Improving Weight Representivity of Fixed Quantity Consumer Price Index Products

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Abstract

The U.S. Bureau of Labor Statistics Consumer Price Index (CPI) program revises fixed quantity weights for products such as the CPI-U, CPI-W, and preliminary C-CPI-U on a biennial basis- each January of even years. The current biennial weight reference period lags from 2-3 years until first use for even index years, and from 3-4 years for odd index years. Reducing this lag via annual revisions improves the timeliness of the fixed quantity weights for index estimation due to the representativeness of household expenditures from a more recent reference period of the Consumer Expenditure Survey. Additionally, annual revisions create a consistent lag across index years, in contrast to the current biennial revision process. Historically, biennial weight revisions were justified in terms of a sufficient sample from the biennial period, effectively mitigating risk of chain drift. The ensuing analysis will demonstrate that the elementary item-area cell coverage of post-processed annual weights is adequate, and that annual weight revisions pose no appreciable risk of chain drift.

Key Words: CPI-U, Consumer Expenditure Survey, weight revisions

1. Introduction

The Consumer Price Index for the urban population (CPI-U) and the Chained Consumer Price Index for the urban population (C-CPI-U) are published by the CPI program to measure consumer price inflation (Bureau of Labor Statistics, 2020, Handbook of Methods: Consumer Price Index). Index estimation is divided into two stages. The first stage, or lower level estimation, processes collected price observations for elementary goods and services, referred to as items, and geography, referred to as areas. The focus of this paper is the second stage, or upper level estimation, which combines the indexes derived from lower level estimation, and expenditure weights derived from the Consumer Expenditure household survey to calculate aggregate indexes. The same lower level indexes are processed for all CPI products.

CPI products differ by the index formula and corresponding weights. The CPI-U uses a Lowe formula defined below, also referred to as a modified Laspeyres formula. The Lowe is an arithmetic average with fixed quantity weights. The CPI-U currently uses biennial weights that are lagged on average three years, and are revised January of even years. The time period between the weight reference period (when a household reports expenditures) and when first used in the index is the lag. Research shows the shorter the lag the more representative, or relevant, the weight quantities are for the index (Balk and Diewert, 2003; Greenlees and Williams 2009; Huang et. al., 2015 and 2015). The annualized average growth rate from December 2001 to December 2020 was 2.1%.

Lowe formula:

$$CPI - U_{t,0}^{LO} = \sum_{k \in j} \left(\left(\hat{P}_{\alpha k} \hat{Q}_{\beta k} \right) \times \frac{(IX_{tk})}{(IX_{vk})} \right)$$

IX_{tk} = Basic (item and area) level index for period t

IX_{vk} = Basic level index for pivot period, as December of an odd year

$\hat{P}_{\alpha k} \hat{Q}_{\beta k}$ = Basic level aggregation weight where α represents an update relative of the pivot month index divided by the 24 month index average of weight reference period, revised each biennial period, and where β represents a biennial weight reference period

$k \in j$ = k basic indexes are elements of j aggregate (Item and Area) index

In contrast, the final C-CPI-U uses a Tornqvist formula defined below as a geometric average where the weights are a 2 month moving average for the corresponding index month (Cage et. al., 2003). The final C-CPI-U is published about 11 months after the reference month given the lag in monthly weight data availability. A preliminary version of the C-CPI-U fills in that gap, via a Constant Elasticity of Substitution formula which uses fixed quantity weights comparable to the CPI-U (Klick, 2018). The annualized average growth rate from December 2001 to September 2020, latest month of final C-CPI-U publication, was 1.8%.

Tornqvist formula:

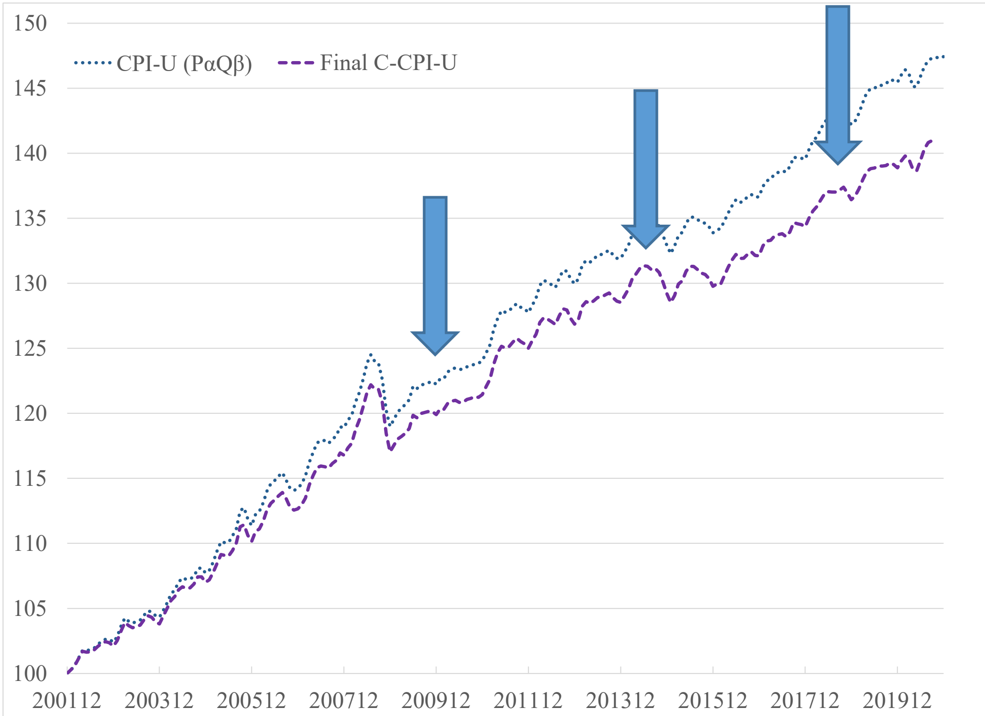
$$Final\ Chained\ CPI - U_{t,0}^{T} = \prod_{k \in j} \left(\frac{IX_{tk}}{IX_{t-1k}} \right)^{\frac{\left(\left(\frac{M_{t-1k}}{\sum_j M_{t-1j}} \right) + \left(\frac{M_{tk}}{\sum_j M_{tj}} \right) \right)}{2}}$$

IX_{t-1k} = Basic level index for period t-1

M_{tk} = Basic level monthly expenditure weight for period t

M_{tj} = Aggregate level monthly expenditure weight for period t

The final C-CPI-U is designed to more closely approximate a cost-of-living measurement objective than the CPI-U. Therefore, there's a motivation to reduce the upper level substitution bias, or the difference between the CPI-U and final C-CPI-U as displayed in Graph 1, which is additive over time. Upper level substitution bias reveals the difference between the CPI-U fixed quantity weight formula, which constrains consumer substitution due to relative price change, and the final C-CPI-U chained monthly weight formula, which reflects consumer substitution due to factors including relative price change. The CPI-U annual average growth rate is 0.3% greater than the final C-CPI-U due to "less timely" fixed quantity weights. This gap can be reduced by decreasing the lag of the fixed quantity weights to be more representative of the index period. Representivity of fixed quantity weights can also be improved by increasing the frequency of weight revisions from biennial to annual revisions, and beyond annual revisions such as semiannual or quarterly revisions, or alternative index formula, both of which are beyond the scope of this paper. Improving fixed quantity weight representivity is subject to a sufficient data quality, and limited risk of chain drift.



Graph 1. CPI-U and Final C-CPI-U; December 2001 = 100

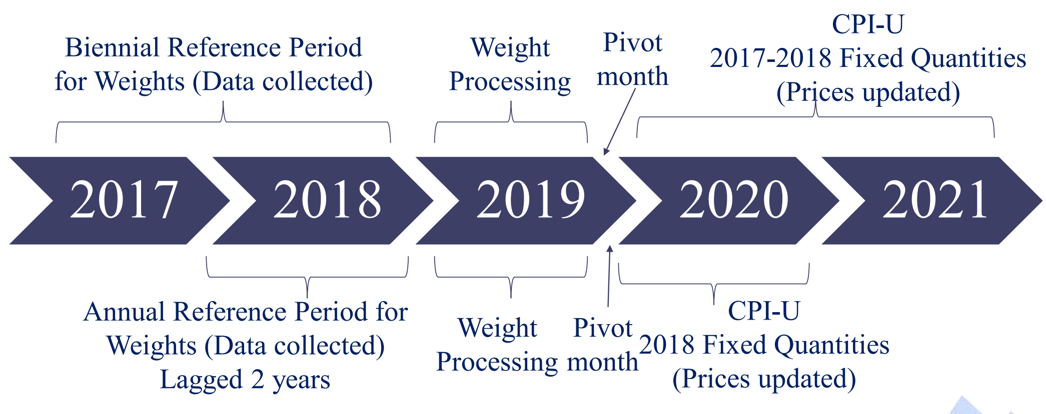
The following section provides an overview of biennial compared to annual weight revisions, and defines the fixed quantity weight formulas evaluated. Next, Section 3 presents results for: data quality, index estimates that includes lags for various fixed quantity weight, upper level substitution bias, index sensitivity to weights, and chain drift. Section 4 presents additional considerations of annual weight revisions. And then Section 5 provides concluding remarks and areas for future research.

2. Overview of weight revisions and fixed quantity weight formulas

The BLS CPI began biennial weight revisions effective with the January 2002 indexes, and prior to that revisions occurred every ten years based on 3 years of CE data (Bureau of Labor Statistics, 2020, Historical-Changes). More frequent revisions are recommended when there is a high level of inflation or sharp changes to household consumption (International Labour Office, 2004, Consumer price index manual: Theory and Practice). Biennial weights are an average of annual data that is composite estimated and raked to smooth across elementary item and area cells to reduce variance (Swanson et. al., 2001). Second stage estimation combines the elementary weights and corresponding indexes to calculate aggregate indexes such as the All Items, which reflects the average weighted price change for all of the items within the CPI market basket.

A general timeline for biennial weight revisions and potential annual weight revisions are outlined in Graph 2. The biennial weight revision process collects data for an odd and even year, referred to as the biennial reference period. The data is processed the following odd year, and then the prices are updated from the reference period as a 24 month index average to the current index period following the pivot month. The weight quantities are fixed for the following 24 months of index estimates. The biennial reference period is lagged 2.5 years for the first index year, and then 3.5 years for the second index year. Biennial weight revisions under this timeline create inconsistent lags of fixed quantity weights. Under biennial weight revisions, the 2022 indexes would have 2019-2020 reference period weights.

A comparable annual weight revision process collects data for a given reference year. Data would be processed the following year, and then prices are updated from the reference period as a 12 month index average to the current index period following the pivot month. The weight quantities could be fixed for the following 12 months of index estimates. The annual reference period is referenced as lagged 2 years prior to index use. Under annual weight revisions, 2021 indexes would have 2019 reference period weights.



Graph 2. Timeline of biennial weight revision vs. annual weight revision

Researching fixed quantity weights allows for review of weight revisions that fit traditional timelines of historical weights for future index periods, as well as untraditional timelines of weights and indexes from the same period as a retrospective analysis. The following formulas modify lagged fixed quantity weights as follows:

1. $P_{\alpha k}Q_{\beta k}$ – Current formula for production of basic level biennial weights as quantities where α represents update relative of pivot month index divided by 24 month index average of weight reference years.
2. $P_{\alpha Rk}Q_{\beta Rk}$ –basic level biennial rolling average weights as quantities where α represents update relative of pivot month index divided by 24 month index average of the rolling biennial period weight reference years.
3. $P_{\alpha(t\text{-}3)k}Q_{A(t\text{-}3)k}$ – basic level annual weights as quantities lagged 3 years prior to index year where α(t-3) represents update relative of pivot month index divided by 12 month index average of weight reference year.
4. $P_{\alpha(t\text{-}2)k}Q_{A(t\text{-}2)k}$ – basic level annual weights as quantities lagged 2 years prior to index year where α(t-2) represents update relative of pivot month index divided by 12 month index average of weight reference year.
5. $P_{\alpha(t\text{-}1)k}Q_{A(t\text{-}1)k}$ – basic level annual weights as quantities lagged 1 years prior to index year where α(t-1) represents update relative of pivot month index divided by 12 month index average of weight reference year. Theoretical target and motivation for future CE survey and CPI improvements to production processing timelines.
6. $P_{\alpha(t\text{-}0)k}Q_{A(t\text{-}0)k}$ – basic level annual weights as quantities lagged 0 years prior to index year where α(t-0) represents update relative of pivot month index divided by 12 month index average of weight reference year. This represents a thought experiment where weights are same period as index year that is consistent with the existing Lowe formula now as a retrospective analysis of lag 0. It is necessary for the fixed quantity

weights to reference a 12 month average price as a single fixed point instead of the January - December price that occurred over the reference year. Requires retrospective 12 months of data, otherwise would require forecasting annual weights and index averages to process when complete 12 months not available.

R = Biennial rolling average

(t-3) = Annual period lagged 3 years prior to index year

(t-2) = Annual period 2 years prior to index year

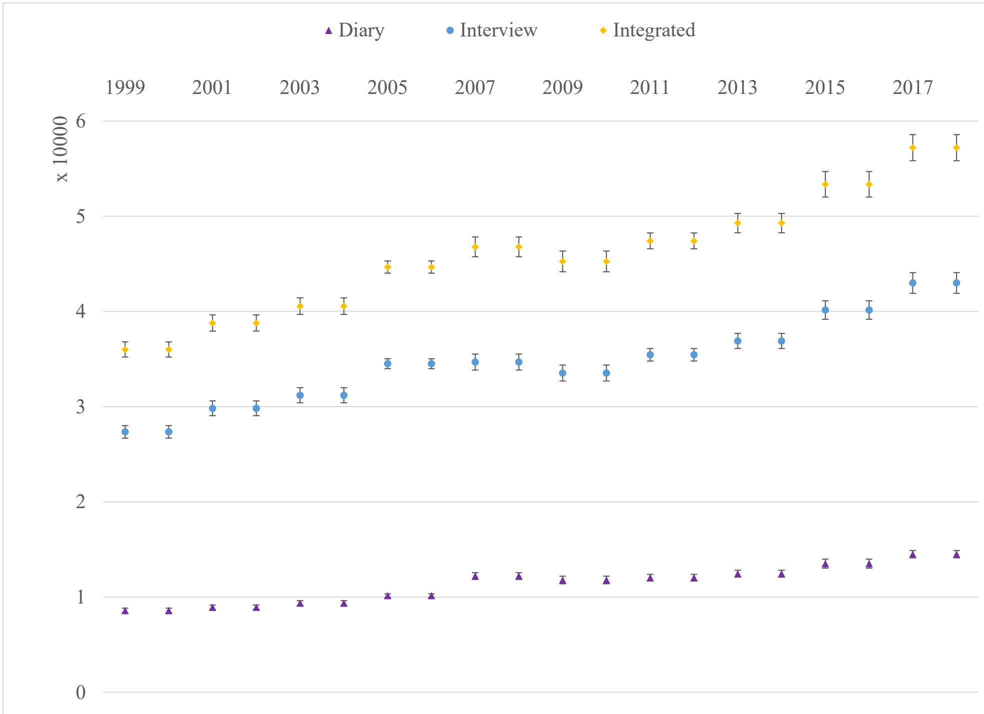
(t-1) = Annual period 1 years prior to index year

(t-0) = Annual period 0 years prior to index year

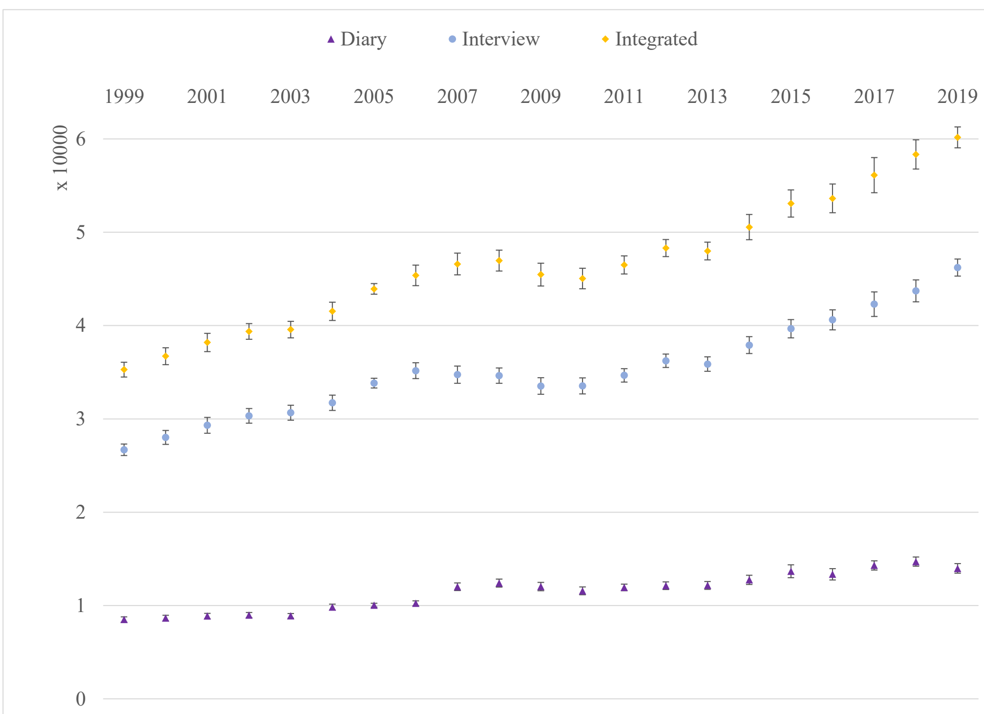
3. Results

A. Data Quality

Data quality is evaluated in terms of average household expenditures and elementary cell coverage as a percent of missing. For 2018 index processing as an example, there were 243 items, and 32 geographic areas, which represent price change and weights for 7776 CPI item-area combinations processed for upper level estimation. Biennial and annual average household expenditures for the CE Interview and Diary Surveys, as well as an integrated form as processed by the CPI, and Balanced Repeated Replication (BRR) 95% confidence intervals are displayed in Graphs 3 and 4. Analysis of the annual average expenditure identifies if an individual year contributes to the variance within the biennial period. Overall, the 95% confidence intervals for biennial estimates from Graph 3 appear relatively consistent across periods, comparable to the 95% confidence intervals for annual data from Graph 4.



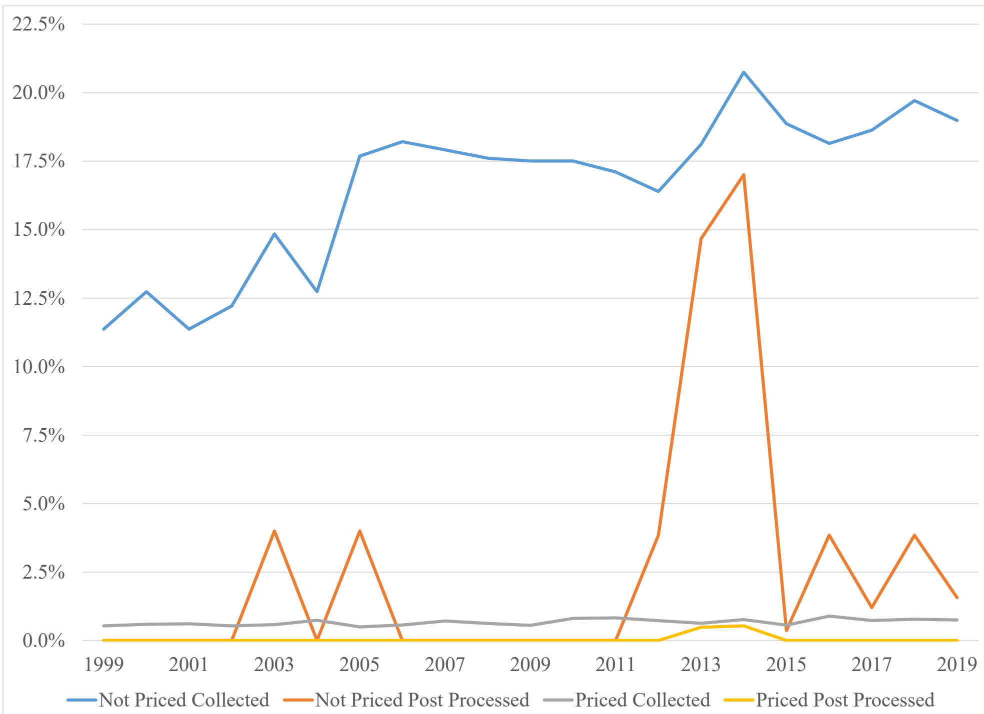
Graph 3. Biennial CPI Household Expenditures and Balanced Repeated Replication 95% Confidence Intervals



Graph 4. Annual CPI Household Expenditures and Balanced Repeated Replication 95% Confidence Intervals

Elementary items can be divided into priced and unsampled (prices not collected due to cost and/or difficulty and therefore are imputed at a higher item level). Examples include:

Unsampled owners' equivalent rent of secondary residence, Unsampled furniture, Unsampled apparel). The quality of coverage can be evaluated via collected data before composite estimation and raking, or via post processed data (Bureau of Labor Statistics, 2020, Consumer Price Index, CPI-U and CPI-W: input basic expenditure weights). Priced items display a low percent of missing elementary cells when evaluating collected data, and an even lower percent after post processing. Unsampled items are missing data in 10-20% of cells when evaluating collected data. Individual year spikes frequently occur due to the household not reporting expenditures for the unsampled item.

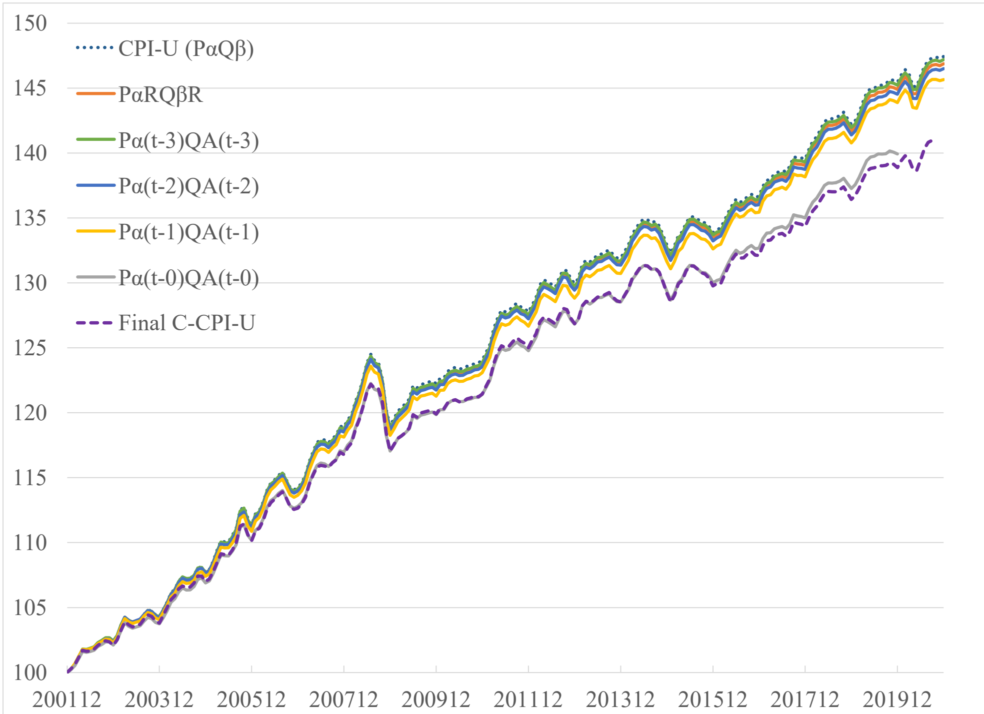


Graph 5. Elementary cell coverage- percent missing

B. Index estimates

Weight representivity can be measured by comparing Lowe indexes to the Final C-CPI-U. The CPI-U currently processes fixed quantity weights from 3 years prior, for a biennial period. As weight representivity improves by decreasing the lag (from 3 to 2 to 1 to 0), the smaller the lag the more closely the CPI-U approximates the Final C-CPI-U as displayed in Graph 6, and as annual growth rates in Table 1. Note that the CPI-U update relative denominator is an index average of the corresponding weight period, so this analysis incorporates both weights and index averages to calculated fixed quantities.

The rolling biennial average weights revised annually is comparable to the CPI-U biennial weights/revisions. The lag of 1 year is included as motivation for future processing considerations. The lag of 0 years provides a theoretical perspective that fixed quantity indexes can perform nearly as well as the final C-CPI-U as a retrospective analysis.



Graph 6. Index Summary of Annual versions of Lowe formula

Table 1. Annual Growth rate comparison to December 2019

	Index Level	CPI-U Less	Annual Growth Rate	CPI-U Less	Less Final C-CPI-U
CPI-U ($P_\alpha Q_\beta$)	145.453	0.000	2.094%	0.000%	0.261%
$P_{\alpha R}Q_{\beta R}$	144.895	0.558	2.072%	0.022%	0.239%
$P_{\alpha(t-3)}Q_{A(t-3)}$	145.222	0.231	2.085%	0.009%	0.252%
$P_{\alpha(t-2)}Q_{A(t-2)}$	144.526	0.927	2.058%	0.036%	0.225%
$P_{\alpha(t-1)}Q_{A(t-1)}$	143.883	1.570	2.032%	0.061%	0.200%
$P_{\alpha(t-0)}Q_{A(t-0)}$	139.928	5.525	1.875%	0.218%	0.042%
Final C-CPI-U	138.879	6.574	1.833%	0.261%	0.000%

Reducing the weight lag reduces the overall, annual, and monthly growth rates as summarized in Table 2. Reducing the lag from 3 to 2 results in a lowered growth rate of 0.025%, and then reducing the lag from 2 years to 1 results in a lowered growth rate of 0.031% as summarized by the gold highlights in Table 3.

As described in the formulas above, there are two components contributing to the fixed quantity lagged estimates: the price update relative effect and the fixed quantity weight effect. The middle portion within Table 2 fixes the quantities at lag of 2 years to evaluate the price update effect. Reducing the lag from 3 to 2 years results in a lowered growth rate of 0.049%, and then reducing the lag from 2 to 1 year results in a lowered growth rate of 0.044% as summarized by the orange highlights in Table 3. A ratio of the price update relative effect to the overall is 1.94 and 1.43 for the respective periods.

The bottom portion within Table 2 fixes the price update relative at lag of 2 to evaluate the weight effect. Reducing the lag from 3 to 2 years results in a lowered growth rate of 0.017%, and then reducing the lag from 2 to 1 year results in a lowered growth rate of -0.021% as summarized by the blue highlights in Table 3. The negative value reducing lag from 2 to 1 years is inconsistent with the results from overall and price update relative effect, and merits further review. A ratio of the weight effect to the overall is 0.68 and -0.67 for the respective periods. The price update relative effect is larger than the weight effect, indicating that updating the average prices to calculated fixed quantity weights is a major factor contributing differences across lags for annual weight revisions.

Table 2. Growth rates as percentage; 229 monthly unless noted otherwise

	CPI-U ($P_\alpha Q_\beta$)	$P_{\alpha R}Q_{\beta R}$	$P_{\alpha(t-3)}Q_{A(t-3)}$	$P_{\alpha(t-2)}Q_{A(t-2)}$	$P_{\alpha(t-1)}Q_{A(t-1)}$	$P_{\alpha(t-0)}Q_{A(t-0)}$*	Final C-CPI-U**
Overall	47.434	46.868	47.190	46.495	45.646	39.928	40.963
Annual average	2.055	2.035	2.046	2.021	1.990	1.875	1.840
Monthly average	0.170	0.168	0.169	0.167	0.164	0.155	0.152
Price Update Relative Effect: $Q_{A(t-2)}$ fixed			$P_{\alpha(t-3)}Q_{A(t-2)}$	$P_{\alpha(t-2)}Q_{A(t-2)}$	$P_{\alpha(t-1)}Q_{A(t-2)}$	$P_{\alpha(t-0)}Q_{A(t-2)}$*	
Overall			47.849	46.495	45.286	39.713	
Annual average			2.070	2.021	1.977	1.768	
Monthly average			0.171	0.167	0.163	0.146	
Weight Effect: $P_{\alpha (t-2)}$ fixed			$P_{\alpha(t-2)}Q_{A(t-3)}$	$P_{\alpha(t-2)}Q_{A(t-2)}$	$P_{\alpha(t-2)}Q_{A(t-1)}$	$P_{\alpha(t-2)}Q_{A(t-0)}$*	
Overall average			46.967	46.495	47.067	45.442	
Annual average			2.038	2.021	2.042	2.093	
Monthly average			0.168	0.167	0.169	0.173	

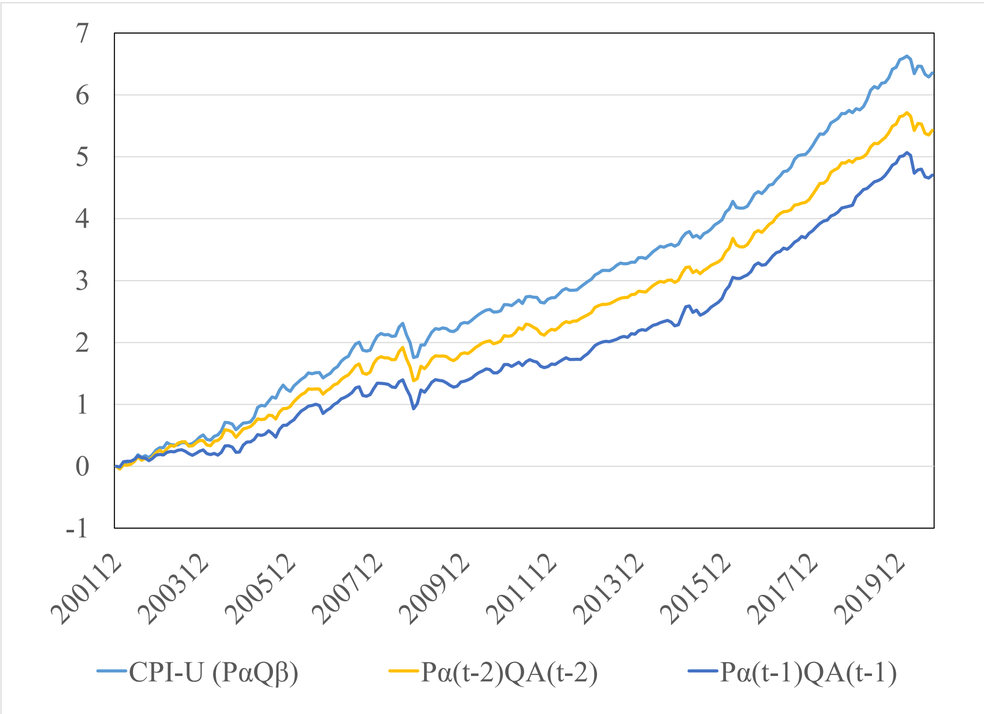
**217 months because 2020 annual weights not yet available for research processing*
***226 months because final C-CPI-U published through September 2020*

Table 3. Annual average growth rate differences as percentage

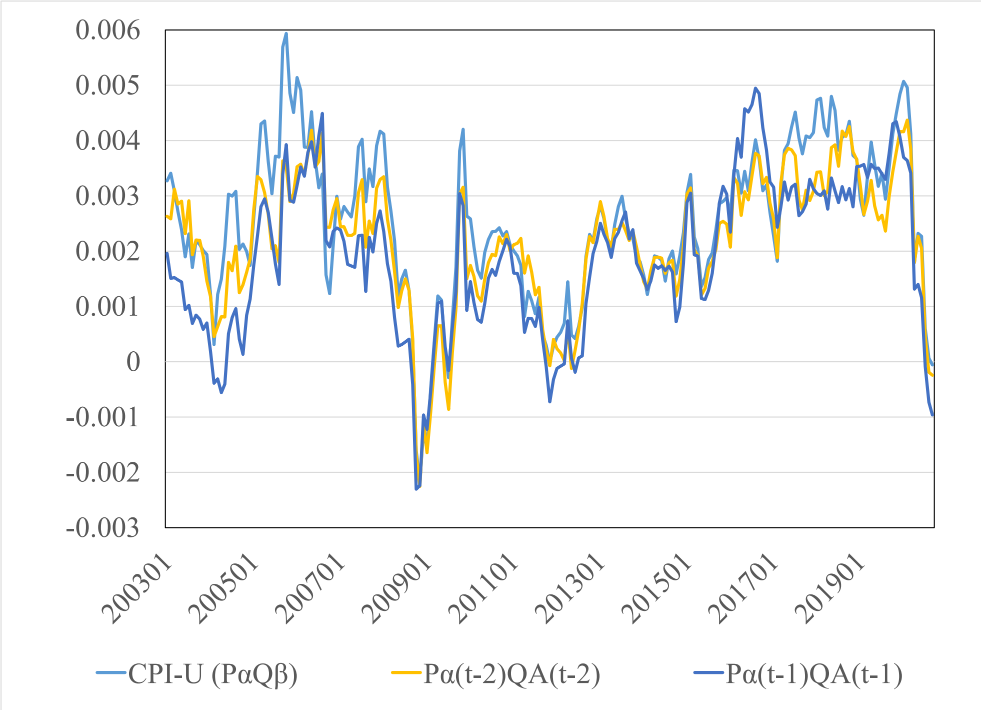
	$P\alpha_{(t-3)}Q_{A(t-3)}$ less $P\alpha_{(t-2)}Q_{A(t-2)}$	$P\alpha_{(t-2)}Q_{A(t-2)}$ less $P\alpha_{(t-1)}Q_{A(t-1)}$
Overall	0.025	0.031
	$P_{\alpha(t-3)}Q_{A(t-2)}$ less $P_{\alpha(t-2)}Q_{A(t-2)}$	$P_{\alpha(t-2)}Q_{A(t-2)}$ less $P_{\alpha(t-1)}Q_{A(t-2)}$
Update Relative effect (PUR)	0.049	0.044
PUR/Overall Ratio	1.94	1.43
	$P_{\alpha(t-2)}Q_{A(t-3)}$ less $P_{\alpha(t-2)}Q_{A(t-2)}$	$P_{\alpha(t-2)}Q_{A(t-2)}$ less $P_{\alpha(t-2)}Q_{A(t-1)}$
Weight effect (W)	0.017	-0.021
W/Overall Ratio	0.68	(0.67)

C. Upper level substitution bias

Substitution bias defined as fixed quantity index estimate less the final C-CPI-U, can be evaluated as an index which is additive over time, or as 12 month average to remove the additive effect as displayed in Graphs 7 and 8 respectively. The substitution bias measure for the lag of 2 is lower than the current CPI-U. The substitution bias measure for lag of 1 is lower than CPI-U and lag 2 as expected. One additional consideration for the 12 month change is that for some months the lag 1 is not the closest to 0 when compared to both the CPI-U and lag 2 due to the numerator and denominator representing different points in time.



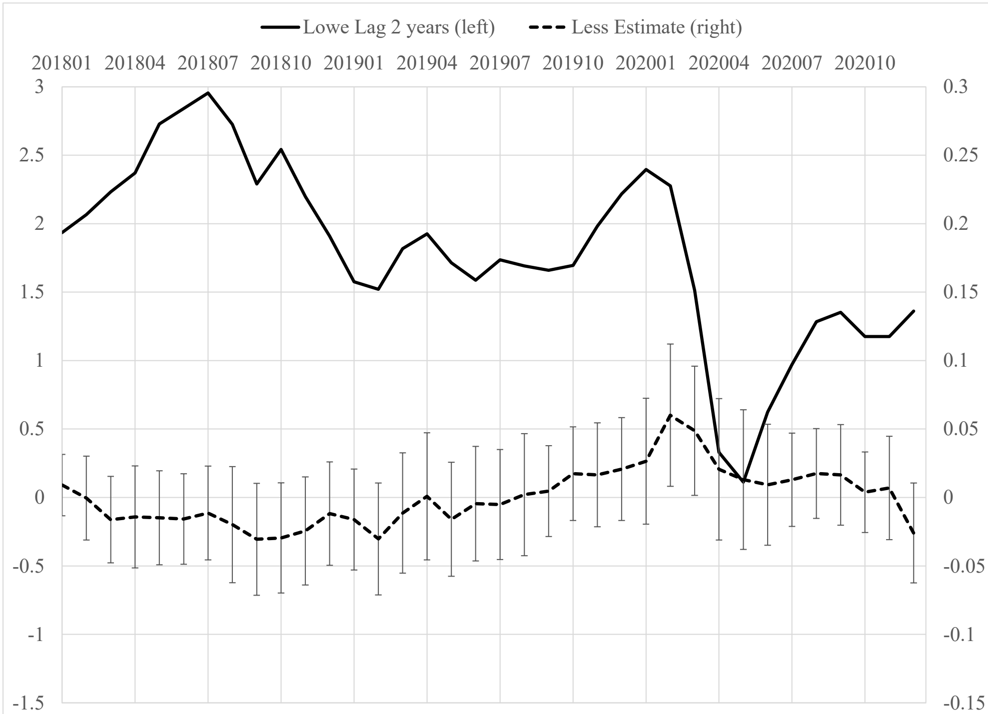
Graph 7. Upper Level Substitution bias- index estimate less final C-CPI-U



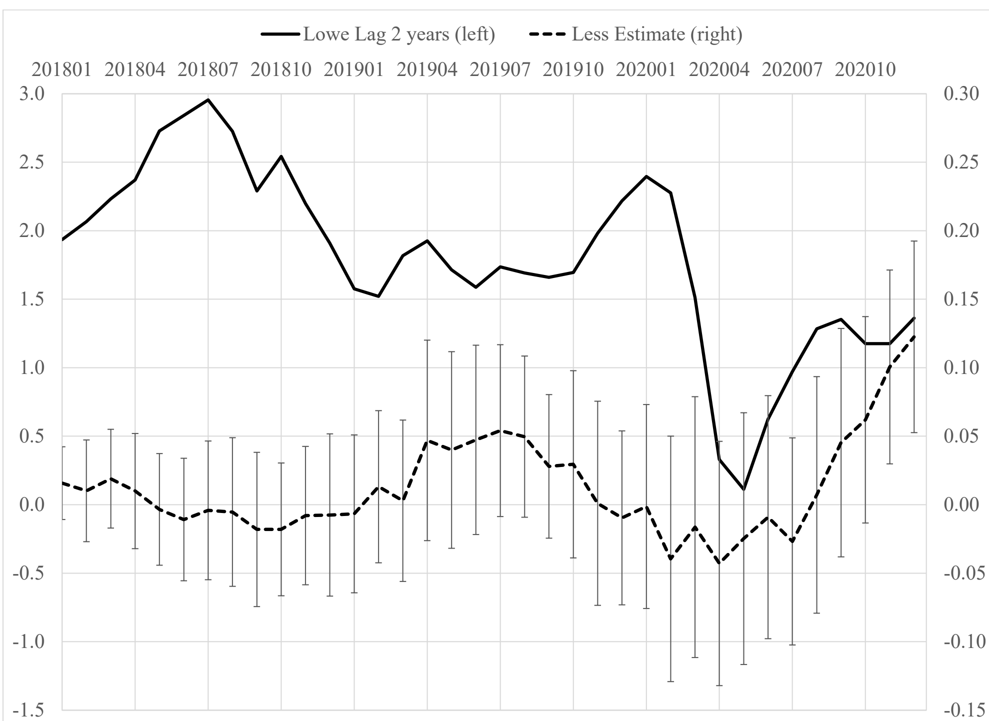
Graph 8. Upper Level Substitution bias- 12 month change estimate less final C-CPI-U

D. Index sensitivity

Index sensitivity to annual weight revisions is evaluated via a sampled proportion of households, and Jackknife standard error differences of the 12 month change (Fitzgerald et. al., 2015; Klick et. al., 2019). Index results derived from a 50% subsample of CE urban households are displayed in Graph 9, and then index results derived from a 25% subsample of CE urban households are displayed in Graph 10. The 12 month change for lag 2 is scaled on the left axis and the 12 month change less sampled proportion estimate is scaled on the right axis. These results depend on weighting from a sampled proportion of households for price change occurring at the elementary level. Sampling proportions of households performs well relative to the full sample, where one would expect significant differences approximately 5% of the time.



Graph 9. Lowe Lag 2 years 12 month change and Less Estimate with 95% Confidence Interval Jackknife standard error: Weights as 50% subsample of CE Household stratified by collection quarter & area



Graph 10. Lowe Lag 2 years 12 month change and Less Estimate with 95% Confidence Interval Jackknife standard error: Weights as 25% of CE Household sample stratified by collection quarter & area

E. Chain drift

The prevalence of risk of chain drift, where the index drifts from the expected trend, is evaluated using a circularity test to test the prevalence of transitivity of long term price change, as a ratio of growth rates + 1 relative to a baseline of 1 (Cage et. al., forthcoming, Klick, 2017). Biennial revision growth rates are compared to annualized revision growth rates as displayed in Table 4. The All Items CPI-U displays a small amount of drift for overall estimates, and then nearly 0 drift for annualized estimates. Fresh Fruits is evaluated because it is traditionally subject to chain drift. Fresh Fruits results are comparable to the all items, indicating that annualized revisions have minimal to no drift when compared to biennial revisions.

Table 4. CPI-U ($P_{\alpha}Q_{\beta}$) Biennial Revision & $P_{\alpha(t-2)}Q_{A(t-2)}$ Annual Revision circularity test as risk of drift (ratio of growth rates + 1); 200112-202012 (229 months)

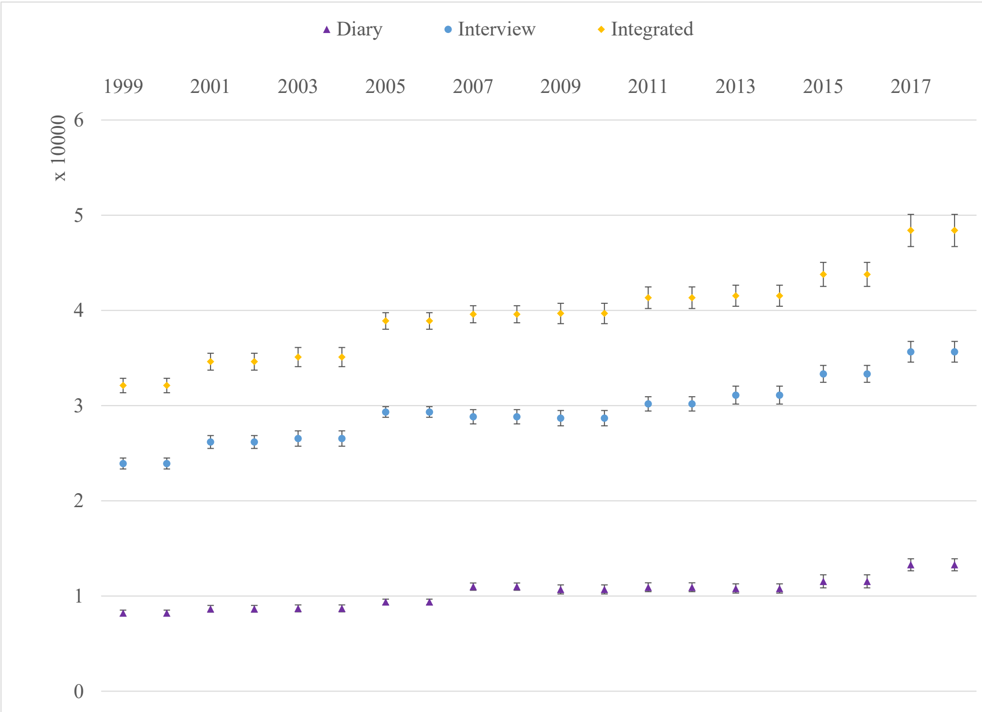
	All Items		Fresh Fruits	
	CPI-U Biennial Revisions	$P_{\alpha(t-2)}Q_{A(t-2)}$ Annual Revisions	CPI-U Biennial Revisions	$P_{\alpha(t-2)}Q_{A(t-2)}$ Annual Revisions
Overall	47.43%	46.5%	32.24	32.53
Overall circularity		0.99		1.01
Annual	2.06%	2.02%	1.48%	1.53%
Annual circularity		1.00		1.00

4. Discussion

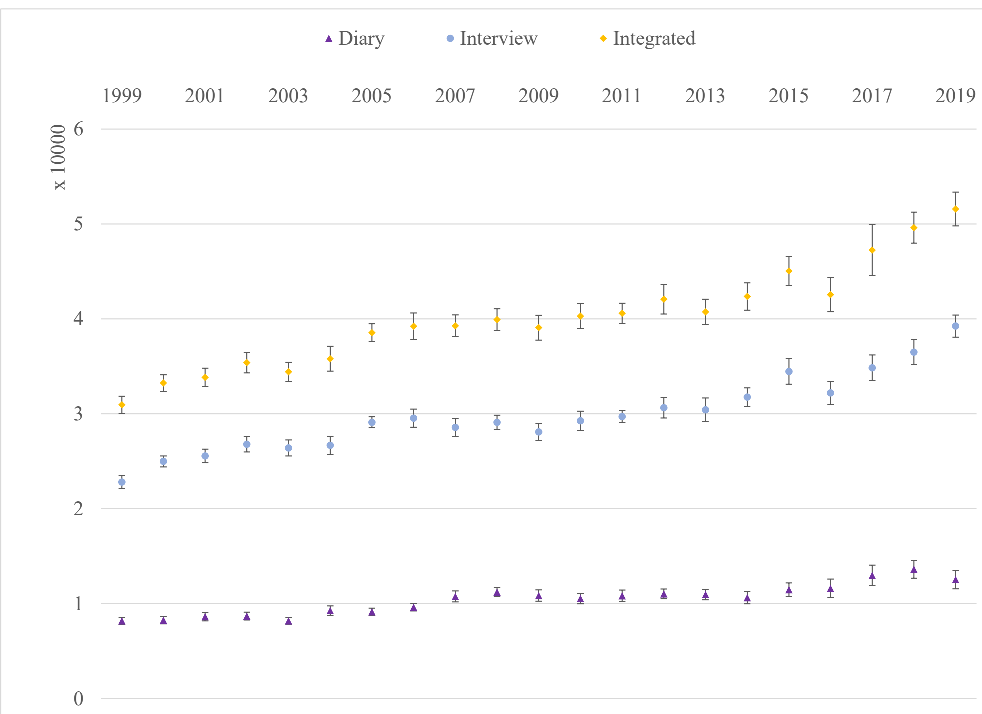
Improving weight representivity of fixed quantity weight indexes by reducing the implementation lag will reduce upper level substitution bias. Additional considerations of annual vs. biennial revisions are summarized below.

One of the other CPI headline products, the Consumer Price Index for Wage Earners (CPI-W) represents about 1/3 the sample size of the urban population. Eligibility for the wage earner population depends on the majority of household income from wage earner related occupations, and at least one member of the household is employed at least 37 weeks for an eligible wage earner occupation.

Biennial and annual average Wage Earner population household expenditures for the CE Interview and Diary Surveys, and integrated form as processed by the CPI, and Balanced Repeated Replication (BRR) 95% confidence intervals are displayed in Graphs 10 and 11. The wage earner population average expenditures are lower for the Diary, Interview, and Integrated estimates than the urban population estimates, with comparable 95% confidence intervals. The wage earner population annual average expenditures are less smooth than biennial estimates, indicating that individual years contribute to the variance of biennial period especially for year 2016 for the 2015-2016 biennial period. Data quality as defined by annual average expenditures of subpopulations should be reviewed further to determine if subpopulation (such as Wage Earners) weight revisions should follow the urban population, or should be evaluated independent of the urban population.



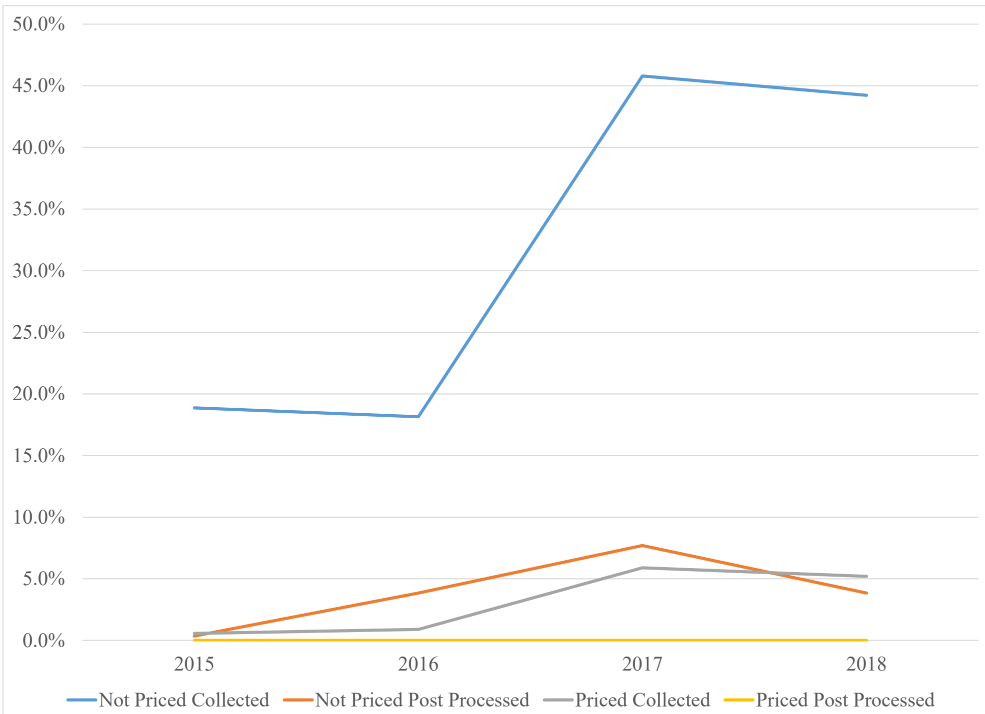
Graph 10. Wage Earner Biennial CPI Household Expenditures and Balanced Repeated Replication 95% Confidence Intervals



Graph 11. Wage Earner Annual CPI Household Expenditures and Balanced Repeated Replication 95% Confidence Intervals

The annual weights elementary cell coverage is expected to be minimally impacted due to smoothing to reduce variance across geography. From 2015-2018, elementary cells for collected data, not priced items is high at about 30%, but the post processed version is less than 10% as displayed in Graph 10. The collected and priced items range from 1% to 5%, and the post processed version is 0% across each of the years.

Therefore, if the above data quality is determined to be sufficient, then annual weight revisions should serve to improve representivity of the CPI-W relative to biennial weight revisions (Casey, 2010). What is not yet defined is a chained CPI-W (C-CPI-W) and if post processing methodology of monthly weights should smooth elementary monthly expenditure data more due to the smaller sample size than the urban population, or possibly develop an elementary cell imputation process. Therefore, attempting to define CPI-W upper level substitution bias relies on monthly weight processing geared for the wage earner population.



Graph 12. Wage Earner Population Elementary Cell Coverage- Percent Missing

Another CPI product to benefit from annual revisions would be the preliminary C-CPI-U, which uses a fixed quantity weight Constant Elasticity of Substitution (CES) formula to set a defined level of substitution occurring due to relative price change. The preliminary C-CPI-U initial version is released concurrent with the CPI-U, and then the interim versions are revised quarterly 3 times until the final is released. The initial and interim versions chain to the terminal final C-CPI-U, which is revised forward each quarter. Initial less final estimates are summarized as revision size.

The CES pivoted expenditure weight for a biennial reference period used for production is:

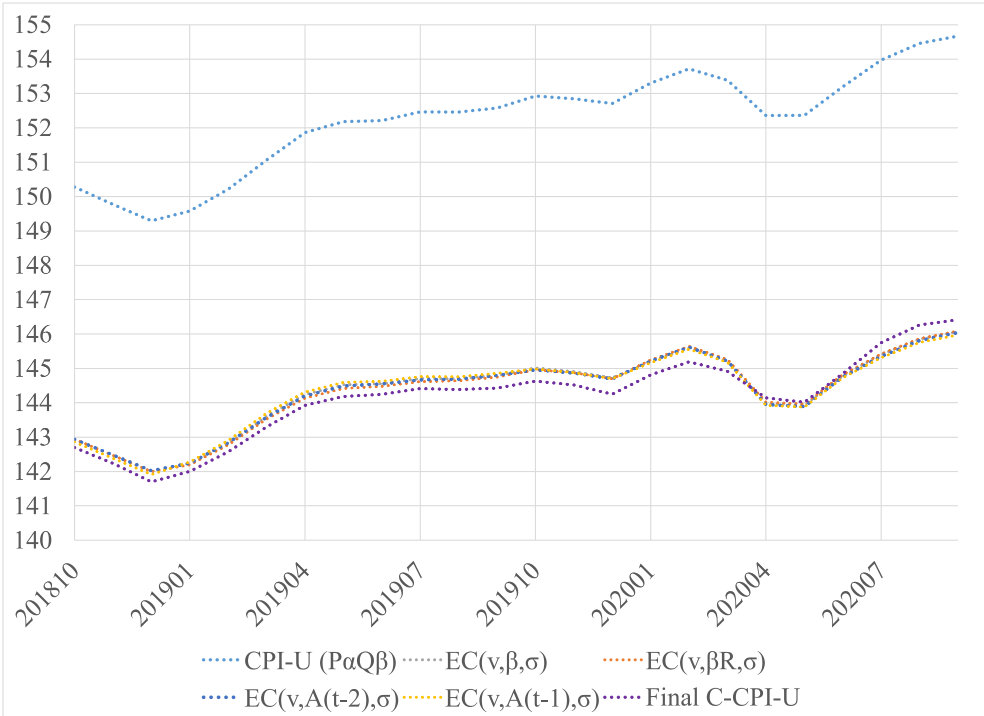
$$E^{C}_{v,\beta,\sigma} = P_\beta Q_\beta * \left(\frac{\mathrm{IX}_v}{\mathrm{IX}_\beta}\right)^{(1-\sigma)}$$

$P_\beta Q_\beta$ is the elementary level annualized expenditure weight from the biennial reference period,
IX_β is the elementary level 24 month average index from the biennial reference period,
σ is the sigma for the biennial reference weight period set at 0.6, and
$E^C_{v,\beta,\sigma}$ is the elementary level annualized expenditure weight from a biennial reference period where prices are from a pivot period adjusted for consumer substitution

The CES formulas evaluated with lags comparable to the CPI-U fixed quantity weight formulas above are defined as:

1. $EC_{(v,\beta,\sigma)}$ Updated CES expenditure share based on biennial period comparable to the CPI-U where prices are from a pivot period adjusted for consumer substitution
2. $EC_{(v,A(t-1),\sigma)}$ Updated CES expenditure share based lag of 1 year comparable to the $P_{\alpha(t-1)}Q_{A(t-1)}$ where prices are from a pivot period adjusted for consumer substitution.
3. $EC_{(v,A(t-2),\sigma)}$ Updated CES expenditure share based lag of 2 years comparable to the $P_{\alpha(t-2)}Q_{A(t-2)}$ where prices are from a pivot period adjusted for consumer substitution.
4. $EC_{(v,\beta R,\sigma)}$ Updated CES expenditure share based rolling biennial period comparable to $P_{\alpha R}Q_{\beta R}$ where prices are from a pivot period adjusted for consumer substitution.

The following evaluates the initial series as an approximation of the final for the versions defined above. Analysis is primarily limited to a quarterly time frame due to quarterly chaining to interims and finals. The different versions of initials provide close approximations of the final C-CPI-U as displayed in Graph 10. The September 2020 final C-CPI-U provides a comparison period to evaluate the annual average growth rates displayed in Table 5, where differences are small at about 0.01%. The revision size is summarized as an absolute value difference in Table 6, where the biennial rolling average performed best in terms the sum, mean, and mean squared error of the absolute value differences though there are small differences between the CES updated weight versions.



Graph 13. Index Summary of Annual versions of Constant Elasticity of Substitution Formula

Table 5. Annual Average Growth Rate Percentages with Constant Elasticity of Substitution Formulas; December 1999 = 100

	CPI-U ($P_\alpha Q_\beta$)	Final C-CPI-U	$EC_{(v,\beta,\sigma)}$	$EC_{(v,A(t-1),\sigma)}$	$EC_{(v,A(t-2),\sigma)}$	$EC_{(v,\beta R,\sigma)}$
Overall	54.672	46.417	46.072	45.975	46.045	46.089%
Annual	2.124	1.855	1.843	1.840	1.842	1.843%

Table 6. Revision Size as absolute different relative to final C-CPI-U

	$EC_{(v,\beta,\sigma)}$	$EC_{(v,A(t-1),\sigma)}$	$EC_{(v,A(t-2),\sigma)}$	$EC_{(v,\beta R,\sigma)}$
Sum	7.237	7.872	7.285	6.486
Mean	0.302	0.328	0.304	0.270
Mean Square Error	0.098	0.108	0.089	0.110

An additional consideration of moving from a biennial weight revision to an annual weight revision is evaluation of periods where consumption is inconsistent with historical trends such as the recent COVID19 pandemic (United Nations Economic Commissions for Europe, 2021: Guide on producing CPI under lockdown). At present, the January 2022 weight revision will be calculated based on household expenditures from 2019-2020, consistent with weight revision methodology beginning in 2002. If annual weight revisions were to be pursued further, criteria would need to be defined regarding the weight revision frequency, and alternatives to remediate the anomalous year of consumer expenditures.

5. Conclusion

The above analysis demonstrates that revising weight revision methodology to improve representivity by decreasing the lag, i.e. decreasing the time between when expenditures occurred and when they are used in index estimation, decreases the upper level substitution bias between the CPI-U and the final C-CPI-U. Additional analysis is needed to determine if annual revisions will decrease the revision size between the initial and final C-CPI-U, which is complicated by quarterly revisions chaining to finals and interims. When an alternative lagged CES formula was processed independent of chaining to finals results indicated that annual revisions lagged by two years performed better than the current biennial revisions, and that annual revisions lagged by one year performed the best.

The above analysis indicates that the data quality of annual weights is sufficient when compared to biennial weights. Additionally, post processing smoothing already occurs at the annual frequency. Also, there is no appreciable risk of chain drift as shown by the circularity test though additional sub-aggregate items with seasonal trends should continue to be evaluated. One additional benefit of annual revisions is that the lag in fixed quantities would remain consistent for future periods, in contrast to the current biennial weight revision process that lags 2.5 years for even index years, and 3.5 years for odd index years, which complicates interpretation of representivity of weights.

There are a number of research opportunities for future improvements to weight revisions. One such project is to evaluate increasing the frequency of fixed quantity weight revisions. Increased weight revision frequency below the annual level will require more rigorous analysis of seasonality and limited risk of chain drift. Another research opportunity is to evaluate improvement of elementary cell coverage, and related improvements to post processing smoothing of weights, as well as any special considerations for processing weights for subpopulations.

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