Robustness of DVC Scores to Alternative Density-Variation and Compactness Measures in Evaluating Redistricting Plans

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Abstract

Partisan gerrymandering has contributed to political polarization in the United States, but enforceable standards to protect against the practice remain elusive. As a one-number summary for evaluating redistricting plans, Belin, Fischer, and Zigler (2011 Statistics, Politics and Policy) proposed a "density-variation / compactness" (DVC) score, where plans with higher scores are expected to have fewer "safe seats". Choices for input components in the framework motivate the present exploration of the robustness of DVC scores to alternative density-variation and compactness measures. In contrast to using mean absolute deviation and Reock compactness (area of district divided by area of circle encompassing district), as in Belin et al. (2011), we calculate DVC scores for multiple states using the standard deviation and a convex-hull compactness measure (area of district divided by area of convex region encompassing district, which yields larger numerical values than the Reock measure). We discuss implications of findings for the ability to provide valuable and timely information into the redistricting process and the prospect of protecting against partisan gerrymandering.

Key Words: redistricting, gerrymandering, seats-votes curve, compactness, shapefile

1. Introduction

Partisan redistricting in the United States, where one party predominantly decides how voters are sectioned into future districts for elections, has provided political parties with the power to create advantages for themselves in upcoming elections. This process, named "gerrymandering" after its infamous abuse by 19th century Massachusetts governor Elbridge Gerry, has become a widespread concern as its practice becomes more prevalent. In conjunction with the increasing availability and depth of voter data, many voters are worried that their ability to influence the outcome of elections is being minimized. When members of their political party are spread thin across many voting districts or packed together in fewer districts, their political representation in official bodies like the House of Representatives is no longer proportional to the votes they are casting.

To illustrate the effect of gerrymandering in a hypothetical election scenario, Table 1 compares the breakdown of voter registration data between two different redistricting plans in the same state. Across the state, voter registration shows that voters are split evenly between the two major political parties, characterized here as the Orange party and the Purple party. In both plans, the state is divided into 8 voting districts, and the only difference between the two plans is how the voters are split up into these districts. Under Plan A, there is no evidence of partisan bias in the initial voter registration distribution. If most voters vote according to their voter registration, each party would have edges of differing degrees in 4 of 8 districts, providing both parties with ample opportunity to receive representation equivalent to the proportion of their voters in the state. Under Plan

B, voters who registered for the Orange party are spread thinly across six of eight districts, and packed into the last two. The degree of the partisan split across all districts in Plan B makes it unrealistic for any candidate in a minority party to be elected. The result would most likely be six representatives for Purple voters and two representatives for Orange voters. Thus, orange voters would receive 25% representation despite having 50% of the votes across the entire state.

Table 1: Voter registration percentages for two hypothetical redistricting plans in a state with 8 districts and a 50-50 statewide voter registration split

	District								
Plan	Party	1	2	3	4	5	6	7	8
A	Orange	43	45	47	49	51	53	55	57
	Purple	57	55	53	51	49	47	45	43
B	Orange	40	40	40	40	40	40	80	80
	Purple	60	60	60	60	60	60	20	20

The process of redistricting itself is not as simple as putting numbers into a table. Redistricting, where district boundary lines are drawn across the geography of each state to split voters evenly between each district, is done at least once every 10 years after the US Census. While this may satisfy the "one person, one vote" rule that ensures there are an equal amount of people in each district across a state, it does not guarantee accurate representation. Despite the geological limitations of drawing these boundaries, as well as additional rules that ensure minority voters also receive proportional representation, imbalanced results are achieved consistently in elections. Aside from requiring the use of census data or a suitable substitute to ensure that districts are of equal population size, there are no national limitations on what data may be used to inform the formation of new district lines. Furthermore, the practice of partisan gerrymandering is still legal at a national level, and in 2019 the Supreme Court ruled that it will no longer examine claims of excessive partisanship that have been elevated from state courts. While severe gerrymandering has been examined as a violation of the 1st and 14th amendment, none of the statistics built to assess the loss of representation for voters have been found in court to prove a strong connection between this loss of representation and the act of partisan redistricting. Additionally, these statistics are typically calculated based on the election results of prior elections, and are thus only able to assess redistricting plans for gerrymandering in hindsight, after the plans have already been put in place and used for an election.

Recognizing the need for a robust and accessible way to assess redistricting plans for evidence of egregious partisan gerrymandering, Belin, Fischer, and Zigler (2011) aimed to combine multiple methods of evaluating gerrymandering into a single statistic, and built the first "density-variation/compactness" score. In theory, these metrics should act as strong indicators with no partisan bias for the manipulation of district boundaries. Due to the significant connection between population density and voter party preference, notable changes in the variation in population density over multiple districts in a state could be attributed to gerrymandering methods highlighted earlier in Table 1 - "packing" voters of the same party into fewer districts to win less seats and "cracking" or breaking them up amongst the others districts to guarantee a heavy advantage in more districts. Compactness plays a similar role but detects irregularities or shifts in the district boundaries themselves, and highlights situations where district lines are drawn to remove some of the communities in a local area to reduce one party's voting power. Combining these two metrics creates a statistic that values fair, competitive, and more localized elections, with the additional benefit that the final result can be calculated using publicly available tools and census data.

The Belin-Fischer-Zigler study also presented a specific DVC score, a function that calculated the shift between two state district plans using average absolute deviation of population density and the average compactness. This original statistic was developed and tested using California and Texas data in the 2010 and 2012 elections for the House of Representatives. With greater access to historical election metrics, census data and district maps, it is now possible to further analyze this original DVC score as well as promising alternatives utilizing data from the last ten years of elections across all states.

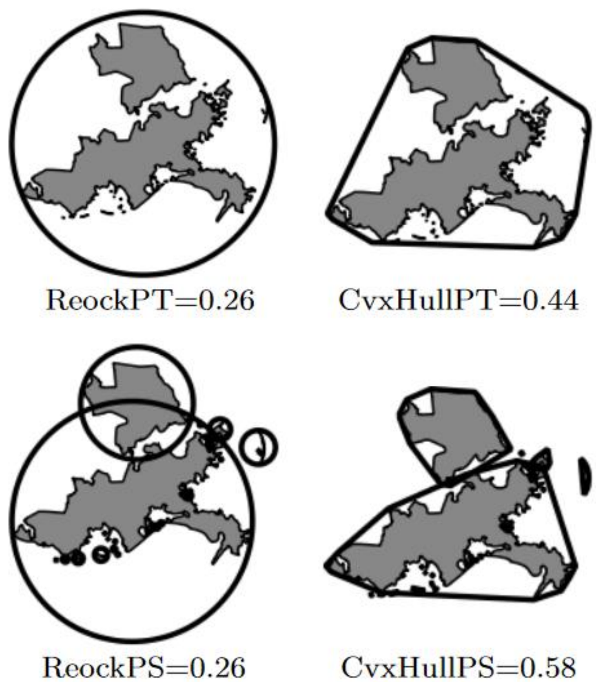
Recent studies have also motivated further investigation into the nature of how compactness can be calculated, as well as the merits and fallbacks of various methods for doing so. The two most prominent of these methods are the Reock degree of compactness and the convex hull method, which both calculate compactness by encapsulating the district of interest in the smallest encompassing shape possible (a circle and a convex hull, respectively), and solve for the ratio of the areas of the district versus its encompassing shape. Due to conflicting reports from multiple studies of the overall accuracy of these two methods, this study aims to test both methods across a large dataset, as well as in the context of a DVC score.

2. Methods

The data for this study was compiled entirely from census and election data, and utilized three major databases - the Tiger/Line database of past US district boundaries, US decennial census redistricting data for population counts, and US Congress block equivalency files to accurately map population counts to districts as district lines shifted. The data used in this study spans all districts in all 50 states for the 2000 election as well as all elections from 2010 to 2020, inclusive. Tiger/Line data is not available in shapefile form for 2000 elections, so the less detailed cartographic boundary file was used instead. The data was joined together from these various sources into one data frame of individual elections, with data on the hosting district's shape and population size. These elections were then grouped up by their corresponding state and the year in which the election occurred.

Compactness is calculated utilizing both methods referenced earlier. The first, the Reock compactness measure, encapsulates each district in the smallest circle possible and then calculates the ratio between the area of the district and the circle. The second, the convex hull measure, uses a convex hull rather than a circle, and is often called the "rubber band" method as the resulting shape takes the same form as a rubber band stretched around the district as tightly as possible. In comparison to the Reock method, this has the benefit of adapting to a much larger range of shapes – a convex hull will perfectly fit triangles, rectangles, any regular polygon, etc. Both methods are demonstrated visually in Figure 2. Both the Reock and convex hull methods result in a decimal between 0 and 1, where a 1 represents no difference between the district and its encapsulating shape. Concerns have also been raised about the validity of calculating the compactness of a district on a thirddimensional sphere using a two-dimensional map, in which situation the convex hull method may result in less overall error.

Figure 2: Two implementations each of the Reock and convex hull compactness methods. In our study, we will not be separating different sections of districts as in the bottom two images, as most districts are already continuous.



The DVC score of Belin, Fischer and Zigler (2011) utilizes data from the districts across a state for two different election cycles - one representing a new redistricting plan, designated by subscript *p*, and the other acting as a reference plan, designated by subscript *ref*, as a point of comparison for the new plan. Let *v* be a measure of the average absolute deviation in population density across districts, and let $\overline{c}$ be a measure of the average Reock compactness of each district. The Belin-Fischer-Zigler DVC score is thus defined as:

$$DVC_p = 15 \times \left[\left(\frac{V_{ref}}{V_p}\right) - 1\right] + 5 \times \left[\left(\frac{\overline{c}_p}{\overline{c}_{ref}}\right) - 1\right]$$

Thus, a new plan that minimizes population density variation, thus theoretically increasing election competitiveness and maximizing compactness to encourage more local elections will earn a higher DVC score. The multipliers 15 and 5 were chosen as appropriate weights for the dataset used in the original study. The original DVC score utilized elections in the year 2000 as a reference plan, hence why that data is also included in the dataset.

This expanded work to build an accessible metric to assess gerrymandering preserves many of the aspects of the original DVC score. Theoretically, population density variation and compactness are both products of the manipulation of district lines, and can be assessed quickly and accurately with US Census data. In order to further improve upon this concept and compare new methods within the same frame of thought, this study also examines the original DVC score with alternative weights on density variation vs. compactness as well as completely new ways of measuring those quantities such as the convex hull method.

In our review of this process, we present a range of descriptive summaries from exploratory analyses.

3. Results

Figure 3 displays a scatterplot of the Reock score versus the convex hull score of the district boundaries for each election in our dataset. An overall positive trend indicates that both methods will result in an approximately similar score, although the amount of variation also shows that certain districts perform better with one method or the other.

Figure 3: Scatterplot of Reock vs Convex Hull Scores for All US House Districts for elections in years 2000, 2010 through 2020 inclusive.

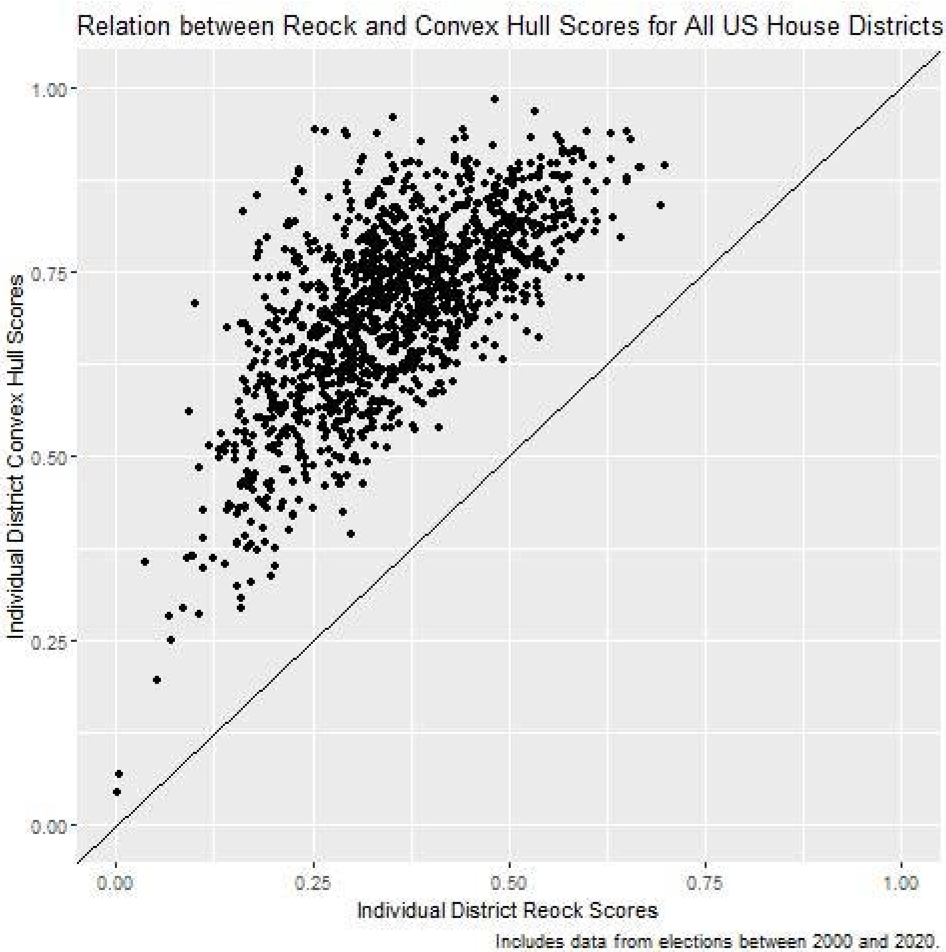


Table 4 and Table 5 examine the districts with the highest differences in Reock and convex hull scores. Note that because the convex hull can take the form of a smallest encompassing circle, the Reock method will never achieve a higher score than the convex hull method.

Table 4: Three states with the greatest difference between Reock and convex hull scores.					
State	Year	District	Reock score	Convex Hull score	Difference
California	2020	36	.265	.940	.674
Minnesota	2010	1	.179	.852	.673
California	2010	45	.233	.887	.654

Table 5: Three states with the smallest difference between Reock and convex hull scores.					
State	Year	District	Reock score	Convex Hull score	Difference
Hawaii	2010	2	.005	.068	.063
Illinois	2010	17	.299	.395	.096
Arizona	2010	2	.288	.426	.137

Figures 6 and 7 display the corresponding congressional districts in Tables 4 and 5. The results appear to be fairly self-explanatory – districts that receive much higher scores using the convex hull over the Reock method tend to have much more rectangular shapes that aren't easily encompassed by a circle, while districts that receive similar scores with both methods tend to have quite low compactnesses overall, typically due either to extreme gerrymandering, irregular edges near a coast, or separate bodies of land such as islands.

Figure 6: California District 36 in 2020, Minnesota District 1 in 2020, and California District 45 in 2010.

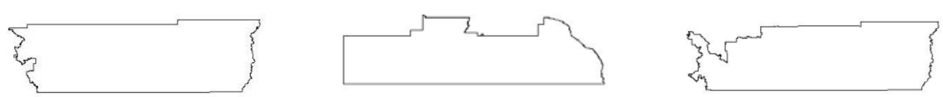
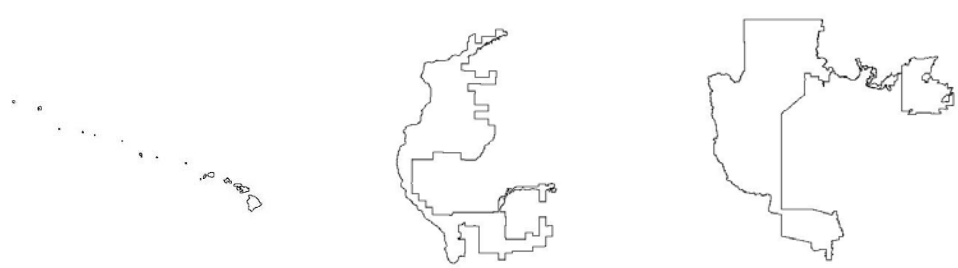
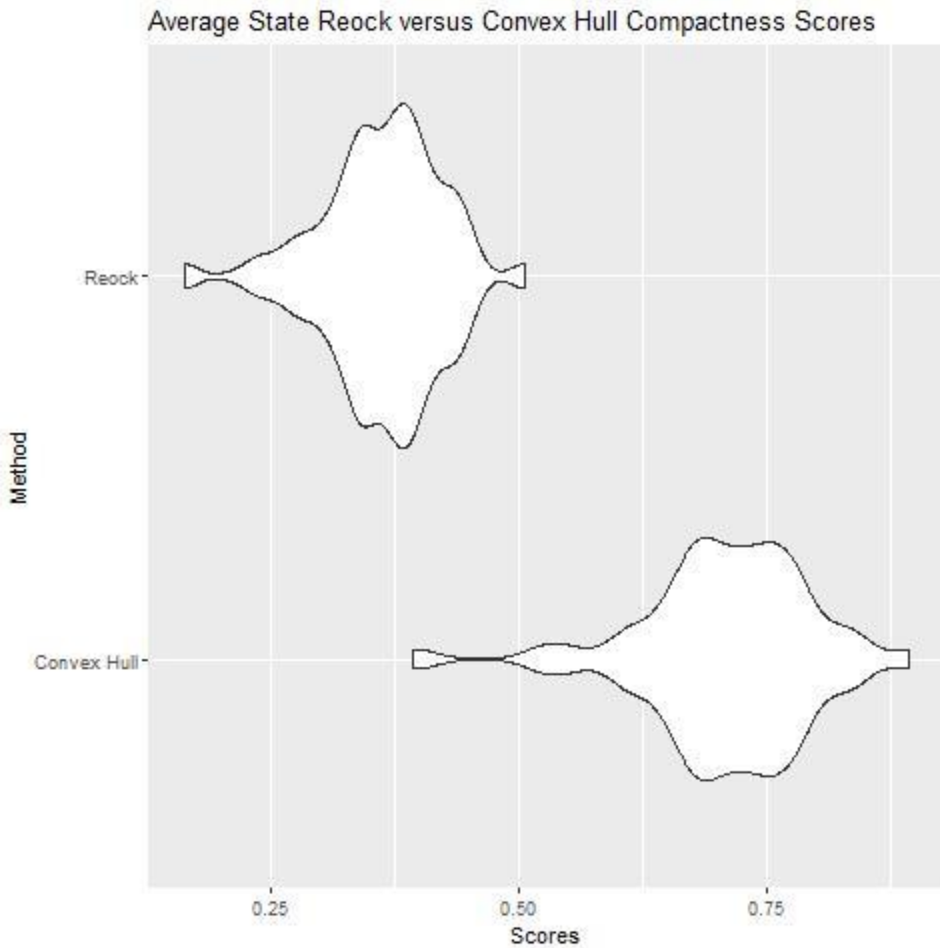


Figure 7: Hawaii District 2 in 2020, Illinois District 17 in 2010, and Arizona District 2 in 2010.



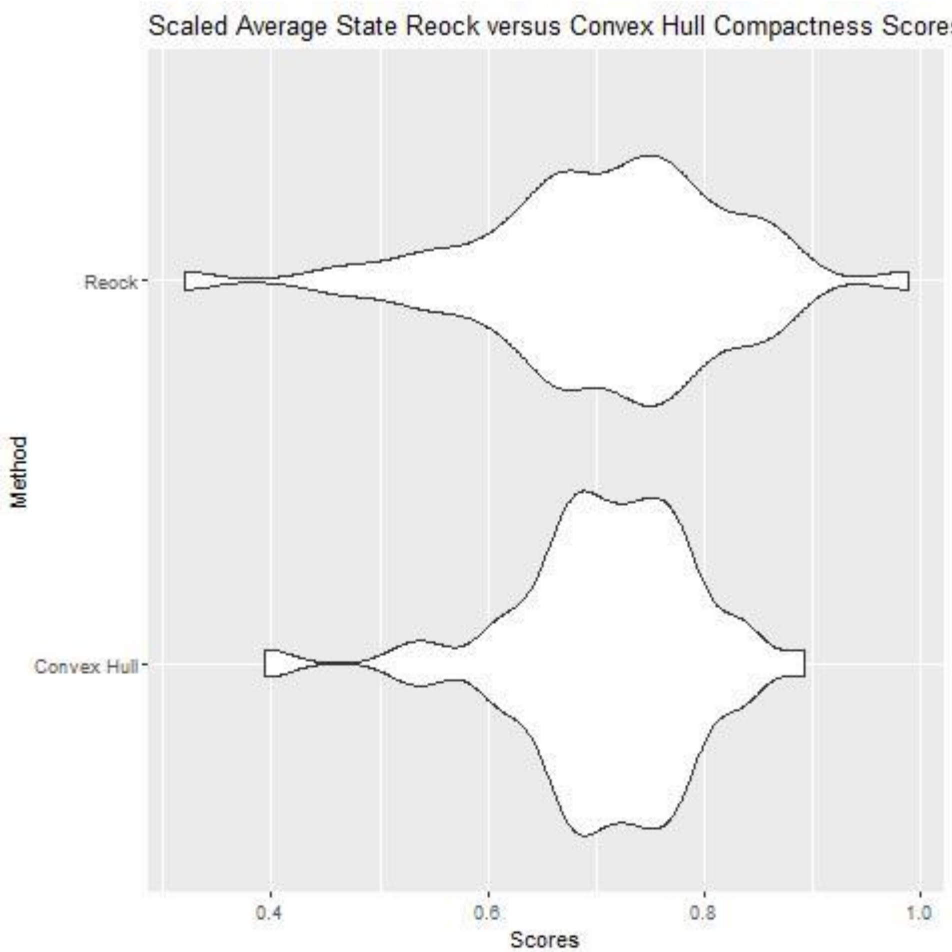
However, this overall variation in compactness appears to be reduced when examining the average compactness of districts across a state. Figure 8 is a comparison of the distributions of each state's average Reock and convex hull compactness scores of all the data in the dataset.

Figure 8: Distribution of all average state Reock scores and all state convex hull scores.



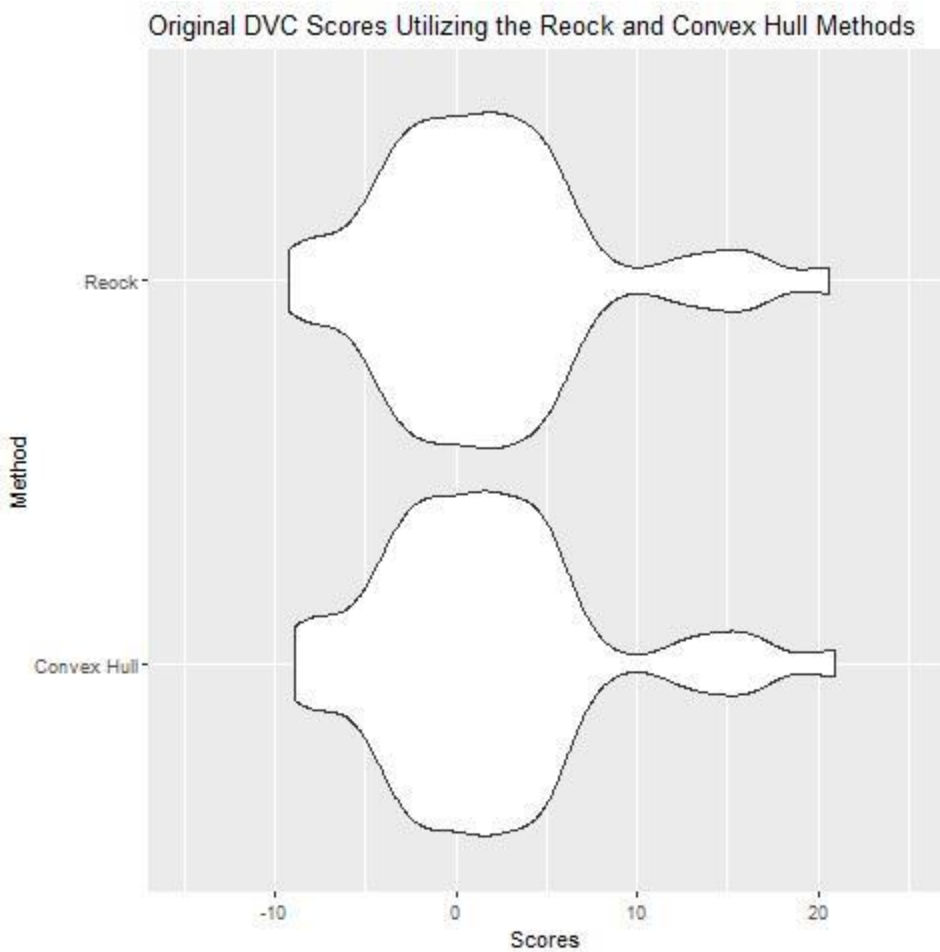
To better compare these distributions, Figure 9 is a representation of the same data, with the average Reock score of each state scaled by the ratio of the average of all convex hull scores in the dataset to the average of all Reock scores in the dataset. Despite the differences in these two methods, the end result when districts are cumulatively examined at the statewide level seems to be minimized.

Figure 9: Distributions of all average state Reock scores and all state convex hull scores, with Reock scores scaled to have the same national average as the convex hull scores.



To further support this point, Figure 10 displays the distributions of the original DVC score using both the Reock and convex hull methods.

Figure 10: Distributions of all state DVC scores from 2010 to 2020 using the Reock and convex hull compactness methods. Reference plan used for each state was from 2000.



4. Discussion

Compactness, like many other current gerrymandering metrics, still struggles to capture the whole picture of what is occurring behind the scenes in the district drawing phase of our elections. However, compactness over time has evolved into a robust method to quantify this concept of "oddness" that drew attention to the gerrymandering issue in the first place, by highlighting districts that appear to have whole chunks taken out of them for apparent political reasons. The convex hull method displayed in this study appears to be another step in the right direction, by creating the capability for this "bounding shape" to more closely hug the outer borders of the encapsulated district, thus more accurately identifying sections that appear to be missing. While the convex hull still struggles with situations where the "gaps" in a district are natural rather than artificial – such as jagged edges along a coastal border – it appears to match or outperform the Reock method in every situation. When other arguments for the convex hull method are taken into account, such as its positive performance when comparing the two-dimensional representations of districts to their actual three-dimensional counterparts, moving from the Reock method to the convex hull seems like a positive addition to the study of compactness and DVC scores.

However, this does not completely invalidate the Reock method – in practice, the Reock method performs nearly identically to the convex hull method. This is especially the case when analyzing entire states rather than one district at a time, which is critical to understanding whether a state has been gerrymandered or not. As seen in Table 1, a few districts outperforming for one party can just as easily result in the party losing seats across the state. In state-level analysis, average Reock and convex hull compactness scores differed mostly in scale, not in distribution. Thus, current and past analyses of compactness utilizing the Reock method should not be discarded and still hold validity, especially when applied to multiple states or across many years of elections. In addition, the Reock also tends to value truly local districts. One might argue that while the longer, skinny rectangular districts don't appear to be actively gerrymandered, by stretching out the voter base across a longer distance they risk losing the communal aspect that is integral to a district election for a local representative.

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