Inference for the Progressively Type-I Censored k-level Step-stress Accelerated Life Tests under Interval Monitoring with the Lifetimes from a Log-location-scale Family

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Abstract
As the field of reliability engineering continues to grow and adapt with time, accelerated life tests (ALT) have progressed from luxury to necessity. The rapid generation of information about the life expectancy of products and devices is crucial to companies wishing to get their products on the market as soon as possible. ALT enables this process by subjecting test units to higher stress levels than normal conditions, thereby generating more failure data in a shorter time period. In this work, we study a progressively Type-I censored k-level step-stress ALT under interval monitoring. In practice, the financial and technical barriers to ascertaining precise failure times of test units could be insurmountable, therefore, it is often practical to collect failure counts at specific points in time during ALT. Here, the latent failure times are assumed to have a log-location-scale distribution. The motivation to use the log-location-scale family other than the simple exponential distribution is that the observed lifetimes may follow Weibull or log-logistic distributions, which are members of the log-location-scale family. Here, we develop the inferential methods for the step-stress ALT under the general log-location-scale family, assuming that the location parameter is linearly linked to the stress level. The methods are illustrated using three popular lifetime distributions: Weibull, and log-logistic.

Key Words: accelerated life tests, Fisher information, interval monitoring, log-location-scale family, order statistics, progressive Type-I censoring, step-stress loading

1. Introduction

With ongoing improvement in manufacturing processes, and technological advances, products and devices that are available today are expected to be rigidly reliable, and last significantly longer than their predecessors. Therefore, the traditional methods of reliability testing are rapidly becoming obsolete. Accelerated life testing (ALT), became a useful tool in cases where test units are subjected to a higher stress environment (e.g. pressure, temperatures, voltage, vibration rate and load, etc.), in order to speed up the failure process. By analyzing the data obtained from these tests, the service life, warranty periods, and maintenance intervals of a product can be decided. The lifetime of the product, presuming normal operating conditions, can then be estimated by extrapolating the data from a stress-response regression model. The step-stress loading scheme is utilized when the stress level remains constant for a predetermined period of time, until the stress level is changed. Step-up stress testing, a subclass of ALT, is similar, though stress levels are gradually increased at fixed time intervals, rather than remaining constant as in standard step-stress testing.

Over the past few decades, there has been increasing focus on step-stress ALT, in particular the inference and design optimization, as evidenced by review of the reliability literature. For example, Miller and Nelson (1983), Bai et al. (1989), Nelson (1990), Meeker and Escobar (1998), Bagdonavicius and Nikulin (2002), Wu et al. (2006), Balakrishnan and Han (2008, 2009), Han and Balakrishnan (2010), Kateri et al. (2010), Han and Ng (2013), Lin et al. (2013), Han and Kundu (2015), and Han (2015). Furthermore, from a practicality standpoint, censored sampling is almost universally used, as it takes less time

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and is more cost-effective. Specifically, a generalized censoring scheme known as progressive Type-I censoring allows for successive withdrawal of functional test units from the life test at some prefixed non-terminal time points. The non-failed units can subsequently be used in other tests, even at different facilities. See Gouno et al. (2004), Han et al. (2006), and Balakrishnan et al. (2010).

Despite the advantages of flexibility, efficiency, and cost, progressively censored sampling remains an unpopular choice, mainly due to increased complexity of analyses. As such, more conventional censoring schemes are still widely employed; see Cohen (1963), Lawless (1982). Inference for the Progressively Type-I Censored ssALT under interval monitoring is less popular, but it is interesting in a pragmatic sense. Due to the fact that it monitors failures at particular time periods, it is useful for designing and managing the life test under technical limitations and budgetary constraints.

In our project, we study the statistical inference for a progressively Type-I censored k-level step-stress accelerated life tests under interval monitoring with the lifetime of a test unit follows a log-location scale family of distributions. Despite simplicity of analysis, there is a relative lack of flexibility to the exponential distribution, due to the constraint of constant hazard rates. Members of log-location-scale family, however, demonstrate both increased numbers of and generally superior model fits in practice. Therefore, our work extends consideration to the general log-location-scale family, and our inferential methods are clarified using two popular lifetime distributions, including Weibull and log-logistic. Assuming that the location parameter is linearly linked to the stress level x_i (i.e., $\mu_i = \alpha + \beta x_i$), where α and β are regression parameters and σ is scale parameter. Fisher's expected information can be derived by having the intermediate censoring take place at the end of each stress level $x_i(viz.,\ \tau_i,\ i = 1, 2, \ldots, k)$. Subsequently the interval estimation is discussed based on these results. Effects of the censoring proportion on inferential performance can also be assessed via computational analysis.

The remainder of the article is organized as follows. The model formulation based on a log-location-scale distribution is discussed in Section 2. The likelihood and MLEs, Fisher information matrix are given in Section 3. The simulation study in Section 4, bias correction method, penalized likelihood method and augmented method we used to overcome bias of MLEs and the results of the computational study of augmented percentile bootstrap are discussed; see Cordeiro and Klein (1994) and Cox and Snell (1968) and Firth (1993).

2. Model Formulation

Here, let n_i be the number of failures at stress level x_i in the time interval $(\tau_{i-1}, \tau_i]$ and c_i denotes the number of censored units at time τ_i, where $i = 1, 2, ..., k$ and $x_1 < x_2 < ... < x_k$, assume $\tau_0 = 0$. N_i denotes the number of units surviving in the beginning of level x_i, such that $N_i = n - \sum_{j=1}^{i-1} n_j - \sum_{j=1}^{i-1} c_j$, where n denotes initial sample size and $N_1 = n$. A progressively Type-I censored k-level step-stress ALT proceeds as follows: Initially, $n \equiv N_1$ number of test units are placed at time $\tau_0 \equiv 0$ and is tested under x_1 stress level until time τ_1. First number of random failures (n_1) are observed during the interval $(\tau_0, \tau_1]$. At τ_1, c_1 number of surviving units are removed and the stress level is increased to x_2. The test continues with the remaining $N_2 = n - n_1 - c_1$ units until time τ_2. Second number of random failures (n_2) are observed during the interval $(\tau_1, \tau_2]$. At τ_2, c_2 number of surviving units are removed and the stress level is increased to x_3. The test continues with the remaining $N_3 = N_1 - n_2 - c_2$ units until time τ_3. This process is continued until k^{th} step. At τ_k, all $c_k = N_k - n_k$ surviving units are removed and the experiment is terminated.

At stress level x_i, the lifetime of a test unit is assumed to have a log-location-scale

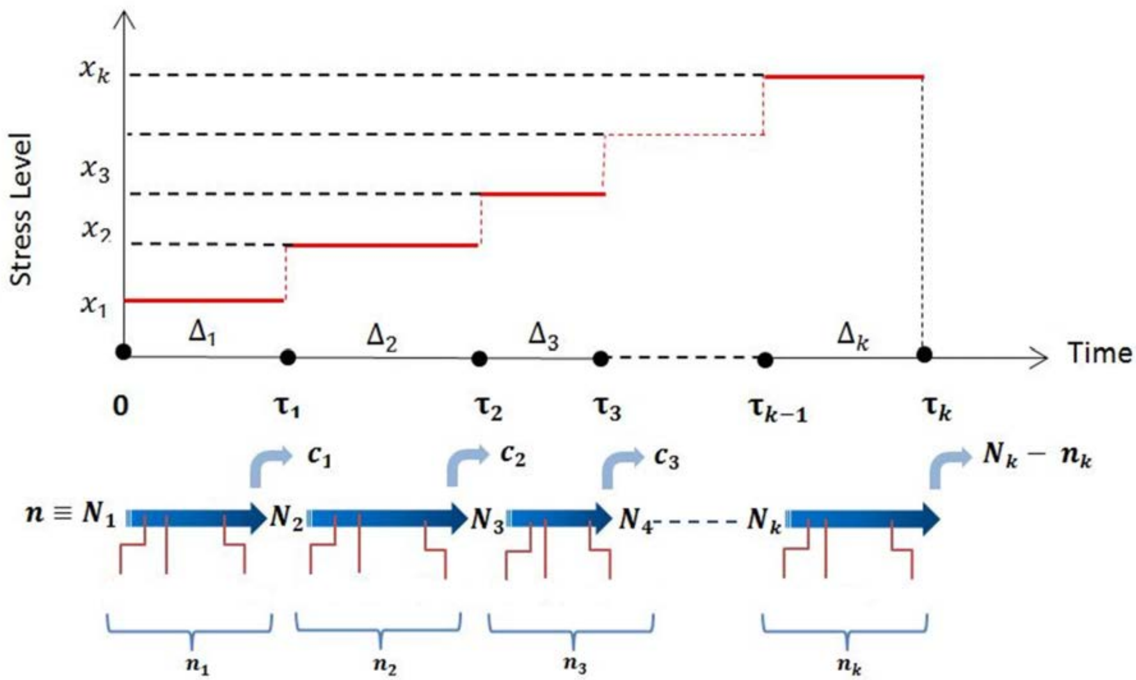

Figure 1: k-level ssALT under progressive Type-I censoring under interval monitoring

distribution with CDF given by

$$F_i(t) = \Phi\left(\frac{\log t - \mu_i}{\sigma}\right), \quad t \geq 0 \tag{2.1}$$

where $\Phi(.)$ is the standard log-location scale CDF, location parameter is $\mu_i = \log\theta_i = \alpha + \beta x_i$ and scale parameter is $\sigma(> 0)$. Further, θ_i is the mean time to failure under the exponential distribution (i.e. when $\sigma = 0$) and α, β are regression parameters, where $\alpha\epsilon(-\infty, \infty)$ and $\beta(< 0)$ are unknown parameters and need to be estimated. Therefore, using the cumulative exposure model, the CDF for a test unit under the step-stress ALT can be obtained as follows:

Step 1: The CDF at the initial stress level x_1 is

$$F_1(t) = \Phi\left(\frac{\log t - \mu_1}{\sigma}\right), \quad 0 < t \leq \tau_1.$$

Step 2: At Δ_1 ($\equiv \tau_1$), $F_1(\tau_1) = F_2(\varepsilon_1)$. Solve for ε_1.

Step 3: Similarly, at τ_2, $F_2(\varepsilon_1 + \Delta_2) = F_3(\varepsilon_2)$, where $\Delta_2 = \tau_2 - \tau_1$. Solve for ε_2.

Step 4: ...

Repeating the above procedure, ε_{i-1} and $F(t)$ are obtained as,

$$\varepsilon_{i-1} = \theta_i \sum_{j=1}^{i-1} \frac{\Delta_j}{\theta_j}$$

$$F(t) = F_i(t - \tau_{i-1} + \varepsilon_{i-1}) = \Phi\left(\frac{\log(t - \tau_{i-1} + \varepsilon_{i-1}) - \mu_i}{\sigma}\right) \tag{2.2}$$

when $\tau_{i-1} < t \leq \tau_i$. The PDF of the lifetime under the step-stress ALT is then obtained as

$$\begin{aligned} f(t) &= f_i(t - \tau_{i-1} + \varepsilon_{i-1}) \\ &= \frac{1}{\sigma(t - \tau_{i-1} + \varepsilon_{i-1})}\phi\left(\frac{\log(t - \tau_{i-1} + \varepsilon_{i-1}) - \mu_i}{\sigma}\right) \end{aligned} \tag{2.3}$$

when $\tau_{i-1} < t \leq \tau_i$ with $\Delta_i = \tau_i - \tau_{i-1}$.

3. Maximum Likelihood Estimation

Using (2.2), the joint distribution of the data $\boldsymbol{n} = (n_1, n_2, ..., n_k)$ is obtained as

$$f(\boldsymbol{n}) = \prod_{i=1}^{k} \binom{N_i}{n_i} [F(\tau_i) - F(\tau_{i-1})]^{n_i} [1 - F(\tau_i)]^{c_i}$$

Furthermore,

$$f(\boldsymbol{n}) = \prod_{i=1}^{k} \binom{N_i}{n_i} [\Phi(\zeta_i) - \Phi(\zeta_{i-1})]^{n_i} [1 - \Phi(\zeta_i)]^{c_i} \tag{3.1}$$

where $F(\tau_i) = \Phi\left(\frac{\log(\Delta_i + \varepsilon_{i-1}) - \mu_i}{\sigma}\right)$ and $\zeta_i = \frac{1}{\sigma}\left(\log(\Delta_i + \varepsilon_{i-1}) - \mu_i\right) = \frac{1}{\sigma} log\left(\sum_{l=1}^{i} \frac{\Delta_i}{\theta_i}\right)$ with $i = 1, 2, ..., k$

After that, using (3.1) the log-likelihood function of (σ, α, β) is obtained as,

$$l = \sum_{i=1}^{k} n_i \log(\Phi(\zeta_i) - \Phi(\zeta_{i-1})) + \sum_{i=1}^{k} c_i \log(1 - \Phi(\zeta_i)) \tag{3.2}$$

Based on (3.2), the MLE $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}$ are solutions to the likelihood equations given by

$$\frac{\partial l}{\partial \sigma} = l'_\sigma = \frac{1}{\sigma}\sum_{i=1}^{k} n_i \frac{-(\zeta_i \phi(\zeta_i) - \zeta_{i-1}\phi(\zeta_{i-1}))}{\Phi(\zeta_i) - \Phi(\zeta_{i-1})} + c_i \zeta_i h_i = 0$$

$$\frac{\partial l}{\partial \alpha} = l'_\alpha = \frac{1}{\sigma}\sum_{i=1}^{k} n_i \frac{-(\phi(\zeta_i) - \phi(\zeta_{i-1}))}{\Phi(\zeta_i) - \Phi(\zeta_{i-1})} + c_i h_i = 0$$

$$\frac{\partial l}{\partial \beta} = l'_\beta = \frac{1}{\sigma}\sum_{i=1}^{k} n_i \frac{-(\omega_i \phi(\zeta_i) - \omega_{i-1}\phi(\zeta_{i-1}))}{\Phi(\zeta_i) - \Phi(\zeta_{i-1})} + c_i \omega_i h_i = 0$$

where, $\omega_i = \omega_i(\zeta_i) = \left(\sum_{l=1}^{k} \frac{\Delta_l}{\theta_l} x_l\right) \Big/ \left(\sum_{l=1}^{k} \frac{\Delta_l}{\theta_l}\right) = \frac{1}{\sigma} e^{-\sigma\zeta_i} \sum_{l=1}^{i} \frac{\Delta_l}{\theta_l} x_l$, $h_i = h(\zeta_i) = \frac{\phi(\zeta_i)}{1 - \Phi(\zeta_i)}$

Also, we define, $\omega_i^{[m]} = \omega_i^{[m]}(\tau_i) = \left(\sum_{l=1}^{k} \frac{\Delta_l}{\theta_l} x_l^m\right) \Big/ \left(\sum_{l=1}^{k} \frac{\Delta_l}{\theta_l}\right) = \frac{1}{\sigma} e^{-\sigma\zeta_i} \sum_{l=1}^{i} \frac{\Delta_l}{\theta_l} x_l^m,$

$m = 1, 2, 3$

which are functions of β only. For notation simplicity, let us define $\omega_i = \omega_i^{[1]}$ and it easy to see that,

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\omega_i^{[m]} = \omega_i\omega_i^{[m]} - \omega_i^{[m+1]}$$

By defining, $\upsilon_i = \dfrac{\mathrm{d}h(\zeta_i)}{\mathrm{d}\zeta_i} = \dfrac{\phi'(\zeta_i)}{1-\Phi(\zeta_i)} + h^2(\zeta_i)$

In particular, we have,

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\omega_i = \omega_i^2 - \omega_i^{[2]} = -\frac{1}{\sigma}\omega_i^2\rho_i < 0, \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}\beta}\omega_i^{[2]} = \omega_i\omega_i^{[2]} - \omega_i^{[3]}$$

where $\rho_i = -\sigma\left(1 - \dfrac{\omega_i^{[2]}}{\omega_i^2}\right)$

Then, the likelihood equations are derived as,

$$0 = l'_\sigma = \frac{1}{\sigma}\sum_{i=1}^{k} n_i G_i(1,0;0) + c_i\zeta_i h_i \tag{3.3}$$

$$0 = l'_\alpha = \frac{1}{\sigma}\sum_{i=1}^{k} n_i G_i(0,0;0) + c_i h_i \tag{3.4}$$

$$0 = l'_\beta = \frac{1}{\sigma}\sum_{i=1}^{k} n_i G_i(0,1;0) + c_i\omega_i h_i \tag{3.5}$$

which $h_i = h(\zeta_i)$ and

$$G_i(p_1,p_2;p_3) = -\frac{\zeta_i^{p_1}\omega_i^{p_2}\phi_i^{(p_3)}(\zeta_i) - \zeta_{i-1}^{p_1}\omega_{i-1}^{p_2}\phi_{i-1}^{(p_3)}(\zeta_{i-1})}{\Phi(\zeta_i) - \Phi(\zeta_{i-1})}$$

for $i = 1, 2, ..., k,\ \ p_1 = 0, 1, 2,\ \ p_2 = 0, 1, 2,$ and $p_3 = 0, 1, 2$ such that $\phi^{(0)}(z) = \phi(z)$, $\phi^{(1)}(z) = \phi'(z)$, and $\phi^{(2)}(z) = \phi''(z)$.

The MLE $(\hat{\sigma}, \hat{\alpha}, \hat{\beta})$ is obtained by solving the above three equations simultaneously.

3.1 Fisher Information Matrix

Here we calculate Fisher information matrix based on observed data $\boldsymbol{n}$. For a feasible progressive censoring, let $c_i = (N_i - n_i)\pi_i^*$, where $\boldsymbol{\pi} = (\pi_1^*, \pi_2^*, ..., \pi_k^*)$ are the predetermined proportions of remaining items to be withdrawn at the end of each stress level x_i, and $c_k = N_k - n_k$ since $\pi_k^* = 1$.

Under the interval monitoring, we have

$$\frac{\partial \zeta_i}{\partial \sigma} = -\frac{1}{\sigma}\zeta_i, \;\; \frac{\partial \zeta_i}{\partial \alpha} = -\frac{1}{\sigma}, \;\; \frac{\partial \zeta_i}{\partial \beta} = -\frac{1}{\sigma}\omega_i$$

$$\frac{\partial^2 \zeta_i}{\partial \sigma^2} = \frac{2}{\sigma}\zeta_i, \;\; \frac{\partial^2 \zeta_i}{\partial \alpha^2} = 0, \;\; \frac{\partial^2 \zeta_i}{\partial \beta^2} = -\frac{1}{\sigma}(\omega_i^2 - \omega_i^{[2]})$$

$$\frac{\partial^2 \zeta_i}{\partial \sigma \partial \alpha} = \frac{1}{\sigma^2}, \;\; \frac{\partial^2 \zeta_i}{\partial \sigma \partial \beta} = \frac{1}{\sigma^2}\omega_i, \;\; \frac{\partial^2 \zeta_i}{\partial \alpha \partial \beta} = 0$$

Then the second partials of likelihood function are given as,

$$\begin{aligned} l''_{\sigma\sigma} =& \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[\frac{n_i}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\big(2\zeta_i\phi(\zeta_i) - 2\zeta_{i-1}\phi(\zeta_{i-1}) + \zeta_i^2\phi'(\zeta_i) - \zeta_{i-1}^2\phi'(\zeta_{i-1})\big) \\ & - \frac{n_i}{\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)^2}\big(\zeta_i\phi(\zeta_i) - \zeta_{i-1}\phi(\zeta_{i-1})\big)^2 - 2c_i\zeta_i h(\zeta_i) - c_i\zeta_i^2\upsilon_i\Bigg] \end{aligned} \tag{3.6}$$

$$l''_{\alpha\alpha} = \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[\frac{n_i}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\big(\phi'(\zeta_i) - \phi'(\zeta_{i-1})\big) - \frac{n_i}{\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)^2}\big(\phi(\zeta_i) - \phi(\zeta_{i-1})\big)^2 - c_i\upsilon_i\Bigg] \tag{3.7}$$

$$\begin{aligned} l''_{\beta\beta} =& \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[\frac{n_i}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\Big(\sigma\phi(\zeta_i)(\omega_i^{[2]} - \omega_i^2) - \sigma\phi(\zeta_{i-1})(\omega_{i-1}^{[2]} - \omega_{i-1}^2) \\ & + \omega_i^2\phi'(\zeta_i) - \omega_{i-1}^2\phi'(\zeta_{i-1})\Big) - \frac{n_i}{\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)^2}\big(\omega_i\phi(\zeta_i) - \omega_{i-1}\phi(\zeta_{i-1})\big)^2 \\ & - \sigma c_i h(\zeta_i)(\omega_i^{[2]} - \omega_i^2) - c_i\omega_i^2\upsilon_i\Bigg] \end{aligned} \tag{3.8}$$

$$\begin{aligned} l''_{\sigma\alpha} =& \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[\frac{n_i}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\big(\phi(\zeta_i) - \phi(\zeta_{i-1}) + \zeta_i\phi'(\zeta_i) - \zeta_{i-1}\phi'(\zeta_{i-1})\big) \\ & - \frac{n_i}{\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)^2}\big(\phi(\zeta_i) - \phi(\zeta_{i-1})\big)\big(\zeta_i\phi(\zeta_i) - \zeta_{i-1}\phi(\zeta_{i-1})\big) - c_i h(\zeta_i) - c_i\zeta_i\upsilon_i\Bigg] \end{aligned} \tag{3.9}$$

$$\begin{aligned} l''_{\sigma\beta} =& \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[\frac{n_i}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\big(\omega_i\phi(\zeta_i) - \omega_{i-1}\phi(\zeta_{i-1}) + \zeta_i\omega_i\phi'(\zeta_i) - \zeta_{i-1}\omega_{i-1}\phi'(\zeta_{i-1})\big) \\ & - \frac{n_i}{\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)^2}\big(\zeta_i\phi(\zeta_i) - \zeta_{i-1}\phi(\zeta_{i-1})\big)\big(\omega_i\phi(\zeta_i) - \omega_{i-1}\phi(\zeta_{i-1})\big) - c_i\omega_i h(\zeta_i) - c_i\zeta_i\omega_i\upsilon_i\Bigg] \end{aligned} \tag{3.10}$$

$$l''_{\alpha\beta} = \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[\frac{n_i}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\big(\omega_i\phi'(\zeta_i)-\omega_{i-1}\phi'(\zeta_{i-1})\big)$$
$$-\frac{n_i}{\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)^2}\big(\phi(\zeta_i)-\phi(\zeta_{i-1})\big)\big(\omega_i\phi(\zeta_i)-\omega_{i-1}\phi(\zeta_{i-1})\big)-c_i\omega_i\upsilon_i\Bigg] \tag{3.11}$$

Now let us define,

$$h'(\zeta_i) = \frac{\mathrm{d}}{\mathrm{d}\zeta_i}h(\zeta_i) = \frac{\phi'(\zeta_i)}{1-\Phi(\zeta_i)} + h^2(\zeta_i)$$

$$h''(\zeta_i) = \frac{\mathrm{d}^2}{\mathrm{d}\zeta_i}h(\zeta_i) = \frac{\phi''(\zeta_i)}{1-\Phi(\zeta_i)} + \frac{3\phi'(\zeta_i)}{1-\Phi(\zeta_i)}h(\zeta_i) + 2h^3(\zeta_i)$$

Using a recursive relationship, it can be also derived that,

$$E(N_i) = nS(\tau_{i-1})\prod_{j=1}^{i-1}(1-\pi_j^*) = n\big(1-\Phi(\zeta_{i-1})\big)\prod_{j=1}^{i-1}(1-\pi_j^*)$$

$$E(n_i) = n\big(F(\tau_i)-F(\tau_{i-1})\big)\prod_{j=1}^{i-1}(1-\pi_j^*) = n\big(\Phi(\zeta_i)-\Phi(\zeta_{i-1})\big)\prod_{j=1}^{i-1}(1-\pi_j^*)$$

$$E(c_i) = nS(\tau_{i-1})\pi_i^*\prod_{j=1}^{i-1}(1-\pi_j^*) = n\big(1-\Phi(\zeta_{i-1})\big)\pi_i^*\prod_{j=1}^{i-1}(1-\pi_j^*)$$

Let $\Theta = (\theta_1, \theta_2, \theta_3)' = (\sigma, \alpha, \beta)'$. Fisher information matrix is expressed as,

$$I(\Theta) = I_n = -E\left[\frac{\partial^2 \log L(\Theta)}{\partial\theta_i\partial\theta_j}\right] = \begin{bmatrix} I_{\sigma\sigma} & I_{\sigma\alpha} & I_{\sigma\beta} \\ I_{\sigma\alpha} & I_{\alpha\alpha} & I_{\alpha\beta} \\ I_{\sigma\beta} & I_{\alpha\beta} & I_{\beta\beta}\end{bmatrix}$$

Using above mention relationships, the entries of Fisher information matrix are obtained as,

$$I_{\sigma\sigma} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\big[(\Phi_i-\Phi_{i-1})G_i^2(1,0;0) + \zeta_i^2\phi_i h_i\pi_i^*\big]\prod_{j=1}^{i-1}(1-\pi_i^*) \tag{3.12}$$

$$I_{\alpha\alpha} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\big[(\Phi_i-\Phi_{i-1})G_i^2(0,0;0) + \phi_i h_i\pi_i^*\big]\prod_{j=1}^{i-1}(1-\pi_i^*) \tag{3.13}$$

$$I_{\beta\beta} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\big[(\Phi_i-\Phi_{i-1})G_i^2(0,1;0) + \omega_i^2\phi_i h_i\pi_i^*\big]\prod_{j=1}^{i-1}(1-\pi_i^*) \tag{3.14}$$

$$I_{\sigma\alpha} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\big[(\Phi_i-\Phi_{i-1})G_i(1,0;0)G_i(0,0;0) + \zeta_i\phi_i h_i\pi_i^*\big]\prod_{j=1}^{i-1}(1-\pi_i^*) \tag{3.15}$$

$$I_{\sigma\beta} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[(\Phi_i - \Phi_{i-1})G_i(1,0;0)G_i(0,1;0) + \zeta_i\omega_i\phi_i h_i\pi_i^*\right]\prod_{j=1}^{i-1}(1-\pi_i^*) \qquad (3.16)$$

$$I_{\alpha\beta} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[(\Phi_i - \Phi_{i-1})G_i(0,0;0)G_i(0,1;0) + \omega_i\phi_i h_i\pi_i^*\right]\prod_{j=1}^{i-1}(1-\pi_i^*) \qquad (3.17)$$

with $\Phi_i = \Phi(\zeta_i)$, $\phi_i = \phi(\zeta_i)$ and $\phi_i' = \phi'(\zeta_i)$ for notational simplicity.

4. Simulation Study

Here we perform a simulation study to understand the performance of MLEs of α, β and σ where the true parameter values are $\sigma = 0.8$, $\alpha = 2$ and $\beta = -1$ and . We use progressively Type-I censored k-level step-stress ALT under interval monitoring for two distributions; Weibull and log-logistic and compared those results with progressively Type-I censored k-level step-stress ALT under interval monitoring results for same settings. Our work focus with $k = 3$ and 4 levels and for each level we consider $0\%, 10\%$, and 20% censoring proportions (π_i^*) under step duration Δ_i. Here k-level step duration Δ_i is uniform. i.e $\Delta_i = \frac{\tau_k}{k}$. Additionally, initial numbers of test units n to be 20 and 100 and total testing time τ_k to be 8.5 and 10.5. Therefore, we have 4 tables from each distributions with different n and τ_k. To interpret this, standardized equi-spaced stress levels between x_1 and x_k are considered for a set number of stress levels.i.e. $x_i = x_1 + \frac{x_k - x_i}{k-1}(i-1)$ for $i = 1, 2, ..., k$. Therefore, $(x_1, x_2, x_3) = (0.1, 0.55, 1.0)$ for $k = 3$ and $(x_1, x_2, x_3, x_4) = (0.1, 0.4, 0.7, 1.0)$ for $k = 4$. The number of items eliminated at the end of x_i (censored items) are decided by $c_i = round((N_i - n_i)\pi_i^*)$, where N_i is number of units surviving on the test in the beginning of level x_i and n_i is number of failures at stress level x_i.

Under above mentioned settings to understand the performance of MLEs of σ, α and β first, we generated 1000 samples and summarized the results. The simulation results show that the original MLEs $(\hat{\sigma}, \hat{\alpha}, \hat{\beta})$ can be extremely biased;(see Table 1-4). When we look at these tables, there is an extreme bias observed in MLEs ($\hat{\sigma}, \hat{\alpha}$ always overestimate the true value and $\hat{\beta}$ always underestimates the true value. When sample size increases from 20 to 100, the bias of ($\hat{\sigma}, \hat{\alpha}$ decreases, but it still does not approach to true value. Also, when sample size increases, bias of $\hat{\beta}$ increases. This happens due to severe loss of information associated with interval monitoring compared to continuous monitoring. To correct this issue we have tried several methods, such as bias correction Method; see Cordeiro and Klein (1994) and Cox and Snell (1968) and penalized Likelihood Method; see Firth (1993). Derivation and coding of bias correction and penalize methods were a little bit cumbersome, but the results were not superior compared to plain MLEs. Then we tried augmented method, the method is random numbers are generated in uniform distribution under n_i, number of failures at stress level x_i. After generating the numbers we treated those values as exact failure times to use an inferential method for the continuous monitoring. This method improved our results tremendously, so we compared augmented method with BCa Bootstrap Method and percentile bootstrap method. Finally, we decided to use augmented percentile bootstrap method for our study.

After using augmented interval monitoring (table 5-20), when sample size is smaller and censoring is not implemented or censoring proportion is lower, the estimators σ, α and β demonstrate means which slightly overestimate the true value, and medians which conversely underestimate the true value. Despite that, mean and median tend to be close to the true values. This is true for both k=3 and k=4, in both Weibull and log-logistics distributions.(see means and medians of both inspections in table 6 when censoring proportion is 10%). As the sample size increases above behavior changes and mean and median get closer to each other. This is true for both continuous and interval monitoring across both distributions.(compare means and medians of table 6 and 10 with same above mentioned setup). Variance of $\hat{\beta}$ always have the highest magnitude, followed by ($\hat{\sigma}$ and then $\hat{\alpha}$. This is true for both continuous and interval monitoring across both distributions. Furthermore, the magnitude of the variance and co-variance of the MLEs tends to increase when the number of stress levels increases. Finally, when censoring is not implemented and sample size is larger, k=4, and all the other settings remain the same, the magnitude of bias of all the estimators of interval monitoring have a larger magnitude compared to continuous monitoring. However, when censoring is implemented and sample size is smaller, this behavior is non-linear.

Next, Table 21-36 consists of the results percentile BCa bootstrap 95% confidence intervals for interval and continuous monitoring. Both methods compare 95% coverage, mean width, lower bound and upper bound. They are also based on two different sample sizes n and three different censoring proportions π_i^* as mentioned before. Here again we generates considered 1000 simulations and $B = 1000$ bootstrap replications. Here we observed, When the sample size is smaller, mean width is wider. When the sample size is larger, mean width gets narrower but still gives good coverage, for all distributions. Despite the sample size, both interval and continuous monitoring maintain good coverage, for all distributions. $\hat{\beta}$ always has the widest mean width, while ($\hat{\sigma}$ has the narrowest mean width, across both monitoring methods and all distributions. Finally, the results of interval and continuous monitoring are quite comparable, as augmentation introduces more information to interval monitoring. Therefore, interval monitoring is improved tremendously, as is continuous monitoring.

5. Conclusion

Based on the simulation process, we can see that due to loss of the information on exact time failures, performance of plain MLEs is extremely poor. Therefore, this is not recommended to use in practice. Performance of bias correction and penalize likelihood methods are slightly better than the original method, but despite slight improvement in bias, the coverage probability is insufficient to warrant recommendation for use in practice. Finally, the augmented method of injecting more stochastic information to the model results in a far superior performance than any other method.

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Table 1: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT under interval monitoring with uniform step duration, $n = 20$, and $\tau = 8.5$ under Weibull distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
3	20%	$\hat{\sigma}$	25.1925	13.57692	24.3925	5163.28707	4572.86604	$cov(\sigma, \alpha)$	8941.04864
		$\hat{\alpha}$	51.90376	33.14149	49.90376	20168.33075	17695.6415	$cov(\alpha, \beta)$	205.31634
		$\hat{\beta}$	-16.11783	-24.49562	-15.11783	422.64778	194.29315	$cov(\sigma, \beta)$	110.28645

Table 2: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT under interval monitoring with uniform step duration, $n = 100$, and $\tau = 8.5$ under Weibull distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
3	20%	$\hat{\sigma}$	16.53411	18.00153	15.73411	305.76653	58.26254	$cov(\sigma, \alpha)$	115.94147
		$\hat{\alpha}$	34.76965	38.1063	32.76965	1306.9426	233.32569	$cov(\alpha, \beta)$	-143.3458
		$\hat{\beta}$	-21.78417	-25.03628	-20.78417	532.72368	100.84281	$cov(\sigma, \beta)$	-69.57132

Table 3: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT under interval monitoring with uniform step duration, $n = 20$, and $\tau = 8.5$ under log-logistic distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
3	20%	$\hat{\sigma}$	11.7596	10.694502	10.9596	449.511011	329.727902	$cov(\sigma, \alpha)$	567.69435
		$\hat{\alpha}$	26.627316	25.385493	24.627316	1589.589888	984.069264	$cov(\alpha, \beta)$	3.371943
		$\hat{\beta}$	-28.129868	-29.898	-27.129868	786.101077	50.121485	$cov(\sigma, \beta)$	7.976661

Table 4: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT under interval monitoring with uniform step duration, $n = 100$, and $\tau = 8.5$ under log-logistic distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
3	20%	$\hat{\sigma}$	10.78357	10.654621	9.98357	102.326326	2.657313	$cov(\sigma, \alpha)$	4.472913
		$\hat{\alpha}$	25.184968	25.259923	23.184968	546.14856	8.614437	$cov(\alpha, \beta)$	-6.901817
		$\hat{\beta}$	-29.572598	-29.777335	-28.572598	825.314383	8.929935	$cov(\sigma, \beta)$	-2.981978

Table 5: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 20$, and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.8058	0.7713	0.0058	0.0707	0.0707	$cov(\sigma,\alpha)$	0.1006
			$\hat{\alpha}$	2.0619	1.9186	0.0619	0.2861	0.2826	$cov(\alpha,\beta)$	-0.5027
			$\hat{\beta}$	-1.0719	-0.8657	-0.0719	1.0935	1.0894	$cov(\sigma,\beta)$	-0.2283
		Interval	$\hat{\sigma}$	0.8076	0.7669	0.0076	0.0806	0.0807	$cov(\sigma,\alpha)$	0.0803
			$\hat{\alpha}$	2.0136	1.8942	0.0136	0.1981	0.1982	$cov(\alpha,\beta)$	-0.3432
			$\hat{\beta}$	-0.9070	-0.7369	0.0930	0.7800	0.7721	$cov(\sigma,\beta)$	-0.1994
	10%	Continuous	$\hat{\sigma}$	0.8075	0.7583	0.0075	0.0790	0.0790	$cov(\sigma,\alpha)$	0.1123
			$\hat{\alpha}$	2.0664	1.9161	0.0664	0.3108	0.3067	$cov(\alpha,\beta)$	-0.5519
			$\hat{\beta}$	-1.0772	-0.7918	-0.0772	1.2137	1.2090	$cov(\sigma,\beta)$	-0.2452
		Interval	$\hat{\sigma}$	0.8384	0.8015	0.0384	0.0855	0.0841	$cov(\sigma,\alpha)$	0.0863
			$\hat{\alpha}$	2.0532	1.9724	0.0532	0.2245	0.2219	$cov(\alpha,\beta)$	-0.3825
			$\hat{\beta}$	-0.9952	-0.8335	0.0048	0.8489	0.8498	$cov(\sigma,\beta)$	-0.2059
	20%	Continuous	$\hat{\sigma}$	0.8215	0.7638	0.0215	0.0919	0.0915	$cov(\sigma,\alpha)$	0.1360
			$\hat{\alpha}$	2.1062	1.9530	0.1062	0.3919	0.3810	$cov(\alpha,\beta)$	-0.6588
			$\hat{\beta}$	-1.1679	-0.8938	-0.1679	1.4270	1.4002	$cov(\sigma,\beta)$	-0.2877
		interval	$\hat{\sigma}$	0.8454	0.8018	0.0454	0.0890	0.0870	$cov(\sigma,\alpha)$	0.0966
			$\hat{\alpha}$	2.0815	1.9558	0.0815	0.2712	0.2648	$cov(\alpha,\beta)$	-0.4570
			$\hat{\beta}$	-1.0681	-0.8586	-0.0681	1.0172	1.0136	$cov(\sigma,\beta)$	-0.2246

Table 6: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 20$, and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8207	0.7583	0.0207	0.1090	0.1087	$cov(\sigma,\alpha)$	0.1855
			$\hat{\alpha}$	2.1085	1.9169	0.1085	0.5038	0.4925	$cov(\alpha,\beta)$	-0.8764
			$\hat{\beta}$	-1.1408	-0.7793	-0.1408	1.7989	1.7808	$cov(\sigma,\beta)$	-0.3765
		Interval	$\hat{\sigma}$	0.8420	0.7871	0.0420	0.0987	0.0970	$cov(\sigma,\alpha)$	0.1257
			$\hat{\alpha}$	2.0918	1.9477	0.0918	0.3227	0.3146	$cov(\alpha,\beta)$	-0.5963
			$\hat{\beta}$	-1.0987	-0.8289	-0.0987	1.3342	1.3258	$cov(\sigma,\beta)$	-0.2934
	10%	Continuous	$\hat{\sigma}$	0.8337	0.7647	0.0337	0.1024	0.1013	$cov(\sigma,\alpha)$	0.1718
			$\hat{\alpha}$	2.1394	1.9636	0.1394	0.4822	0.4632	$cov(\alpha,\beta)$	-0.8682
			$\hat{\beta}$	-1.2869	-0.9847	-0.2869	2.0121	1.9318	$cov(\sigma,\beta)$	-0.3670
		Interval	$\hat{\sigma}$	0.8383	0.7860	0.0383	0.1008	0.0994	$cov(\sigma,\alpha)$	0.1367
			$\hat{\alpha}$	2.1043	1.9327	0.1043	0.3526	0.3421	$cov(\alpha,\beta)$	-0.6535
			$\hat{\beta}$	-1.1870	-0.9208	-0.1870	1.5475	1.5141	$cov(\sigma,\beta)$	-0.3222
	20%	Continuous	$\hat{\sigma}$	0.8136	0.7502	0.0136	0.0896	0.0895	$cov(\sigma,\alpha)$	0.1539
			$\hat{\alpha}$	2.1033	1.8857	0.1033	0.4657	0.4555	$cov(\alpha,\beta)$	-0.8670
			$\hat{\beta}$	-1.1868	-0.7550	-0.1868	2.0036	1.9707	$cov(\sigma,\beta)$	-0.3339
		Interval	$\hat{\sigma}$	0.8878	0.8179	0.0878	0.1540	0.1464	$cov(\sigma,\alpha)$	0.2481
			$\hat{\alpha}$	2.1935	1.9747	0.1935	0.6688	0.6320	$cov(\alpha,\beta)$	-1.0905
			$\hat{\beta}$	-1.3336	-0.9435	-0.3336	2.3373	2.2282	$cov(\sigma,\beta)$	-0.4828

Table 7: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 20$, and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.8047	0.7549	0.0047	0.0650	0.0650	$cov(\sigma,\alpha)$	0.0775
			$\hat{\alpha}$	2.0561	1.9473	0.0561	0.2126	0.2097	$cov(\alpha,\beta)$	-0.4003
			$\hat{\beta}$	-1.1107	-0.9308	-0.1107	1.0353	1.0240	$cov(\sigma,\beta)$	-0.2063
		Interval	$\hat{\sigma}$	0.8023	0.7649	0.0023	0.0632	0.0632	$cov(\sigma,\alpha)$	0.0477
			$\hat{\alpha}$	2.0016	1.9184	0.0016	0.1329	0.1330	$cov(\alpha,\beta)$	-0.2435
			$\hat{\beta}$	-0.8764	-0.7077	0.1236	0.7044	0.6898	$cov(\sigma,\beta)$	-0.1598
	10%	Continuous	$\hat{\sigma}$	0.8081	0.7731	0.0081	0.0616	0.0616	$cov(\sigma,\alpha)$	0.0788
			$\hat{\alpha}$	2.0637	1.9501	0.0637	0.2255	0.2216	$cov(\alpha,\beta)$	-0.4262
			$\hat{\beta}$	-1.1301	-0.9692	-0.1301	1.1070	1.0912	$cov(\sigma,\beta)$	-0.2058
		Interval	$\hat{\sigma}$	0.8200	0.7870	0.0200	0.0669	0.0666	$cov(\sigma,\alpha)$	0.0603
			$\hat{\alpha}$	2.0197	1.9233	0.0197	0.1782	0.1780	$cov(\alpha,\beta)$	-0.3275
			$\hat{\beta}$	-0.9232	-0.7597	0.0768	0.8463	0.8412	$cov(\sigma,\beta)$	-0.1816
	20%	Continuous	$\hat{\sigma}$	0.8076	0.7721	0.0076	0.0579	0.0579	$cov(\sigma,\alpha)$	0.0719
			$\hat{\alpha}$	2.0623	1.9605	0.0623	0.2199	0.2162	$cov(\alpha,\beta)$	-0.4382
			$\hat{\beta}$	-1.1397	-0.9606	-0.1397	1.2109	1.1926	$cov(\sigma,\beta)$	-0.1966
		Interval	$\hat{\sigma}$	0.8143	0.7816	0.0143	0.0643	0.0642	$cov(\sigma,\alpha)$	0.0510
			$\hat{\alpha}$	2.0178	1.9465	0.0178	0.1441	0.1439	$cov(\alpha,\beta)$	-0.2752
			$\hat{\beta}$	-0.9243	-0.7724	0.0757	0.8022	0.7973	$cov(\sigma,\beta)$	-0.1629

Table 8: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 20$, and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8240	0.7681	0.0240	0.0914	0.0910	$cov(\sigma,\alpha)$	0.1457
			$\hat{\alpha}$	2.1019	1.9478	0.1019	0.3883	0.3783	$cov(\alpha,\beta)$	-0.7360
			$\hat{\beta}$	-1.2089	-0.9371	-0.2089	1.7582	1.7163	$cov(\sigma,\beta)$	-0.3338
		Interval	$\hat{\sigma}$	0.8418	0.7951	0.0418	0.0869	0.0853	$cov(\sigma,\alpha)$	0.0892
			$\hat{\alpha}$	2.0666	1.9548	0.0666	0.2207	0.2165	$cov(\alpha,\beta)$	-0.4580
			$\hat{\beta}$	-1.1032	-0.8451	-0.1032	1.2483	1.2389	$cov(\sigma,\beta)$	-0.2643
	10%	Continuous	$\hat{\sigma}$	0.8284	0.7765	0.0284	0.0909	0.0902	$cov(\sigma,\alpha)$	0.1385
			$\hat{\alpha}$	2.1088	1.9406	0.1088	0.3805	0.3690	$cov(\alpha,\beta)$	-0.7446
			$\hat{\beta}$	-1.2520	-0.8846	-0.2520	1.8666	1.8049	$cov(\sigma,\beta)$	-0.3220
		Interval	$\hat{\sigma}$	0.8646	0.8217	0.0646	0.0863	0.0822	$cov(\sigma,\alpha)$	0.0895
			$\hat{\alpha}$	2.0960	1.9780	0.0960	0.2468	0.2378	$cov(\alpha,\beta)$	-0.4974
			$\hat{\beta}$	-1.2103	-0.9508	-0.2103	1.4146	1.3718	$cov(\sigma,\beta)$	-0.2612
	20%	Continuous	$\hat{\sigma}$	0.8198	0.7699	0.0198	0.0877	0.0874	$cov(\sigma,\alpha)$	0.1392
			$\hat{\alpha}$	2.1338	1.9651	0.1338	0.4200	0.4025	$cov(\alpha,\beta)$	-0.7935
			$\hat{\beta}$	-1.3082	-0.9895	-0.3082	2.0407	1.9476	$cov(\sigma,\beta)$	-0.3220
		Interval	$\hat{\sigma}$	0.8522	0.8090	0.0522	0.0896	0.0869	$cov(\sigma,\alpha)$	0.1073
			$\hat{\alpha}$	2.1366	1.9918	0.1366	0.3308	0.3125	$cov(\alpha,\beta)$	-0.6308
			$\hat{\beta}$	-1.2962	-1.0552	-0.2962	1.7499	1.6638	$cov(\sigma,\beta)$	-0.2792

Table 9: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 100$, and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.7994	0.7980	-0.0006	0.0167	0.0167	$cov(\sigma,\alpha)$	0.0230
			$\hat{\alpha}$	2.0095	1.9808	0.0095	0.0562	0.0561	$cov(\alpha,\beta)$	-0.1142
			$\hat{\beta}$	-1.0013	-0.9721	-0.0013	0.2669	0.2672	$cov(\sigma,\beta)$	-0.0566
		Interval	$\hat{\sigma}$	0.8169	0.8089	0.0169	0.0180	0.0177	$cov(\sigma,\alpha)$	0.0177
			$\hat{\alpha}$	2.0034	1.9916	0.0034	0.0417	0.0418	$cov(\alpha,\beta)$	-0.0811
			$\hat{\beta}$	-0.9224	-0.9129	0.0777	0.1977	0.1919	$cov(\sigma,\beta)$	-0.0472
	10%	Continuous	$\hat{\sigma}$	0.7984	0.7820	-0.0016	0.0174	0.0174	$cov(\sigma,\alpha)$	0.0246
			$\hat{\alpha}$	2.0147	1.9902	0.0147	0.0584	0.0583	$cov(\alpha,\beta)$	-0.1180
			$\hat{\beta}$	-1.0083	-0.9515	-0.0083	0.2785	0.2787	$cov(\sigma,\beta)$	-0.0596
		Interval	$\hat{\sigma}$	0.8349	0.8302	0.0349	0.0187	0.0175	$cov(\sigma,\alpha)$	0.0204
			$\hat{\alpha}$	2.0335	2.0164	0.0335	0.0491	0.0481	$cov(\alpha,\beta)$	-0.0912
			$\hat{\beta}$	-0.9853	-0.9788	0.0147	0.2115	0.2115	$cov(\sigma,\beta)$	-0.0506
	20%	Continuous	$\hat{\sigma}$	0.7977	0.7822	-0.0023	0.0188	0.0188	$cov(\sigma,\alpha)$	0.0282
			$\hat{\alpha}$	2.0191	1.9876	0.0191	0.0684	0.0682	$cov(\alpha,\beta)$	-0.1378
			$\hat{\beta}$	-1.0158	-0.9789	-0.0158	0.3229	0.3230	$cov(\sigma,\beta)$	-0.0660
		Interval	$\hat{\sigma}$	0.8417	0.8344	0.0417	0.0220	0.0203	$cov(\sigma,\alpha)$	0.0233
			$\hat{\alpha}$	2.0485	2.0294	0.0485	0.0560	0.0537	$cov(\alpha,\beta)$	-0.1042
			$\hat{\beta}$	-1.0210	-1.0118	-0.0210	0.2455	0.2453	$cov(\sigma,\beta)$	-0.0579

Table 10: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 100$, and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8016	0.7887	0.0016	0.0192	0.0192	$cov(\sigma,\alpha)$	0.0312
			$\hat{\alpha}$	2.0148	1.9813	0.0148	0.0795	0.0794	$cov(\alpha,\beta)$	-0.1617
			$\hat{\beta}$	-1.0154	-0.9712	-0.0154	0.3674	0.3676	$cov(\sigma,\beta)$	-0.0732
		Interval	$\hat{\sigma}$	0.8344	0.8296	0.0344	0.0230	0.0219	$cov(\sigma,\alpha)$	0.0284
			$\hat{\alpha}$	2.0412	2.0240	0.0412	0.0681	0.0665	$cov(\alpha,\beta)$	-0.1371
			$\hat{\beta}$	-1.0480	-1.0439	-0.0480	0.3273	0.3253	$cov(\sigma,\beta)$	-0.0727
	10%	Continuous	$\hat{\sigma}$	0.7986	0.7821	-0.0014	0.0188	0.0188	$cov(\sigma,\alpha)$	0.0306
			$\hat{\alpha}$	2.0057	1.9812	0.0057	0.0783	0.0784	$cov(\alpha,\beta)$	-0.1643
			$\hat{\beta}$	-0.9901	-0.9615	0.0099	0.3930	0.3933	$cov(\sigma,\beta)$	-0.0728
		Interval	$\hat{\sigma}$	0.8462	0.8370	0.0462	0.0250	0.0228	$cov(\sigma,\alpha)$	0.0306
			$\hat{\alpha}$	2.0564	2.0285	0.0564	0.0742	0.0711	$cov(\alpha,\beta)$	-0.1456
			$\hat{\beta}$	-1.0822	-1.0604	-0.0822	0.3566	0.3502	$cov(\sigma,\beta)$	-0.0749
	20%	Continuous	$\hat{\sigma}$	0.7945	0.7821	-0.0055	0.0187	0.0187	$cov(\sigma,\alpha)$	0.0324
			$\hat{\alpha}$	2.0068	1.9738	0.0068	0.0880	0.0881	$cov(\alpha,\beta)$	-0.1864
			$\hat{\beta}$	-0.9882	-0.9726	0.0118	0.4463	0.4466	$cov(\sigma,\beta)$	-0.0774
		Interval	$\hat{\sigma}$	0.8730	0.8664	0.0730	0.0311	0.0258	$cov(\sigma,\alpha)$	0.0373
			$\hat{\alpha}$	2.1070	2.0863	0.1070	0.0992	0.0879	$cov(\alpha,\beta)$	-0.1795
			$\hat{\beta}$	-1.1934	-1.1834	-0.1934	0.4603	0.4233	$cov(\sigma,\beta)$	-0.0886

Table 11: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 100$, and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.8013	0.7925	0.0013	0.0137	0.0137	$cov(\sigma, \alpha)$	0.0156
			$\hat{\alpha}$	2.0174	2.0047	0.0174	0.0412	0.0409	$cov(\alpha, \beta)$	-0.0906
			$\hat{\beta}$	-1.0306	-1.0532	-0.0306	0.2460	0.2453	$cov(\sigma, \beta)$	-0.0465
		Interval	$\hat{\sigma}$	0.8044	0.7979	0.0044	0.0138	0.0138	$cov(\sigma, \alpha)$	0.0121
			$\hat{\alpha}$	1.9839	1.9732	-0.0161	0.0309	0.0307	$cov(\alpha, \beta)$	-0.0650
			$\hat{\beta}$	-0.8246	-0.8117	0.1754	0.2095	0.1789	$cov(\sigma, \beta)$	-0.0400
	10%	Continuous	$\hat{\sigma}$	0.8034	0.7970	0.0034	0.0137	0.0137	$cov(\sigma, \alpha)$	0.0162
			$\hat{\alpha}$	2.0048	1.9892	0.0048	0.0410	0.0410	$cov(\alpha, \beta)$	-0.0922
			$\hat{\beta}$	-1.0167	-1.0043	-0.0167	0.2585	0.2585	$cov(\sigma, \beta)$	-0.0483
		Interval	$\hat{\sigma}$	0.8272	0.8172	0.0272	0.0148	0.0141	$cov(\sigma, \alpha)$	0.0122
			$\hat{\alpha}$	1.9948	1.9804	-0.0052	0.0321	0.0321	$cov(\alpha, \beta)$	-0.0664
			$\hat{\beta}$	-0.8868	-0.8575	0.1132	0.1969	0.1842	$cov(\sigma, \beta)$	-0.0393
	20%	Continuous	$\hat{\sigma}$	0.7986	0.7957	-0.0014	0.0135	0.0135	$cov(\sigma, \alpha)$	0.0153
			$\hat{\alpha}$	2.0028	1.9957	0.0028	0.0393	0.0393	$cov(\alpha, \beta)$	-0.0893
			$\hat{\beta}$	-0.9938	-0.9896	0.0062	0.2613	0.2615	$cov(\sigma, \beta)$	-0.0471
		Interval	$\hat{\sigma}$	0.8290	0.8195	0.0290	0.0155	0.0147	$cov(\sigma, \alpha)$	0.0136
			$\hat{\alpha}$	2.0044	1.9879	0.0044	0.0341	0.0341	$cov(\alpha, \beta)$	-0.0720
			$\hat{\beta}$	-0.8996	-0.8805	0.1004	0.2159	0.2060	$cov(\sigma, \beta)$	-0.0433

Table 12: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 100$, and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8008	0.7933	0.0008	0.0151	0.0151	$cov(\sigma, \alpha)$	0.0198
			$\hat{\alpha}$	2.0144	2.0010	0.0144	0.0503	0.0502	$cov(\alpha, \beta)$	-0.1182
			$\hat{\beta}$	-1.0227	-1.0138	-0.0227	0.3246	0.3244	$cov(\sigma, \beta)$	-0.0585
		Interval	$\hat{\sigma}$	0.8363	0.8323	0.0363	0.0179	0.0166	$cov(\sigma, \alpha)$	0.0194
			$\hat{\alpha}$	2.0362	2.0162	0.0362	0.0481	0.0468	$cov(\alpha, \beta)$	-0.1056
			$\hat{\beta}$	-1.0336	-1.0333	-0.0336	0.2864	0.2856	$cov(\sigma, \beta)$	-0.0574
	10%	Continuous	$\hat{\sigma}$	0.7996	0.7920	-0.0004	0.0152	0.0152	$cov(\sigma, \alpha)$	0.0217
			$\hat{\alpha}$	2.0041	1.9745	0.0041	0.0565	0.0565	$cov(\alpha, \beta)$	-0.1330
			$\hat{\beta}$	-0.9935	-0.9424	0.0065	0.3642	0.3645	$cov(\sigma, \beta)$	-0.0620
		Interval	$\hat{\sigma}$	0.8532	0.8500	0.0532	0.0219	0.0191	$cov(\sigma, \alpha)$	0.0224
			$\hat{\alpha}$	2.0492	2.0452	0.0492	0.0557	0.0533	$cov(\alpha, \beta)$	-0.1234
			$\hat{\beta}$	-1.0753	-1.0754	-0.0753	0.3508	0.3455	$cov(\sigma, \beta)$	-0.0673
	20%	Continuous	$\hat{\sigma}$	0.8023	0.7974	0.0023	0.0167	0.0167	$cov(\sigma, \alpha)$	0.0243
			$\hat{\alpha}$	2.0132	1.9765	0.0132	0.0650	0.0649	$cov(\alpha, \beta)$	-0.1499
			$\hat{\beta}$	-1.0306	-0.9738	-0.0306	0.4129	0.4124	$cov(\sigma, \beta)$	-0.0685
		Interval	$\hat{\sigma}$	0.8495	0.8393	0.0495	0.0225	0.0201	$cov(\sigma, \alpha)$	0.0259
			$\hat{\alpha}$	2.0535	2.0232	0.0535	0.0656	0.0628	$cov(\alpha, \beta)$	-0.1430
			$\hat{\beta}$	-1.0910	-1.0608	-0.0910	0.4014	0.3935	$cov(\sigma, \beta)$	-0.0740

Table 13: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 20$, and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.8102	0.7474	0.0102	0.1016	0.1016	$cov(\sigma,\alpha)$	0.1754
			$\hat{\alpha}$	2.0992	1.9062	0.0992	0.5585	0.549232	$cov(\alpha,\beta)$	-0.7465
			$\hat{\beta}$	-1.0737	-0.7580	-0.0737	1.4643	1.460327	$cov(\sigma,\beta)$	-0.3087
		Interval	$\hat{\sigma}$	0.8042	0.7509	0.0042	0.0862	0.0862	$cov(\sigma,\alpha)$	0.1054
			$\hat{\alpha}$	2.0502	1.9147	0.0502	0.3356	0.3334	$cov(\alpha,\beta)$	-0.4520
			$\hat{\beta}$	-0.9330	-0.6963	0.0670	0.9863	0.9828	$cov(\sigma,\beta)$	-0.2132
	10%	Continuous	$\hat{\sigma}$	0.8289	0.7749	0.0289	0.1033	0.1026	$cov(\sigma,\alpha)$	0.1909
			$\hat{\alpha}$	2.1474	1.9776	0.1474	0.6432	0.6221	$cov(\alpha,\beta)$	-0.8471
			$\hat{\beta}$	-1.1916	-0.8742	-0.1916	1.6630	1.6279	$cov(\sigma,\beta)$	-0.3240
		Interval	$\hat{\sigma}$	0.8163	0.7446	0.0163	0.1086	0.1085	$cov(\sigma,\alpha)$	0.1430
			$\hat{\alpha}$	2.0858	1.9316	0.0858	0.4254	0.4184	$cov(\alpha,\beta)$	-0.5721
			$\hat{\beta}$	-1.0116	-0.7228	-0.0116	1.2148	1.2159	$cov(\sigma,\beta)$	-0.2785
	20%	Continuous	$\hat{\sigma}$	0.7983	0.7449	-0.0017	0.0891	0.0892	$cov(\sigma,\alpha)$	0.1449
			$\hat{\alpha}$	2.0913	1.9183	0.0913	0.4649	0.4570	$cov(\alpha,\beta)$	-0.7061
			$\hat{\beta}$	-1.0901	-0.7554	-0.0901	1.5930	1.5864	$cov(\sigma,\beta)$	-0.2863
		Interval	$\hat{\sigma}$	0.8238	0.7782	0.0238	0.1022	0.1018	$cov(\sigma,\alpha)$	0.1387
			$\hat{\alpha}$	2.0968	1.9429	0.0968	0.4382	0.4293	$cov(\alpha,\beta)$	-0.6076
			$\hat{\beta}$	-1.0367	-0.7382	-0.0367	1.3406	1.3406	$cov(\sigma,\beta)$	-0.2711

Table 14: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 20$, and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8384	0.7650	0.0384	0.1236	0.1223	$cov(\sigma,\alpha)$	0.2398
			$\hat{\alpha}$	2.1576	1.9029	0.1576	0.7522	0.7281	$cov(\alpha,\beta)$	-1.1500
			$\hat{\beta}$	-1.2371	-0.8229	-0.2371	2.3603	2.3063	$cov(\sigma,\beta)$	-0.4455
		Interval	$\hat{\sigma}$	0.8308	0.7790	0.0308	0.1056	0.1047	$cov(\sigma,\alpha)$	0.1566
			$\hat{\alpha}$	2.1025	1.9238	0.1025	0.4937	0.4837	$cov(\alpha,\beta)$	-0.7226
			$\hat{\beta}$	-1.0948	-0.7346	-0.0948	1.5664	1.5589	$cov(\sigma,\beta)$	-0.3131
	10%	Continuous	$\hat{\sigma}$	0.8462	0.7561	0.0462	0.1552	0.1532	$cov(\sigma,\alpha)$	0.3411
			$\hat{\alpha}$	2.1781	1.8689	0.1781	1.0932	1.0626	$cov(\alpha,\beta)$	-1.5598
			$\hat{\beta}$	-1.2581	-0.6759	-0.2581	2.9184	2.8546	$cov(\sigma,\beta)$	-0.5549
		Interval	$\hat{\sigma}$	0.8699	0.7813	0.0699	0.1564	0.1517	$cov(\sigma,\alpha)$	0.2721
			$\hat{\alpha}$	2.1670	1.9433	0.1670	0.7961	0.7689	$cov(\alpha,\beta)$	-1.1223
			$\hat{\beta}$	-1.2442	-0.8288	-0.2442	2.2785	2.2210	$cov(\sigma,\beta)$	-0.4748
	20%	Continuous	$\hat{\sigma}$	0.8268	0.7611	0.0268	0.1081	0.1075	$cov(\sigma,\alpha)$	0.2093
			$\hat{\alpha}$	2.1277	1.8994	0.1277	0.7015	0.6859	$cov(\alpha,\beta)$	-1.0623
			$\hat{\beta}$	-1.2039	-0.6312	-0.2039	2.3375	2.2982	$cov(\sigma,\beta)$	-0.3781
		Interval	$\hat{\sigma}$	0.8586	0.7867	0.0586	0.1332	0.1299	$cov(\sigma,\alpha)$	0.2108
			$\hat{\alpha}$	2.1470	1.9352	0.1470	0.6438	0.6228	$cov(\alpha,\beta)$	-0.9582
			$\hat{\beta}$	-1.2110	-0.6989	-0.2110	2.2000	2.1577	$cov(\sigma,\beta)$	-0.3909

Table 15: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 20$, and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.8127	0.7553	0.0127	0.0799	0.0798	$cov(\sigma,\alpha)$	0.1138
			$\hat{\alpha}$	2.0906	1.9727	0.0906	0.3512	0.3433	$cov(\alpha,\beta)$	-0.5271
			$\hat{\beta}$	-1.1137	-0.9028	-0.1137	1.2927	1.2811	$cov(\sigma,\beta)$	-0.2474
		Interval	$\hat{\sigma}$	0.7883	0.7558	-0.0117	0.0741	0.0741	$cov(\sigma,\alpha)$	0.0675
			$\hat{\alpha}$	2.0251	1.9554	0.0251	0.2285	0.2281	$cov(\alpha,\beta)$	-0.3185
			$\hat{\beta}$	-0.8821	-0.6882	0.1179	0.8862	0.8732	$cov(\sigma,\beta)$	-0.1805
	10%	Continuous	$\hat{\sigma}$	0.8002	0.7640	0.0002	0.0709	0.0710	$cov(\sigma,\alpha)$	0.0966
			$\hat{\alpha}$	2.0462	1.9309	0.0462	0.3281	0.3263	$cov(\alpha,\beta)$	-0.4929
			$\hat{\beta}$	-1.0502	-0.7928	-0.0502	1.2601	1.2588	$cov(\sigma,\beta)$	-0.2147
		Interval	$\hat{\sigma}$	0.7936	0.7539	-0.0064	0.0747	0.0747	$cov(\sigma,\alpha)$	0.0847
			$\hat{\alpha}$	2.0219	1.9206	0.0219	0.2721	0.2719	$cov(\alpha,\beta)$	-0.3854
			$\hat{\beta}$	-0.8777	-0.5967	0.1223	0.9587	0.9447	$cov(\sigma,\beta)$	-0.1881
	20%	Continuous	$\hat{\sigma}$	0.8144	0.7706	0.0144	0.0776	0.0775	$cov(\sigma,\alpha)$	0.1071
			$\hat{\alpha}$	2.0705	1.9509	0.0705	0.3370	0.3323	$cov(\alpha,\beta)$	-0.5216
			$\hat{\beta}$	-1.1414	-0.8824	-0.1414	1.3974	1.3788	$cov(\sigma,\beta)$	-0.2313
		Interval	$\hat{\sigma}$	0.8038	0.7429	0.0038	0.0915	0.0916	$cov(\sigma,\alpha)$	0.1186
			$\hat{\alpha}$	2.0474	1.9160	0.0474	0.3449	0.3430	$cov(\alpha,\beta)$	-0.4944
			$\hat{\beta}$	-0.9645	-0.6781	0.0356	1.1976	1.1975	$cov(\sigma,\beta)$	-0.2418

Table 16: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 20$, and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8294	0.7621	0.0294	0.1146	0.1139	$cov(\sigma,\alpha)$	0.2084
			$\hat{\alpha}$	2.1300	1.9597	0.1300	0.6170	0.6007	$cov(\alpha,\beta)$	-0.8869
			$\hat{\beta}$	-1.2206	-0.8110	-0.2206	1.9933	1.9466	$cov(\sigma,\beta)$	-0.3758
		Interval	$\hat{\sigma}$	0.8300	0.7694	0.0300	0.1031	0.1023	$cov(\sigma,\alpha)$	0.1286
			$\hat{\alpha}$	2.0911	1.9450	0.0911	0.3608	0.3529	$cov(\alpha,\beta)$	-0.5864
			$\hat{\beta}$	-1.1115	-0.7460	-0.1115	1.5666	1.5557	$cov(\sigma,\beta)$	-0.3144
	10%	Continuous	$\hat{\sigma}$	0.8166	0.7650	0.0166	0.0933	0.0932	$cov(\sigma,\alpha)$	0.1552
			$\hat{\alpha}$	2.0646	1.9031	0.0646	0.4964	0.4927	$cov(\alpha,\beta)$	-0.8339
			$\hat{\beta}$	-1.1165	-0.6207	-0.1165	2.0055	1.9939	$cov(\sigma,\beta)$	-0.3287
		Interval	$\hat{\sigma}$	0.8395	0.7793	0.0395	0.0994	0.0979	$cov(\sigma,\alpha)$	0.1261
			$\hat{\alpha}$	2.0637	1.9100	0.0637	0.3768	0.3731	$cov(\alpha,\beta)$	-0.6251
			$\hat{\beta}$	-1.0856	-0.7090	-0.0856	1.6374	1.6317	$cov(\sigma,\beta)$	-0.3022
	20%	Continuous	$\hat{\sigma}$	0.8157	0.7654	0.0157	0.0868	0.0866	$cov(\sigma,\alpha)$	0.1482
			$\hat{\alpha}$	2.1155	1.9320	0.1155	0.5146	0.5017	$cov(\alpha,\beta)$	-0.8690
			$\hat{\beta}$	-1.2790	-0.8112	-0.2790	2.3392	2.2636	$cov(\sigma,\beta)$	-0.3274
		Interval	$\hat{\sigma}$	0.8367	0.7707	0.0367	0.1377	0.1365	$cov(\sigma,\alpha)$	0.1941
			$\hat{\alpha}$	2.1087	1.9684	0.1087	0.5183	0.5070	$cov(\alpha,\beta)$	-0.8559
			$\hat{\beta}$	-1.2221	-0.8646	-0.2221	2.2441	2.1969	$cov(\sigma,\beta)$	-0.4193

Table 17: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 100$, and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.8037	0.7939	0.0037	0.0194	0.0194	$cov(\sigma, \alpha)$	0.0300
			$\hat{\alpha}$	2.0155	1.9767	0.0155	0.0839	0.0837	$cov(\alpha, \beta)$	-0.1399
			$\hat{\beta}$	-0.9974	-0.9799	0.0026	0.3161	0.3164	$cov(\sigma, \beta)$	-0.0629
		Interval	$\hat{\sigma}$	0.8074	0.7923	0.0074	0.0191	0.0191	$cov(\sigma, \alpha)$	0.0239
			$\hat{\alpha}$	2.0078	1.9794	0.0078	0.0664	0.0664	$cov(\alpha, \beta)$	-0.1070
			$\hat{\beta}$	-0.9065	-0.8837	0.0935	0.2620	0.2535	$cov(\sigma, \beta)$	-0.0541
	10%	Continuous	$\hat{\sigma}$	0.7971	0.7892	-0.0029	0.0176	0.0177	$cov(\sigma, \alpha)$	0.0275
			$\hat{\alpha}$	2.0012	1.9771	0.0012	0.0797	0.0798	$cov(\alpha, \beta)$	-0.1348
			$\hat{\beta}$	-0.9904	-0.9818	0.0096	0.3207	0.3209	$cov(\sigma, \beta)$	-0.0595
		Interval	$\hat{\sigma}$	0.8078	0.7997	0.0078	0.0200	0.0200	$cov(\sigma, \alpha)$	0.0251
			$\hat{\alpha}$	2.0041	1.9823	0.0041	0.0705	0.0706	$cov(\alpha, \beta)$	-0.1184
			$\hat{\beta}$	-0.9225	-0.8893	0.0775	0.2993	0.2936	$cov(\sigma, \beta)$	-0.0601
	20%	Continuous	$\hat{\sigma}$	0.8091	0.8023	0.0091	0.0183	0.0182	$cov(\sigma, \alpha)$	0.0298
			$\hat{\alpha}$	2.0256	1.9980	0.0256	0.0892	0.0886	$cov(\alpha, \beta)$	-0.1547
			$\hat{\beta}$	-1.0207	-0.9657	-0.0207	0.3744	0.3743	$cov(\sigma, \beta)$	-0.0639
		Interval	$\hat{\sigma}$	0.8188	0.8082	0.0188	0.0214	0.0211	$cov(\sigma, \alpha)$	0.0269
			$\hat{\alpha}$	2.0278	2.0045	0.0278	0.0738	0.0731	$cov(\alpha, \beta)$	-0.1255
			$\hat{\beta}$	-0.9495	-0.9322	0.0505	0.3163	0.3141	$cov(\sigma, \beta)$	-0.0612

Table 18: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 100$, and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.7985	0.7823	-0.0015	0.0216	0.0216	$cov(\sigma, \alpha)$	0.0351
			$\hat{\alpha}$	2.0063	1.9607	0.0063	0.0946	0.0947	$cov(\alpha, \beta)$	-0.1814
			$\hat{\beta}$	-0.9818	-0.9081	0.0182	0.4375	0.4376	$cov(\sigma, \beta)$	-0.0805
		Interval	$\hat{\sigma}$	0.8316	0.8166	0.0316	0.0247	0.0237	$cov(\sigma, \alpha)$	0.0339
			$\hat{\alpha}$	2.0492	2.0224	0.0492	0.0907	0.0884	$cov(\alpha, \beta)$	-0.1634
			$\hat{\beta}$	-1.0430	-1.0056	-0.0430	0.3911	0.3897	$cov(\sigma, \beta)$	-0.0770
	10%	Continuous	$\hat{\sigma}$	0.7967	0.7850	-0.0033	0.0227	0.0227	$cov(\sigma, \alpha)$	0.0386
			$\hat{\alpha}$	2.0035	1.9559	0.0035	0.1120	0.1120	$cov(\alpha, \beta)$	-0.2069
			$\hat{\beta}$	-0.9912	-0.9027	0.0088	0.4869	0.4873	$cov(\sigma, \beta)$	-0.0868
		Interval	$\hat{\sigma}$	0.8376	0.8238	0.0376	0.0258	0.0244	$cov(\sigma, \alpha)$	0.0370
			$\hat{\alpha}$	2.0586	2.0163	0.0586	0.1079	0.1046	$cov(\alpha, \beta)$	-0.1918
			$\hat{\beta}$	-1.0797	-1.0378	-0.0797	0.4692	0.4633	$cov(\sigma, \beta)$	-0.0862
	20%	Continuous	$\hat{\sigma}$	0.8033	0.7867	0.0033	0.0228	0.0229	$cov(\sigma, \alpha)$	0.0404
			$\hat{\alpha}$	2.0202	1.9627	0.0202	0.1176	0.1173	$cov(\alpha, \beta)$	-0.2261
			$\hat{\beta}$	-0.9954	-0.9277	0.0046	0.5552	0.5558	$cov(\sigma, \beta)$	-0.0893
		Interval	$\hat{\sigma}$	0.8508	0.8375	0.0508	0.0316	0.0290	$cov(\sigma, \alpha)$	0.0460
			$\hat{\alpha}$	2.0882	2.0481	0.0882	0.1281	0.1205	$cov(\alpha, \beta)$	-0.2351
			$\hat{\beta}$	-1.1122	-1.0729	-0.1122	0.5995	0.5875	$cov(\sigma, \beta)$	-0.1053

Table 19: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 3$, $n = 100$, and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
3	0%	Continuous	$\hat{\sigma}$	0.7927	0.7900	-0.0073	0.0151	0.0151	$cov(\sigma,\alpha)$	0.0183
			$\hat{\alpha}$	2.0203	1.9926	0.0203	0.0566	0.0562	$cov(\alpha,\beta)$	-0.0994
			$\hat{\beta}$	-1.0018	-0.9842	-0.0018	0.2821	0.2824	$cov(\sigma,\beta)$	-0.0504
		Interval	$\hat{\sigma}$	0.7809	0.7725	-0.0191	0.0166	0.0162	$cov(\sigma,\alpha)$	0.0175
			$\hat{\alpha}$	2.0007	1.9757	0.0007	0.0507	0.0508	$cov(\alpha,\beta)$	-0.0861
			$\hat{\beta}$	-0.8372	-0.8075	0.1628	0.2702	0.2440	$cov(\sigma,\beta)$	-0.0489
	10%	Continuous	$\hat{\sigma}$	0.7915	0.7838	-0.0085	0.0144	0.0143	$cov(\sigma,\alpha)$	0.0183
			$\hat{\alpha}$	1.9845	1.9562	-0.0155	0.0532	0.0530	$cov(\alpha,\beta)$	-0.0937
			$\hat{\beta}$	-0.9587	-0.9422	0.0413	0.2802	0.2788	$cov(\sigma,\beta)$	-0.0479
		Interval	$\hat{\sigma}$	0.7895	0.7844	-0.0105	0.0152	0.0151	$cov(\sigma,\alpha)$	0.0157
			$\hat{\alpha}$	1.9774	1.9539	-0.0226	0.0460	0.0456	$cov(\alpha,\beta)$	-0.0792
			$\hat{\beta}$	-0.8381	-0.8196	0.1619	0.2675	0.2415	$cov(\sigma,\beta)$	-0.0461
	20%	Continuous	$\hat{\sigma}$	0.7983	0.7922	-0.0017	0.0153	0.0153	$cov(\sigma,\alpha)$	0.0209
			$\hat{\alpha}$	2.0039	1.9703	0.0039	0.0637	0.0638	$cov(\alpha,\beta)$	-0.1152
			$\hat{\beta}$	-0.9962	-0.9644	0.0038	0.3341	0.3344	$cov(\sigma,\beta)$	-0.0539
		Interval	$\hat{\sigma}$	0.7997	0.7925	-0.0003	0.0167	0.0167	$cov(\sigma,\alpha)$	0.0187
			$\hat{\alpha}$	1.9994	1.9815	-0.0006	0.0560	0.0561	$cov(\alpha,\beta)$	-0.0956
			$\hat{\beta}$	-0.8731	-0.8528	0.1269	0.2839	0.2680	$cov(\sigma,\beta)$	-0.0485

Table 20: Performance measure of the MLE with their empirical variance based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $k = 4$, $n = 100$, and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Mean	Median	Bias	MSE	Empirical		
								Var	Cov	
4	0%	Continuous	$\hat{\sigma}$	0.8021	0.7882	0.0021	0.0176	0.0176	$cov(\sigma,\alpha)$	0.0267
			$\hat{\alpha}$	2.0102	1.9742	0.0102	0.0772	0.0771	$cov(\alpha,\beta)$	-0.1514
			$\hat{\beta}$	-0.9926	-0.9764	0.0074	0.4040	0.4043	$cov(\sigma,\beta)$	-0.0684
		Interval	$\hat{\sigma}$	0.8219	0.8121	0.0219	0.0201	0.0196	$cov(\sigma,\alpha)$	0.0254
			$\hat{\alpha}$	2.0284	1.9979	0.0284	0.0700	0.0693	$cov(\alpha,\beta)$	-0.1352
			$\hat{\beta}$	-0.9837	-0.9344	0.0163	0.3661	0.3662	$cov(\sigma,\beta)$	-0.0681
	10%	Continuous	$\hat{\sigma}$	0.7934	0.7822	-0.0066	0.0175	0.0175	$cov(\sigma,\alpha)$	0.0277
			$\hat{\alpha}$	2.0019	1.9688	0.0019	0.0810	0.0811	$cov(\alpha,\beta)$	-0.1612
			$\hat{\beta}$	-0.9820	-0.9218	0.0180	0.4443	0.4444	$cov(\sigma,\beta)$	-0.0698
		Interval	$\hat{\sigma}$	0.8259	0.8099	0.0259	0.0227	0.0220	$cov(\sigma,\alpha)$	0.0299
			$\hat{\alpha}$	2.0395	2.0114	0.0395	0.0817	0.0802	$cov(\alpha,\beta)$	-0.1590
			$\hat{\beta}$	-1.0267	-0.9825	-0.0267	0.4458	0.4455	$cov(\sigma,\beta)$	-0.0791
	20%	Continuous	$\hat{\sigma}$	0.7999	0.7888	-0.0001	0.0187	0.0187	$cov(\sigma,\alpha)$	0.0286
			$\hat{\alpha}$	2.0133	1.9691	0.0133	0.0843	0.0842	$cov(\alpha,\beta)$	-0.1741
			$\hat{\beta}$	-0.9935	-0.9388	0.0065	0.5114	0.5119	$cov(\sigma,\beta)$	-0.0764
		Interval	$\hat{\sigma}$	0.8321	0.8206	0.0321	0.0238	0.0228	$cov(\sigma,\alpha)$	0.0299
			$\hat{\alpha}$	2.0502	2.0094	0.0502	0.0871	0.0847	$cov(\alpha,\beta)$	-0.1626
			$\hat{\beta}$	-1.0320	-1.0174	-0.0320	0.4602	0.4596	$cov(\sigma,\beta)$	-0.0773

Table 21: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 20$ and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9850	1.0662	0.4121	1.4783
			$\hat{\alpha}$	0.9930	2.1541	1.4358	3.5899
			$\hat{\beta}$	0.9920	3.7594	-3.7916	-0.0322
		Interval	$\hat{\sigma}$	1.0000	1.0591	0.4023	1.4614
			$\hat{\alpha}$	0.9960	1.7992	1.4368	3.2360
			$\hat{\beta}$	1.0000	3.0498	-3.0498	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9870	1.0883	0.4090	1.4973
			$\hat{\alpha}$	0.9890	2.2357	1.4330	3.6687
			$\hat{\beta}$	0.9890	3.9181	-3.9515	-0.0334
		Interval	$\hat{\sigma}$	1.0000	1.0972	0.4039	1.5011
			$\hat{\alpha}$	0.9970	1.9073	1.4319	3.3392
			$\hat{\beta}$	1.0000	3.2314	-3.2314	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9800	1.1354	0.4038	1.5393
			$\hat{\alpha}$	0.9850	2.3930	1.4349	3.8280
			$\hat{\beta}$	0.9840	4.2284	-4.2694	-0.0410
		Interval	$\hat{\sigma}$	1.0000	1.1348	0.3963	1.5311
			$\hat{\alpha}$	0.9980	2.0321	1.4307	3.4628
			$\hat{\beta}$	1.0000	3.4869	-3.4869	0.0000

Table 22: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 20$ and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9900	1.2125	0.4140	1.6265
			$\hat{\alpha}$	0.9890	2.6584	1.4270	4.0853
			$\hat{\beta}$	0.9870	4.7271	-4.7566	-0.0295
		Interval	$\hat{\sigma}$	1.0000	1.1763	0.4063	1.5826
			$\hat{\alpha}$	1.0000	2.2273	1.4198	3.6470
			$\hat{\beta}$	1.0000	3.9863	-3.9863	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9990	1.2707	0.4051	1.6759
			$\hat{\alpha}$	0.9930	2.8775	1.4146	4.2921
			$\hat{\beta}$	0.9910	5.2355	-5.2667	-0.0312
		Interval	$\hat{\sigma}$	1.0000	1.2300	0.3939	1.6239
			$\hat{\alpha}$	1.0000	2.4171	1.4097	3.8268
			$\hat{\beta}$	1.0000	4.4220	-4.4220	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9980	1.2759	0.3953	1.6712
			$\hat{\alpha}$	0.9920	3.0024	1.3990	4.4015
			$\hat{\beta}$	0.9930	5.5061	-5.5286	-0.0225
		Interval	$\hat{\sigma}$	1.0000	1.2984	0.3930	1.6914
			$\hat{\alpha}$	0.9990	2.6699	1.4022	4.0722
			$\hat{\beta}$	1.0000	4.8813	-4.8813	0.0000

Table 23: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 20$ and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9700	0.9649	0.4306	1.3954
			$\hat{\alpha}$	0.9850	1.8062	1.4853	3.2915
			$\hat{\beta}$	0.9910	3.6834	-3.7287	-0.0453
		Interval	$\hat{\sigma}$	1.0000	0.9561	0.4254	1.3815
			$\hat{\alpha}$	0.9900	1.4694	1.4810	2.9504
			$\hat{\beta}$	1.0000	2.8167	-2.8167	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9820	0.9828	0.4273	1.4102
			$\hat{\alpha}$	0.9820	1.8636	1.4849	3.3485
			$\hat{\beta}$	0.9860	3.8163	-3.8661	-0.0498
		Interval	$\hat{\sigma}$	1.0000	0.9827	0.4265	1.4092
			$\hat{\alpha}$	0.9900	1.5489	1.4750	3.0239
			$\hat{\beta}$	1.0000	2.9644	-2.9644	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9850	0.9940	0.4243	1.4184
			$\hat{\alpha}$	0.9890	1.9233	1.4792	3.4025
			$\hat{\beta}$	0.9880	4.0207	-4.0659	-0.0453
		Interval	$\hat{\sigma}$	1.0000	1.0177	0.4225	1.4402
			$\hat{\alpha}$	0.9940	1.6312	1.4786	3.1098
			$\hat{\beta}$	1.0000	3.2051	-3.2051	0.0000

Table 24: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 20$ and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9950	1.0906	0.4346	1.5252
			$\hat{\alpha}$	0.9890	2.2347	1.4737	3.7083
			$\hat{\beta}$	0.9920	4.6534	-4.6924	-0.0390
		Interval	$\hat{\sigma}$	1.0000	1.0566	0.4266	1.4833
			$\hat{\alpha}$	0.9990	1.8231	1.4601	3.2832
			$\hat{\beta}$	1.0000	3.7725	-3.7725	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9900	1.1275	0.4273	1.5549
			$\hat{\alpha}$	0.9860	2.3733	1.4608	3.8341
			$\hat{\beta}$	0.9900	5.0006	-5.0305	-0.0299
		Interval	$\hat{\sigma}$	1.0000	1.1000	0.4252	1.5252
			$\hat{\alpha}$	0.9970	1.9663	1.4502	3.4165
			$\hat{\beta}$	1.0000	4.1096	-4.1096	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9940	1.1645	0.4109	1.5754
			$\hat{\alpha}$	0.9900	2.5622	1.4636	4.0258
			$\hat{\beta}$	0.9930	5.4483	-5.4793	-0.0311
		Interval	$\hat{\sigma}$	1.0000	1.1550	0.4089	1.5638
			$\hat{\alpha}$	0.9970	2.1814	1.4614	3.6428
			$\hat{\beta}$	1.0000	4.6355	-4.6355	0.0000

Table 25: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 100$ and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9370	0.4864	0.5797	1.0661
			$\hat{\alpha}$	0.9560	0.8889	1.6420	2.5309
			$\hat{\beta}$	0.9380	1.8505	-2.0423	-0.1918
		Interval	$\hat{\sigma}$	1.0000	0.5123	0.5919	1.1042
			$\hat{\alpha}$	0.9930	0.8211	1.6460	2.4671
			$\hat{\beta}$	1.0000	1.7095	-1.7701	-0.0606
	10%	Continuous	$\hat{\sigma}$	0.9500	0.4921	0.5766	1.0687
			$\hat{\alpha}$	0.9640	0.9119	1.6408	2.5527
			$\hat{\beta}$	0.9530	1.9084	-2.0912	-0.1827
		Interval	$\hat{\sigma}$	1.0000	0.5268	0.5962	1.1230
			$\hat{\alpha}$	0.9980	0.8633	1.6505	2.5138
			$\hat{\beta}$	0.9990	1.8011	-1.8593	-0.0582
	20%	Continuous	$\hat{\sigma}$	0.9350	0.4978	0.5745	1.0723
			$\hat{\alpha}$	0.9350	0.9352	1.6411	2.5763
			$\hat{\beta}$	0.9490	1.9716	-2.1526	-0.1810
		Interval	$\hat{\sigma}$	1.0000	0.5413	0.5990	1.1403
			$\hat{\alpha}$	0.9990	0.9026	1.6557	2.5583
			$\hat{\beta}$	1.0000	1.8864	-1.9558	-0.0694

Table 26: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 100$ and $\tau = 8.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9560	0.5260	0.5735	1.0994
			$\hat{\alpha}$	0.9570	1.0289	1.6090	2.6379
			$\hat{\beta}$	0.9810	2.1698	-2.3004	-0.1306
		Interval	$\hat{\sigma}$	1.0000	0.5711	0.5879	1.1590
			$\hat{\alpha}$	0.9980	1.0081	1.6193	2.6273
			$\hat{\beta}$	1.0000	2.1682	-2.2099	-0.0417
	10%	Continuous	$\hat{\sigma}$	0.9600	0.5321	0.5708	1.1029
			$\hat{\alpha}$	0.9680	1.0585	1.5995	2.6580
			$\hat{\beta}$	0.9820	2.2438	-2.3557	-0.1119
		Interval	$\hat{\sigma}$	1.0000	0.5909	0.5931	1.1840
			$\hat{\alpha}$	0.9990	1.0712	1.6175	2.6888
			$\hat{\beta}$	1.0000	2.2997	-2.3413	-0.0417
	10%	Continuous	$\hat{\sigma}$	0.9710	0.5386	0.5664	1.1050
			$\hat{\alpha}$	0.9690	1.1038	1.5940	2.6978
			$\hat{\beta}$	0.9820	2.3584	-2.4584	-0.1000
		Interval	$\hat{\sigma}$	1.0000	0.6141	0.5977	1.2117
			$\hat{\alpha}$	1.0000	1.1507	1.6222	2.7729
			$\hat{\beta}$	1.0000	2.4737	-2.5164	-0.0427

Table 27: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 100$ and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9490	0.4446	0.5977	1.0423
			$\hat{\alpha}$	0.9420	0.7600	1.6947	2.4547
			$\hat{\beta}$	0.9410	1.8028	-2.0351	-0.2323
		Interval	$\hat{\sigma}$	1.0000	0.4611	0.6076	1.0687
			$\hat{\alpha}$	0.9930	0.6773	1.6845	2.3619
			$\hat{\beta}$	0.9960	1.5537	-1.5848	-0.0311
	10%	Continuous	$\hat{\sigma}$	0.9410	0.4471	0.5996	1.0466
			$\hat{\alpha}$	0.9530	0.7694	1.6813	2.4507
			$\hat{\beta}$	0.9510	1.8526	-2.0625	-0.2098
		Interval	$\hat{\sigma}$	1.0000	0.4689	0.6142	1.0831
			$\hat{\alpha}$	0.9920	0.6980	1.6765	2.3744
			$\hat{\beta}$	0.9980	1.6096	-1.6426	-0.0330
	20%	Continuous	$\hat{\sigma}$	0.9510	0.4486	0.5955	1.0441
			$\hat{\alpha}$	0.9570	0.7814	1.6789	2.4604
			$\hat{\beta}$	0.9610	1.9039	-2.0871	-0.1832
		Interval	$\hat{\sigma}$	1.0000	0.4824	0.6167	1.0991
			$\hat{\alpha}$	0.9920	0.7319	1.6820	2.4139
			$\hat{\beta}$	0.9990	1.6978	-1.7324	-0.0346

Table 28: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 100$ and $\tau = 10.5$ under Weibull distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9610	0.4790	0.5898	1.0688
			$\hat{\alpha}$	0.9670	0.8829	1.6578	2.5408
			$\hat{\beta}$	0.9860	2.1328	-2.2787	-0.1459
		Interval	$\hat{\sigma}$	1.0000	0.5146	0.6067	1.1213
			$\hat{\alpha}$	0.9980	0.8354	1.6646	2.5000
			$\hat{\beta}$	1.0000	2.0267	-2.0639	-0.0372
	10%	Continuous	$\hat{\sigma}$	0.9570	0.4835	0.5893	1.0728
			$\hat{\alpha}$	0.9570	0.9086	1.6473	2.5559
			$\hat{\beta}$	0.9820	2.1978	-2.3275	-0.1297
		Interval	$\hat{\sigma}$	1.0000	0.5309	0.6133	1.1442
			$\hat{\alpha}$	0.9960	0.8879	1.6593	2.5472
			$\hat{\beta}$	1.0000	2.1484	-2.1841	-0.0357
	20%	Continuous	$\hat{\sigma}$	0.9510	0.4952	0.5883	1.0835
			$\hat{\alpha}$	0.9710	0.9536	1.6455	2.5990
			$\hat{\beta}$	0.9780	2.3444	-2.4758	-0.1314
		Interval	$\hat{\sigma}$	1.0000	0.5486	0.6137	1.1623
			$\hat{\alpha}$	0.9980	0.9442	1.6606	2.6048
			$\hat{\beta}$	1.0000	2.3120	-2.3492	-0.0372

Table 29: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 20$ and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9830	1.2167	0.3869	1.6037
			$\hat{\alpha}$	0.9810	2.9075	1.2995	4.2070
			$\hat{\beta}$	0.9920	4.2976	-4.3180	-0.0204
		Interval	$\hat{\sigma}$	1.0000	1.1764	0.3643	1.5407
			$\hat{\alpha}$	0.9940	2.4532	1.3091	3.7624
			$\hat{\beta}$	0.9990	3.5181	-3.5181	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9870	1.2732	0.3810	1.6542
			$\hat{\alpha}$	0.9820	3.0836	1.2986	4.3822
			$\hat{\beta}$	0.9930	4.6525	-4.6711	-0.0186
		Interval	$\hat{\sigma}$	1.0000	1.2130	0.3547	1.5677
			$\hat{\alpha}$	0.9900	2.5797	1.3107	3.8904
			$\hat{\beta}$	1.0000	3.7842	-3.7842	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9800	1.2449	0.3709	1.6158
			$\hat{\alpha}$	0.9820	3.0668	1.2925	4.3594
			$\hat{\beta}$	0.9950	4.7075	-4.7257	-0.0182
		Interval	$\hat{\sigma}$	1.0000	1.2504	0.3513	1.6017
			$\hat{\alpha}$	0.9860	2.6762	1.3083	3.9844
			$\hat{\beta}$	1.0000	3.9706	-3.9706	0.0000

Table 30: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 20$ and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9950	1.3866	0.3889	1.7755
			$\hat{\alpha}$	0.9850	3.5041	1.2785	4.7826
			$\hat{\beta}$	0.9940	5.4905	-5.5071	-0.0166
		Interval	$\hat{\sigma}$	1.0000	1.2941	0.3718	1.6659
			$\hat{\alpha}$	0.9940	2.9167	1.2815	4.1983
			$\hat{\beta}$	1.0000	4.5501	-4.5501	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9940	1.4357	0.3798	1.8154
			$\hat{\alpha}$	0.9890	3.6993	1.2563	4.9556
			$\hat{\beta}$	0.9980	5.8364	-5.8462	-0.0098
		Interval	$\hat{\sigma}$	1.0000	1.3542	0.3654	1.7196
			$\hat{\alpha}$	0.9990	3.1241	1.2614	4.3855
			$\hat{\beta}$	1.0000	4.9543	-4.9543	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9940	1.4492	0.3679	1.8171
			$\hat{\alpha}$	0.9920	3.8481	1.2404	5.0885
			$\hat{\beta}$	1.0000	6.2496	-6.2517	-0.0021
		Interval	$\hat{\sigma}$	1.0000	1.4174	0.3578	1.7752
			$\hat{\alpha}$	0.9950	3.3672	1.2510	4.6181
			$\hat{\beta}$	1.0000	5.4169	-5.4169	0.0000

Table 31: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 20$ and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9800	1.1031	0.4071	1.5102
			$\hat{\alpha}$	0.9780	2.4677	1.3377	3.8055
			$\hat{\beta}$	0.9930	4.0702	-4.0929	-0.0227
		Interval	$\hat{\sigma}$	1.0000	1.0551	0.3795	1.4347
			$\hat{\alpha}$	0.9920	2.0457	1.3577	3.4034
			$\hat{\beta}$	1.0000	3.2222	-3.2222	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9680	1.0954	0.4024	1.4978
			$\hat{\alpha}$	0.9740	2.4744	1.3109	3.7853
			$\hat{\beta}$	0.9920	4.1603	-4.1802	-0.0198
		Interval	$\hat{\sigma}$	1.0000	1.0808	0.3787	1.4596
			$\hat{\alpha}$	0.9900	2.1086	1.3369	3.4454
			$\hat{\beta}$	1.0000	3.3747	-3.3747	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9820	1.1315	0.3996	1.5311
			$\hat{\alpha}$	0.9760	2.5641	1.3116	3.8757
			$\hat{\beta}$	0.9910	4.4421	-4.4642	-0.0221
		Interval	$\hat{\sigma}$	1.0000	1.1075	0.3718	1.4793
			$\hat{\alpha}$	0.9900	2.1840	1.3354	3.5194
			$\hat{\beta}$	1.0000	3.6031	-3.6031	0.0000

Table 32: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 20$ and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9860	1.2408	0.4045	1.6453
			$\hat{\alpha}$	0.9770	2.9269	1.3176	4.2444
			$\hat{\beta}$	0.9960	5.1192	-5.1339	-0.014
		Interval	$\hat{\sigma}$	1.0000	1.1649	0.3860	1.5509
			$\hat{\alpha}$	0.9900	2.4380	1.3268	3.7648
			$\hat{\beta}$	1.0000	4.2207	-4.2207	0.0000
	10%	Continuous	$\hat{\sigma}$	0.9940	1.2361	0.4020	1.6380
			$\hat{\alpha}$	0.9880	2.9662	1.2802	4.2464
			$\hat{\beta}$	0.9960	5.2685	-5.2827	-0.0143
		Interval	$\hat{\sigma}$	1.0000	1.2052	0.3881	1.5932
			$\hat{\alpha}$	0.9970	2.5717	1.2901	3.8618
			$\hat{\beta}$	1.0000	4.4976	-4.4976	0.0000
	20%	Continuous	$\hat{\sigma}$	0.9870	1.2875	0.3840	1.6716
			$\hat{\alpha}$	0.9900	3.2109	1.2910	4.5019
			$\hat{\beta}$	0.9940	5.9482	-5.9637	-0.0155
		Interval	$\hat{\sigma}$	1.0000	1.2666	0.3683	1.6349
			$\hat{\alpha}$	0.9970	2.7863	1.3052	4.0915
			$\hat{\beta}$	1.0000	5.0999	-5.0999	0.0000

Table 33: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 100$ and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9440	0.5230	0.5720	1.0950
			$\hat{\alpha}$	0.9480	1.0872	1.5783	2.6655
			$\hat{\beta}$	0.9690	2.0558	-2.1904	-0.1346
		Interval	$\hat{\sigma}$	1.0000	0.5421	0.5636	1.1056
			$\hat{\alpha}$	0.9960	1.0073	1.5873	2.5946
			$\hat{\beta}$	0.9990	1.8714	-1.8953	-0.0239
	10%	Continuous	$\hat{\sigma}$	0.9460	0.5215	0.5672	1.0887
			$\hat{\alpha}$	0.9450	1.0894	1.5668	2.6562
			$\hat{\beta}$	0.9810	2.1050	-2.2256	-0.1205
		Interval	$\hat{\sigma}$	1.0000	0.5514	0.5651	1.1164
			$\hat{\alpha}$	0.9950	1.0344	1.5818	2.6162
			$\hat{\beta}$	0.9990	1.9503	-1.9766	-0.0263
	20%	Continuous	$\hat{\sigma}$	0.9610	0.5375	0.5728	1.1103
			$\hat{\alpha}$	0.9630	1.1340	1.5774	2.7114
			$\hat{\beta}$	0.9790	2.2076	-2.3341	-0.1266
		Interval	$\hat{\sigma}$	1.0000	0.5659	0.5683	1.1342
			$\hat{\alpha}$	0.9960	1.0805	1.5895	2.6700
			$\hat{\beta}$	0.9980	2.0421	-2.0693	-0.0272

Table 34: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 100$ and $\tau = 8.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9510	0.5534	0.5643	1.1176
			$\hat{\alpha}$	0.9750	1.2085	1.5454	2.7539
			$\hat{\beta}$	0.9870	2.3359	-2.4311	-0.0952
		Interval	$\hat{\sigma}$	1.0000	0.6003	0.5680	1.1683
			$\hat{\alpha}$	0.9990	1.2006	1.5639	2.7644
			$\hat{\beta}$	1.0000	2.3443	-2.3641	-0.0198
	10%	Continuous	$\hat{\sigma}$	0.9610	0.5610	0.5608	1.1218
			$\hat{\alpha}$	0.9710	1.2480	1.5343	2.7823
			$\hat{\beta}$	0.9880	2.4550	-2.5403	-0.0854
		Interval	$\hat{\sigma}$	1.0000	0.6176	0.5697	1.1874
			$\hat{\alpha}$	0.9960	1.2651	1.5577	2.8228
			$\hat{\beta}$	1.0000	2.4949	-2.5142	-0.0193
	20%	Continuous	$\hat{\sigma}$	0.9630	0.5786	0.5620	1.1406
			$\hat{\alpha}$	0.9720	1.3099	1.5387	2.8486
			$\hat{\beta}$	0.9910	2.6015	-2.6756	-0.0741
		Interval	$\hat{\sigma}$	1.0000	0.6397	0.5720	1.2117
			$\hat{\alpha}$	1.0000	1.3372	1.5645	2.9017
			$\hat{\beta}$	1.0000	2.6509	-2.6667	-0.0159

Table 35: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 3$, $n = 100$ and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
3	0%	Continuous	$\hat{\sigma}$	0.9460	0.4735	0.5803	1.0538
			$\hat{\alpha}$	0.9630	0.9470	1.6266	2.5736
			$\hat{\beta}$	0.9660	1.9834	-2.1362	-0.1528
		Interval	$\hat{\sigma}$	1.0000	0.4934	0.5686	1.0620
			$\hat{\alpha}$	0.9940	0.8773	1.6406	2.5178
			$\hat{\beta}$	0.9970	1.7663	-1.7914	-0.0252
	10%	Continuous	$\hat{\sigma}$	0.9470	0.4706	0.5817	1.0524
			$\hat{\alpha}$	0.9650	0.9413	1.5979	2.5392
			$\hat{\beta}$	0.9810	2.0140	-2.1355	-0.1214
		Interval	$\hat{\sigma}$	1.0000	0.4968	0.5736	1.0705
			$\hat{\alpha}$	0.9910	0.8853	1.6149	2.5002
			$\hat{\beta}$	0.9990	1.8081	-1.8255	-0.0174
	20%	Continuous	$\hat{\sigma}$	0.9510	0.4813	0.5842	1.0655
			$\hat{\alpha}$	0.9550	0.9712	1.6075	2.5786
			$\hat{\beta}$	0.9820	2.1123	-2.2431	-0.1307
		Interval	$\hat{\sigma}$	1.0000	0.5083	0.5753	1.0837
			$\hat{\alpha}$	0.9920	0.9172	1.6219	2.5391
			$\hat{\beta}$	0.9960	1.8924	-1.9127	-0.0203

Table 36: Percentile bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $k = 4$, $n = 100$ and $\tau = 10.5$ under log-logistic distribution

k	π^*	Inspection	MLE	Percentile bootstrap 95% CI			
				% Coverage	Mean Width	LB	UB
4	0%	Continuous	$\hat{\sigma}$	0.9600	0.5085	0.5822	1.0907
			$\hat{\alpha}$	0.9630	1.0615	1.5888	2.6503
			$\hat{\beta}$	0.9850	2.2727	-2.3765	-0.1038
		Interval	$\hat{\sigma}$	1.0000	0.5420	0.5810	1.1231
			$\hat{\alpha}$	0.9940	1.0293	1.6038	2.6331
			$\hat{\beta}$	1.0000	2.1922	-2.2100	-0.0178
	10%	Continuous	$\hat{\sigma}$	0.9580	0.5117	0.5746	1.0863
			$\hat{\alpha}$	0.9650	1.0851	1.5800	2.6651
			$\hat{\beta}$	0.9890	2.3712	-2.4628	-0.0916
		Interval	$\hat{\sigma}$	1.0000	0.5584	0.5810	1.1394
			$\hat{\alpha}$	0.9960	1.0795	1.6026	2.6821
			$\hat{\beta}$	1.0000	2.3446	-2.3654	-0.0208
	20%	Continuous	$\hat{\sigma}$	0.9590	0.5247	0.5768	1.1016
			$\hat{\alpha}$	0.9720	1.1344	1.5812	2.7156
			$\hat{\beta}$	0.9860	2.5149	-2.5984	-0.0835
		Interval	$\hat{\sigma}$	1.0000	0.5753	0.5832	1.1585
			$\hat{\alpha}$	0.9990	1.1372	1.6038	2.7410
			$\hat{\beta}$	1.0000	2.4844	-2.5010	-0.0165