Inference for k-level Step-stress Accelerated Life Tests under Progressive Type-I Censoring with the Lifetimes from a Log-location-scale Family

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Abstract
In reliability engineering, the accelerated life test is not only getting increasingly popular but also absolutely necessary as it rapidly yields information about the lifetime distribution of a highly reliable product and device in a shorter period of time by conducting the life test at more extreme stress levels than normal operating conditions. Through extrapolation, the lifetime distribution at the usage stress can be estimated with an appropriate regression model. In this work, we investigate the statistical inference for a progressively Type-I censored k-level step-stress accelerated life test when the lifetime of a test unit follows a log-location-scale family of distributions. Although simple and analytical, the popular exponential distribution lacks the model flexibility demanded in practice due to its constraint of constant hazard rates. In practice, Weibull or lognormal distributions, which are members of the log-location-scale family, demonstrate more superior model fits. Therefore, our study is extended to consider the general log-location-scale family, and our inferential methods are illustrated using three popular lifetime distributions, including Weibull, lognormal, and log-logistic. Assuming that the location parameter is linearly linked to the (transformed) stress level ($i.e.$, $\mu_i = \alpha + \beta x_i$), an iterative algorithm is developed to estimate the regression parameters α and β along with the scale parameter σ. Allowing the intermediate censoring to take place at the end of each stress level x_i ($viz.$, τ_i, $i = 1, 2, \ldots, k$), the Fisher's expected information is derived, and the interval estimation is discussed based on the results. The effect of the intermediate censoring proportion on the inferential performance is also assessed computationally.

Key Words: accelerated life tests, Fisher information, log-location-scale family, order statistics, progressive Type-I censoring, step-stress loading

1. Introduction

Thanks to the continual improvement in manufacturing process and technology, products are becoming increasingly reliable with substantially long life-spans, which makes the standard life tests under normal usage conditions very difficult if not impossible. This difficulty is overcome by accelerated life test (ALT) where the units are subjected to higher stress levels than normal operating conditions so that more failures can be collected in a shorter period of time. The lifetime at the normal operating condition is then estimated through extrapolation using a stress-response regression model. The (step-up) step-stress test is a special class of ALT where the stress levels are gradually increased at some fixed time points during the experiment. During the past decades, the inference and design optimization for the step-stress ALT have attracted great attention in the reliability literature; see, for example, Miller and Nelson (1983), Bai et al. (1989), Nelson (1990), Meeker and Escobar (1998), Bagdonavicius and Nikulin (2002), Wu et al. (2006), Balakrishnan and Han (2008, 2009), Han and Balakrishnan (2010), Kateri et al. (2010), Han and Ng (2013), Lin et al. (2013), Han and Kundu (2015), and Han (2015).

Moreover, due to time and cost constraints, censored sampling is usually unavoidable in practice, and in particular, a generalized censoring scheme known as progressive Type-I censoring allows functional test units to be withdrawn successively from the life test at some prefixed non-terminal time points. Withdrawn unfailed units can be used in other

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tests in the same or at a different facility; see, for instance, Gouno et al. (2004), Han et al. (2006), and Balakrishnan et al. (2010). Despite its flexibility and efficient utilization of the available resources, progressively censored sampling has not gained much popularity in ALT due to its analytical complexity compared to the conventional censoring schemes; see Cohen (1963) and Lawless (1982). In particular, understanding the mean completion time of a life test under progressive Type-I censoring is of great practical interest in order to design and manage the life test optimally under frequent budgetary and time constraints.

In this work, we investigate the statistical inference for a progressively Type-I censored k-level step-stress accelerated life test when the lifetime of a test unit follows a log-location-scale family of distributions. Although simple and analytical, the popular exponential distribution lacks the model flexibility demanded in practice due to its constraint of constant hazard rates. In practice, Weibull or lognormal distributions, which are members of the log-location-scale family, demonstrate more superior model fits. Therefore, our study is extended to consider the general log-location-scale family, and our inferential methods are illustrated using three popular lifetime distributions, including Weibull, lognormal, and log-logistic. Assuming that the location parameter is linearly linked to the (transformed) stress level ($i.e.$, $\mu_i = \alpha + \beta x_i$), an iterative algorithm is developed to estimate the regression parameters α and β along with the scale parameter σ. Allowing the intermediate censoring to take place at the end of each stress level x_i ($viz.$, τ_i, $i = 1, 2, \ldots, k$), the Fisher's expected information is derived, and the interval estimation is discussed based on the results. The effect of the intermediate censoring proportion on the inferential performance is also assessed computationally.

The remainder of the article is organized as follows. The model formulation based on a log-location-scale distribution is discussed in Section 2. The likelihood and MLEs, iterative algorithm and Fisher information matrix are presented in Section 3. Using the method which we developed in this article is used for simulation studies in Section 4, and the results of the computational study are discussed.

2. Model Formulation

Let n_i be the number of failures at stress level x_i in the time interval $(\tau_{i-1}, \tau_i]$ and c_i denotes the number of censored units at time τ_i, where $i = 1, 2, ..., k$ and $x_1 < x_2 < ... < x_k$, assume $\tau_0 = 0$. Also $y_{i,j}$ denotes the jth ordered failure time during the n_i failed units at x_i, where $j = 1, 2, ..., n_i$ and N_i denotes the number of units surviving in the beginning of level x_i, such that $N_i = n - \sum_{j=1}^{i-1} n_j - \sum_{j=1}^{i-1} c_j$, where n denotes initial sample size and $N_1 = n$. A progressively Type-I censored k-level step-stress ALT proceeds as follows: Initially, $n \equiv N_1$ number of test units are placed at time $\tau_0 \equiv 0$ and is tested under x_1 stress level until time τ_1. The random failure times $(y_{1,1}, y_{1,2}, \ldots, y_{1,n_1})$ are observed during the interval $(\tau_0, \tau_1]$. At τ_1, c_1 number of surviving units are removed and the stress level is increased to x_2. The test continues with the remaining $N_2 = n - n_1 - c_1$ units until time τ_2. The random failure times $(y_{2,1}, y_{2,2}, \ldots, y_{2,n_2})$ are observed during the interval $(\tau_1, \tau_2]$. At τ_2, c_2 number of surviving units are removed and the stress level is increased to x_3. The test continues with the remaining $N_3 = N_1 - n_2 - c_2$ units until time τ_3 and so on. At τ_k, all $c_k = N_k - n_k$ surviving units are removed and the experiment is terminated.

At stress level x_i, the lifetime Y of a test unit is assumed to have a log-location-scale distribution with CDF,

$$F_i(y) = \Phi\left(\frac{\log y - \mu_i}{\sigma}\right), y \geq 0 \tag{2.1}$$

where $\Phi(.)$ is the standard log-location scale CDF, location parameter is $\mu_i = ln\theta_i =$

$\alpha + \beta x_i$ and scale parameter is $\sigma(> 0)$. Further, θ_i is the mean time to failure under the exponential distribution (i.e. when $\sigma = 0$) and α, β are regression parameters, where $\alpha\epsilon(-\infty, \infty)$ and $\beta(< 0)$ are unknown parameters and need to be estimated using an iterative algorithm. Therefore, using the cumulative exposure model, the CDF for a test unit under the step-stress ALT can be obtained as follows:

Step 1: The CDF at the initial stress level x_1 is

$$F_1(y) = \Phi\left(\frac{\log y - \mu_1}{\sigma}\right), \quad 0 < y \leq \tau_1.$$

Step 2: At Δ_1 ($\equiv \tau_1$), $F_1(\tau_1) = F_2(\varepsilon_1)$. Solve for ε_1.

Step 3: Similarly, at τ_2, $F_2(\varepsilon_1 + \Delta_2) = F_3(\varepsilon_2)$, where $\Delta_2 = \tau_2 - \tau_1$. Solve for ε_2.

Step 4: ...

Repeating the above procedure, ε_{i-1} and $F(y)$ are obtained as,

$$\varepsilon_{i-1} = \theta_i \sum_{j=1}^{i-1} \frac{\Delta_j}{\theta_j}$$

$$F(y) = F_i(y - \tau_{i-1} + \varepsilon_{i-1}) = \Phi\left(\frac{\log(y - \tau_{i-1} + \varepsilon_{i-1}) - \mu_i}{\sigma}\right) \tag{2.2}$$

when $\tau_{i-1} < y \leq \tau_i$. The PDF of the lifetime under the step-stress ALT is then obtained as

$$\begin{aligned} f(y) &= f_i(y - \tau_{i-1} + \varepsilon_{i-1}) \\ &= \frac{1}{\sigma(y - \tau_{i-1} + \varepsilon_{i-1})}\phi\left(\frac{\log(y - \tau_{i-1} + \varepsilon_{i-1}) - \mu_i}{\sigma}\right) \end{aligned} \tag{2.3}$$

when $\tau_{i-1} < y \leq \tau_i$ with $\Delta_i = \tau_i - \tau_{i-1}$.

3. Maximum Likelihood Estimation

Upon using (2.2) and (2.3) the joint PDF of observed data $\boldsymbol{n} = (n_1, n_2, ..., n_k)$, $\boldsymbol{y} = (\boldsymbol{y_1}, \boldsymbol{y_2}, ..., \boldsymbol{y_k})$ and $\boldsymbol{y}_i = (y_{i,1}, y_{i,2}, ..., y_{i,n_i})$ is obtained as

$$f(\boldsymbol{y}, \boldsymbol{n}) = \left(\prod_{i=1}^{k} \frac{N_i!}{(N_i - n_i)!}\right)\left(\prod_{i=1}^{k}\prod_{j=1}^{n_i} f(y_{i,j})\right)\left(\prod_{i=1}^{k}(1 - F(\tau_i))^{c_i}\right) \tag{3.1}$$

where $F(\tau_i) = \Phi\left(\frac{\log(\Delta_i+\varepsilon_{i-1})-\mu_i}{\sigma}\right)$ and $f(y_{i,j}) = \phi(z_{i,j})\frac{\partial z_{i,j}}{\partial y_{i,j}}$, with $z_{i,j} = \frac{\log(y_{i,j}-\tau_{i-1}+\varepsilon_{i-1})-\mu_i}{\sigma} = \frac{1}{\sigma}\log\left(\frac{y_{i,j}-\tau_{i-1}}{\theta_i} + \sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}\right)$, $i = 1, 2, ..., k$ and $j = 1, 2, ..., n_i$. Then, using (3.1) the log-likelihood function of (α,β,σ) is derived as,

$$\begin{aligned} l = &-\left(\sum_{i=1}^{k} n_i\right)\log\sigma + \sum_{i=1}^{k}\sum_{j=1}^{n_i}\log\phi(z_{i,j}) \\ &- \sum_{i=1}^{k}\sum_{j=1}^{n_i}\log(y_{i,j} + \varepsilon_{i-1} - \tau_{i-1}) + \sum_{i=1}^{k} c_i\log(1 - \Phi(\zeta_i)) \end{aligned} \tag{3.2}$$

where $\zeta_i = \frac{\log(\Delta_i+\varepsilon_{i-1})-\mu_i}{\sigma} = \frac{1}{\sigma}\log\left(\sum_{l=1}^{i}\frac{\Delta_l}{\theta_l}\right)$. Based on (3.2), the MLE $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}$ are solutions to the likelihood equations given by

$$\frac{\partial l}{\partial \alpha} = -\frac{1}{\sigma}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{\phi'(z_{i,j})}{\phi(z_{i,j})} + \frac{1}{\sigma}\sum_{i=1}^{k} c_i h(\zeta_i) c_i = 0 \tag{3.3}$$

$$\frac{\partial l}{\partial \beta} = -\sigma\sum_{i=1}^{k} n_i x_i + \sum_{i=1}^{k}\left(\sum_{j=1}^{n_i}\frac{-\phi'(z_{i,j})}{\phi(z_{i,j})} + \sigma\right) w_i(y_{i,j}) + \sum_{i=1}^{k} h(\zeta_i) w_i(\tau_i) c_i = 0 \tag{3.4}$$

$$\frac{\partial l}{\partial \sigma} = \frac{1}{\sigma}\sum_{i=1}^{k} n_i - \frac{1}{\sigma}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{\phi'(z_{i,j})}{\phi(z_{i,j})} z_{i,j} + \frac{1}{\sigma}\sum_{i=1}^{k} c_i h(\zeta_i)\zeta_i = 0 \tag{3.5}$$

where
$\frac{\partial z_{i,j}}{\partial \sigma} = -\frac{1}{\sigma} z_{i,j}$, $\frac{\partial z_{i,j}}{\partial \alpha} = -\frac{1}{\sigma} z_{i,j}$, $\frac{\partial l}{\partial \beta} = -\frac{1}{\sigma} e^{-\sigma\zeta_i}\sum_{l=1}^{i}\frac{\Delta_l}{\theta_l} x_l$, $h(\zeta_i) = \frac{\phi(\zeta_i)}{1-\Phi(\zeta_i)}$
$w_i(y) = \left(\frac{\frac{y-\tau_{i-1}}{\theta_i} x_i + \sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l} x_l}{\frac{y-\tau_{i-1}}{\theta i} + \sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}}\right)$, function of β only. Here we use a iterative algorithm to evaluate $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}$, because these equations are cannot be solved analytically.

3.1 Iterative Algorithm for $\hat{\alpha}, \hat{\beta}, \hat{\sigma}$

To evaluate initial values, consider the dataset composed of $y_{11}, y_{12}, ..., y_{1n_1}$ and $n - n_1$ number of τ_1's. Let $\widetilde{\mu_1}$ = median of log-transformed data, and $\widetilde{\sigma^2}$ = sample variance with median as center. Then, with $\sigma_0 = \widetilde{\sigma}$ and $\alpha_0 = \widetilde{\mu_1} - \beta x_1$, use (3.4) to estimate β. Use the estimated β to find α_0. Using (3.4) again with α_0 and σ_0, obtain β_0. Finally, using these initial values, start the iteration to estimate $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}$.

The algorithm proceeds as follows:

Initialization: set $m = 0$

Step 1: Given α_m, β_m, use (3.5) to estimate σ_{m+1}.

Step 2: Given σ_{m+1}, β_m, use (2.3) to estimate α_{m+1}.

Step 3: Given $\alpha_{m+1}, \sigma_{m+1}$, use (3.4) to estimate β_{m+1}.

Step 4: Check convergence, $i.e.$,
$|\alpha_{m+1} - \alpha_m| < \epsilon$
$|\beta_{m+1} - \beta_m| < \epsilon$
$|\sigma_{m+1} - \sigma_m| < \epsilon$
for some $\epsilon > 0$. If converged, stop. Otherwise, repeat the above three steps with $m = m + 1$.

3.2 Fisher Information Matrix

Here we calculate Fisher information matrix based on observed data $\boldsymbol{n}$. For a feasible progressive censoring, let $c_i = (N_i - n_i)\pi_i^*$, where $\boldsymbol{\pi} = (\pi_1^*, \pi_2^*, ..., \pi_k^*)$ are the predetermined proportions of remaining items to be withdrawn at the end of each stress level x_i, and $c_k = N_k - n_k$ since $\pi_k^* = 1$. Next, the following properties are used to derive the Fisher information matrix:

1. $n_i|N_i \sim Binomial\left(N_i, \frac{F(\tau_i)-F(\tau_{i-1})}{1-F(\tau_{i-1})}\right)$, where $\frac{F(\tau_i)-F(\tau_{i-1})}{1-F(\tau_{i-1})} \equiv \frac{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}{1-\Phi(\zeta_{i-1})}$, $i = 1, 2, ..., k$. Note that $n_1 \sim Binomial(n, F(\tau_1))$.

2. $(y_{i,1}, \ldots, y_{i,n_i})|n_i$ form the order statistics from a random sample of size n_i from a left-right truncated distribution at τ_{i-1} and τ_i.

The second partials of Log-likelihood function are given as:

$$\frac{\partial^2 l}{\partial \sigma^2} = -\left[\frac{2}{\sigma}\frac{\partial l}{\partial \sigma} + \frac{1}{\sigma^2}\sum_{i=1}^{k}\left(\sum_{j=1}^{n_i} u_{i,j} z_{i,j}^2 + \upsilon_i \zeta_i^2 c_i + n_i\right)\right] \tag{3.6}$$

$$\frac{\partial^2 l}{\partial \alpha^2} = -\left[\frac{1}{\sigma^2}\sum_{i=1}^{k}\left(\sum_{j=1}^{n_i} u_{i,j} + \upsilon_i c_i\right)\right] \tag{3.7}$$

$$\frac{\partial^2 l}{\partial \beta^2} = -\frac{1}{\sigma^2}\sum_{i=1}^{k}\left[\sum_{j=1}^{n_i}\left(u_{i,j} w_i^2(y_{i,j}) + \sigma\left(\frac{\phi'(z_{i,j})}{\phi(z_{i,j})} - \sigma\right) w_i^\beta(y_{i,j})\right)\right.$$
$$\left. + \upsilon_i w_i^2(\tau_i) c_i - \sigma h(\zeta_i) w_i^\beta(\tau_i) c_i)\right] \tag{3.8}$$

$$\frac{\partial^2 l}{\partial \alpha \partial \beta} = -\frac{1}{\sigma^2}\sum_{i=1}^{k}\left[\sum_{j=1}^{n_i} u_{i,j} w_i(y_{i,j}) + \upsilon_i w_i(\tau_i) c_i\right] \tag{3.9}$$

$$\frac{\partial^2 l}{\partial \alpha \partial \sigma} = -\frac{1}{\sigma^2}\sum_{i=1}^{k}\left[\sum_{j=1}^{n_i} u_{i,j} z_{i,j} + \upsilon_i \zeta_i c_i\right] \tag{3.10}$$

$$\frac{\partial^2 l}{\partial \beta \partial \sigma} = -\left[\frac{1}{\sigma}\frac{\partial l}{\partial \beta} + \frac{1}{\sigma^2}\sum_{i=1}^{k}\left(\sum_{j=1}^{n_i}(u_{i,j} z_{i,j} - \sigma) w_i(y_{i,j}) + \upsilon_i \zeta_i w_i(\tau_i) c_i + \sigma n_i x_i\right)\right] \tag{3.11}$$

where
$u_{i,j} = -\frac{\phi''(z_{i,j})}{\phi(z_{i,j})} + \left[\frac{\phi'(z_{i,j})}{\phi(z_{i,j})}\right]^2 = -\frac{\mathrm{d}^2}{\mathrm{d}z_{i,j}^2} ln\phi(z_{i,j})$, $\upsilon_i = \frac{\phi'(\zeta_i)}{1-\Phi(\zeta_i)} + h^2(\zeta_i) = \frac{\mathrm{d}}{\mathrm{d}\zeta_i} h(\zeta_i)$,
$w_i^\beta(y) = \frac{\partial w_i(y)}{\partial \beta} = w_i^2(y) - \left(\frac{\frac{y-\tau_{i-1}}{\theta_i} x_i^2 + \sum_{l=1}^{i-1} \frac{\Delta_l}{\theta_l} x_l^2}{\frac{y-\tau_{i-1}}{\theta i} + \sum_{l=1}^{i-1} \frac{\Delta_l}{\theta_l}}\right) = w_i^2(y) - w_i^{[2]}(y)$.
Note that, using above mentioned two properties, we get

$$E(n_i) = n(\Phi(\zeta_i) - \Phi(\zeta_{i-1}))\prod_{j=1}^{i-1}(1 - \pi_j^*)$$

and we can show that $E\left(\frac{\partial l}{\partial \beta}\right) = E\left(\frac{\partial l}{\partial \sigma}\right) = 0$, since expected value of score function. Also, by property 2, define

$$H_i(r, \sigma) = \int_{\zeta_{i-1}}^{\zeta_i} z^r e^{-\sigma z}\left(\frac{\phi'(z)}{\phi(z)}\right)^2 \phi(z) dz$$

where $r = 0, 1, 2, \sigma \geq 0$ with $\zeta_0 \equiv -\infty$, and

$$H_i(r, \sigma; 2) = \int_{\zeta_{i-1}}^{\zeta_i} z^r e^{-\sigma z} \phi''(z) dz \ \text{, o.w. } 0 \quad \text{if } r < 0$$
$$H_i(r, \sigma; 2) = \zeta_i^r e^{-\sigma \zeta_i} \phi'(\zeta_i) - \zeta_{i-1}^r e^{-\sigma \zeta_{i-1}} \phi'(\zeta_{i-1}) - rH(r-1, \sigma; 1) + \sigma H(r, \sigma; 1)$$

where

$$H_i(r-1,\sigma;1) = \int_{\zeta_{i-1}}^{\zeta_i} z^{r-1} e^{-\sigma z} \phi'(z) dz$$

and

$$H_i(r,\sigma;1) = \int_{\zeta_{i-1}}^{\zeta_i} z^{r} e^{-\sigma z} \phi'(z) dz$$

with

$$\begin{aligned} H_i(r-1,\sigma;1) &= \zeta_i^{r-1} e^{-\sigma\zeta_i}\phi(\zeta_i) - \zeta_{i-1}^{r-1} e^{-\sigma\zeta_{i-1}}\phi(\zeta_{i-1}) - (r-1)H_i(r-2,\sigma;0) + \sigma H_i(r-1,\sigma;0) \\ H_i(r,\sigma;1) &= \zeta_i^{r} e^{-\sigma\zeta_i}\phi(\zeta_i) - \zeta_{i-1}^{r} e^{-\sigma\zeta_{i-1}}\phi(\zeta_{i-1}) - rH_i(r-1,\sigma;0) + \sigma H_i(r,\sigma;0) \end{aligned}$$

and

$$H_i(r,\sigma;0) = \int_{\zeta_{i-1}}^{\zeta_i} z^{r} e^{-\sigma z} \phi(z) dz \ \text{, o.w. } 0 \quad \text{if } r < 0$$

$$H(0,0;0) = \int_{\zeta_{i-1}}^{\zeta_i} \phi(z) dz = \Phi(\zeta_i) - \Phi(\zeta_{i-1})$$

Note that

$$\begin{aligned} \sum_{i=1}^{k} H_i(r,\sigma;2)\prod_{j=1}^{i-1}(1-\pi_j^*) = \sum_{i=1}^{k} \Bigg[& \zeta_i^r e^{-\sigma\zeta_i}\phi(\zeta_i)\left(\frac{\phi'(\zeta_i)}{\phi(\zeta_i} - \frac{r}{\zeta_i} + \sigma\right)\pi_i^* + r(r-1)H_i(r-2,\sigma;0) \\ & - 2r\sigma H_i(r-1,\sigma;0) + \sigma^2 H_i(r,\sigma;0)\Bigg]\prod_{j=1}^{i-1}(1-\pi_j^*), \quad \text{with } \pi_i^* \equiv 1 \end{aligned}$$

and

$$\begin{aligned} E(u_{i,j} z_{i,j}^r | n_i) &= \frac{1}{\Phi(\zeta_i) - \Phi(\zeta_{i-1})} \int_{\zeta_{i-1}}^{\zeta_i} \left(-\frac{\phi''(z_{i,j})}{\phi(z_{i,j})} + \left[\frac{\phi'(z_{i,j})}{\phi(z_{i,j})}\right]^2 \right) z_{i,j}^r \ \phi(z_{i,j}) dz_{i,j} \\ &= \frac{1}{\Phi(\zeta_i) - \Phi(\zeta_{i-1})}\big[-H_i(r,0;2) + H_i(r,0)\big] \end{aligned}$$

Note that, we can define

$$\begin{aligned} w_i(y_{i,j}) = \frac{\left(e^{\sigma z_{i,j}} - \sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}\right)x_i + \sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}x_l}{e^{\sigma z_{i,j}}} &= \quad x_i - e^{-\sigma z_{i,j}}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l) \\ &= \quad x_i - \frac{\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l)}{\frac{y_{i,j}-\tau_{i-1}}{\theta_i} + \sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}}. \end{aligned}$$

Therefore, using above formula we can derive

$$w_i(\tau_i) = x_i - e^{-\sigma\zeta_i}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l) = x_i - \frac{\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l)}{\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}} = x_i - \frac{\sum_{l=1}^{i}\frac{\Delta_l}{\theta_l}(x_i - x_l)}{\sum_{l=1}^{i}\frac{\Delta_l}{\theta_l}}$$

$$w_i(\tau_i) = \frac{\sum_{l=1}^{i}\Delta_l x_l e^{-\beta x_l}}{\sum_{l=1}^{i}\Delta_l e^{-\beta x_l}}.$$

Similarly,

$$w_i^{[2]}(y_{i,j}) = \frac{\frac{y_{i,j}-\tau_i}{\theta_i} + \sum_{l=1}^{i}\frac{\Delta_l}{\theta_l}x_l^2}{\frac{y_{i,j}-\tau_i}{\theta_i} + \sum_{l=1}^{i}\frac{\Delta_l}{\theta_l}} = x_i^2 - e^{-\sigma z_{i,j}}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i^2 - x_l^2)$$

and

$$w_i^{[2]}(\tau_i) = x_i^2 - e^{-\sigma\zeta i}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i^2 - x_l^2).$$

Furthermore, we can show

$$w_i^{[2]}(y_{i,j}) = x_i^2 - e^{-\sigma z_{i,j}}e^{-\sigma\zeta i}(x_i^2 - w_i^{[2]}(\tau_i))$$

$$\begin{aligned} w_i^2(y_{i,j}) &= \left(x_i - e^{-\sigma z_{i,j}}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l)\right)^2 \\ &= x_i^2 - 2x_i e^{-\sigma z_{i,j}}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l) + e^{-2\sigma z_{i,j}}\left(\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l)\right)^2 \end{aligned}$$

$$w_i^{\beta}(y_{i,j}) = w_i^2(y_{i,j}) + w_i^{[2]}(y_{i,j}))$$

Here $w_i(y_{i,j}), w_i(\tau_i), w_i^{[2]}(y_{i,j})$ and $w_i^{[2]}(\tau_i)$ are functions of β only.

Let $\Theta = (\theta_1, \theta_2, \theta_3)' = (\alpha, \beta, \sigma)'$. Fisher information matrix is expressed as

$$I(\Theta) = -E\left[\frac{\partial^2 \log L(\Theta)}{\partial\theta_i\partial\theta_j}\right] = \begin{bmatrix} I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha\sigma} \\ I_{\alpha\beta} & I_{\beta\beta} & I_{\beta\sigma} \\ I_{\alpha\sigma} & I_{\beta\sigma} & I_{\sigma\sigma} \end{bmatrix}$$

Therefore, using above mentioned properties we can derive elements of Fisher information matrix as

$$\begin{aligned} I_{\sigma\sigma} = E\left[-\frac{\partial^2 l}{\partial\sigma^2}\right] &= \frac{1}{\sigma^2}\sum_{i=1}^{k}\left[E(n_i E(u_{i,j}z_{i,j}^2|n_i)) + \upsilon_i\zeta_i^2 E(c_i) + E(n_i)\right] \\ &= \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[-H_i(2,0;2) + H_i(2,0) + \upsilon_i\zeta_i^2(1-\Phi(\zeta_i))\pi_i^* + \Phi(\zeta_i) - \Phi(\zeta_{i-1})\right]\prod_{j=1}^{i-1}(1-\pi_j^*) \end{aligned}$$

After simplification,

$$I_{\sigma\sigma} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[\zeta_i\Big(2+\zeta_i h(\zeta_i)\Big)\phi(\zeta_i)\pi_i^* - \Phi(\zeta_i)\pi_i^* + H_i(2,0)\right]\prod_{j=1}^{i-1}(1-\pi_j^*) \qquad (3.12)$$

$$\begin{aligned}I_{\alpha\sigma} = E\left[-\frac{\partial^2 l}{\partial\alpha\partial\sigma}\right] &= \frac{1}{\sigma^2}\sum_{i=1}^{k}\left[E(n_i E(u_{i,j}z_{i,j}|n_i)) + \upsilon_i\zeta_i E(c_i))\right]\\ &= \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[-H_i(1,0;2) + H_i(1,0) + \upsilon_i\zeta_i(1-\Phi(\zeta_i))\pi_i^*\right]\prod_{j=1}^{i-1}(1-\pi_j^*)\end{aligned}$$

After simplification,

$$I_{\alpha\sigma} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[(1+\zeta_i h(\zeta_i))\phi(\zeta_i)\pi_i^* + H_i(1,0)\right]\prod_{j=1}^{i-1}(1-\pi_j^*) \qquad (3.13)$$

$$\begin{aligned}I_{\alpha\alpha} = E\left[-\frac{\partial^2 l}{\partial\alpha^2}\right] &= \frac{1}{\sigma^2}\sum_{i=1}^{k}\Big[E(E(u_{i,j}|n_i)n_i) + \upsilon_i E(c_i))\Big]\\ &= \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[-H_i(0,0;2) + H_i(0,0) + \upsilon_i(1-\Phi(\zeta_i))\pi_i^*\right]\prod_{j=1}^{i-1}(1-\pi_j^*)\end{aligned}$$

After simplification,

$$I_{\alpha\alpha} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\left[h(\zeta_i)\phi(\zeta_i)\pi_i^* + H_i(0,0)\right]\prod_{j=1}^{i-1}(1-\pi_j^*) \qquad (3.14)$$

$$I_{\alpha\beta} = E\left[-\frac{\partial^2 l}{\partial\alpha\partial\beta}\right] = \frac{1}{\sigma^2}\sum_{i=1}^{k}\left[E(n_i E(u_{i,j}w_i(y_{i,j})|n_i)) + \upsilon_i w_i(\tau_i)E(c_i))\right]$$

where

$$E(u_{i,j}w_i(y_{i,j})|n_i) = E(u_{i,j}|n_i)x_i - E(u_{i,j}e^{-\sigma z_{i,j}}|n_i)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l)$$

$$= \frac{1}{\Phi(\zeta_i)-\Phi(\zeta_{i-1})}\Big[\big(-H_i(0,0;2)+H_i(0,0)\big)x_i + \Big(H_i(0,\sigma;2) - H_i(0,\sigma)\Big)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i - x_l)\Big]$$

Now using previously introduced formulas it can be written as

$$\begin{aligned}I_{\alpha\beta} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[\big(-H_i(0,0;2)+H_i(0,0)\big)x_i + \Big(H_i(0,\sigma;2)-H_i(0,\sigma)\Big)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l)\\ &+ \Big(\upsilon_i x_i - \upsilon_i e^{-\sigma\zeta_i}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l)\Big)(1-\phi(\zeta_i))\pi^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*)\end{aligned}$$

Further simplification we can get final formula as

$$\begin{aligned} I_{\alpha\beta} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[x_i\big(H_i(0,0)+\sigma\phi(\zeta_i)-\sigma\phi(\zeta_{i-1})\big) \\ &+\big(x_i-w_i(\tau_i)\big)e^{\sigma\zeta_i}\big(\sigma^2 H_i(0,\sigma;0)-H_i(0,\sigma)\big) \\ &+w_i(\tau_i)\phi(\zeta_i)\big(h(\zeta_i)-\sigma\big)\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \end{aligned} \tag{3.15}$$

$$\begin{aligned} I_{\beta\sigma} = E\left[-\frac{\partial^2 l}{\partial\beta\partial\sigma}\right] =& \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[E(n_iE(u_{i,j}z_{i,j}w_i(y_{i,j})|n_i))-\sigma E[n_iE(w_i(y_{i,j})|n_i)] \\ &+\upsilon_i\zeta_i w_i(\tau_i)E(c_i)+\sigma x_i E(n_i)\Bigg] \end{aligned}$$

Then it can be written as

$$\begin{aligned} I_{\beta\sigma} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[x_i\big(-H_i(1,0;2)+H_i(1,0)\big)-\Big(-H_i(1,\sigma;2)+H_i(1,\sigma)\Big)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l) \\ &+\sigma\big(H_i(0,\sigma;0)\big)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l) \\ &+\zeta_i\Big(\phi'(\zeta_i)+h(\zeta_i)\phi(\zeta_i)\Big)\Big(x_i-e^{-\sigma\zeta_i}\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l)\Big)\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \end{aligned}$$

Using similar technique as previous formula, it is simplified to be

$$\begin{aligned} I_{\beta\sigma} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[x_i\big(H_i(1,0)+\sigma(\zeta_i\phi(\zeta_i)-\zeta_{i-1}\phi(\zeta_{i-1}))\big) \\ &+\big(x_i-w_i(\tau_i)\big)e^{\sigma\zeta_i}\big(\sigma^2 H_i(1,\sigma;0)-\sigma H_i(0,\sigma;0)-H_i(1,\sigma)\big) \\ &+w_i(\tau_i)\phi(\zeta_i)\big(1-\sigma\zeta_i+\zeta_i h(\zeta_i)\big)\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \end{aligned} \tag{3.16}$$

$$\begin{aligned} I_{\beta\beta} = E\left[-\frac{\partial^2 l}{\partial\beta^2}\right] =& \frac{1}{\sigma^2}\sum_{i=1}^{k}\Bigg[E(n_iE(u_{i,j}w_i^2(y_{i,j})|n_i))-\sigma E\Big(n_iE\Big(\frac{\phi'(z_{i,j})}{\phi(z_{i,j})}-\sigma\Big)w_i^\beta(y_{i,j})|n_i\Big) \\ &+\upsilon_i w_i^2(\tau_i)E(c_i)-\sigma h(\tau_i)w_i^\beta E(c_i)\Bigg] \end{aligned}$$

Now using previously introduced formulas it can be written as

$$
\begin{aligned}
I_{\beta\beta} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Bigg[x_i^2\big(-H_i(0,0;2)+H_i(0,0)+(\phi'(\zeta_i)+h(\zeta_i)\phi(\zeta_i))\pi_i^*\big) \\
&- 2x_i\Big(-H_i(0,\sigma;2)+H_i(0,\sigma)+\pi_i^*\big(\phi'(\zeta_i)e^{-\sigma\zeta_i}+h(\zeta_i)\phi(\zeta_i)e^{-\sigma\zeta_i}\big)\Big)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l) \\
&+ \Big(-H_i(0,\sigma;1)+\sigma H_i(0,\sigma;0)+\pi_i^* e^{-\sigma\zeta_i}\sigma\phi(\zeta_i)\Big)\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l)^2 \\
&+ \Big(-H_i(0,2\sigma;2)+H_i(0,2\sigma)+H_i(0,2\sigma;1)-\sigma H_i(0,2\sigma;0)+\big(\phi'(\zeta_i)+h(\zeta_i)\phi(\zeta_i)\big)\pi_i^* e^{-2\sigma\zeta_i} \\
&- \sigma\pi_i^*\phi(\zeta_i)e^{-2\sigma\zeta_i}\Big)\Big[\sum_{l=1}^{i-1}\frac{\Delta_l}{\theta_l}(x_i-x_l)\Big]^2\Bigg]\prod_{j=1}^{i-1}(1-\pi_j^*)
\end{aligned}
$$

After simplification,

$$
\begin{aligned}
I_{\beta\beta} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[-2\sigma\big(x_i-x_{i-1})w_{i-1}(\tau_{i-1}\phi(\zeta_{i-1})\big) \\
&+ x_i^2\big(H_i(0,0)+h(\zeta_i)\phi(\zeta_i)\pi_i^*\big) \\
&- 2x_i(x_i-w_i(\tau_i))\big[e^{\sigma\zeta_i}H_i(0,\sigma)-\sigma^2 e^{\sigma\zeta_i}H_i(0,\sigma,0)-(\sigma-h(\zeta_i))\phi(\zeta_i)\pi_i^*\big] \\
&+ (x_i-w_i(\tau_i))^2\big(e^{2\sigma\zeta_i}H_i(0,2\sigma)+\sigma(1-4\sigma)e^{2\sigma\zeta_i}H_i(0,2\sigma;0)-(2\sigma-h(\zeta_i))\phi(\zeta_i)\pi_i^*\big) \\
&+ (1-\sigma)w_i^{\beta}(\tau_i)\phi(\zeta_i)\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*)
\end{aligned}
\tag{3.17}
$$

where $w_i^{\beta}(\tau_i)=w_i^2(\tau_i)-w_i^{[2]}(\tau_i)$. For further simplicity, let

$$
G_i^*(r,\sigma)=e^{\sigma\zeta_i}H_i(r,\sigma;0)=\int_{\zeta_{i-1}}^{\zeta_i} z^r e^{\sigma(\zeta_i-z)}\phi(z)dz
$$

$$
G_i^{\#}(r,\sigma)=e^{\sigma\zeta_i}H_i(r,\sigma)=\int_{\zeta_{i-1}}^{\zeta_i} z^r e^{\sigma(\zeta_i-z)}\Big(\frac{\phi'(z)}{\phi(z)}\Big)^2\phi(z)dz
$$

with $h_i=h(\zeta_i)$, $\phi_i=\phi(\zeta_i)$, $r_i=x_i-w_i(\tau_i)$, $r_i^{\beta}=-w_i^{\beta}(\tau_i)$ and $\Phi_i=\Phi(\zeta_i)$. Therefore, final simplified elements of Fisher information are as follows:

$$
I_{\sigma\sigma}=\frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[G_i^{\#}(2,0)-\Phi_i\pi_i^*+\zeta_i\Big(2+\zeta_i h_i\Big)\phi_i\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \tag{3.18}
$$

$$
I_{\alpha\sigma}=\frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[G_i^{\#}(1,0)+\Big(1+\zeta_i h_i\Big)\phi_i\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \tag{3.19}
$$

$$
\begin{aligned}
I_{\beta\sigma} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[-\sigma(x_i-x_{i-1})\zeta_{i-1}\phi_{i-1}+x_i\big(G_i^{\#}(1,0)+(1+\zeta_i h_i)\phi_i\pi_i^*\big) \\
&- r_i\big(G_i^{\#}(1,\sigma)+\sigma G_i^{\#}(0,\sigma)-\sigma^2 G_i^*(1,\sigma)-(1-\sigma\zeta_i+\zeta_i h_i)\phi_i\pi_i^*\big)\Big]\prod_{j=1}^{i-1}(1-\pi_j^*)
\end{aligned}
\tag{3.20}
$$

$$I_{\alpha\alpha} = \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[G_i^{\#}(0,0) + h_i\phi_i\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \tag{3.21}$$

$$\begin{aligned} I_{\alpha\beta} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[-\sigma(x_i - x_{i-1})\phi_{i-1} + x_i\big(G_i^{\#}(0,0) + h_i\phi_i\pi_i^*\big) \\ & - r_i\big(G_i^{\#}(0,\sigma) - \sigma^2 G_i^*(0,\sigma) - (\sigma - h_i)\big)\phi_i\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \end{aligned} \tag{3.22}$$

$$\begin{aligned} I_{\beta\beta} =& \frac{n}{\sigma^2}\sum_{i=1}^{k}\Big[2\sigma(x_i - x_{i-1})(r_{i-1} - x_{i-1})\phi_{i-1} + x_i^2\big(G_i^{\#}(0,0) + h_i\phi_i\pi_i^*\big) \\ & - 2x_i r_i\big(G_i^{\#}(0,0) - \sigma^2 G_i^*(0,\sigma) - (\sigma - h_i)\phi_i\pi_i^*\big) \\ & + r_i^2\big(G_i^{\#}(0,2\sigma) + \sigma(1-4\sigma)G_i^*(0,2\sigma) - (2\sigma - h_i)\phi_i\pi_i^*\big) \\ & - (1-\sigma)r_i^{\beta}\phi_i\pi_i^*\Big]\prod_{j=1}^{i-1}(1-\pi_j^*) \end{aligned} \tag{3.23}$$

4. Simulation Study

Here we carry out a simulation study to understand the performance of MLEs of α, β and σ where the true parameter values are $\alpha = 2$, $\beta = -1$ and $\sigma = 0.8$. We consider progressively Type-I censored k-level step-stress ALT for three distributions; Weibull, log-normal and log-logistic. Our study encompasses $k = 2, 3, 4$ levels and for each level we consider $0\%, 10\%$, and 20% censoring proportions (π_i^*) under step duration Δ_i. Here k-level step duration Δ_i is uniform. i.e $\Delta_i = \frac{\tau_k}{k}$. Furthermore, initial numbers of test units n to be 20 and 100 and total testing time τ_k to be 20 and 40. Therefore, we have 4 tables from each distributions with different n and τ_k. To depict this, standardized equi-spaced stress levels between x_1 and x_k are considered for a set number of stress levels.i.e. $x_i = x_1 + \frac{x_k - x_i}{k-1}(i-1)$ for $i = 1, 2, ..., k$. Therefore, $(x_1, x_2) = (0.1, 1.0)$ for $k = 2$, $(x_1, x_2, x_3) = (0.1, 0.55, 1.0)$ for $k = 3$ and $(x_1, x_2, x_3, x_4) = (0.1, 0.4, 0.7, 1.0)$ for $k = 4$. The number of items eliminated at the end of x_i (censored items) are determined by $c_i = round((N_i - n_i)\pi_i^*)$, where N_i is number of units surviving on the test in the begging of level x_i and n_i is number of failures at stress level x_i.

Under above setup, we generated 1000 samples and summarized the results in Table 1-24. Table 1-12 consist of the mean, median, bias, mean squared error (MSE), empirical variance and covariance from the sampling distribution of MLE. It is observed that for simple stress level mean and median tend to have similar values, but when k increases mean and median are not comparable. This nature is true for all MLEs of all three distributions. The mean values of MLEs of Weibull and lognormal are always comparable with true values while giving near-overestimated or underestimated values, but in some cases (log-logistic distribution, Table 9, 4-levels, means of β) the mean of $\hat{\beta}$ overestimate the true value and skew to the right significantly. The variance of $\hat{\beta}$ always has the highest values and $\hat{\alpha}$ and $\hat{\sigma}$ follows respectively. Furthermore, when sample size is large $(n = 100)$ the magnitude of the empirical variances and covariances of MLEs tend to increase when the number of stress levels are increased or censoring proportions are increased. On the other hand this behavior is non-linear when we have a small sample size ($n = 20$). Moreover, when sample size is large, empirical variances and variances based on $I_{obs}(\alpha, \beta, \sigma)$ have comparable values, but when sample size is small variances and covariances based on $I_{obs}(\alpha, \beta, \sigma)$ are extremely skewed to the right, especially for the slop parameter β (it is much more sensitive). When τ_k increases with higher censoring proportions, this will worsen as it gets more difficult to estimate β and α. Increased variance will widen the confidence interval width considerably. Ex: Table Table 1, 4-levels β variances based on $I_{obs}(\alpha, \beta, \sigma)$ and Wald-type confidence intervals, Table 2, 2-levels α andβ variances and covariances based on $I_{obs}(\alpha, \beta, \sigma)$ and Wald-type confidence intervals. The $cov(\sigma, \alpha)$ values are always positive and other two covariance values are always negative. This behavior is true for all three distributions under all conditions. In addition, when sample size n increases the bias and MSE decreases. That means when sample size increases the precision of MLEs are improved.

Next, Table 13-24 consists of the results of Wald-type asymptotic 95% confidence intervals and BCa bootstrap 95% confidence intervals. Both methods compare % coverage, mean width, lower bound and upper bound. They are also based on two different sample sizes n and three different censoring proportions π_i^* as mentioned before. Here again we generates considered 1000 simulations and $B = 1000$ bootstrap replications. Here we observe the Wald-type asymptotic 95% confidence intervals and BCa bootstrap 95% confidence intervals give similar coverage. When sample size n is small, $\hat{\alpha}$ and $\hat{\beta}$ have better average coverage probability than $\hat{\sigma}$, but when sample size n is large, all three estimates have better average coverage probability. In addition, $\hat{\beta}$ always has the widest mean width and $\hat{\sigma}$ always has the narrowest mean width throughout all three distributions in both methods. Furthermore, as censoring is increasing both mean widths tend to be increased for all three estimators. In addition, as sample size is increasing the mean width is getting narrower.

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Table 1: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 20$ and $\tau = 20$ under Weibull distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7680	0.7530	-0.0320	0.0271	0.0261	$cov(\sigma, \alpha)$	0.0068
		$\hat{\alpha}$	2.0266	2.0284	0.0266	0.0544	0.0538	$cov(\alpha, \beta)$	-0.0842
		$\hat{\beta}$	-1.1848	-1.1428	-0.1848	0.4971	0.4634	$cov(\sigma, \beta)$	-0.0599
	10%	$\hat{\sigma}$	0.7700	0.7579	-0.0300	0.0267	0.0259	$cov(\sigma, \alpha)$	0.0048
		$\hat{\alpha}$	2.0264	2.0269	0.0264	0.0516	0.0510	$cov(\alpha, \beta)$	-0.0731
		$\hat{\beta}$	-1.2024	-1.1775	-0.2024	0.4809	0.4404	$cov(\sigma, \beta)$	-0.0553
	20%	$\hat{\sigma}$	0.7722	0.7573	-0.0278	0.0287	0.0279	$cov(\sigma, \alpha)$	0.0053
		$\hat{\alpha}$	2.0207	2.0185	0.0207	0.0527	0.0523	$cov(\alpha, \beta)$	-0.0833
		$\hat{\beta}$	-1.2079	-1.1622	-0.2079	0.5754	0.5327	$cov(\sigma, \beta)$	-0.0586
3	0%	$\hat{\sigma}$	0.7845	0.7732	-0.0155	0.0369	0.0367	$cov(\sigma, \alpha)$	0.0230
		$\hat{\alpha}$	2.0658	2.0333	0.0658	0.0940	0.0898	$cov(\alpha, \beta)$	-0.2163
		$\hat{\beta}$	-1.4228	-1.3434	-0.4228	1.1980	1.0202	$cov(\sigma, \beta)$	-0.1428
	10%	$\hat{\sigma}$	0.8008	0.7801	0.0008	0.0412	0.0412	$cov(\sigma, \alpha)$	0.0349
		$\hat{\alpha}$	2.0959	2.0589	0.0959	0.1151	0.1060	$cov(\alpha, \beta)$	-0.2754
		$\hat{\beta}$	-1.5827	-1.4864	-0.5827	1.5524	1.2140	$cov(\sigma, \beta)$	-0.1671
	20%	$\hat{\sigma}$	0.7975	0.7736	-0.0025	0.0409	0.0410	$cov(\sigma, \alpha)$	0.0297
		$\hat{\alpha}$	2.0835	2.0456	0.0835	0.1149	0.1080	$cov(\alpha, \beta)$	-0.2613
		$\hat{\beta}$	-1.5807	-1.4855	-0.5807	1.4491	1.1130	$cov(\sigma, \beta)$	-0.1501
4	0%	$\hat{\sigma}$	0.8211	0.7885	0.0211	0.0526	0.0522	$cov(\sigma, \alpha)$	0.0560
		$\hat{\alpha}$	2.1513	2.0962	0.1513	0.1855	0.1628	$cov(\alpha, \beta)$	-0.4535
		$\hat{\beta}$	-1.7835	-1.6135	-0.7835	2.4155	1.8034	$cov(\sigma, \beta)$	-0.2486
	10%	$\hat{\sigma}$	0.8230	0.7989	0.0230	0.0543	0.0538	$cov(\sigma, \alpha)$	0.0517
		$\hat{\alpha}$	2.1671	2.1196	0.1671	0.1709	0.1431	$cov(\alpha, \beta)$	-0.4043
		$\hat{\beta}$	-1.8773	-1.7674	-0.8773	2.4591	1.6910	$cov(\sigma, \beta)$	-0.2417
	20%	$\hat{\sigma}$	0.8332	0.8006	0.0332	0.0518	0.0507	$cov(\sigma, \alpha)$	0.0540
		$\hat{\alpha}$	2.1912	2.1596	0.1912	0.2055	0.1691	$cov(\alpha, \beta)$	-0.4535
		$\hat{\beta}$	-2.0963	-1.9846	-1.0963	2.9296	1.7294	$cov(\sigma, \beta)$	-0.2295

Table 2: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 20$ and $\tau = 40$ under Weibull distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7619	0.7548	-0.0381	0.0199	0.0185	$cov(\sigma,\alpha)$	-0.0015
		$\hat{\alpha}$	1.9935	1.9872	-0.0065	0.0440	0.0440	$cov(\alpha,\beta)$	-0.0587
		$\hat{\beta}$	-1.0132	-0.8892	-0.0132	0.3364	0.3366	$cov(\sigma,\beta)$	-0.0445
	10%	$\hat{\sigma}$	0.7702	0.7577	-0.0298	0.0220	0.0211	$cov(\sigma,\alpha)$	-0.0022
		$\hat{\alpha}$	1.9990	2.0041	-0.0010	0.0463	0.0464	$cov(\alpha,\beta)$	-0.0676
		$\hat{\beta}$	-1.0205	-0.8889	-0.0205	0.4250	0.4250	$cov(\sigma,\sigma)$	-0.0546
	20%	$\hat{\sigma}$	0.7679	0.7581	-0.0321	0.0213	0.0203	$cov(\sigma,\alpha)$	-0.0002
		$\hat{\alpha}$	1.9940	2.0009	-0.0060	0.0478	0.0478	$cov(\alpha,\beta)$	-0.0674
		$\hat{\beta}$	-1.0182	-0.8880	-0.0182	0.3943	0.3944	$cov(\sigma,\beta)$	-0.0552
3	0%	$\hat{\sigma}$	0.7845	0.7700	-0.0155	0.0236	0.0233	$cov(\sigma,\alpha)$	0.0063
		$\hat{\alpha}$	2.0072	2.0184	0.0072	0.0554	0.0554	$cov(\alpha,\beta)$	-0.1373
		$\hat{\beta}$	-1.1766	-1.1473	-0.1766	1.0185	0.9883	$cov(\sigma,\beta)$	-0.0944
	10%	$\hat{\sigma}$	0.7716	0.7619	-0.0284	0.0227	0.0219	$cov(\sigma,\alpha)$	0.0072
		$\hat{\alpha}$	2.0144	2.0165	0.0144	0.0535	0.0534	$cov(\alpha,\beta)$	-0.1326
		$\hat{\beta}$	-1.1515	-1.1246	-0.1515	0.9989	0.9770	$cov(\sigma,\beta)$	-0.0956
	20%	$\hat{\sigma}$	0.7751	0.7609	-0.0249	0.0238	0.0232	$cov(\sigma,\alpha)$	0.0037
		$\hat{\alpha}$	2.0120	2.0156	0.0120	0.0483	0.0482	$cov(\alpha,\beta)$	-0.1141
		$\hat{\beta}$	-1.1551	-1.0877	-0.1551	1.0023	0.9792	$cov(\sigma,\beta)$	-0.0973
4	0%	$\hat{\sigma}$	0.7901	0.7713	-0.0099	0.0281	0.0280	$cov(\sigma,\alpha)$	0.0149
		$\hat{\alpha}$	2.0194	2.0030	0.0194	0.0692	0.0689	$cov(\alpha,\beta)$	-0.2193
		$\hat{\beta}$	-1.1956	-1.2033	-0.1956	1.5847	1.5480	$cov(\sigma,\beta)$	-0.1501
	10%	$\hat{\sigma}$	0.7741	0.7541	-0.0259	0.0260	0.0253	$cov(\sigma,\alpha)$	0.0098
		$\hat{\alpha}$	2.0007	1.9893	0.0007	0.0669	0.0669	$cov(\alpha,\beta)$	-0.1965
		$\hat{\beta}$	-1.0743	-1.1058	-0.0743	1.4068	1.4027	$cov(\sigma,\beta)$	-0.1282
	20%	$\hat{\sigma}$	0.7885	0.7725	-0.0115	0.0270	0.0269	$cov(\sigma,\alpha)$	0.0124
		$\hat{\alpha}$	2.0244	2.0247	0.0244	0.0729	0.0724	$cov(\alpha,\beta)$	-0.2155
		$\hat{\beta}$	-1.2486	-1.2141	-0.2486	1.4940	1.4337	$cov(\sigma,\beta)$	-0.1375

Table 3: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 100$ and $\tau = 20$ under Weibull distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7916	0.7905	-0.0084	0.0055	0.0054	$cov(\sigma, \alpha)$	0.0005
		$\hat{\alpha}$	2.0030	2.0043	0.0030	0.0103	0.0103	$cov(\alpha, \beta)$	-0.0175
		$\hat{\beta}$	-1.0135	-1.0168	-0.0135	0.0994	0.0993	$cov(\sigma, \beta)$	-0.0124
	10%	$\hat{\sigma}$	0.7942	0.7951	-0.0058	0.0051	0.0051	$cov(\sigma, \alpha)$	0.0008
		$\hat{\alpha}$	2.0017	1.9969	0.0017	0.0114	0.0114	$cov(\alpha, \beta)$	-0.0198
		$\hat{\beta}$	-1.0095	-1.0031	-0.0095	0.1106	0.1106	$cov(\sigma, \beta)$	-0.0121
	20%	$\hat{\sigma}$	0.7921	0.7913	-0.0079	0.0052	0.0052	$cov(\sigma, \alpha)$	0.0007
		$\hat{\alpha}$	1.9990	1.9948	-0.0010	0.0103	0.0103	$cov(\alpha, \beta)$	-0.0192
		$\hat{\beta}$	-1.0139	-0.9906	-0.0139	0.1156	0.1155	$cov(\sigma, \beta)$	-0.0124
3	0%	$\hat{\sigma}$	0.8039	0.8003	0.0039	0.0070	0.0070	$cov(\sigma, \alpha)$	0.0039
		$\hat{\alpha}$	2.0211	2.0170	0.0211	0.0180	0.0176	$cov(\alpha, \beta)$	-0.0444
		$\hat{\beta}$	-1.1350	-1.1079	-0.1350	0.2220	0.2039	$cov(\sigma, \beta)$	-0.0258
	10%	$\hat{\sigma}$	0.8013	0.7959	0.0013	0.0071	0.0071	$cov(\sigma, \alpha)$	0.0044
		$\hat{\alpha}$	2.0272	2.0207	0.0272	0.0179	0.0172	$cov(\alpha, \beta)$	-0.0471
		$\hat{\beta}$	-1.1474	-1.1212	-0.1474	0.2432	0.2217	$cov(\sigma, \beta)$	-0.0276
	20%	$\hat{\sigma}$	0.8031	0.8015	0.0031	0.0073	0.0073	$cov(\sigma, \alpha)$	0.0040
		$\hat{\alpha}$	2.0288	2.0222	0.0288	0.0174	0.0166	$cov(\alpha, \beta)$	-0.0444
		$\hat{\beta}$	-1.1741	-1.1499	-0.1741	0.2455	0.2154	$cov(\sigma, \beta)$	-0.0266
4	0%	$\hat{\sigma}$	0.8014	0.7975	0.0014	0.0087	0.0087	$cov(\sigma, \alpha)$	0.0078
		$\hat{\alpha}$	2.0414	2.0269	0.0414	0.0266	0.0249	$cov(\alpha, \beta)$	-0.0725
		$\hat{\beta}$	-1.2239	-1.1583	-0.2239	0.3499	0.3001	$cov(\sigma, \beta)$	-0.0390
	10%	$\hat{\sigma}$	0.8094	0.8030	0.0094	0.0096	0.0095	$cov(\sigma, \alpha)$	0.0088
		$\hat{\alpha}$	2.0659	2.0609	0.0659	0.0310	0.0266	$cov(\alpha, \beta)$	-0.0786
		$\hat{\beta}$	-1.3471	-1.3111	-0.3471	0.4463	0.3261	$cov(\sigma, \beta)$	-0.0428
	20%	$\hat{\sigma}$	0.8237	0.8199	0.0237	0.0095	0.0089	$cov(\sigma, \alpha)$	0.0079
		$\hat{\alpha}$	2.0932	2.0854	0.0932	0.0352	0.0266	$cov(\alpha, \beta)$	-0.0752
		$\hat{\beta}$	-1.5251	-1.4762	-0.5251	0.5846	0.3091	$cov(\sigma, \beta)$	-0.0390

Table 4: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 100$ and $\tau = 40$ under Weibull distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7912	0.7895	-0.0088	0.0037	0.0036	$cov(\sigma, \alpha)$	-0.0005
		$\hat{\alpha}$	2.0100	2.0106	0.0100	0.0109	0.0108	$cov(\alpha, \beta)$	-0.0340
		$\hat{\beta}$	-1.1110	-1.1037	-0.1110	0.3353	0.3233	$cov(\sigma, \beta)$	-0.0099
	10%	$\hat{\sigma}$	0.7931	0.7908	-0.0070	0.0039	0.0039	$cov(\sigma, \alpha)$	-0.0007
		$\hat{\alpha}$	2.0085	2.0097	0.0085	0.0107	0.0106	$cov(\alpha, \beta)$	-0.0335
		$\hat{\beta}$	-1.0807	-1.0463	-0.0807	0.3415	0.3353	$cov(\sigma, \beta)$	-0.0107
	20%	$\hat{\sigma}$	0.7924	0.7932	-0.0076	0.0040	0.0039	$cov(\sigma, \alpha)$	-0.0001
		$\hat{\alpha}$	2.0140	2.0158	0.0140	0.0118	0.0117	$cov(\alpha, \beta)$	-0.0410
		$\hat{\beta}$	-1.1414	-1.0956	-0.1414	0.3927	0.3731	$cov(\sigma, \beta)$	-0.0138
3	0%	$\hat{\sigma}$	0.7978	0.7952	-0.0022	0.0045	0.0045	$cov(\sigma, \alpha)$	0.0011
		$\hat{\alpha}$	2.0031	2.0023	0.0031	0.0146	0.0146	$cov(\alpha, \beta)$	-0.0620
		$\hat{\beta}$	-1.0535	-1.0997	-0.0535	0.5172	0.5148	$cov(\sigma, \beta)$	-0.0213
	10%	$\hat{\sigma}$	0.7912	0.7907	-0.0088	0.0047	0.0047	$cov(\sigma, \alpha)$	0.0012
		$\hat{\alpha}$	2.0086	2.0114	0.0086	0.0156	0.0155	$cov(\alpha, \beta)$	-0.0647
		$\hat{\beta}$	-1.0764	-1.1689	-0.0764	0.5465	0.5412	$cov(\sigma, \beta)$	-0.0229
	20%	$\hat{\sigma}$	0.7967	0.7953	-0.0033	0.0048	0.0048	$cov(\sigma, \alpha)$	0.0014
		$\hat{\alpha}$	1.9949	1.9958	-0.0051	0.0148	0.0148	$cov(\alpha, \beta)$	-0.0690
		$\hat{\beta}$	-1.0218	-1.1044	-0.0218	0.6122	0.6124	$cov(\sigma, \beta)$	-0.0266
4	0%	$\hat{\sigma}$	0.7940	0.7942	-0.0060	0.0053	0.0053	$cov(\sigma, \alpha)$	0.0034
		$\hat{\alpha}$	2.0054	2.0097	0.0054	0.0186	0.0186	$cov(\alpha, \beta)$	-0.0873
		$\hat{\beta}$	-1.0568	-1.0958	-0.0568	0.6584	0.6558	$cov(\sigma, \beta)$	-0.0357
	10%	$\hat{\sigma}$	0.7926	0.7918	-0.0074	0.0051	0.0051	$cov(\sigma, \alpha)$	0.0024
		$\hat{\alpha}$	2.0110	2.0166	0.0110	0.0186	0.0184	$cov(\alpha, \beta)$	-0.0812
		$\hat{\beta}$	-1.0872	-1.1127	-0.0872	0.6287	0.6217	$cov(\sigma, \beta)$	-0.0302
	20%	$\hat{\sigma}$	0.7946	0.7920	-0.0054	0.0051	0.0050	$cov(\sigma, \alpha)$	0.0025
		$\hat{\alpha}$	2.0100	2.0121	0.0100	0.0178	0.0177	$cov(\alpha, \beta)$	-0.0796
		$\hat{\beta}$	-1.0765	-1.1222	-0.0765	0.6364	0.6311	$cov(\sigma, \beta)$	-0.0314

Table 5: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 20$ and $\tau = 20$ under lognormal distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7607	0.7469	-0.0393	0.0254	0.0238	$cov(\sigma,\alpha)$	0.0118
		$\hat{\alpha}$	2.0014	1.9956	0.0014	0.0536	0.0537	$cov(\alpha,\beta)$	-0.0956
		$\hat{\beta}$	-1.1168	-1.1214	-0.1168	0.4741	0.4609	$cov(\sigma,\beta)$	-0.0631
	10%	$\hat{\sigma}$	0.7599	0.7502	-0.0401	0.0240	0.0224	$cov(\sigma,\alpha)$	0.0091
		$\hat{\alpha}$	1.9825	1.9739	-0.0175	0.0452	0.0449	$cov(\alpha,\beta)$	-0.0770
		$\hat{\beta}$	-1.0999	-1.1102	-0.0999	0.5186	0.5091	$cov(\sigma,\beta)$	-0.0566
	20%	$\hat{\sigma}$	0.7479	0.7363	-0.0521	0.0264	0.0237	$cov(\sigma,\alpha)$	0.0090
		$\hat{\alpha}$	1.9689	1.9725	-0.0311	0.0422	0.0412	$cov(\alpha,\beta)$	-0.0801
		$\hat{\beta}$	-1.0730	-1.0475	-0.0730	0.5305	0.5257	$cov(\sigma,\beta)$	-0.0594
3	0%	$\hat{\sigma}$	0.7730	0.7612	-0.0270	0.0354	0.0347	$cov(\sigma,\alpha)$	0.0330
		$\hat{\alpha}$	2.0365	2.0034	0.0365	0.0898	0.0885	$cov(\alpha,\beta)$	-0.2449
		$\hat{\beta}$	-1.2387	-1.1430	-0.2387	1.1839	1.1280	$cov(\sigma,\beta)$	-0.1513
	10%	$\hat{\sigma}$	0.7699	0.7527	-0.0301	0.0349	0.0340	$cov(\sigma,\alpha)$	0.0277
		$\hat{\alpha}$	1.9972	1.9696	-0.0028	0.0701	0.0701	$cov(\alpha,\beta)$	-0.2090
		$\hat{\beta}$	-1.2217	-1.1406	-0.2217	1.1652	1.1172	$cov(\sigma,\beta)$	-0.1424
	20%	$\hat{\sigma}$	0.7430	0.7331	-0.0570	0.0379	0.0347	$cov(\sigma,\alpha)$	0.0253
		$\hat{\alpha}$	1.9328	1.9060	-0.0672	0.0654	0.0609	$cov(\alpha,\beta)$	-0.2041
		$\hat{\beta}$	-1.0396	-0.9822	-0.0396	1.2143	1.2139	$cov(\sigma,\beta)$	-0.1465
4	0%	$\hat{\sigma}$	0.7938	0.7751	-0.0062	0.0493	0.0493	$cov(\sigma,\alpha)$	0.0630
		$\hat{\alpha}$	2.0833	2.0207	0.0833	0.1603	0.1535	$cov(\alpha,\beta)$	-0.4875
		$\hat{\beta}$	-1.4167	-1.2583	-0.4167	2.2655	2.0939	$cov(\sigma,\beta)$	-0.2630
	10%	$\hat{\sigma}$	0.7847	0.7588	-0.0153	0.0473	0.0471	$cov(\sigma,\alpha)$	0.0529
		$\hat{\alpha}$	2.0108	1.9738	0.0108	0.1268	0.1268	$cov(\alpha,\beta)$	-0.4137
		$\hat{\beta}$	-1.2924	-1.1457	-0.2924	1.9531	1.8695	$cov(\sigma,\beta)$	-0.2334
	20%	$\hat{\sigma}$	0.7660	0.7418	-0.0340	0.0497	0.0486	$cov(\sigma,\alpha)$	0.0533
		$\hat{\alpha}$	1.9357	1.8964	-0.0643	0.1282	0.1242	$cov(\alpha,\beta)$	-0.4508
		$\hat{\beta}$	-1.1357	-0.9846	-0.1357	2.2834	2.2672	$cov(\sigma,\beta)$	-0.2599

Table 6: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 20$ and $\tau = 40$ under lognormal distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7746	0.7674	-0.0254	0.0189	0.0183	$cov(\sigma,\alpha)$	0.0049
		$\hat{\alpha}$	2.0278	2.0174	0.0278	0.0483	0.0476	$cov(\alpha,\beta)$	-0.1219
		$\hat{\beta}$	-1.2361	-1.0362	-0.2361	1.1257	1.0710	$cov(\sigma,\beta)$	-0.0366
	10%	$\hat{\sigma}$	0.7642	0.7572	-0.0358	0.0188	0.0175	$cov(\sigma,\alpha)$	0.0038
		$\hat{\alpha}$	2.0235	2.0208	0.0235	0.0444	0.0439	$cov(\alpha,\beta)$	-0.1270
		$\hat{\beta}$	-1.2571	-1.0192	-0.2571	1.2195	1.1546	$cov(\sigma,\beta)$	-0.0312
	20%	$\hat{\sigma}$	0.7680	0.7643	-0.0320	0.0185	0.0175	$cov(\sigma,\alpha)$	0.0036
		$\hat{\alpha}$	2.0247	2.0109	0.0247	0.0447	0.0441	$cov(\alpha,\beta)$	-0.1336
		$\hat{\beta}$	-1.2635	-1.0508	-0.2635	1.2814	1.2132	$cov(\sigma,\beta)$	-0.0390
3	0%	$\hat{\sigma}$	0.7797	0.7697	-0.0203	0.0224	0.0220	$cov(\sigma,\alpha)$	0.0121
		$\hat{\alpha}$	2.0325	2.0234	0.0325	0.0589	0.0579	$cov(\alpha,\beta)$	-0.2332
		$\hat{\beta}$	-1.3026	-1.1793	-0.3026	2.1833	2.0938	$cov(\sigma,\beta)$	-0.0862
	10%	$\hat{\sigma}$	0.7823	0.7754	-0.0177	0.0212	0.0209	$cov(\sigma,\alpha)$	0.0123
		$\hat{\alpha}$	2.0287	2.0312	0.0287	0.0647	0.0640	$cov(\alpha,\beta)$	-0.2750
		$\hat{\beta}$	-1.3739	-1.2234	-0.3739	2.5158	2.3784	$cov(\sigma,\beta)$	-0.0825
	20%	$\hat{\sigma}$	0.7672	0.7522	-0.0328	0.0203	0.0193	$cov(\sigma,\alpha)$	0.0090
		$\hat{\alpha}$	2.0232	2.0182	0.0232	0.0671	0.0667	$cov(\alpha,\beta)$	-0.3009
		$\hat{\beta}$	-1.4359	-1.2851	-0.4359	2.9545	2.7673	$cov(\sigma,\beta)$	-0.0622
4	0%	$\hat{\sigma}$	0.7893	0.7794	-0.0107	0.0254	0.0253	$cov(\sigma,\alpha)$	0.0219
		$\hat{\alpha}$	2.0729	2.0607	0.0729	0.0855	0.0803	$cov(\alpha,\beta)$	-0.3593
		$\hat{\beta}$	-1.5049	-1.3814	-0.5049	2.9675	2.7152	$cov(\sigma,\beta)$	-0.1486
	10%	$\hat{\sigma}$	0.7861	0.7696	-0.0139	0.0259	0.0257	$cov(\sigma,\alpha)$	0.0220
		$\hat{\alpha}$	2.0539	2.0347	0.0539	0.0836	0.0808	$cov(\alpha,\beta)$	-0.3575
		$\hat{\beta}$	-1.4232	-1.2620	-0.4232	2.8861	2.7098	$cov(\sigma,\beta)$	-0.1456
	20%	$\hat{\sigma}$	0.7772	0.7650	-0.0228	0.0262	0.0257	$cov(\sigma,\alpha)$	0.0218
		$\hat{\alpha}$	2.0357	2.0316	0.0357	0.0871	0.0859	$cov(\alpha,\beta)$	-0.4341
		$\hat{\beta}$	-1.4413	-1.3326	-0.4413	3.8697	3.6787	$cov(\sigma,\beta)$	-0.1568

Table 7: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 100$ and $\tau = 20$ under lognormal distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7811	0.7788	-0.0189	0.0050	0.0047	$cov(\sigma, \alpha)$	0.0022
		$\hat{\alpha}$	1.9911	1.9898	-0.0089	0.0092	0.0091	$cov(\alpha, \beta)$	-0.0156
		$\hat{\beta}$	-1.1045	-1.0984	-0.1045	0.0915	0.0807	$cov(\sigma, \beta)$	-0.0131
	10%	$\hat{\sigma}$	0.7791	0.7769	-0.0209	0.0052	0.0048	$cov(\sigma, \alpha)$	0.0024
		$\hat{\alpha}$	1.9863	1.9858	-0.0137	0.0101	0.0099	$cov(\alpha, \beta)$	-0.0181
		$\hat{\beta}$	-1.0968	-1.0833	-0.0968	0.0978	0.0886	$cov(\sigma, \beta)$	-0.0135
	20%	$\hat{\sigma}$	0.7690	0.7659	-0.0310	0.0057	0.0047	$cov(\sigma, \alpha)$	0.0023
		$\hat{\alpha}$	1.9683	1.9721	-0.0317	0.0111	0.0101	$cov(\alpha, \beta)$	-0.0176
		$\hat{\beta}$	-1.0633	-1.0589	-0.0633	0.0961	0.0922	$cov(\sigma, \beta)$	-0.0123
3	0%	$\hat{\sigma}$	0.7938	0.7889	-0.0062	0.0074	0.0073	$cov(\sigma, \alpha)$	0.0063
		$\hat{\alpha}$	2.0250	2.0225	0.0250	0.0157	0.0151	$cov(\alpha, \beta)$	-0.0437
		$\hat{\beta}$	-1.2634	-1.2564	-0.2634	0.2822	0.2130	$cov(\sigma, \beta)$	-0.0309
	10%	$\hat{\sigma}$	0.7766	0.7735	-0.0234	0.0068	0.0062	$cov(\sigma, \alpha)$	0.0045
		$\hat{\alpha}$	1.9711	1.9655	-0.0289	0.0128	0.0120	$cov(\alpha, \beta)$	-0.0340
		$\hat{\beta}$	-1.1660	-1.1658	-0.1660	0.2070	0.1797	$cov(\sigma, \beta)$	-0.0246
	20%	$\hat{\sigma}$	0.7655	0.7594	-0.0345	0.0073	0.0061	$cov(\sigma, \alpha)$	0.0035
		$\hat{\alpha}$	1.9240	1.9200	-0.0760	0.0141	0.0083	$cov(\alpha, \beta)$	-0.0303
		$\hat{\beta}$	-1.0595	-1.0508	-0.0595	0.2004	0.1971	$cov(\sigma, \beta)$	-0.0249
4	0%	$\hat{\sigma}$	0.8027	0.8018	0.0027	0.0100	0.0100	$cov(\sigma, \alpha)$	0.0106
		$\hat{\alpha}$	2.0438	2.0347	0.0438	0.0261	0.0242	$cov(\alpha, \beta)$	-0.0778
		$\hat{\beta}$	-1.3434	-1.3184	-0.3434	0.4633	0.3457	$cov(\sigma, \beta)$	-0.0492
	10%	$\hat{\sigma}$	0.7846	0.7767	-0.0154	0.0096	0.0093	$cov(\sigma, \alpha)$	0.0111
		$\hat{\alpha}$	1.9769	1.9687	-0.0231	0.0264	0.0259	$cov(\alpha, \beta)$	-0.0844
		$\hat{\beta}$	-1.2156	-1.1950	-0.2156	0.4139	0.3678	$cov(\sigma, \beta)$	-0.0489
	20%	$\hat{\sigma}$	0.7669	0.7669	-0.0331	0.0100	0.0089	$cov(\sigma, \alpha)$	0.0090
		$\hat{\alpha}$	1.8885	1.8848	-0.1115	0.0348	0.0224	$cov(\alpha, \beta)$	-0.0769
		$\hat{\beta}$	-1.0643	-1.0607	-0.0643	0.3897	0.3860	$cov(\sigma, \beta)$	-0.0467

Table 8: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 100$ and $\tau = 40$ under lognormal distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical Var	Empirical Cov	
2	0%	$\hat{\sigma}$	0.7920	0.7917	-0.0080	0.0035	0.0035	$cov(\sigma, \alpha)$	0.0010
		$\hat{\alpha}$	2.0096	2.0111	0.0096	0.0095	0.0094	$cov(\alpha, \beta)$	-0.0254
		$\hat{\beta}$	-1.1166	-1.1053	-0.1166	0.2462	0.2328	$cov(\sigma, \beta)$	-0.0092
	10%	$\hat{\sigma}$	0.7856	0.7841	-0.0144	0.0037	0.0035	$cov(\sigma, \alpha)$	0.0013
		$\hat{\alpha}$	2.0013	2.0033	0.0013	0.0096	0.0096	$cov(\alpha, \beta)$	-0.0267
		$\hat{\beta}$	-1.0971	-1.0902	-0.0971	0.2477	0.2385	$cov(\sigma, \beta)$	-0.0093
	20%	$\hat{\sigma}$	0.7802	0.7787	-0.0198	0.0036	0.0032	$cov(\sigma, \alpha)$	0.0006
		$\hat{\alpha}$	1.9890	1.9902	-0.0110	0.0099	0.0097	$cov(\alpha, \beta)$	-0.0311
		$\hat{\beta}$	-1.0823	-1.0766	-0.0823	0.2976	0.2911	$cov(\sigma, \beta)$	-0.0067
3	0%	$\hat{\sigma}$	0.7921	0.7889	-0.0079	0.0041	0.0040	$cov(\sigma, \alpha)$	0.0025
		$\hat{\alpha}$	2.0151	2.0095	0.0151	0.0123	0.0120	$cov(\alpha, \beta)$	-0.0450
		$\hat{\beta}$	-1.1258	-1.0762	-0.1258	0.3664	0.3509	$cov(\sigma, \beta)$	-0.0184
	10%	$\hat{\sigma}$	0.7872	0.7856	-0.0128	0.0043	0.0041	$cov(\sigma, \alpha)$	0.0026
		$\hat{\alpha}$	1.9976	1.9932	-0.0024	0.0133	0.0134	$cov(\alpha, \beta)$	-0.0497
		$\hat{\beta}$	-1.1009	-1.0864	-0.1009	0.3881	0.3783	$cov(\sigma, \beta)$	-0.0202
	0%	$\hat{\sigma}$	0.7789	0.7796	-0.0211	0.0047	0.0042	$cov(\sigma, \alpha)$	0.0020
		$\hat{\alpha}$	1.9883	1.9840	-0.0117	0.0132	0.0131	$cov(\alpha, \beta)$	-0.0512
		$\hat{\beta}$	-1.0915	-1.0883	-0.0915	0.4234	0.4154	$cov(\sigma, \beta)$	-0.0195
4	0%	$\hat{\sigma}$	0.7983	0.7934	-0.0017	0.0055	0.0055	$cov(\sigma, \alpha)$	0.0044
		$\hat{\alpha}$	2.0251	2.0227	0.0251	0.0168	0.0162	$cov(\alpha, \beta)$	-0.0679
		$\hat{\beta}$	-1.2065	-1.1823	-0.2065	0.5353	0.4932	$cov(\sigma, \beta)$	-0.0328
	10%	$\hat{\sigma}$	0.7928	0.7896	-0.0072	0.0051	0.0050	$cov(\sigma, \alpha)$	0.0041
		$\hat{\alpha}$	2.0105	2.0107	0.0105	0.0163	0.0162	$cov(\alpha, \beta)$	-0.0700
		$\hat{\beta}$	-1.1887	-1.1561	-0.1887	0.5382	0.5031	$cov(\sigma, \beta)$	-0.0310
	20%	$\hat{\sigma}$	0.7821	0.7792	-0.0179	0.0053	0.0049	$cov(\sigma, \alpha)$	0.0043
		$\hat{\alpha}$	1.9939	1.9895	-0.0061	0.0177	0.0176	$cov(\alpha, \beta)$	-0.0800
		$\hat{\beta}$	-1.1471	-1.1282	-0.1471	0.6125	0.5914	$cov(\sigma, \beta)$	-0.0312

Table 9: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 20$ and $\tau = 20$ under log-logistic distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7980	0.7790	-0.0020	0.0367	0.0367	$cov(\sigma, \alpha)$	0.0197
		$\hat{\alpha}$	2.0106	1.9930	0.0106	0.1271	0.1271	$cov(\alpha, \beta)$	-0.1232
		$\hat{\beta}$	-1.1357	-1.0635	-0.1357	0.4935	0.4756	$cov(\sigma, \beta)$	-0.0597
	10%	$\hat{\sigma}$	0.7906	0.7704	-0.0094	0.0359	0.0358	$cov(\sigma, \alpha)$	0.0219
		$\hat{\alpha}$	2.0221	1.9778	0.0221	0.1384	0.1381	$cov(\alpha, \beta)$	-0.1306
		$\hat{\beta}$	-1.1270	-1.0525	-0.1270	0.5457	0.5301	$cov(\sigma, \beta)$	-0.0620
	20%	$\hat{\sigma}$	0.7902	0.7794	-0.0098	0.0330	0.0329	$cov(\sigma, \alpha)$	0.0173
		$\hat{\alpha}$	2.0114	1.9914	0.0114	0.1290	0.1290	$cov(\alpha, \beta)$	-0.1207
		$\hat{\beta}$	-1.1332	-1.0884	-0.1332	0.5830	0.5658	$cov(\sigma, \beta)$	-0.0582
3	0%	$\hat{\sigma}$	0.8617	0.8404	0.0617	0.0494	0.0456	$cov(\sigma, \alpha)$	0.0483
		$\hat{\alpha}$	2.0858	2.0359	0.0858	0.2288	0.2216	$cov(\alpha, \beta)$	-0.3016
		$\hat{\beta}$	-1.7037	-1.5231	-0.7037	1.3947	0.9004	$cov(\sigma, \beta)$	-0.0997
	10%	$\hat{\sigma}$	0.8638	0.8409	0.0638	0.0525	0.0484	$cov(\sigma, \alpha)$	0.0504
		$\hat{\alpha}$	2.0909	2.0407	0.0909	0.2155	0.2074	$cov(\alpha, \beta)$	-0.2875
		$\hat{\beta}$	-1.6727	-1.5444	-0.6727	1.3365	0.8849	$cov(\sigma, \beta)$	-0.0979
	20%	$\hat{\sigma}$	0.8575	0.8303	0.0575	0.0490	0.0457	$cov(\sigma, \alpha)$	0.0529
		$\hat{\alpha}$	2.1224	2.0615	0.1224	0.2339	0.2191	$cov(\alpha, \beta)$	-0.3395
		$\hat{\beta}$	-1.7361	-1.5707	-0.7361	1.7204	1.1796	$cov(\sigma, \beta)$	-0.1141
4	0%	$\hat{\sigma}$	0.9383	0.9194	0.1383	0.0764	0.0573	$cov(\sigma, \alpha)$	0.0704
		$\hat{\alpha}$	2.1938	2.1141	0.1938	0.3216	0.2843	$cov(\alpha, \beta)$	-0.4600
		$\hat{\beta}$	-2.2760	-2.1003	-1.2760	2.9653	1.3384	$cov(\sigma, \beta)$	-0.1522
	10%	$\hat{\sigma}$	0.9203	0.8863	0.1203	0.0732	0.0588	$cov(\sigma, \alpha)$	0.0751
		$\hat{\alpha}$	2.1801	2.1226	0.1801	0.3156	0.2834	$cov(\alpha, \beta)$	-0.4791
		$\hat{\beta}$	-2.2347	-2.0547	-1.2347	2.9718	1.4487	$cov(\sigma, \beta)$	-0.1605
	20%	$\hat{\sigma}$	0.9240	0.8878	0.1240	0.0809	0.0656	$cov(\sigma, \alpha)$	0.0923
		$\hat{\alpha}$	2.2250	2.1463	0.2250	0.3699	0.3196	$cov(\alpha, \beta)$	-0.5674
		$\hat{\beta}$	-2.3439	-2.1061	-1.3439	3.5967	1.7925	$cov(\sigma, \beta)$	-0.1918

Table 10: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 20$ and $\tau = 40$ under log-logistic distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7855	0.7726	-0.0145	0.0298	0.0296	$cov(\sigma, \alpha)$	0.0077
		$\hat{\alpha}$	2.0371	2.0341	0.0371	0.1131	0.1118	$cov(\alpha, \beta)$	-0.1029
		$\hat{\beta}$	-1.1370	-1.0401	-0.1370	0.9440	0.9261	$cov(\sigma, \beta)$	-0.0404
	10%	$\hat{\sigma}$	0.7769	0.7627	-0.0231	0.0266	0.0261	$cov(\sigma, \alpha)$	0.0081
		$\hat{\alpha}$	2.0190	2.0313	0.0190	0.1168	0.1166	$cov(\alpha, \beta)$	-0.0966
		$\hat{\beta}$	-1.1323	-1.0948	-0.1323	0.8712	0.8545	$cov(\sigma, \beta)$	-0.0373
	20%	$\hat{\sigma}$	0.7847	0.7726	-0.0153	0.0258	0.0256	$cov(\sigma, \alpha)$	0.0087
		$\hat{\alpha}$	2.0340	2.0197	0.0340	0.1155	0.1145	$cov(\alpha, \beta)$	-0.1305
		$\hat{\beta}$	-1.1873	-1.1052	-0.1873	1.1891	1.1552	$cov(\sigma, \beta)$	-0.0406
3	0%	$\hat{\sigma}$	0.8029	0.7910	0.0029	0.0274	0.0274	$cov(\sigma, \alpha)$	0.0120
		$\hat{\alpha}$	2.0374	2.0317	0.0374	0.1264	0.1251	$cov(\alpha, \beta)$	-0.2016
		$\hat{\beta}$	-1.5207	-1.3110	-0.5207	1.7676	1.4979	$cov(\sigma, \beta)$	-0.0568
	10%	$\hat{\sigma}$	0.8019	0.7851	0.0019	0.0311	0.0311	$cov(\sigma, \alpha)$	0.0186
		$\hat{\alpha}$	2.0556	2.0457	0.0556	0.1297	0.1267	$cov(\alpha, \beta)$	-0.2218
		$\hat{\beta}$	-1.5496	-1.3679	-0.5496	1.8637	1.5632	$cov(\sigma, \beta)$	-0.0778
	20%	$\hat{\sigma}$	0.7957	0.7785	-0.0043	0.0298	0.0298	$cov(\sigma, \alpha)$	0.0179
		$\hat{\alpha}$	2.0613	2.0358	0.0613	0.1414	0.1378	$cov(\alpha, \beta)$	-0.2492
		$\hat{\beta}$	-1.5537	-1.3394	-0.5537	2.1349	1.8302	$cov(\sigma, \beta)$	-0.0550
4	0%	$\hat{\sigma}$	0.8313	0.8098	0.0313	0.0364	0.0355	$cov(\sigma, \alpha)$	0.0267
		$\hat{\alpha}$	2.1081	2.0878	0.1081	0.1787	0.1672	$cov(\alpha, \beta)$	-0.3187
		$\hat{\beta}$	-1.9167	-1.7015	-0.9167	2.7594	1.9211	$cov(\sigma, \beta)$	-0.1057
	10%	$\hat{\sigma}$	0.8316	0.8043	0.0316	0.0413	0.0404	$cov(\sigma, \alpha)$	0.0287
		$\hat{\alpha}$	2.1140	2.0916	0.1140	0.1721	0.1593	$cov(\alpha, \beta)$	-0.3479
		$\hat{\beta}$	-1.8946	-1.5924	-0.8946	2.9819	2.1838	$cov(\sigma, \beta)$	-0.1241
	0%	$\hat{\sigma}$	0.8288	0.8046	0.0288	0.0382	0.0374	$cov(\sigma, \alpha)$	0.0273
		$\hat{\alpha}$	2.1056	2.0786	0.1056	0.1833	0.1724	$cov(\alpha, \beta)$	-0.3800
		$\hat{\beta}$	-1.9583	-1.6994	-0.9583	3.4619	2.5461	$cov(\sigma, \beta)$	-0.0912

Table 11: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 100$ and $\tau = 20$ under log-logistic distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7919	0.7890	-0.0081	0.0074	0.0073	$cov(\sigma, \alpha)$	0.0039
		$\hat{\alpha}$	2.0021	1.9999	0.0021	0.0255	0.0255	$cov(\alpha, \beta)$	-0.0270
		$\hat{\beta}$	-0.9845	-0.9744	0.0155	0.1403	0.1402	$cov(\sigma, \beta)$	-0.0185
	10%	$\hat{\sigma}$	0.7914	0.7835	-0.0086	0.0071	0.0070	$cov(\sigma, \alpha)$	0.0043
		$\hat{\alpha}$	2.0004	1.9954	0.0004	0.0268	0.0268	$cov(\alpha, \beta)$	-0.0292
		$\hat{\beta}$	-0.9705	-0.9622	0.0295	0.1390	0.1383	$cov(\sigma, \beta)$	-0.0175
	20%	$\hat{\sigma}$	0.7942	0.7908	-0.0058	0.0071	0.0070	$cov(\sigma, \alpha)$	0.0039
		$\hat{\alpha}$	2.0049	2.0054	0.0049	0.0243	0.0243	$cov(\alpha, \beta)$	-0.0294
		$\hat{\beta}$	-0.9908	-1.0019	0.0092	0.1468	0.1469	$cov(\sigma, \beta)$	-0.0166
3	0%	$\hat{\sigma}$	0.8320	0.8251	0.0320	0.0087	0.0077	$cov(\sigma, \alpha)$	0.0060
		$\hat{\alpha}$	2.0305	2.0285	0.0305	0.0309	0.0300	$cov(\alpha, \beta)$	-0.0414
		$\hat{\beta}$	-1.2602	-1.2064	-0.2602	0.2209	0.1533	$cov(\sigma, \beta)$	-0.0187
	10%	$\hat{\sigma}$	0.8236	0.8145	0.0236	0.0083	0.0078	$cov(\sigma, \alpha)$	0.0065
		$\hat{\alpha}$	2.0219	2.0183	0.0219	0.0341	0.0336	$cov(\alpha, \beta)$	-0.0509
		$\hat{\beta}$	-1.2247	-1.1890	-0.2247	0.2305	0.1802	$cov(\sigma, \beta)$	-0.0184
	20%	$\hat{\sigma}$	0.8172	0.8100	0.0172	0.0085	0.0082	$cov(\sigma, \alpha)$	0.0057
		$\hat{\alpha}$	2.0254	2.0181	0.0254	0.0327	0.0321	$cov(\alpha, \beta)$	-0.0475
		$\hat{\beta}$	-1.1861	-1.1454	-0.1861	0.2323	0.1979	$cov(\sigma, \beta)$	-0.0193
4	0%	$\hat{\sigma}$	0.8775	0.8714	0.0775	0.0148	0.0088	$cov(\sigma, \alpha)$	0.0094
		$\hat{\alpha}$	2.1097	2.0922	0.1097	0.0561	0.0441	$cov(\alpha, \beta)$	-0.0666
		$\hat{\beta}$	-1.6746	-1.6271	-0.6746	0.6245	0.1696	$cov(\sigma, \beta)$	-0.0190
	0%	$\hat{\sigma}$	0.8679	0.8604	0.0679	0.0137	0.0091	$cov(\sigma, \alpha)$	0.0113
		$\hat{\alpha}$	2.0960	2.0748	0.0960	0.0571	0.0480	$cov(\alpha, \beta)$	-0.0759
		$\hat{\beta}$	-1.6192	-1.5493	-0.6192	0.5868	0.2036	$cov(\sigma, \beta)$	-0.0222
	0%	$\hat{\sigma}$	0.8511	0.8441	0.0511	0.0115	0.0089	$cov(\sigma, \alpha)$	0.0108
		$\hat{\alpha}$	2.0826	2.0658	0.0826	0.0540	0.0472	$cov(\alpha, \beta)$	-0.0898
		$\hat{\beta}$	-1.6006	-1.5264	-0.6006	0.6498	0.2893	$cov(\sigma, \beta)$	-0.0271

Table 12: Performance measure of the MLE with their empirical variances and covariances based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, $n = 100$ and $\tau = 40$ under log-logistic distribution

k	π^*	MLE	Mean	Median	Bias	MSE	Empirical		
							Var	Cov	
2	0%	$\hat{\sigma}$	0.7894	0.7843	-0.0106	0.0053	0.0052	$cov(\sigma,\alpha)$	0.0018
		$\hat{\alpha}$	2.0045	2.0091	0.0045	0.0227	0.0227	$cov(\alpha,\beta)$	-0.0290
		$\hat{\beta}$	-0.9868	-0.9948	0.0132	0.2112	0.2112	$cov(\sigma,\beta)$	-0.0122
	10%	$\hat{\sigma}$	0.7926	0.7879	-0.0074	0.0053	0.0052	$cov(\sigma,\alpha)$	0.0021
		$\hat{\alpha}$	2.0009	2.0054	0.0009	0.0244	0.0244	$cov(\alpha,\beta)$	-0.0284
		$\hat{\beta}$	-0.9759	-0.9869	0.0241	0.2035	0.2032	$cov(\sigma,\beta)$	-0.0131
	20%	$\hat{\sigma}$	0.7903	0.7839	-0.0097	0.0051	0.0050	$cov(\sigma,\alpha)$	0.0018
		$\hat{\alpha}$	1.9975	2.0001	-0.0025	0.0227	0.0227	$cov(\alpha,\beta)$	-0.0304
		$\hat{\beta}$	-0.9674	-0.9795	0.0326	0.2466	0.2457	$cov(\sigma,\beta)$	-0.0129
3	0%	$\hat{\sigma}$	0.8012	0.7961	0.0012	0.0064	0.0064	$cov(\sigma,\alpha)$	0.0031
		$\hat{\alpha}$	2.0067	2.0121	0.0067	0.0238	0.0238	$cov(\alpha,\beta)$	-0.0462
		$\hat{\beta}$	-1.0823	-1.0442	-0.0823	0.3412	0.3348	$cov(\sigma,\beta)$	-0.0234
	10%	$\hat{\sigma}$	0.7969	0.7934	-0.0031	0.0067	0.0067	$cov(\sigma,\alpha)$	0.0041
		$\hat{\alpha}$	2.0077	2.0058	0.0077	0.0261	0.0261	$cov(\alpha,\beta)$	-0.0573
		$\hat{\beta}$	-1.0614	-1.0359	-0.0614	0.3998	0.3964	$cov(\sigma,\beta)$	-0.0264
	20%	$\hat{\sigma}$	0.7951	0.7924	-0.0049	0.0058	0.0058	$cov(\sigma,\alpha)$	0.0024
		$\hat{\alpha}$	1.9999	2.0021	-0.0001	0.0281	0.0281	$cov(\alpha,\beta)$	-0.0621
		$\hat{\beta}$	-1.0369	-0.9964	-0.0369	0.4431	0.4421	$cov(\sigma,\beta)$	-0.0184
4	0%	$\hat{\sigma}$	0.8069	0.8025	0.0069	0.0069	0.0069	$cov(\sigma,\alpha)$	0.0045
		$\hat{\alpha}$	2.0257	2.0183	0.0257	0.0296	0.0290	$cov(\alpha,\beta)$	-0.0604
		$\hat{\beta}$	-1.2046	-1.1348	-0.2046	0.4395	0.3981	$cov(\sigma,\beta)$	-0.0263
	10%	$\hat{\sigma}$	0.7991	0.7963	-0.0009	0.0065	0.0065	$cov(\sigma,\alpha)$	0.0043
		$\hat{\alpha}$	2.0126	2.0083	0.0126	0.0313	0.0312	$cov(\alpha,\beta)$	-0.0656
		$\hat{\beta}$	-1.1219	-1.0681	-0.1219	0.4431	0.4287	$cov(\sigma,\beta)$	-0.0236
	20%	$\hat{\sigma}$	0.8025	0.7953	0.0025	0.0064	0.0064	$cov(\sigma,\alpha)$	0.0038
		$\hat{\alpha}$	2.0114	2.0086	0.0114	0.0292	0.0291	$cov(\alpha,\beta)$	-0.0692
		$\hat{\beta}$	-1.1265	-1.0354	-0.1265	0.5280	0.5125	$cov(\sigma,\beta)$	-0.0237

Table 13: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 20$ and $\tau = 20$ under Weibull distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9120	0.7163	0.4099	1.1262	0.9040	0.6808	0.5488	1.2296
		$\hat{\alpha}$	0.9620	0.9008	1.5762	2.4770	0.9100	0.9918	1.5924	2.5843
		$\hat{\beta}$	0.9580	3.0585	-2.7140	0.3444	0.9430	2.5934	-2.4509	0.1425
	10%	$\hat{\sigma}$	0.9090	0.7030	0.4185	1.1214	0.9220	0.6750	0.5413	1.2163
		$\hat{\alpha}$	0.9630	0.9048	1.5740	2.4788	0.9210	0.9658	1.6012	2.5670
		$\hat{\beta}$	0.9670	3.1331	-2.7689	0.3642	0.9440	2.5844	-2.4331	0.1514
	20%	$\hat{\sigma}$	0.9120	0.7584	0.3930	1.1515	0.9200	0.6785	0.5475	1.2260
		$\hat{\alpha}$	0.9680	0.9431	1.5492	2.4923	0.9250	0.9333	1.6393	2.5725
		$\hat{\beta}$	0.9500	3.4832	-2.9495	0.5337	0.9420	2.6297	-2.5346	0.0950
3	0%	$\hat{\sigma}$	0.9070	0.9189	0.3250	1.2439	0.9120	0.8291	0.5048	1.3340
		$\hat{\alpha}$	0.9640	1.3079	1.4118	2.7197	0.9410	1.3997	1.5088	2.9084
		$\hat{\beta}$	0.9920	5.2212	-4.0334	1.1878	0.9510	4.6522	-3.6805	0.9717
	10%	$\hat{\sigma}$	0.9210	0.9490	0.3263	1.2754	0.9190	0.8452	0.5149	1.3600
		$\hat{\alpha}$	0.9720	1.3718	1.4100	2.7818	0.9430	1.4445	1.5356	2.9802
		$\hat{\beta}$	0.9860	5.5430	-4.3542	1.1888	0.9370	4.7461	-3.8620	0.8842
	20%	$\hat{\sigma}$	0.9380	1.0092	0.2929	1.3021	0.9160	0.8280	0.5135	1.3415
		$\hat{\alpha}$	0.9790	1.4398	1.3636	2.8034	0.9420	1.3819	1.5706	2.9525
		$\hat{\beta}$	0.9920	6.1504	-4.6559	1.4945	0.9310	4.6339	-3.8136	0.8204
4	0%	$\hat{\sigma}$	0.9270	1.1480	0.2471	1.3951	0.9020	0.9546	0.4949	1.4495
		$\hat{\alpha}$	0.9700	1.8344	1.2341	3.0685	0.9360	1.7720	1.4434	3.2154
		$\hat{\beta}$	1.0000	7.3191	-5.4431	1.8760	0.9400	5.9395	-4.6244	1.3151
	10%	$\hat{\sigma}$	0.9290	1.2488	0.1986	1.4474	0.9130	0.9723	0.4893	1.4616
		$\hat{\alpha}$	0.9860	2.0064	1.1639	3.1703	0.9320	1.7609	1.4503	3.2111
		$\hat{\beta}$	1.0000	8.3126	-6.0336	2.2790	0.9240	6.0716	-4.6810	1.3906
	20%	$\hat{\sigma}$	0.9700	1.3989	0.1337	1.5327	0.9190	1.0102	0.4822	1.4923
		$\hat{\alpha}$	0.9920	2.2949	1.0438	3.3387	0.9210	1.8203	1.4673	3.2876
		$\hat{\beta}$	1.0000	9.8819	-7.0373	2.8446	0.9280	6.4317	-4.9655	1.4662

Table 14: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 20$ and $\tau = 40$ under Weibull distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.8970	0.5385	0.4926	1.0311	0.8820	0.5691	0.5804	1.1496
		$\hat{\alpha}$	0.9820	1.7505	1.1182	2.8687	0.8990	0.8207	1.6092	2.4299
		$\hat{\beta}$	0.9920	14.4752	-8.2508	6.2244	0.9730	1.6940	-1.8045	-0.1105
	10%	$\hat{\sigma}$	0.8970	0.5569	0.4917	1.0486	0.8850	0.5678	0.5806	1.1484
		$\hat{\alpha}$	0.9810	6.5743	-1.2881	5.2862	0.9260	0.8165	1.6068	2.4233
		$\hat{\beta}$	0.9910	62.7051	-32.3731	30.3321	0.9760	1.6630	-1.7394	-0.0763
	20%	$\hat{\sigma}$	0.9010	0.5449	0.4954	1.0403	0.8720	0.5592	0.5702	1.1294
		$\hat{\alpha}$	0.9810	2.0752	0.9565	3.0316	0.9080	0.8085	1.6136	2.4221
		$\hat{\beta}$	0.9940	17.8336	-9.9350	7.8986	0.9770	1.5877	-1.6614	-0.0737
3	0%	$\hat{\sigma}$	0.9200	0.6078	0.4806	1.0884	0.9080	0.6312	0.5624	1.1936
		$\hat{\alpha}$	0.9890	1.1830	1.4157	2.5987	0.8830	0.9676	1.5872	2.5548
		$\hat{\beta}$	1.0000	7.8925	-5.1228	2.7697	0.9610	3.9175	-3.2420	0.6756
	10%	$\hat{\sigma}$	0.9140	0.6089	0.4672	1.0761	0.8970	0.6501	0.5575	1.2076
		$\hat{\alpha}$	0.9920	1.2048	1.4120	2.6168	0.8910	0.9688	1.5887	2.5575
		$\hat{\beta}$	1.0000	8.3188	-5.3109	3.0080	0.9489	3.8600	-3.2777	0.5823
	20%	$\hat{\sigma}$	0.9170	0.6238	0.4632	1.0870	0.8830	0.6439	0.5647	1.2086
		$\hat{\alpha}$	0.9960	1.2490	1.3875	2.6365	0.8260	0.9356	1.5942	2.5298
		$\hat{\beta}$	1.0000	8.8948	-5.6025	3.2923	0.9560	3.9278	-3.2356	0.6922
4	0%	$\hat{\sigma}$	0.9330	0.6738	0.4532	1.1270	0.8860	0.7064	0.5508	1.2572
		$\hat{\alpha}$	1.0000	1.2664	1.3862	2.6526	0.8920	1.1443	1.5382	2.6825
		$\hat{\beta}$	1.0000	7.9846	-5.1879	2.7968	0.9310	5.3764	-3.9621	1.4143
	10%	$\hat{\sigma}$	0.9320	0.6912	0.4285	1.1198	0.8530	0.6877	0.5506	1.2383
		$\hat{\alpha}$	0.9990	1.2991	1.3511	2.6502	0.8830	1.1651	1.5200	2.6851
		$\hat{\beta}$	1.0000	8.6474	-5.3981	3.2494	0.9310	5.5061	-3.8472	1.6589
	0%	$\hat{\sigma}$	0.9410	0.7371	0.4199	1.1570	0.8750	0.6808	0.5615	1.2423
		$\hat{\alpha}$	1.0000	1.3876	1.3306	2.7181	0.8870	1.0772	1.5673	2.6445
		$\hat{\beta}$	1.0000	9.4583	-5.9777	3.4806	0.9290	5.0913	-3.8390	1.2523

Table 15: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 100$ and $\tau = 20$ under Weibull distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9460	0.3142	0.6345	0.9487	0.9480	0.2971	0.6706	0.9677
		$\hat{\alpha}$	0.9430	0.3983	1.8039	2.2022	0.9410	0.4124	1.8009	2.2132
		$\hat{\beta}$	0.9650	1.2628	-1.6449	-0.3821	0.9760	1.2479	-1.6348	-0.3869
	10%	$\hat{\sigma}$	0.9540	0.3188	0.6348	0.9536	0.9470	0.2996	0.6717	0.9713
		$\hat{\alpha}$	0.9380	0.4037	1.7998	2.2036	0.9380	0.4005	1.8086	2.2091
		$\hat{\beta}$	0.9620	1.3223	-1.6706	-0.3483	0.9660	1.2815	-1.6489	-0.3674
	20%	$\hat{\sigma}$	0.9540	0.3113	0.6364	0.9477	0.9390	0.2959	0.6731	0.9690
		$\hat{\alpha}$	0.9550	0.4012	1.7983	2.1996	0.9220	0.3865	1.8104	2.1969
		$\hat{\beta}$	0.9610	1.3417	-1.6848	-0.3430	0.9490	1.3140	-1.6648	-0.3508
3	0%	$\hat{\sigma}$	0.9750	0.4484	0.5798	1.0281	0.9460	0.3495	0.6526	1.0021
		$\hat{\alpha}$	0.9670	0.5919	1.7251	2.3171	0.9440	0.5445	1.7573	2.3018
		$\hat{\beta}$	0.9710	2.4432	-2.3566	0.0866	0.9700	1.9653	-2.0554	-0.0901
	10%	$\hat{\sigma}$	0.9750	0.4886	0.5570	1.0456	0.9500	0.3495	0.6483	0.9977
		$\hat{\alpha}$	0.9730	0.6332	1.7106	2.3438	0.9380	0.5317	1.7714	2.3031
		$\hat{\beta}$	0.9740	2.7491	-2.5220	0.2271	0.9670	1.9898	-2.0792	-0.0895
	20%	$\hat{\sigma}$	0.9820	0.5290	0.5386	1.0676	0.9380	0.3519	0.6550	1.0070
		$\hat{\alpha}$	0.9850	0.6822	1.6877	2.3699	0.9240	0.5260	1.7797	2.3056
		$\hat{\beta}$	0.9830	3.0872	-2.7177	0.3694	0.9620	2.0615	-2.1459	-0.0845
4	0%	$\hat{\sigma}$	0.9750	0.5409	0.5309	1.0718	0.9420	0.3884	0.6418	1.0302
		$\hat{\alpha}$	0.9710	0.7895	1.6467	2.4362	0.9470	0.6552	1.7190	2.3742
		$\hat{\beta}$	0.9710	3.3784	-2.9131	0.4653	0.9720	2.4473	-2.3720	0.0754
	0%	$\hat{\sigma}$	0.9860	0.6635	0.4776	1.1411	0.9440	0.3837	0.6370	1.0207
		$\hat{\alpha}$	0.9860	0.9661	1.5828	2.5489	0.9430	0.6471	1.7199	2.3670
		$\hat{\beta}$	0.9830	4.2931	-3.4937	0.7994	0.9550	2.5168	-2.3728	0.1440
	0%	$\hat{\sigma}$	0.9940	0.8041	0.4216	1.2257	0.9470	0.3870	0.6381	1.0252
		$\hat{\alpha}$	0.9950	1.1885	1.4989	2.6875	0.9320	0.6488	1.7214	2.3702
		$\hat{\beta}$	0.9900	5.4621	-4.2562	1.2059	0.9700	2.6288	-2.4472	0.1816

Table 16: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 100$ and $\tau = 40$ under Weibull distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9490	0.2497	0.6663	0.9160	0.9520	0.2577	0.6878	0.9455
		$\hat{\alpha}$	0.9720	0.4696	1.7752	2.2448	0.9070	0.3812	1.8234	2.2045
		$\hat{\beta}$	0.9930	3.1504	-2.6862	0.4642	0.9190	1.4183	-1.8455	-0.4272
	10%	$\hat{\sigma}$	0.9370	0.2503	0.6679	0.9182	0.9420	0.2567	0.6881	0.9449
		$\hat{\alpha}$	0.9760	0.4789	1.7690	2.2479	0.8830	0.3822	1.8238	2.2060
		$\hat{\beta}$	0.9910	3.2759	-2.7187	0.5572	0.9120	1.4424	-1.8344	-0.3919
	20%	$\hat{\sigma}$	0.9390	0.2498	0.6675	0.9173	0.9500	0.2543	0.6916	0.9459
		$\hat{\alpha}$	0.9670	0.4948	1.7666	2.2615	0.8850	0.3752	1.8208	2.1960
		$\hat{\beta}$	0.9890	3.4941	-2.8884	0.6056	0.9400	1.4016	-1.8307	-0.4291
3	0%	$\hat{\sigma}$	0.9510	0.2721	0.6617	0.9338	0.9320	0.2755	0.6787	0.9542
		$\hat{\alpha}$	0.9580	0.4821	1.7620	2.2442	0.9130	0.4631	1.7719	2.2350
		$\hat{\beta}$	0.9820	2.9710	-2.5390	0.4319	0.8760	2.5815	-2.1930	0.3885
	10%	$\hat{\sigma}$	0.9390	0.2731	0.6547	0.9278	0.9400	0.2735	0.6813	0.9548
		$\hat{\alpha}$	0.9560	0.4905	1.7633	2.2539	0.9010	0.4504	1.7851	2.2355
		$\hat{\beta}$	0.9860	3.1352	-2.6440	0.4912	0.8510	2.5248	-2.2399	0.2849
	20%	$\hat{\sigma}$	0.9430	0.2765	0.6585	0.9350	0.9390	0.2722	0.6797	0.9519
		$\hat{\alpha}$	0.9730	0.5032	1.7433	2.2466	0.8890	0.4374	1.7853	2.2227
		$\hat{\beta}$	0.9930	3.3237	-2.6836	0.6400	0.8470	2.5571	-2.2237	0.3334
4	0%	$\hat{\sigma}$	0.9400	0.2917	0.6481	0.9398	0.9430	0.2975	0.6704	0.9680
		$\hat{\alpha}$	0.9570	0.5390	1.7359	2.2749	0.9220	0.5207	1.7545	2.2752
		$\hat{\beta}$	0.9950	3.2966	-2.7051	0.5915	0.8830	2.8698	-2.4295	0.4402
	10%	$\hat{\sigma}$	0.9490	0.3057	0.6397	0.9454	0.9420	0.3005	0.6724	0.9730
		$\hat{\alpha}$	0.9630	0.5622	1.7299	2.2921	0.9000	0.5089	1.7636	2.2725
		$\hat{\beta}$	0.9950	3.5972	-2.8858	0.7114	0.8820	2.8927	-2.4457	0.4470
	20%	$\hat{\sigma}$	0.9620	0.3241	0.6326	0.9567	0.9240	0.3003	0.6757	0.9760
		$\hat{\alpha}$	0.9820	0.5905	1.7147	2.3053	0.8880	0.4950	1.7753	2.2704
		$\hat{\beta}$	0.9990	3.9555	-3.0543	0.9012	0.8690	2.8898	-2.4601	0.4297

Table 17: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 20$ and $\tau = 20$ under lognormal distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.8650	0.5582	0.4815	1.0398	0.8920	0.6036	0.5394	1.1430
		$\hat{\alpha}$	0.9340	0.8204	1.5912	2.4116	0.9260	0.9486	1.5625	2.5110
		$\hat{\beta}$	0.9620	2.5706	-2.4021	0.1685	0.9390	3.0406	-2.6075	0.4331
	10%	$\hat{\sigma}$	0.8730	0.5540	0.4829	1.0369	0.9200	0.6048	0.5338	1.1387
		$\hat{\alpha}$	0.9420	0.8308	1.5671	2.3979	0.9370	0.9336	1.5210	2.4546
		$\hat{\beta}$	0.9610	2.7136	-2.4567	0.2569	0.9470	3.3535	-2.6427	0.7108
	20%	$\hat{\sigma}$	0.8370	0.5421	0.4768	1.0190	0.8920	0.6046	0.5367	1.1412
		$\hat{\alpha}$	0.9470	0.8314	1.5532	2.3846	0.9300	0.9130	1.5209	2.4339
		$\hat{\beta}$	0.9620	2.8547	-2.5004	0.3543	0.9370	3.3579	-2.6490	0.7089
3	0%	$\hat{\sigma}$	0.8590	0.6340	0.4560	1.0900	0.9130	0.7813	0.5012	1.2825
		$\hat{\alpha}$	0.9250	1.0299	1.5215	2.5514	0.9660	1.2575	1.5471	2.8045
		$\hat{\beta}$	0.9840	3.9866	-3.2320	0.7546	0.9550	4.2873	-3.7105	0.5767
	10%	$\hat{\sigma}$	0.8570	0.6259	0.4570	1.0829	0.9100	0.8129	0.5003	1.3131
		$\hat{\alpha}$	0.9340	1.0380	1.4782	2.5162	0.9450	1.2558	1.5340	2.7898
		$\hat{\beta}$	0.9860	4.2238	-3.3336	0.8902	0.9540	4.5074	-3.8251	0.6823
	20%	$\hat{\sigma}$	0.8030	0.5990	0.4435	1.0425	0.8950	0.7841	0.4910	1.2752
		$\hat{\alpha}$	0.9410	1.0201	1.4227	2.4428	0.9500	1.1884	1.5092	2.6977
		$\hat{\beta}$	0.9880	4.4816	-3.2804	1.2012	0.9630	4.6392	-3.7879	0.8513
4	0%	$\hat{\sigma}$	0.8660	0.7330	0.4273	1.1603	0.9270	1.0488	0.4761	1.5249
		$\hat{\alpha}$	0.9270	1.3032	1.4316	2.7349	0.9560	1.7844	1.4717	3.2561
		$\hat{\beta}$	0.9930	5.2197	-4.0266	1.1931	0.9600	6.1866	-5.1126	1.0740
	10%	$\hat{\sigma}$	0.8660	0.7238	0.4228	1.1466	0.9090	1.0561	0.4572	1.5133
		$\hat{\alpha}$	0.9340	1.3117	1.3549	2.6667	0.9600	1.7494	1.4228	3.1722
		$\hat{\beta}$	0.9930	5.6345	-4.1097	1.5248	0.9610	6.4236	-5.0903	1.3332
	0%	$\hat{\sigma}$	0.8570	0.7579	0.3870	1.1449	0.8950	1.0113	0.4317	1.4431
		$\hat{\alpha}$	0.9190	1.4023	1.2346	2.6369	0.9240	1.6669	1.3609	3.0278
		$\hat{\beta}$	0.9900	6.5966	-4.4340	2.1625	0.9770	6.7029	-4.9803	1.7226

Table 18: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 20$ and $\tau = 40$ under lognormal distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.8940	0.5017	0.5237	1.0255	0.9070	0.5296	0.5805	1.1101
		$\hat{\alpha}$	0.9510	0.8639	1.5959	2.4598	0.8990	0.8289	1.5983	2.4273
		$\hat{\beta}$	0.9280	4.6593	-3.5658	1.0935	0.9500	2.4929	-2.2296	0.2633
	10%	$\hat{\sigma}$	0.8900	0.4965	0.5160	1.0125	0.9090	0.5270	0.5822	1.1093
		$\hat{\alpha}$	0.9610	0.8805	1.5832	2.4637	0.9100	0.8442	1.5975	2.4417
		$\hat{\beta}$	0.9420	4.9440	-3.7290	1.2149	0.9540	2.6343	-2.3120	0.3224
	20%	$\hat{\sigma}$	0.8970	0.5034	0.5163	1.0197	0.9250	0.5177	0.5828	1.1005
		$\hat{\alpha}$	0.9710	0.9014	1.5740	2.4754	0.8870	0.8453	1.5885	2.4338
		$\hat{\beta}$	0.9380	5.1492	-3.8381	1.3111	0.9780	2.5586	-2.2133	0.3453
3	0%	$\hat{\sigma}$	0.8800	0.5311	0.5141	1.0452	0.9100	0.5983	0.5699	1.1683
		$\hat{\alpha}$	0.9740	0.9765	1.5443	2.5208	0.9170	1.0549	1.5503	2.6052
		$\hat{\beta}$	0.9570	5.8003	-4.2028	1.5976	0.9380	5.3307	-4.1113	1.2194
	10%	$\hat{\sigma}$	0.8940	0.5349	0.5149	1.0498	0.9260	0.6010	0.5619	1.1629
		$\hat{\alpha}$	0.9700	1.0161	1.5206	2.5367	0.9180	1.0526	1.5375	2.5901
		$\hat{\beta}$	0.9500	6.2261	-4.4869	1.7392	0.9390	5.6133	-4.2530	1.3603
	20%	$\hat{\sigma}$	0.8950	0.5249	0.5048	1.0296	0.9280	0.6052	0.5645	1.1697
		$\hat{\alpha}$	0.9800	1.0385	1.5040	2.5424	0.9220	1.0716	1.5117	2.5833
		$\hat{\beta}$	0.9450	6.6405	-4.7562	1.8844	0.9470	5.8089	-4.3280	1.4809
4	0%	$\hat{\sigma}$	0.9010	0.5759	0.5013	1.0772	0.9090	0.6819	0.5496	1.2315
		$\hat{\alpha}$	0.9650	1.1058	1.5200	2.6258	0.9470	1.2401	1.5086	2.7487
		$\hat{\beta}$	0.9570	6.5484	-4.7791	1.7693	0.9580	6.8011	-5.2865	1.5146
	10%	$\hat{\sigma}$	0.8940	0.5717	0.5002	1.0719	0.9330	0.6879	0.5496	1.2375
		$\hat{\alpha}$	0.9770	1.1432	1.4823	2.6255	0.9530	1.2238	1.5012	2.7250
		$\hat{\beta}$	0.9680	7.0215	-4.9339	2.0876	0.9510	7.0605	-5.4197	1.6408
	20%	$\hat{\sigma}$	0.8830	0.5642	0.4951	1.0593	0.9130	0.6614	0.5392	1.2006
		$\hat{\alpha}$	0.9710	1.1872	1.4420	2.6293	0.9110	1.1983	1.4982	2.6965
		$\hat{\beta}$	0.9710	7.7003	-5.2915	2.4089	0.9300	7.2323	-5.4982	1.7342

Table 19: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 100$ and $\tau = 20$ under lognormal distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9160	0.2528	0.6547	0.9075	0.9230	0.2654	0.6615	0.9269
		$\hat{\alpha}$	0.9400	0.3690	1.8066	2.1756	0.9380	0.3839	1.8076	2.1915
		$\hat{\beta}$	0.9430	1.1220	-1.6655	-0.5436	0.9330	1.1162	-1.6829	-0.5667
	10%	$\hat{\sigma}$	0.9030	0.2516	0.6533	0.9049	0.9260	0.2698	0.6625	0.9323
		$\hat{\alpha}$	0.9310	0.3747	1.7989	2.1736	0.9430	0.3796	1.8033	2.1829
		$\hat{\beta}$	0.9500	1.1767	-1.6852	-0.5085	0.9430	1.1501	-1.7182	-0.5681
	20%	$\hat{\sigma}$	0.8690	0.2468	0.6456	0.8924	0.9220	0.2668	0.6594	0.9263
		$\hat{\alpha}$	0.9240	0.3761	1.7803	2.1564	0.9280	0.3733	1.7894	2.1627
		$\hat{\beta}$	0.9600	1.2420	-1.6843	-0.4424	0.9480	1.1901	-1.7212	-0.5311
3	0%	$\hat{\sigma}$	0.8970	0.2806	0.6535	0.9341	0.9490	0.3307	0.6528	0.9835
		$\hat{\alpha}$	0.9400	0.4517	1.7992	2.2509	0.9510	0.5047	1.7929	2.2976
		$\hat{\beta}$	0.9230	1.7351	-2.1310	-0.3959	0.9140	1.7545	-2.1885	-0.4341
	10%	$\hat{\sigma}$	0.8830	0.2719	0.6406	0.9125	0.9330	0.3175	0.6392	0.9567
		$\hat{\alpha}$	0.9400	0.4491	1.7465	2.1956	0.9450	0.4575	1.7633	2.2209
		$\hat{\beta}$	0.9650	1.8311	-2.0816	-0.2505	0.9560	1.7409	-2.0670	-0.3262
	20%	$\hat{\sigma}$	0.8440	0.2662	0.6324	0.8985	0.8920	0.3126	0.6307	0.9433
		$\hat{\alpha}$	0.9310	0.4519	1.6980	2.1499	0.8800	0.4085	1.7568	2.1653
		$\hat{\beta}$	0.9800	1.9622	-2.0406	-0.0784	0.9560	1.7498	-2.0390	-0.2891
4	0%	$\hat{\sigma}$	0.8640	0.3053	0.6501	0.9554	0.9320	0.3886	0.6478	1.0365
		$\hat{\alpha}$	0.9280	0.5348	1.7764	2.3111	0.9250	0.6351	1.7857	2.4208
		$\hat{\beta}$	0.9260	2.1693	-2.4281	-0.2588	0.8860	2.2616	-2.6364	-0.3748
	10%	$\hat{\sigma}$	0.8740	0.3040	0.6326	0.9366	0.9330	0.3842	0.6279	1.0120
		$\hat{\alpha}$	0.8940	0.5450	1.7044	2.2494	0.9450	0.6301	1.6994	2.3295
		$\hat{\beta}$	0.9700	2.3688	-2.4000	-0.0312	0.9430	2.3674	-2.5103	-0.1429
	20%	$\hat{\sigma}$	0.8340	0.2976	0.6181	0.9157	0.9140	0.3731	0.6042	0.9772
		$\hat{\alpha}$	0.8010	0.5495	1.6137	2.1633	0.8800	0.6122	1.6237	2.2359
		$\hat{\beta}$	0.9870	2.6053	-2.3670	0.2384	0.9620	2.4557	-2.3597	0.0960

Table 20: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 100$ and $\tau = 40$ under lognormal distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9320	0.2263	0.6789	0.9051	0.9350	0.2354	0.6920	0.9275
		$\hat{\alpha}$	0.9440	0.3756	1.8218	2.1974	0.9450	0.3953	1.8087	2.2039
		$\hat{\beta}$	0.9400	1.8593	-2.0463	-0.1869	0.9410	1.9572	-2.0911	-0.1339
	10%	$\hat{\sigma}$	0.9300	0.2260	0.6726	0.8986	0.9410	0.2350	0.6911	0.9260
		$\hat{\alpha}$	0.9490	0.3834	1.8097	2.1930	0.9570	0.3956	1.8039	2.1995
		$\hat{\beta}$	0.9510	1.9803	-2.0873	-0.1070	0.9290	2.0810	-2.1617	-0.0807
	20%	$\hat{\sigma}$	0.9210	0.2262	0.6671	0.8933	0.9340	0.2314	0.6849	0.9163
		$\hat{\alpha}$	0.9540	0.3930	1.7925	2.1855	0.9390	0.4069	1.7746	2.1815
		$\hat{\beta}$	0.9520	2.1339	-2.1493	-0.0154	0.9530	2.2466	-2.1720	0.0746
3	0%	$\hat{\sigma}$	0.9260	0.2374	0.6734	0.9108	0.9330	0.2582	0.6892	0.9474
		$\hat{\alpha}$	0.9550	0.4281	1.8011	2.2292	0.9420	0.4425	1.8059	2.2484
		$\hat{\beta}$	0.9610	2.4117	-2.3317	0.0800	0.9490	2.2664	-2.3683	-0.1019
	10%	$\hat{\sigma}$	0.9120	0.2367	0.6688	0.9055	0.9380	0.2568	0.6794	0.9362
		$\hat{\alpha}$	0.9470	0.4411	1.7770	2.2181	0.9430	0.4408	1.7854	2.2262
		$\hat{\beta}$	0.9670	2.5911	-2.3965	0.1946	0.9510	2.3707	-2.3869	-0.0162
	20%	$\hat{\sigma}$	0.8980	0.2351	0.6613	0.8964	0.9310	0.2547	0.6758	0.9305
		$\hat{\alpha}$	0.9490	0.4541	1.7613	2.2154	0.9450	0.4457	1.7715	2.2172
		$\hat{\beta}$	0.9620	2.7850	-2.4841	0.3010	0.9530	2.5097	-2.4365	0.0733
4	0%	$\hat{\sigma}$	0.9120	0.2538	0.6714	0.9252	0.9470	0.2822	0.6792	0.9614
		$\hat{\alpha}$	0.9400	0.4810	1.7846	2.2656	0.9290	0.4980	1.7846	2.2826
		$\hat{\beta}$	0.9550	2.7779	-2.5954	0.1825	0.9430	2.6635	-2.6070	0.0565
	10%	$\hat{\sigma}$	0.9210	0.2522	0.6667	0.9189	0.9430	0.2802	0.6748	0.9550
		$\hat{\alpha}$	0.9520	0.4974	1.7618	2.2592	0.9390	0.4958	1.7684	2.2642
		$\hat{\beta}$	0.9660	2.9954	-2.6864	0.3090	0.9490	2.7600	-2.6330	0.1270
	20%	$\hat{\sigma}$	0.8950	0.2488	0.6577	0.9065	0.9380	0.2748	0.6653	0.9401
		$\hat{\alpha}$	0.9470	0.5140	1.7369	2.2509	0.9340	0.4982	1.7467	2.2450
		$\hat{\beta}$	0.9670	3.2575	-2.7759	0.4816	0.9400	2.9702	-2.7562	0.2140

Table 21: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 20$ and $\tau = 20$ under log-logistic distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9530	1.0025	0.2967	1.2992	0.9050	0.7502	0.5169	1.2671
		$\hat{\alpha}$	0.9660	1.5449	1.2381	2.7830	0.9250	1.6248	1.2781	2.9029
		$\hat{\beta}$	0.9860	4.1282	-3.1998	0.9284	0.9580	4.3161	-2.9555	1.3607
	10%	$\hat{\sigma}$	0.9480	1.0083	0.2864	1.2947	0.8880	0.7277	0.5252	1.2529
		$\hat{\alpha}$	0.9660	1.5644	1.2399	2.8043	0.9320	1.6409	1.2389	2.8798
		$\hat{\beta}$	0.9720	4.2739	-3.2640	1.0099	0.9480	4.7776	-2.9504	1.8272
	20%	$\hat{\sigma}$	0.9520	0.9636	0.3084	1.2720	0.8850	0.7069	0.5218	1.2287
		$\hat{\alpha}$	0.9690	1.5271	1.2479	2.7750	0.9440	1.6347	1.2109	2.8457
		$\hat{\beta}$	0.9830	4.2792	-3.2728	1.0065	0.9610	5.0888	-3.0050	2.0837
3	0%	$\hat{\sigma}$	0.9780	1.6101	0.0567	1.6668	0.9160	0.9300	0.4823	1.4124
		$\hat{\alpha}$	0.9860	2.4657	0.8530	3.3186	0.9410	1.9666	1.2977	3.2643
		$\hat{\beta}$	0.9810	8.4053	-5.9064	2.4989	0.9570	5.3571	-4.1244	1.2327
	10%	$\hat{\sigma}$	0.9830	1.5013	0.1132	1.6144	0.9020	0.9302	0.4858	1.4160
		$\hat{\alpha}$	0.9850	2.3637	0.9091	3.2728	0.9490	1.9617	1.3181	3.2798
		$\hat{\beta}$	0.9860	8.1079	-5.7266	2.3813	0.9520	5.5869	-4.3197	1.2672
	20%	$\hat{\sigma}$	0.9770	1.5743	0.0703	1.6446	0.9020	0.9056	0.4873	1.3929
		$\hat{\alpha}$	0.9820	2.5119	0.8664	3.3784	0.9330	2.0427	1.2578	3.3005
		$\hat{\beta}$	0.9790	8.9674	-6.2199	2.7476	0.9460	6.5582	-4.4816	2.0766
4	0%	$\hat{\sigma}$	0.9900	1.9296	-0.0265	1.9031	0.9240	1.1021	0.4623	1.5644
		$\hat{\alpha}$	0.9890	3.2100	0.5888	3.7989	0.9500	2.3954	1.2625	3.6579
		$\hat{\beta}$	0.9960	11.1053	-7.8287	3.2766	0.9460	6.4823	-5.1783	1.3040
	10%	$\hat{\sigma}$	0.9900	1.9586	-0.0590	1.8997	0.9060	1.1075	0.4590	1.5664
		$\hat{\alpha}$	0.9900	3.3202	0.5200	3.8402	0.9510	2.4500	1.2852	3.7353
		$\hat{\beta}$	0.9960	11.9944	-8.2319	3.7624	0.9480	6.8035	-5.3528	1.4507
	20%	$\hat{\sigma}$	0.9860	1.9096	-0.0308	1.8788	0.9120	1.0609	0.4588	1.5197
		$\hat{\alpha}$	0.9890	3.3848	0.5326	3.9174	0.9360	2.4635	1.2554	3.7189
		$\hat{\beta}$	0.9930	12.3587	-8.5232	3.8355	0.9430	7.7828	-5.7276	2.0552

Table 22: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 20$ and $\tau = 40$ under log-logistic distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9390	0.7717	0.3996	1.1713	0.9020	0.6349	0.5543	1.1892
		$\hat{\alpha}$	0.9520	1.3550	1.3596	2.7146	0.9300	1.8094	0.9122	2.7216
		$\hat{\beta}$	0.9550	4.4581	-3.3661	1.0920	0.9450	6.9868	-2.4756	4.5113
	10%	$\hat{\sigma}$	0.9270	0.9320	0.3109	1.2429	0.9030	0.6262	0.5551	1.1813
		$\hat{\alpha}$	0.9380	1.4903	1.2738	2.7642	0.9160	1.7976	0.9166	2.7143
		$\hat{\beta}$	0.9610	5.6455	-3.9551	1.6905	0.9490	6.7386	-2.3587	4.3799
	20%	$\hat{\sigma}$	0.9340	0.7865	0.3915	1.1779	0.8790	0.6051	0.5606	1.1657
		$\hat{\alpha}$	0.9520	1.3938	1.3371	2.7309	0.9090	1.7982	0.8884	2.6866
		$\hat{\beta}$	0.9460	5.0582	-3.7164	1.3418	0.9560	7.7149	-2.1305	5.5844
3	0%	$\hat{\sigma}$	0.9560	0.9384	0.3337	1.2721	0.9250	0.7347	0.5355	1.2703
		$\hat{\alpha}$	0.9760	1.5913	1.2418	2.8331	0.9420	1.8559	1.0912	2.9471
		$\hat{\beta}$	0.9610	6.8747	-4.9581	1.9166	0.9540	7.2668	-4.3352	2.9316
	10%	$\hat{\sigma}$	0.9490	0.9401	0.3319	1.2720	0.9220	0.7346	0.5369	1.2715
		$\hat{\alpha}$	0.9780	1.6312	1.2401	2.8712	0.9490	1.8905	1.0603	2.9508
		$\hat{\beta}$	0.9640	7.3890	-5.2442	2.1449	0.9179	7.8078	-4.3722	3.4356
	20%	$\hat{\sigma}$	0.9580	0.9231	0.3342	1.2572	0.9080	0.7148	0.5396	1.2543
		$\hat{\alpha}$	0.9700	1.6368	1.2429	2.8797	0.9250	2.0215	0.9105	2.9320
		$\hat{\beta}$	0.9630	7.6246	-5.3660	2.2586	0.9240	9.7843	-4.3478	5.4365
4	0%	$\hat{\sigma}$	0.9610	1.1241	0.2693	1.3934	0.9260	0.8029	0.5134	1.3163
		$\hat{\alpha}$	0.9760	1.9209	1.1477	3.0685	0.9580	1.8486	1.2507	3.0993
		$\hat{\beta}$	0.9740	8.9307	-6.3820	2.5487	0.9110	6.8851	-5.0384	1.8467
	10%	$\hat{\sigma}$	0.9640	1.1070	0.2781	1.3851	0.9210	0.8109	0.5202	1.3311
		$\hat{\alpha}$	0.9870	1.9513	1.1384	3.0897	0.9470	1.8915	1.2590	3.1505
		$\hat{\beta}$	0.9770	9.4619	-6.6255	2.8363	0.9120	7.4048	-5.3751	2.0298
	20%	$\hat{\sigma}$	0.9730	1.1020	0.2778	1.3798	0.9100	0.7877	0.5283	1.3159
		$\hat{\alpha}$	0.9820	2.0090	1.1011	3.1101	0.9340	2.1500	0.9725	3.1225
		$\hat{\beta}$	0.9700	10.2599	-7.0882	3.1717	0.9250	10.2410	-5.6434	4.5976

Table 23: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 100$ and $\tau = 20$ under log-logistic distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9810	0.4353	0.5743	1.0096	0.9340	0.3255	0.6593	0.9848
		$\hat{\alpha}$	0.9610	0.6618	1.6712	2.3330	0.9280	0.6042	1.7151	2.3193
		$\hat{\beta}$	0.9810	1.7544	-1.8617	-0.1073	0.9620	1.3652	-1.7290	-0.3638
	10%	$\hat{\sigma}$	0.9750	0.4116	0.5856	0.9972	0.9330	0.3269	0.6604	0.9873
		$\hat{\alpha}$	0.9470	0.6477	1.6765	2.3242	0.9390	0.5908	1.7195	2.3103
		$\hat{\beta}$	0.9850	1.7122	-1.8267	-0.1144	0.9580	1.4161	-1.7461	-0.3300
	20%	$\hat{\sigma}$	0.9760	0.4062	0.5911	0.9973	0.9210	0.3272	0.6606	0.9878
		$\hat{\alpha}$	0.9680	0.6509	1.6795	2.3304	0.9200	0.5757	1.7311	2.3068
		$\hat{\beta}$	0.9850	1.7571	-1.8693	-0.1122	0.9650	1.4459	-1.7714	-0.3256
3	0%	$\hat{\sigma}$	0.9980	0.8776	0.3932	1.2709	0.9330	0.3853	0.6412	1.0266
		$\hat{\alpha}$	0.9940	1.1888	1.4361	2.6249	0.9420	0.7303	1.6916	2.4219
		$\hat{\beta}$	0.9890	4.5096	-3.5150	0.9946	0.9460	1.9305	-2.0797	-0.1492
	10%	$\hat{\sigma}$	0.9990	0.8251	0.4110	1.2362	0.9400	0.3784	0.6374	1.0158
		$\hat{\alpha}$	0.9860	1.1596	1.4421	2.6017	0.9340	0.7101	1.6955	2.4056
		$\hat{\beta}$	0.9880	4.4477	-3.4486	0.9992	0.9580	1.9774	-2.0765	-0.0992
	20%	$\hat{\sigma}$	0.9960	0.7457	0.4443	1.1900	0.9440	0.3797	0.6389	1.0186
		$\hat{\alpha}$	0.9920	1.0906	1.4801	2.5707	0.9180	0.7001	1.7055	2.4056
		$\hat{\beta}$	0.9950	4.2283	-3.3002	0.9281	0.9440	2.0688	-2.1436	-0.0748
4	0%	$\hat{\sigma}$	0.9960	1.1850	0.2850	1.4700	0.9530	0.4137	0.6280	1.0417
		$\hat{\alpha}$	0.9980	1.7359	1.2418	2.9777	0.9460	0.8187	1.6574	2.4761
		$\hat{\beta}$	0.9900	6.7489	-5.0491	1.6999	0.9560	2.2485	-2.2570	-0.0085
	10%	$\hat{\sigma}$	1.0000	1.1613	0.2873	1.4486	0.9340	0.4185	0.6262	1.0448
		$\hat{\alpha}$	0.9960	1.7617	1.2152	2.9768	0.9460	0.8175	1.6602	2.4777
		$\hat{\beta}$	0.9910	7.0116	-5.1250	1.8866	0.9410	2.4039	-2.3456	0.0584
	20%	$\hat{\sigma}$	0.9970	1.0771	0.3125	1.3896	0.9240	0.4192	0.6270	1.0462
		$\hat{\alpha}$	0.9980	1.7086	1.2283	2.9369	0.9410	0.8117	1.6571	2.4688
		$\hat{\beta}$	0.9860	7.0349	-5.1181	1.9168	0.9330	2.5511	-2.4151	0.1360

Table 24: Wald-type asymptotic 95% CI and BCa bootstrap 95% CI of the MLE based on 1000 simulations for a progressively Type-I censored k-level step-stress ALT with uniform step duration, B=1000, $n = 100$ and $\tau = 40$ under log-logistic distribution

k	π^*	MLE	Wald-type asymptotic 95% CI				BCa bootstrap 95% CI			
			% Coverage	Mean Width	LB	UB	% Coverage	Mean Width	LB	UB
2	0%	$\hat{\sigma}$	0.9600	0.3197	0.6296	0.9493	0.9270	0.2828	0.6778	0.9606
		$\hat{\alpha}$	0.9420	0.5866	1.7112	2.2978	0.9470	0.5878	1.7133	2.3011
		$\hat{\beta}$	0.9610	1.7693	-1.8714	-0.1022	0.9400	1.5722	-1.8609	-0.2887
	10%	$\hat{\sigma}$	0.9710	0.3148	0.6352	0.9500	0.9280	0.2801	0.6783	0.9584
		$\hat{\alpha}$	0.9380	0.5883	1.7067	2.2950	0.9360	0.5781	1.7225	2.3006
		$\hat{\beta}$	0.9700	1.8182	-1.8851	-0.0668	0.9490	1.6519	-1.8933	-0.2414
	20%	$\hat{\sigma}$	0.9680	0.3143	0.6332	0.9475	0.9230	0.2734	0.6803	0.9536
		$\hat{\alpha}$	0.9600	0.5909	1.7021	2.2930	0.9380	0.5712	1.7146	2.2857
		$\hat{\beta}$	0.9720	1.9217	-1.9282	-0.0065	0.9460	1.7092	-1.9293	-0.2200
3	0%	$\hat{\sigma}$	0.9760	0.4096	0.5964	1.0060	0.9490	0.3131	0.6694	0.9825
		$\hat{\alpha}$	0.9750	0.6830	1.6652	2.3482	0.9440	0.6502	1.7027	2.3528
		$\hat{\beta}$	0.9680	2.8906	-2.5276	0.3631	0.9180	2.0109	-2.1478	-0.1370
	10%	$\hat{\sigma}$	0.9720	0.4159	0.5889	1.0048	0.9380	0.3084	0.6679	0.9764
		$\hat{\alpha}$	0.9590	0.7007	1.6573	2.3580	0.9470	0.6372	1.7003	2.3375
		$\hat{\beta}$	0.9700	3.1323	-2.6276	0.5048	0.9260	2.1242	-2.1831	-0.0589
	20%	$\hat{\sigma}$	0.9750	0.3877	0.6012	0.9889	0.9480	0.3115	0.6735	0.9850
		$\hat{\alpha}$	0.9640	0.6858	1.6570	2.3428	0.9370	0.6250	1.7157	2.3407
		$\hat{\beta}$	0.9740	3.1139	-2.5938	0.5201	0.9160	2.2326	-2.2697	-0.0371
4	0%	$\hat{\sigma}$	0.9880	0.5310	0.5414	1.0724	0.9470	0.3328	0.6593	0.9921
		$\hat{\alpha}$	0.9810	0.8357	1.6078	2.4435	0.9590	0.6915	1.6721	2.3636
		$\hat{\beta}$	0.9740	4.1248	-3.2670	0.8578	0.9360	2.2811	-2.2740	0.0071
	10%	$\hat{\sigma}$	0.9840	0.5665	0.5159	1.0824	0.9390	0.3333	0.6606	0.9938
		$\hat{\alpha}$	0.9810	0.9014	1.5618	2.4633	0.9490	0.6864	1.6958	2.3822
		$\hat{\beta}$	0.9810	4.7444	-3.4941	1.2503	0.9200	2.4577	-2.3998	0.0579
	20%	$\hat{\sigma}$	0.9900	0.5057	0.5496	1.0553	0.9230	0.3320	0.6633	0.9953
		$\hat{\alpha}$	0.9870	0.8625	1.5801	2.4426	0.9180	0.6782	1.6987	2.3769
		$\hat{\beta}$	0.9850	4.5769	-3.4149	1.1620	0.9180	2.6580	-2.5067	0.1513