Updating Demand Elasticities by means of Administrative Records

Luis Frank*

Abstract

The article presents a method based on linear programming to estimate and update demand elasticities. The method is particularly useful for updating estimates of elasticities by means of administrative records. In fact, the author uses this method to update the elasticities of 47 classes of items (more than 95% of the consumption basket of Argentine households) through tax records and to compute new estimates that satisfy conditions of homogeneity and aggregation imposed by the economic theory on demand systems. The new elasticities are used to compute demand indices which in turn are compared with their counterparts from the supply-side of the National Accounts System in order to evaluate the performance of the updating method.

Key Words: demand elasticities, demand system, administrative records, linear programming, National Accounts, official statistics

1. Introduction

Estimating household consumption is one of the greatest challenges in National Accounts offices due to the lack of information on the consumption of many items. In Argentina, for example, continuous series on the consumption of many relevant items, such as fruits and vegetables, repair of household appliances, meals out, laundry and dry cleaning services, personal care, taxi trips, etc. are not available. To overcome this drawback, it is common to estimate the consumption of these concepts through demand indices [2, 7, 8], i.e. quantity indices which are function of the own-price of the item, of the consumer income and, whenever possible, the price of substitute and complementary items. The bibliography (see e.g. [12, 15]) has proposed different functions - some of them very sophisticated - to represent the demand for goods in general. In those functions, the parameters are (or can easily be transformed in) demand elasticities, which are constant at any consumption level. However, studies on data from the 2004 National Household Expenditure Survey (ENGHo'04) [5, 6] found that none of these functions represented adequately the demand of Argentine households at least at the level of single items. Instead, the linear function was the one that best represented the demand for goods and was used in refered papers to compute the demand elasticities of more than 100 classes of items. Although linear functions are mathematically attractive due to their simplicity, they are rarely used in practice to model demand because the implicit price and income elasticities are not constant. This disadvantage, however, is only apparent, since expressing prices and income in the form of index-numbers, the "parameters" of the function are in fact price and income elasticities evaluated in the index-base year and are are constant at any level of consumption. To better visualize it we consider a (linear) Marshallian demand function

$$\begin{aligned} q_i &= \mu_i + \beta_{i1}\, p_1 + \cdots + \beta_{ii}\, p_i + \cdots + \beta_{in}\, p_n + \beta_{i,\,n+1}\, y, \quad i = 1, \ldots, n \\ &= \mu_i + \beta_{ii}\, p_i + \boldsymbol{\beta}'_{i(i)}\, \mathbf{p}_{(i)} + \beta_{i,\,n+1}\, y \end{aligned} \tag{1}$$

*Universidad de Buenos Aires, Av. San Martín 4453, C1417DSE. E-mail: frank@agro.uba.ar

where q_i is the quantity of the i-th item consumed by a representative agent, p_i is the (real) price of the item, y is the agent's (real) income, and $\mathbf{p}_{(i)}$ is a vector of prices of other items, substitutes and complementary. Mathematically, the coefficients $\mu_i, \beta_{i1}, \ldots, \beta_{i,n+1}$ are fixed parameters of the function but conditional to a certain consumption basket. Logically, q, p and $y \in \mathbb{R}^+$, $\beta_{ii} \leq 0$, and usually $\beta_{i,n+1} \geq 0$. Then, dividing both sides of the equation by the quantity consumed in the base year q_0, the demand function of the i-th item can be rewritten as

$$\begin{aligned} \frac{q_i}{q_{i0}} &= \frac{\mu_i}{q_{i0}} + \left(\beta_{i1}\,\frac{p_{10}}{q_{i0}}\right)\frac{p_1}{p_{10}} + \cdots + \left(\beta_{i,n+1}\,\frac{y_0}{q_{i0}}\right)\frac{y}{y_0} \\ &= \mu_i^* + \lambda_{ii}\,\frac{p_i}{p_{i0}} + \boldsymbol{\lambda}'_{i(i)}\,\mathbf{p}^*_{(i)} + \lambda_{i,n+1}\,\frac{y}{y_0}. \end{aligned} \tag{2}$$

where we can see that the new parameters $\lambda_{i1}, \ldots, \lambda_{in}$ are price elasticities and $\lambda_{i,n+1}$ is the income elasticity in the base year ($t = 0$) and are constant at any price and income level. It is expected that in the base year all the variables of the function, including the quantity index, will be set to 1 so that the function constant and the elasticities add to the unit. We will call this last condition the "constant consistency" condition. In the appendix we show other properties of the function (2).

The implicit assumption in (2) that elasticities are constant may be reasonable in the short run, but it is undoubtedly false in the long run due to the appearance of new articles that modify consumer preferences. As a consequence, the demand indices calculated from (2) may represent reasonably well the evolution of demand for a certain item in years close to the base year, but they will become increasingly imprecise as we move away from that year, or more precisely, as the consumption basket is renewed. In practice, this problem is solved by reviewing periodically the elasticities of demand. However, experience shows that the periods between consecutive ENGHo can be irregular and that these surveys do not always match the System of National Accounts (SNA) base years. Therefore it is necessary to devise a method for updating demand elasticities based on continuous information sources such as administrative records.

2. Objectives

The article has three main objectives. The first, to propose a procedure to update demand elasticities from the taxable sales registry of the Federal Agency of Public Revenues (AFIP) of Argentina. Second, to review demand elasticities calculated previously by [5, 6] with data from ENGHo'04 and to update those elasticities that might be outdated. Third, construct a *demand system* and carry out an estimation exercise that leads to elasticities compatible with the restrictions imposed by the economic theory on consumption. To meet these objectives we will use three sources of information: (a) a presumably outdated database of price and income elasticities computed from the ENGHo'04; (b) an unpublished collection of elasticities in the arch calculated from various sources (AFIP, FAO, EIM, etc.) or extracted from the bibliography (see publications cited by [6]); and (c) a database of implicit price indices (period 2014-2018) computed from sales records taxed with the Value Added Tax (VAT) [1]. We chose the period 2014-2018 to work with an homogeneous classification of activities since in 2014 AFIP moved from the classifier ISIC revision 3.1 to ISIC revision 4 [18, 19], and to evaluate the robustness of the estimation procedure as this period was particularly unstable for the Argentine economy.

3. The Demand System

The n demand functions (2) can be arranged in a linear system that includes the complete basket of items consumed by households, so that at any period t the $n \times 1$ vector $\mathbf{q}_t^*$ of quantity indices will be expressed as a function of an $(n+1) \times 1$ vector of price and income indices $\mathbf{p}_t^*$ transformed by a matrix of $n \times (n+1)$ elasticities called $\mathbf{\Lambda}$. To normalize the notation we shall call $p_{n+1,\,t}^*$ to the income index previously called y_t^*. We use asterisks to indicate indices instead of absolute magnitudes. Notice that we call each vector or matrix with the same letter of their elements. The extension of this linear system to $t = 1, \ldots, T$ periods is straightforward and takes the form

$$\begin{bmatrix} q_{11}^* & \cdots & q_{1T}^* \\ \vdots & \ddots & \vdots \\ q_{n1}^* & \cdots & q_{nT}^* \end{bmatrix} = \begin{bmatrix} \mu_1^* & \lambda_{11} & \cdots & \lambda_{1n} & \lambda_{1\,n+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mu_n^* & \lambda_{n1} & \cdots & \lambda_{nn} & \lambda_{n\,n+1} \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ p_{11}^* & \cdots & p_{1T}^* \\ \vdots & \ddots & \vdots \\ p_{n+1,\,1}^* & \cdots & p_{n+1,\,T}^* \end{bmatrix}$$

or,

$$\mathbf{Q}^* = \mathbf{M} + \mathbf{\Lambda}\mathbf{P}^*, \tag{3}$$

where $\mathbf{Q}\ ^*$ is an array of indices of quantities of $n \times T$ (n items times T períodos), $\mathbf{\Lambda}$ is an array of $n \times (n+1)$ constant elasticities in time, and $\mathbf{P}^*$ is a matrix of $(n+1) \times T$ price indices. Like any demand system, this system must satisfy the conditions of homogeneity, Engle's aggregation and symmetry (also called the Slutsky condition) [4, ch. 3]. The first condition (homogeneity) states that the price and income elasticities of the i-th item must add up to zero. Note that if we combine this condition with the constant-consistency condition, then it must be true that $\mu_i^* = 1$ or in matrix notation $\mathbf{M} = \mathbf{1}_n\mathbf{1}_T' = \mathbf{J}$. The second condition (Engle's aggregation) states that the weighted sum (weighted by the share of each item in the overall consumer expenditure) of income elasticities must add up to unity. The third condition (i.e. symmetry) states that the cross price elasticity between two items is proportional to their relative share in the overall consumer expenditure. Then the complete demand system is written

$$\mathbf{Q}^* = \mathbf{J} + \mathbf{\Lambda}\mathbf{P}^* \quad \text{subject to} \quad \begin{cases} \mathbf{\Lambda}\mathbf{1}_{n+1} = \mathbf{0}_n \\ \\ \mathbf{w}'\boldsymbol{\lambda}_{n+1} = 1, \quad (\text{for all } \mathbf{w}'\mathbf{1}_n = 1) \\ \\ \boldsymbol{\delta}_i' \left(\mathbf{\Lambda} - \frac{w_j}{w_i}\mathbf{\Lambda}'\right)\boldsymbol{\delta}_j = w_j\left(\lambda_{jy} - \lambda_{iy}\right) \end{cases} \tag{4}$$

Model (4) represents a complete demand system whose "parameters" are the n constants, the $n(n+1)/2$ price elasticities and the n income elasticities. That is, a total of $n(n+5)/2$ parameters, all of them unknown.To estimate such a large number of parameters we would require at least the same amount of equations which in practice are not available.[1] Clearly, the system needs to be reduced to a more tractable size.

[1] Just for comparison purposes, INDEC's monthly CPI report presents 81 classes of articles for 6 regions [11], which would imply the calculation of 20898 elasticities.

3.1 Dimension reduction

To reduce the dimension of the demand system we propose to summarize the cross-elasticity terms in a single term that be a function of the general consumer price index. Logically, this transformation would imply a loss of information that must be minimized to ensure that the reduced system does not deviate too much from the complete system. To find a reduced form of (4) let us first recall the demand function (2) evaluated at period t

$$q_{it}^* = \mu_i^* + \lambda_i \, p_{it}^* + \boldsymbol{\lambda}_{(i)}' \mathbf{p}_{(i)t}^* + \lambda_{n+1} \, y_t^* . \tag{5}$$

The subscript in parentheses in (5) means that the vector encompasses all items in the consumption basket except the i-th one. Besides, let us define the general price index $\bar{p}_t^* = \mathbf{w}'\mathbf{p}_t^*$, where $\mathbf{w}$ is a weighting vector whose i-th element is the share of item i in the consumption basket at the base year ($t = 0$) and $\mathbf{p}_t^*$ is a vector of price indices equal to unity in the base year. Now, each term of $\boldsymbol{\lambda}_{(i)}\mathbf{p}_{(i)t}^*$, let's say the j-th term, is proportional to the difference between the general price index and the rest of the terms, where the proprionality factor is the inverse of the j-th weight. That is,

$$p_{jt}^* = \frac{1}{w_j} \, \bar{p}_t^* - \frac{1}{w_j} \, \mathbf{w}_{(j)}' \mathbf{p}_{(j)t}^* \;\Rightarrow\; \boldsymbol{\lambda}_{(i)}' \mathbf{p}_{(i)t}^* = \sum_{j \neq i} \frac{\lambda_j}{w_j} \, p_t^* - \sum_{j \neq i} \frac{\lambda_j}{w_j} \, \mathbf{w}_{(j)}' \mathbf{p}_{(j)t}^* .$$

Then, introducing this last expression in (5), calling R_{it} to the sum of terms that involve the products $\mathbf{w}_{(j)}' \mathbf{p}_{(j)t}^*$ and rearranging terms conveniently, equation (5) may be rewritten as

$$\begin{aligned} q_{it}^* &= \mu_i^* + \lambda_i \, p_{it}^* + \left(\sum_{j \neq i} \frac{\lambda_j}{w_j} \right) \bar{p}_t^* - \sum_{j \neq i} \frac{\lambda_j}{w_j} \, \mathbf{w}_{(j)}' \mathbf{p}_{(j)t}^* + \lambda_{n+1} \, y_t^* \\ &= (\mu_i^* + R_{i0}) + \lambda_i \, p_{it}^* + \left(\sum_{j \neq i} \frac{\lambda_j}{w_j} \right) \bar{p}_t^* + \lambda_{n+1} \, y_t^* + (R_{it} - R_{i0}) . \end{aligned} \tag{6}$$

In this fashion the cross-price elasticities terms in (5) are replaced by a single term plus a "discrepancy", while the constant term appears rescaled. The discrepancy term is presumably small (as will be seen below) because (i) most of the elasticities λ_j are null as most the items are neither substitutes nor complementary of i, (ii) in those cases where $\lambda_j \neq 0$ the discrepancies will represent the sum of a sequence of positive and negative terms - depending on whether they are substitute or complementary items - that partly cancel each other, and (iii) each of these terms also depend on the difference between the actual price of the item and its price in the base year (see below), a difference that will normally oscillate around zero.

$$\begin{aligned} R_{it} - R_{i0} &= \sum_{j \neq i} \frac{\lambda_j}{w_j} \, \mathbf{w}_{(j)}' \left(\mathbf{p}_{(j)t}^* - \mathbf{1}_{n-1} \right) \\ \mu_i^* + R_{i0} &= 1 + \sum_{j \neq i} \lambda_j \left(\frac{1 - w_j}{w_j} \right) , \end{aligned} \tag{7}$$

Expression (6) can be reformulated into a function of four parameters plus a discrepancy term, like the one transcribed below. For reasons that will become evident soon, we decompose the discrepancy term in two terms according to its sign.

$$q_{it}^* = \theta_{i0} + \theta_{ii} \, p_{it}^* + \theta_{ip} \, p_t^* + \theta_{iy} \, y_t^* + \Delta R_{it}^- - \Delta R_{it}^+ \tag{8}$$

Then the demand system that arises from reformulating (6) in (8) has $4n$ unknown parameters plus a discrepancy between the original function and the reduced version. Written in matrix form, the new system is similar to (3) except for the price matrix that now has an additional row of general price indices. The expanded form of this system is

$$\begin{bmatrix} q_{11}^* & \dots & q_{1T}^* \\ \vdots & \ddots & \vdots \\ q_{n1}^* & \dots & q_{nT}^* \end{bmatrix} = \begin{bmatrix} \theta_{10} & \theta_{11} & \dots & 0 & \theta_{1\bar{p}} & \theta_{1y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \theta_{n0} & 0 & \dots & \theta_{nn} & \theta_{n\bar{p}} & \theta_{ny} \end{bmatrix} \begin{bmatrix} 1 & \dots & 1 \\ p_{11}^* & \dots & p_{1T}^* \\ \vdots & \ddots & \vdots \\ p_{n1}^* & \dots & p_{nT}^* \\ \bar{p}_1^* & \dots & \bar{p}_T^* \\ y_1^* & \dots & y_T^* \end{bmatrix} + \mathbf{R}^- - \mathbf{R}^+$$

$$\mathbf{Q}^* = \mathbf{\Theta}_{n\times(n+3)}\widetilde{\mathbf{P}}^{*\prime}_{(n+3)\times T} + \mathbf{R}^-_{n\times T} - \mathbf{R}^+_{n\times T}. \tag{9}$$

It is worth noting that the only observable variables of (9) are the series of prices and quantities. Neither elasticities nor discrepancies are observable. Then it is necessary to set a criterion to choose a set of matrices $\mathbf{\Theta}$ and $\{\mathbf{R}^-, \mathbf{R}^+\}$ among those that satisfy equation (9). The obvious choice would be the set of elasticities that minimizes the sum of absolute discrepancies $|R_{it} - R_{i0}|$, i.e.the distance between the original demand system and the reducedversion measured through the unit norm. Then, if we vectorize and reformulate the expression (9) as follows

$$\begin{aligned} \text{vec}(\mathbf{Q}^*) &= \left(\widetilde{\mathbf{P}}^{*\prime} \otimes \mathbf{I}_n\right) \text{vec}\left(\mathbf{\Theta}\right) + \text{vec}\left(\mathbf{R}^-\right) - \text{vec}\left(\mathbf{R}^+\right) \\ &= \begin{bmatrix} \left(\widetilde{\mathbf{P}}^{*\prime} \otimes \mathbf{I}_n\right) & \mathbf{I}_{nT} & -\mathbf{I}_{nT} \end{bmatrix} \begin{bmatrix} \text{vec}\left(\mathbf{\Theta}\right) \\ \text{vec}\left(\mathbf{R}^-\right) \\ \text{vec}\left(\mathbf{R}^+\right) \end{bmatrix}, \end{aligned} \tag{10}$$

the desired set of elasticities will be the solution that solves the optimization problem

$$\min_{\mathbf{\Theta},\mathbf{R}} \left\{ \mathbf{0}'_{nT}\text{vec}\left(\mathbf{\Theta}\right) + \mathbf{1}'_{nT}\text{vec}\left(\mathbf{R}^-\right) + \mathbf{1}'_{nT}\text{vec}\left(\mathbf{R}^+\right) \right\} \quad \text{subject to}$$

$$\text{vec}(\mathbf{Q}^*) - \begin{bmatrix} \left(\widetilde{\mathbf{P}}^{*\prime} \otimes \mathbf{I}_n\right) & \mathbf{I}_{nT} & -\mathbf{I}_{nT} \end{bmatrix} \begin{bmatrix} \text{vec}\left(\tilde{\mathbf{\Theta}}\right) \\ \text{vec}\left(\mathbf{R}^-\right) \\ \text{vec}\left(\mathbf{R}^+\right) \end{bmatrix} = \mathbf{0}_{nT}, \quad \begin{bmatrix} \text{vec}\left(\mathbf{R}^-\right) \\ \text{vec}\left(\mathbf{R}^+\right) \end{bmatrix} \geq \mathbf{0}_{2nT}. \tag{11}$$

To interpret the formulas of the linear program (11) it is necessary to introduce some notation. The vec(.) operator is a function that rearranges the columns of the matrix in the argument by placing each one below the previous. The symbol $\otimes$ refers to the Kronecker product. The subscripts at the bottom of each matrix indicate its dimension in rows by columns, except in the case of square matrices such as the identity matrix $\mathbf{I}$ or vectors in which the dimension may be unambiguously defined by the number of rows. The inequality signs indicate that the relationship is satisfied element by element. The optimization problem (11) is a linear program whose objective function is the distance between the observed demand indices and the indices calculated with the solution vec($\tilde{\mathbf{\Theta}}$). However, the solution needs to be restricted to additional constraints to guarantee its belonging to the parametric space suggested by the theory. In the next section we will deal with this point.

3.2 Additional restrictions

Throughout the preceding development we focused on the construction of a demand system compatible with the linear function of indices, omitting certain restrictions necessary to restrict the results to the parametric space suggested by the economic theory. Let's see what those restrictions are.

- Parameter signs. The intercept θ_{i0} must be positive to exclude the possibility of negative consumptions as p_{it} and y_i tend to zero. The price elasticity must be negative under the assumption that the share of Giffen goods in the representative agent's basket is completely irrelevant. Income elasticity must be positive assuming that practically the entire consumption basket is made up of normal goods.[2] However, the sign of $\theta_{i\bar{p}}$ must be left unbounded because it is a weighted sum of cross-price elasticities whose result cannot be established in advance. So, for each item the solution of the linear program must satisfy the following inequality

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_{i0} \\ \theta_{ii} \\ \theta_{ip} \\ \theta_{iy} \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{12}$$

  while for the complete demand system, the matrix of elasticities must satisfy the inequality

$$\begin{bmatrix} \theta_{10} & \theta_{11} & \dots & 0 & \theta_{1\bar{p}} & \theta_{1y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \theta_{n0} & 0 & \dots & \theta_{nn} & \theta_{n\bar{p}} & \theta_{ny} \end{bmatrix} \begin{bmatrix} -1 & \mathbf{0}_n' & 0 & 0 \\ \mathbf{0}_n & \mathbf{I}_n & \mathbf{0}_n & \mathbf{0}_n \\ 0 & \mathbf{0}_n' & 0 & 0 \\ 0 & \mathbf{0}_n' & 0 & -1 \end{bmatrix} \leq \begin{bmatrix} \mathbf{0}_n & \dots & \mathbf{0}_n \end{bmatrix}. \tag{13}$$

- Relationship between parameters. Expressions (6) to (8) implicitly establish relationships among parameters that must be made explicit in the program (11). The first one arises directly from combining the expression (7) with the homogeneity condition. Replacing μ_i^* by 1, and the sum of cross-price elasticities that results from distributing the factor in parentheses by $\theta_{ii} + \theta_{iy}$ (homogeneity condition), we conclude that R_{i0} must be equal to the sum of the parameters θ_{ii}, $\theta_{i\bar{p}}$ and θ_{iy}. The second relation arises from adding, member by member, the previous restriction with the constant-consistency condition (the sum of all the parameters plus R_{i0} must be equal to the unit) imposed on (8). The third relation is obtained by replacing R_{i0} in the definition (7), that is $\theta_{i0} = 1 + R_{i0}$, by the first relation. As a result, the difference between θ_{i0} and the other parameters is equal to unity. For the i-th item the three restrictions make up the system

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \theta_{i0} \\ \theta_{ii} \\ \theta_{i\bar{p}} \\ \theta_{iy} \end{bmatrix} + \begin{bmatrix} R_{i0}^- \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} R_{i0}^+ \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

[2] In practice, we deal with classes of goods instead of songle items for which these assumptions are undoubtedly true.

And, for the whole demand system, the system of restrictions is as follows. For expository reasons we separate the second and third conditions from the first one.

$$\begin{bmatrix} \theta_{10} & \theta_{11} & \dots & 0 & \theta_{1\bar{p}} & \theta_{1y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \theta_{n0} & 0 & \dots & \theta_{nn} & \theta_{n\bar{p}} & \theta_{ny} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \mathbf{2}_n & -\mathbf{1}_n \\ 2 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} \mathbf{1}_n & \mathbf{1}_n \end{bmatrix}, \tag{14}$$

and

$$\begin{bmatrix} \theta_{10} & \theta_{11} & \dots & 0 & \theta_{1\bar{p}} & \theta_{1y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \theta_{n0} & 0 & \dots & \theta_{nn} & \theta_{n\bar{p}} & \theta_{ny} \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{1}_n \\ 1 \\ 1 \end{bmatrix} + \mathbf{r}_0^- - \mathbf{r}_0^+ = \mathbf{0}_n. \tag{15}$$

- Prior information. We often have preliminary estimates of the demand elasticities and wish to incorporate them into the estimation to enrich the final result. If, for example, we had a complete database of previous estimates, we could arrange them as follows

$$\begin{bmatrix} \theta_{10} & \theta_{11} & \dots & 0 & \theta_{1\bar{p}} & \theta_{1y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \theta_{n0} & 0 & \dots & \theta_{nn} & \theta_{n\bar{p}} & \theta_{ny} \end{bmatrix} \begin{bmatrix} 0 & \mathbf{0}_2' \\ \mathbf{1}_n & \mathbf{0}_n \\ \mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix} + \mathbf{E}^- - \mathbf{E}^+ = \begin{bmatrix} \hat{\theta}_{11} & \hat{\theta}_{1\bar{p}} & \hat{\theta}_{1y} \\ \vdots & \vdots & \vdots \\ \hat{\theta}_{nn} & \hat{\theta}_{n\bar{p}} & \hat{\theta}_{ny} \end{bmatrix}.$$

  The matrices $\mathbf{E}^-$ and $\mathbf{E}^+$ are discrepancy matrices similar to $\mathbf{R}^-$ and $\mathbf{R}^+$ but between elasticities known *a priori* and those to be computed. Of course, it is possible to have more than one database of previous estimates, in which case the constraint system will have the form

$$\begin{array}{ccccccccccccc} \boldsymbol{\Theta} & + & \mathbf{E}_1^- & - & \mathbf{E}_1^+ & + & \mathbf{0} & + & \dots & + & \mathbf{0} & = & \hat{\boldsymbol{\Theta}}_1 \\ \vdots & & & & & & & & & & & & \vdots \\ \boldsymbol{\Theta} & + & \mathbf{0} & + & \dots & + & \mathbf{0} & + & \mathbf{E}_s^- & - & \mathbf{E}_s^+ & = & \hat{\boldsymbol{\Theta}}_s \end{array} \tag{16}$$

- Engle aggregation. As mentioned at the beginning of the section, the weighted sum of the income values must equal unity. Algebraically, this condition is expressed

$$\mathbf{w}' \begin{bmatrix} \theta_{10} & \theta_{11} & \dots & 0 & \theta_{1\bar{p}} & \theta_{1y} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \theta_{n0} & 0 & \dots & \theta_{nn} & \theta_{n\bar{p}} & \theta_{ny} \end{bmatrix} \begin{bmatrix} \mathbf{0}_{n+2} \\ 1 \end{bmatrix} = 1 \tag{17}$$

  Although this is a bilinear form, it is possible to incorporate this restriction in the program (11) in a verctorized form, as will be shown later.

Simple inspection of constraints (10) and (13) to (16) suggests that they may be arranged in a general system. Note that calling $\mathbf{A}$ to the matrix that post-multiplies $\boldsymbol{\Theta}$ these constraints have the general form

$$\left(\mathbf{A}' \otimes \mathbf{I}_n\right) \mathrm{vec}\left(\boldsymbol{\Theta}\right) + \mathbf{D}_{mn} \mathrm{vec}\left(\mathbf{H}^-\right) - \mathbf{D}_{mn} \mathrm{vec}\left(\mathbf{H}^+\right) \leq \mathrm{vec}\left(\mathbf{B}\right) \tag{18}$$

where $\mathbf{D}_{mn} = \mathbf{0}_{mn}$ if the solution satisfies the equality exactly and $\mathbf{D} = \mathbf{I}_{mn}$ if it satisfies the equality approximately, in which case matrices $\mathbf{H}^-$ y $\mathbf{H}^+$ are matrices of discrepancies just as $\mathbf{R}^-$ and $\mathbf{R}^+$ or $\mathbf{E}^-$ and $\mathbf{E}^+$ as appropriate. $\mathbf{A}$ has dimension $(n+3)\times m$ and $\mathrm{vec}(\mathbf{\Theta})$ has dimension $n(n+3)$, so each inequality term would have dimension $mn \times 1$. In (18) we use the inequality sign loosely, to indicate both equality and strict inequality. However, in formulating the linear program we will distinguish between equalities and inequalities more rigorously. Expression (18) can be written more compactly as

$$\left[\begin{array}{cc} \mathbf{A}'\otimes\mathbf{I}_n & \mathbf{u}'\otimes\mathbf{D}_{nm} \end{array}\right] \left[\begin{array}{c} \mathrm{vec}(\mathbf{\Theta}) \\ \mathrm{vec}(\mathbf{H}^-) \\ \mathrm{vec}(\mathbf{H}^+) \end{array}\right] \leq \mathrm{vec}(\mathbf{B}) \quad \text{donde} \quad \mathbf{u}' = [1, -1]. \tag{19}$$

The vectorized form of Engel's aggregation restriction is less evident than the others and is obtained by two successive vectorizations

$$\mathbf{w}'\mathbf{\Theta}\,\mathbf{a}_{\mathrm{w}} = \mathbf{a}_{\mathrm{w}}'\mathrm{vec}\left(\mathbf{w}'\mathbf{\Theta}\right) = \mathbf{a}_{\mathrm{w}}'\left(\mathbf{I}_{n+3}\otimes\mathbf{w}'\right)\mathrm{vec}\left(\mathbf{\Theta}\right). \tag{20}$$

With this new notation we are able to complete the linear program (11). For more regularity in the notation we will call $\mathbf{A}_{\mathrm{I}}$ to $\mathbf{P}^{*\prime}$, $\mathbf{A}_{\mathrm{II}}$ to the matrix that pre-multiplies the right side of the inequality (13), $\mathbf{A}_{\mathrm{III}}$ and $\mathbf{A}_{\mathrm{IV}}$ to the exact and approximate equality constraints (14) and (15), respectively; and $\mathbf{A}_{\mathrm{V}}$ to the matrices associated with *a priori* elasticities.

$$\min_{\tilde{\mathbf{\Theta}},\mathbf{R},\mathbf{E}} \left\{\mathbf{0}'_{n(n+3)}\mathrm{vec}\left(\tilde{\mathbf{\Theta}}\right) + \mathbf{1}'_{nT}\mathrm{vec}\left(\mathbf{R}^-\right) + \mathbf{1}'_{nT}\mathrm{vec}\left(\mathbf{R}^+\right) + \cdots + \mathbf{1}'_{3n}\mathrm{vec}\left(\mathbf{E}_1^-\right) + \mathbf{1}'_{3n}\mathrm{vec}\left(\mathbf{E}_1^+\right) + \dots\right\}$$

subject to

$$\left[\begin{array}{c:cc:ccc} \mathbf{A}'_{\mathrm{I}}\otimes\mathbf{I}_n & \mathbf{u}'\otimes\mathbf{I}_{nT} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}'_{\mathrm{III}}\otimes\mathbf{I}_n & \mathbf{0} & \mathbf{0} & \vdots & \ddots & \vdots \\ \mathbf{A}'_{\mathrm{IV}}\otimes\mathbf{I}_n & \mathbf{0} & \mathbf{u}'\otimes\mathbf{I}_n & \mathbf{0} & \dots & \mathbf{0} \\ \hdashline \mathbf{A}'_{\mathrm{V}}\otimes\mathbf{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{u}'\otimes\mathbf{I}_{3n} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}'_{\mathrm{V}}\otimes\mathbf{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{u}'\otimes\mathbf{I}_{3n} \\ \hdashline \mathbf{a}'_{\mathrm{w}}(\mathbf{I}_{n+3}\otimes\mathbf{w}') & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{array}\right] \left[\begin{array}{c} \mathrm{vec}(\tilde{\mathbf{\Theta}}) \\ \hdashline \mathrm{vec}(\mathbf{R}^-) \\ \mathrm{vec}(\mathbf{R}^+) \\ \mathrm{vec}(\mathbf{r}_0^-) \\ \mathrm{vec}(\mathbf{r}_0^+) \\ \hdashline \mathrm{vec}(\mathbf{E}_1^-) \\ \mathrm{vec}(\mathbf{E}_1^+) \\ \vdots \\ \mathrm{vec}(\mathbf{E}_s^-) \\ \mathrm{vec}(\mathbf{E}_s^+) \end{array}\right] = \left[\begin{array}{c} \mathrm{vec}(\mathbf{Q}^*) \\ \mathbf{1}_{2n} \\ \mathbf{0}_n \\ \hdashline \mathrm{vec}(\hat{\mathbf{\Theta}}_1) \\ \vdots \\ \mathrm{vec}(\hat{\mathbf{\Theta}}_s) \\ \hdashline 1 \end{array}\right],$$

$$\left[\begin{array}{c} \mathrm{vec}\left(\mathbf{R}^-\right) \\ \mathrm{vec}\left(\mathbf{R}^+\right) \\ \mathrm{vec}\left(\mathbf{r}_0^-\right) \\ \mathrm{vec}\left(\mathbf{r}_0^+\right) \\ \mathrm{vec}(\mathbf{E}_1^-) \\ \mathrm{vec}(\mathbf{E}_1^+) \\ \vdots \\ \mathrm{vec}(\mathbf{E}_s^-) \\ \mathrm{vec}(\mathbf{E}_s^+) \end{array}\right] \geq \mathbf{0} \quad \text{and} \quad \left(\mathbf{A}'_{\mathrm{II}}\otimes\mathbf{I}_n\right)\,\mathrm{vec}(\tilde{\mathbf{\Theta}}) \leq \mathbf{0}_{mn} \tag{21}$$

4. Computational procedure

Next we describe the procedure followed to solve the linear program (11) using data from AFIP. As already said, the period studied was 2014-2018. We chose this period for several reasons. The first, due to the need to work with a period in which the classification of economic activities was homogeneous. Remember that before 2014 AFIP used the ISIC rev. 3.1 and in 2014 it was replaced by ISIC rev. 4. Although there are conversion tables between both classifications, theose tables were elaborated considering all the products each branch could offer worldwide instead of the basket of products actually offered in Argentina, which leads to frequent errors. The second reason was to use a sufficiently long period to avoid conjuncture situations that would obscure the true behavior of the consumer. The third reason was to evaluate the goodness of the method with data from a particularly unstable period due to the different exchange regimes that were adopted in the country.

(1) The first step consisted in selecting from AFIP's sales database those branches of activity mainly oriented to the supply of goods for household consumption. To that end, we compared the ISIC rev. 4 classifier [19] at three digits level with the item's description of INDEC's CPI in orden to identify the producing or retailing branches in ISIC. The selection criteria was mainly descriptive and only in a few cases we checked with the 2004 Supply and Use Tables [9] to corroborate the relevance of the branch in the provision of final good in the CPI basket. In the case of strongly integrated industries, such as the extraction and distribution of natural gas, both the producing and distribution branches were selected and treated as a unit to avoid the distorting effect of own transfer-prices since it is known that these are set according to the royalties agreed with provincial governments and do not reflect true market prices. Of the 223 ISIC branches from AFIP's database, 78 were selected in first instance, although this amount was later reduced to 47 by grouping branches into COICOP categories [17, 10] at two or three digits.

(2) In the second step we computed *per capita* sales indices for the 47 COICOP branches and for the total AFIP's database sales. These indices, which we shall call IV_{it}, are simply value indices scaled to unity in 2014. Besides, we computed a general volume index (IVF_t) of sales (also in *per capita* terms) from the global supply at constant prices, net of exports, from the SNA. Then, we computed an implicit price index $\mathrm{IPI}_t = \mathrm{IV}_t/\mathrm{IVF}_t$ to deflate current sales by branch of activity (IV_{it}) and obtain an IVF_{it} as a proxy of the quantity variable q^*_{it} defined in previous sections. There were, however, a few exceptions. The IVF_{it} of branches related to public services, for example, were replaced by specific indices of INDEC's Synthetic Indicator of Public Services (ISSP), or by indices computed from the gross value of production (VBP) at constant SNA prices. Recall that during the 2014-2018 period these branches of activity were heavily subsidized, so deflated sales do not necessarily reflect the true evolution of demand in physical quantities.

(3) Thirs, we computed implicit price indices by branch of activity (IPI_{it}) to represent the price indices p^*_{it}. The calculation method was indirect, based on the relationship

$$\frac{p_{it}}{p_{i0}} = \left(\frac{w_{it}}{w_{i0}}\right)\left(\frac{q_{i0}}{q_{it}}\right)\left(\frac{\mathbf{p}_t'\mathbf{q}_t}{\mathbf{p}_0'\mathbf{q}_0}\right),$$

which can be transcribed empirically as

$$\mathrm{IPI}_{it} = \left(\frac{w_{it}}{w_{i0}}\right)\left(\frac{\mathrm{IVF}_{i0}}{\mathrm{IVF}_{it}}\right)\left(\frac{\mathrm{IV}_{it}}{\mathrm{IV}_{i0}}\right) = \left(\frac{w_{it}}{w_{i0}}\right)\left(\frac{\mathrm{IV}_{it}}{\mathrm{IVF}_{it}}\right).$$

As proxy variables of p_{it}^* and p_t^* we use the IPI_{it} and IPI_t deflated with INDEC's salary index (IS_t), while as a proxy of real income y_t^* we used an index of average income that included people with and without income (IMe_t) with data from EPH deflated with the IPI_t, so that the empirical demand function was defined as given below. The justification for deflating the different IPIs with IS_t can be found in [5] and will not be discussed again in this paper.

$$\mathrm{IVF}_{it} \approx \theta_{i0} + \left(\frac{\theta_{ii}}{\mathrm{IS}_t}\right)\mathrm{IPI}_{it} + \left(\frac{\theta_{ip}}{\mathrm{IS}_t}\right)\mathrm{IPI}_t + \left(\frac{\theta_{iy}}{\mathrm{IPI}_t}\right)\mathrm{IMe}_t \tag{22}$$

To reduce numerical error in the calculation of elasticities, we preferred to express the IVF_{it} in terms of nominal price indices instead of real price indices. That is why the parameters θ_{ii}, θ_{ip} and θ_{iy} appear divided by the corresponding deflactor in equation (22). Expressing the demand function in this fashion required minor changes of the constraint system in (11). Those changes were the following: (a) *a priori* price and income elasticities were divided by IS_t and IPI_t, respectively; (b) non-null elements of the arrays $\mathbf{A}_{\mathrm{III}}$ and $\mathbf{A}_{\mathrm{IV}}$ were multiplied by IS_t and IPI_t, as well as the weights w_i which were multiplied by IPI_t. This transformation required, of course, anti-transformation of the solution by multiplying by IS_t or IPI_t in order to return to the original scale.

(4) Fourth, we incorporated *a priori* information from two exogenous sources. The first source was an unpublished compilation of price and income elasticities in the arc computed by the author from the apparent consumption of items or classes of items at market prices. The second source were price and income elasticities from the period 2004-2006 which arise from the demand indices computed by [5] using data from INDEC's ENGHo'04.

(5) Fifth, we solved the linear program (11) through the built-in simplex algorithm in Euler Math Toolbox.[3] Actually, we restated the program to an equivalent but computationally more efficient form. In the restated version, restrictions corresponding to the same item were grouped in blocks which were in turn located diagonally in the general restriction matrix, except for the Engel aggregation restriction which was located at the end. The advantage of this formulation was that null rows due to vectorization of sparse matrices such as $\mathbf{\Theta}$ could be identified and eliminated easily in order to reduce the dimension of the grand matrix.

5. Results

Table 1 shows the solution of the program $\mathrm{vec}(\tilde{\mathbf{\Theta}})$. For comparative purposes we attach (see Table 2) minimum, maximum and average demand elasticities reported by six authors between 2002 and 2014. Simple inspection of the results shows that computed price and income elasticities are in general in line with those reported in the bibliography. This observation was also corroborated the comparison of our elasticities (reclassified in the categories of table 2) with the mean price and income elasticities of table 2 through Wilcoxon's test, since the null hypothesis could not be rejected with 5% probability of type I error. This result suggests (a) that the

[3] Free software downloadable from `http://euler-math-toolbox.de/`.

Table 1: Estimated parameters of the demand system. θii is the price elasticity and θ_{iy} the income elasticity. Weights of INDEC's CPI, December 2016.

Code	Description	Weight	θ_{ii}	θ_{ip}	θ_{iy}
01.1.1	Bread and cereals	0,0475	-0,4083	0,0710	0,3373
01.1.2	Meat and related products	0,0892	-0,7108	0,2490	0,4618
01.1.3	Fish and shellfish	0,0047	-0,8357	-2,0676	2,9033
01.1.4	Dairy and eggs	0,0361	-0,8488	0,2981	0,5507
01.1.5	Oil, fat and butter	0,0065	-0,5185	-1,5754	2,0939
01.1.6/7	Fruits and vegetables	0,0401	-0,5252	0,3504	0,1748
01.1.8/9	Sugar, chocolate, candies, etc.	0,0144	-0,9310	-2,1596	3,0906
01.2/02.1	Infusions and beverages	0,0470	-0,8999	0,5508	0,3491
02.2	Cigarettes and tobacco	0,0191	-0,2785	-1,4384	1,7169
03.1.1	Textiles, cloth and spinning	0,0009	-1,1677	-3,7100	4,8777
03.1.2/3	Garments and accessories	0,0698	-0,8388	0,3142	0,5246
03.2.1	Shoes and other footwear	0,0262	-2,0777	1,5140	0,5637
03.1.4/ 2.2/ 12.1.1	Dry cleaning, shoe repair; hairdressing, personal care	0,0096	-1,9878	-7,1299	9,1177
04.3	Home maintenance	0,0115	-0,9053	-1,5472	2,4525
04.4	Water supply	0,0077	0,0000	-0,0137	0,0137
04.5.1	Electricity	0,0123	0,0000	-0,2712	0,2712
04.5.2	Gas supply and cylinder gas	0,0165	-0,5246	-1,2649	1,7895
05.1	Furniture and rugs	0,0060	-1,3475	-4,4465	5,7940
05.3	Home appliances	0,0124	-1,1207	-3,3146	4,4353
06.1.1/2	Pharmaceutical products	0,0392	-1,0609	0,2657	0,7952
06.1.3	Therapeutic devices and equipment	0,0027	-1,1902	0,8894	0,3008
06.2/3/4	Health services	0,0800	-1,0117	0,4539	0,5578
07.1	Motor vehicles	0,0271	-1,2416	0,4853	0,7563
07.2 exc. 07.2.2	Automotive maintenance	0,0109	-1,0836	-4,7478	5,8314
07.2.2	Fuels and lubricants	0,0424	-0,3063	0,0922	0,2141
07.3.1	Rail transport services	0,0260	-0,0797	-0,0712	0,1509
07.3.2	Bus services	0,0018	-0,0986	-1,1392	1,2378
07.3.3	Air transport services	0,0014	-0,2835	0,0863	0,1972
07.3.6	Other transportation services	0,0003	-0,0661	-1,2396	1,3057
08.1.1	Postal services	0,0002	-0,2517	0,2517	0,0000
08.2/09.1.1	Phone devices	0,0054	-1,6448	0,4963	1,1485
08.3.1/2	Telephone service	0,0201	-0,0366	-0,7422	0,7788
08.3.3	Internet connection service	0,0073	-0,8720	0,5698	0,3022
09.1.2	Photographic and cinematographic equipment and optical instruments	0,0015	-1,4294	-3,1243	4,5537
09.1.3/4	Computer equipment	0,0065	-1,4294	-3,1243	4,5537
09.3.1/2	Games, toys and sports equipment	0,0045	-1,3562	-5,1503	6,5065
09.3.4	Pets and related products	0,0055	-1,3562	-3,4231	4,7793
09.4	Recreational and cultural services	0,0288	-1,8696	0,9763	0,8933
09.5.1/2	Books and publications	0,0120	-0,7343	-2,5612	3,2955
09.5.4	Paper and office supplies	0,0038	-0,6098	-2,6857	3,2955
09.6	Tourist packages	0,0052	-1,8060	1,4911	0,3149
10	Education (initial and primary)	0,0231	-0,4595	-0,1000	0,5595
11.1	Restaurantes	0,0860	-1,3256	1,0493	0,2763
11.2	Hotels	0,0037	-2,4904	0,5100	1,9804
12.1.3	Other items for personal care	0,0222	-2,0285	-1,6343	3,6628
12.5	Insurance	0,0036	-2,6713	1,9835	0,6878
12.7	Other services	0,0021	-0,0273	-0,5948	0,6221

Table 2: Minimum, maximum and average price and income elasticities in the 2002-2014 period. Compilation made from figures [3], [16], [12], [15], [13] and [14].

Código	Concepto	λ_{ii}			$\lambda_{i\,n+1}$		
		mín.	máx.	prom.	mín.	máx.	prom.
01.1.1	Bread and cereals	-0,8460	0,3640	-0,2410	0,1060	1,0010	0,5535
01.1.2	Meat and related products	-1,0280	-0,0920	-0,5600	0,1470	1,2020	0,6745
01.1.3	Fish and shellfish	-1,5997	-0,3890	-0,9944	0,1477	0,9995	0,5736
01.1.4	Dairy (inc. butter)	-1,2290	-0,0890	-0,6590	0,1320	0,9140	0,5230
01.1.4.3	Huevos	-0,8420	-0,4480	-0,6450	0,2370	0,9300	0,5835
01.1.5	Oil and fat (excl. butter)	-1,0690	0,0850	-0,4920	0,1620	1,1410	0,6515
01.1.7	Vegetables and legumes	-0,9440	-0,7640	-0,8540	0,3040	1,2410	0,7725
01.1.6	Fruits	-1,0240	-0,0340	-0,5290	0,1560	1,6610	0,9085
01.1.8	Sugar, marmelade and candies	-1,0180	0,0000	-0,5090	0,0530	1,2630	0,6580
01.2.1/1.9	Infusions and spices	-1,0660	-1,0070	-1,0365	0,4220	0,7050	0,5635
01.2.2	Non-alcoholic beverages	-1,0380	-0,9570	-0,9975	0,3410	0,6350	0,4880
02.1	Alcoholic beverages	-0,4760	-0,4040	-0,4400	0,4190	0,7030	0,5610
03	Clothing and footwear	-0,7520	-0,6710	-0,7115	0,8450	0,9650	0,9050
06.2	Medical care	-1,0980	-0,8800	-0,9890	1,2900	1,3120	1,3010
07/08	Transport and comunication	-1,0340	-0,8310	-0,9325	1,1590	1,4090	1,2840
09	Recreation	-1,1640	-0,6570	-0,9105	0,9210	1,3880	1,1545
10	Education	-0,8770	-0,7090	-0,7930	1,0130	1,1550	1,0840
11.1.1.1	Ready-to-go meals	-0,7320	-0,6480	-0,6900	0,5970	1,1460	0,8715

analytical form of the underlying demand function is not decisive for the computation of the elasticities, (b) that computing demand elasticities through a demand system does not lead to results notoriously different from those obtained through isolated demand functions, and (c) that the optimallity criterion followed to estimate the demand elasticities does not play an important role in the final results. However, the large dispersion observed in the elasticities of different sources may obscure these claims.

In figure 1, we compare demand indices of four classes of items of the CPI's consumption basket with their counterparts on the supply side of the SCN as an indirect check of the accuracy of our estimations. These classes were selected among those for which continuous production series were available, which is not always the case in Argentina's Statistical System. The demand indices represented in figure 1 are Laspeyres chained indices and should match the supply indices under the walrasian principle of market clearing if our estimates are reasonably accurate and of course model (2) is true. Note however that demand and supply indices will not match perfectly due to the conversion of production series by activity to items consumed by households, and because demand elasticities do not remain constant when items are aggregated into classes. Notwithstanding these caveats, it can be seen that for the four selected classes supply and demand match fairly good, better in unregulated goods and services and worse in those classes with regulated prices. In fact, we observe that the fast rise in gas prices since 2016 caused demand indices to fall to zero in 2017 and 2018, which revealed a certain lack of realism in our demand function.

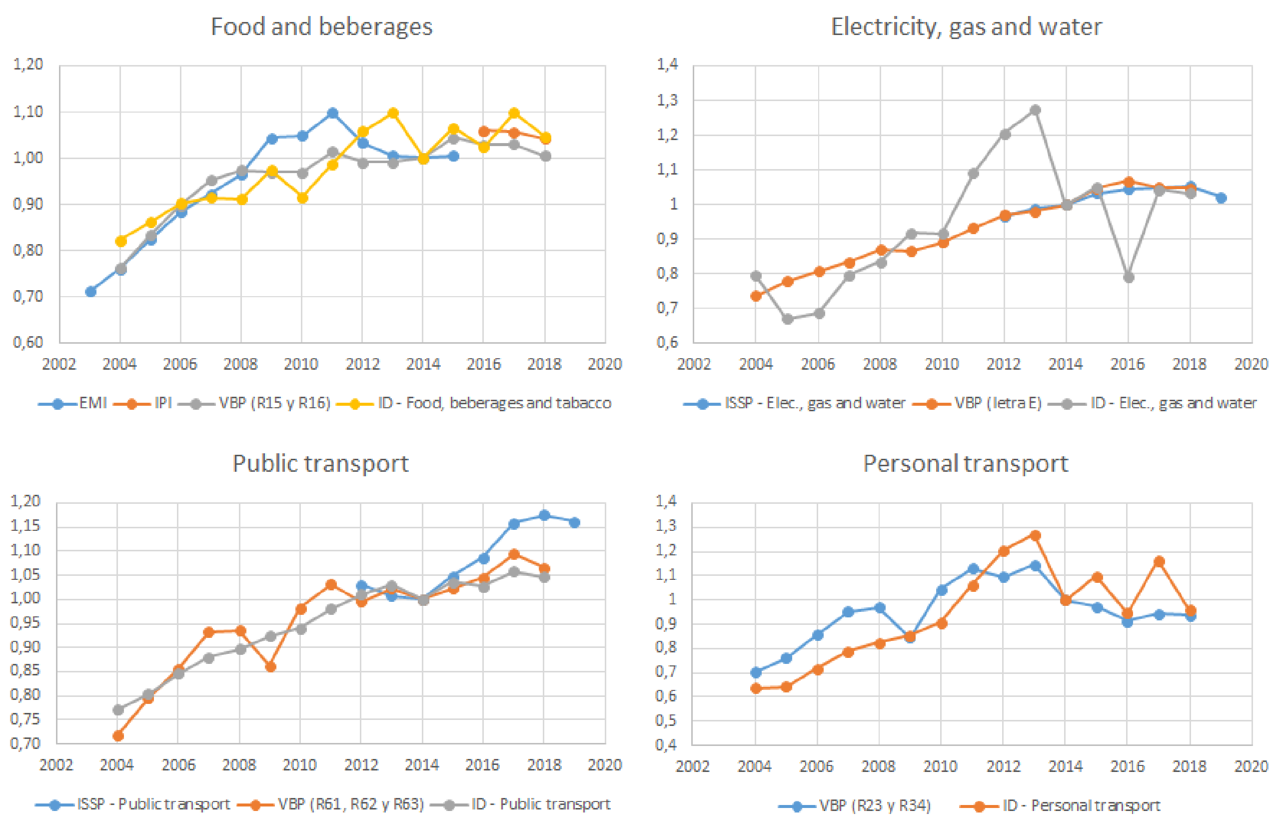

Figure 1: Supply vs demand indices (ID) of four classes of item.

6. Concluding remarks

In the article we presented a method based on linear programming to estimate and update demand-elasticities from adminsitartive records. In fact, we used this method to update the elasticities of 47 classes of items (more than 95% of the consumption basket of Argentine households) through tax records and to compute new estimates that satisfy conditions of homogeneity and aggregation (but not symmetry) imposed by the economic theory on demand systems. Neverthless, the comparison of the newly computed elasticities with those reported in the bibliography (computed through different models) did not show significant differences. Besides, the demand indices computed with the new elasticities were close to their counterparts from the supply-side of the National Accounts System, somehow validating the analytical form of the demand functions underlying the elasticity-estimates.

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