the Normal Distribution N($\theta = \mu,\ 2$) for the specification $\alpha = \frac{1}{8}$, d = 2. So, L($\theta,\ \sqrt{2}$) and N($\theta = \mu,\ 2$) are the two specifications of the same Generalized Laplace Double Exponential Distribution. As a result, the traditional test procedures were unsuccessful in inferring the samples. So, for several shaped distributions under each family of distributions, the sequential tests can play correct roles compared to the existing ones.

## 6. Conclusion

The exiting traditional test statistic as well as the discrimination procedure for the real data cannot ensure the correct decision about the genesis for two sampled data sets if these samples are drawn from two almost identical shaped real distributions. The present likelihood-ratio and Kolmogorov Smirnov test based sequential discrimination procedure can properly decide the origin of the two types of samples. For the computations and the related graphs, we have used R package. This test can be extended to more than two types of samples.

## References

Adnan, M. A. S. and Roy, S. (2016). A Sequential Discrimination Procedure for Two Almost Identically Shaped Real Distributions." Journal of Applied Statistics. DOI: 10.1080/02664763.2016.1189516.

Adnan, M. A. S. and Kiser, H. (2011). A Generalization of the Family of Exponential and Beta Distributions. JSM Proc., Statistical Computing Section. Alexandria, VA: American Statistical Association. 2336 – 2347.

Adnan, M. A. S. and Kiser, H. (2010). A Two Parameter Generalized Double Exponential Distribution. JSM Proc., Statistical Computing Section. Alexandria, VA: American Statistical Association. 4247-4261.

Casella, G. and Berger, R. (2002). Statistical Inference. Duxbury Publisher's. 2nd Edition.

Devore, J. L. (2016). Probability and Statistics for Engineering and the Sciences. CENGAGE Publishers. Ninth Edition.

Kundu D (2005). Discriminating between the normal and the Laplace distribution. Advances in Ranking and Selection, Multiple Comparisons, and Reliability. 65-79. Springer link DOI: 10.1007/0-8176-4422-9_4.

Lehmann, E. (2006). Nonparametrics: Statistical methods based on ranks. ISBN: 978-0387352121.

Mood, A. M., Graybill, F. A, and Boes, D. C. (1973). Introduction to the Theory of Statistics. McGraw-Hill. 3rd Edition.

Roy S, Adnan M A S (2012). Wrapped Weighted Exponential Distribution. Statistics & Probability Letters. 82(1), 77-83.