Prediction of Shrimp Size Distribution Reared Inside Submersible Sea Cages.

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Abstract

We developed a non-normal regression model to characterize the growth of shrimp and the shrimp's weight distributions throughout the fattening process. The experiment was conducted in the Baja California Gulf in 2010 where shrimp producers built submersible sea cages to breed shrimp.
During the fattening process, producers took samples and performed biometric analyses in order to observe characteristics of the shrimp in the growth process. We analyzed the information from the samples using several distributions of probability to study the behavior of shrimp growth. To find the best mathematical model to characterize this behavior, we applied techniques that are used in reliability engineering and data lifetime studies.

Key Words: Non-Normal Regression, Extreme Value Distribution, Submersible Sea Cages.

1. Literature Review

For shrimp producers, predicting shrimp growth is fundamental to make adequate business decisions (Tian et al., 1993).

Scholars have conducted qualitative (Carvajal, 1993) and quantitative studies that model the average shrimp weight. Quantitative methods model the average weight through statistical regression techniques using several functional forms. Yu et al. (2006) compared eight functional forms to model the average weight of shrimps. Katsanevakis (2006) did something similar with fish populations.

Because shrimp is one of the species whose weight-marketing price depends on the shrimp size, not only the average weight prediction is required, but also the distribution of the weight and size.

2. Rationale

In 2010, shrimp producers in the Baja California Gulf experimented with how to breed shrimp inside of submersible sea cages. White shrimp post-larvae PL8 were seeded at the beginning of June 2010 (time zero) and harvested on October 25th (time = 135 days). Each day, scuba divers went to feed them inside sea cages.

In order to monitor shrimp growth during this fattening period, samples were taken on June 28th, July 05th and 19th, August 02nd and 16th, September 13th and 27th, and on October 11th and 25th. For each sample, a biometry was performed to obtain individual information about the weight, size, and other shrimp quality characteristics.

For producers, it is important to forecast the distribution of sizes of the final product: it is not just enough to know the average weights because shrimp market prices depend on the sizes of shrimps.

A balanced harvest point—between continuing feeding shrimps and the final harvest to commercialize them—is desirable. Producers seek to optimize the time of harvest to maximize growth, while minimizing the risk of losing money feeding shrimp that cannot grow anymore.

3. Rationale

The goal of this research was to characterize the growth of shrimp throughout the fattening process. The specific objective was to model the distribution of the weight of shrimp at different times before and after the harvest. For producers, it was important to predict the distribution of sizes of the final product, because these factors determine the project feasibility, marketing conditions, their final price in the market, and the return on investment of this new form of shrimp production. To achieve this goal, we applied non-regression models to these data.

Given that the dates to collect the samples were selected according to weather conditions and human resources availability, this study is observational.

The variables considered in this work (see Figure 1) are the following:

Independent variable: the number of days since larva was seeded.
Dependent variable: the size of the shrimp.

4. Methodology

We propose a model that describes shrimp size distribution as a function of the day variable $Pr(Y \leq y{:}\, x) = F(y, x) = F(y)$ where Y is the shrimp weight at time t and x represents the day variable. The mean of the distribution has only one explainer variable $\mu_i = \beta_0 + \beta_1 x_i$. The simplest distribution model for normal, logistic, and extreme value distributions is the following:

$$Pr(Y \leq y) = F(y; \mu, \sigma) = F(y; \beta_0, \beta_1, \sigma) = \Phi((y - \mu)/\sigma),$$

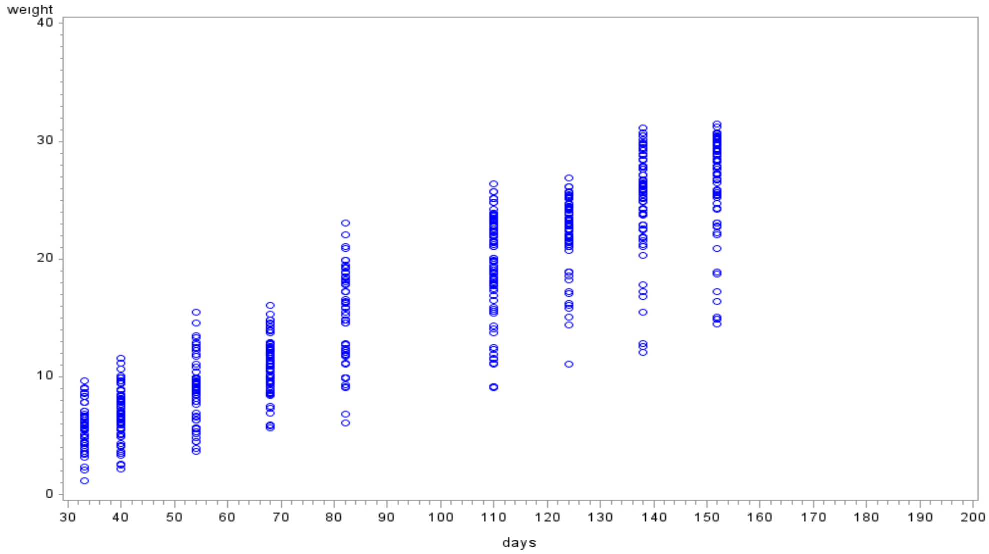

Figure 1: Weight of shrimp as a function of time. Shrimp were bred in submersible sea cages.

Where $\mu_i = \beta_0 + \beta_1 x_i$. and σ does not depend on x, the explainer variable. The quantile function for this model is as follows:

$$y_p = \mu + \Phi^{-1}(p)\sigma = \beta_0 + \beta_1\, x + \Phi^{-1}(p)\sigma.$$

The localization and scale parameters estimation of this regression model can be done by the maximum likelihood estimation for, β_0, β_1 "and σ such that the sample likelihood function is maximum. The likelihood function is given by

$$L(\beta_0, \beta_1, \sigma; x) = \prod_{i=1}^{n} L_i\,(\beta_0, \beta_1, \sigma; x, data_i)$$

$$= \prod_{i=1}^{n} \left[\frac{1}{\sigma}\Phi\left(\frac{x_i - \mu_i}{\sigma}\right)\right]^{\delta_i} \left[1 - \Phi\left(\frac{x_i - \mu_i}{\sigma}\right)\right]^{1-\delta_i}$$

Numerical methods are used to maximize the log of this function, in which the first derivatives with respect to β_0, β_1, σ of the likelihood function are set equal to zero and then the equations are solved for to β_0, β_1, σ.

This procedure can be executed using statistical software. We used the “Proc reliability” and “Proc Lifereg” procedures in SAS. The quantile function can be used to calculate size weight percentiles for the shrimp size distribution for a specific value of x (days). The initial step is to identify what kind of distribution the variable of interest follows for each level of the explanatory variable.

Then, the location parameter and scale (or shape) parameter are fitted separately for each level (Figure 2).

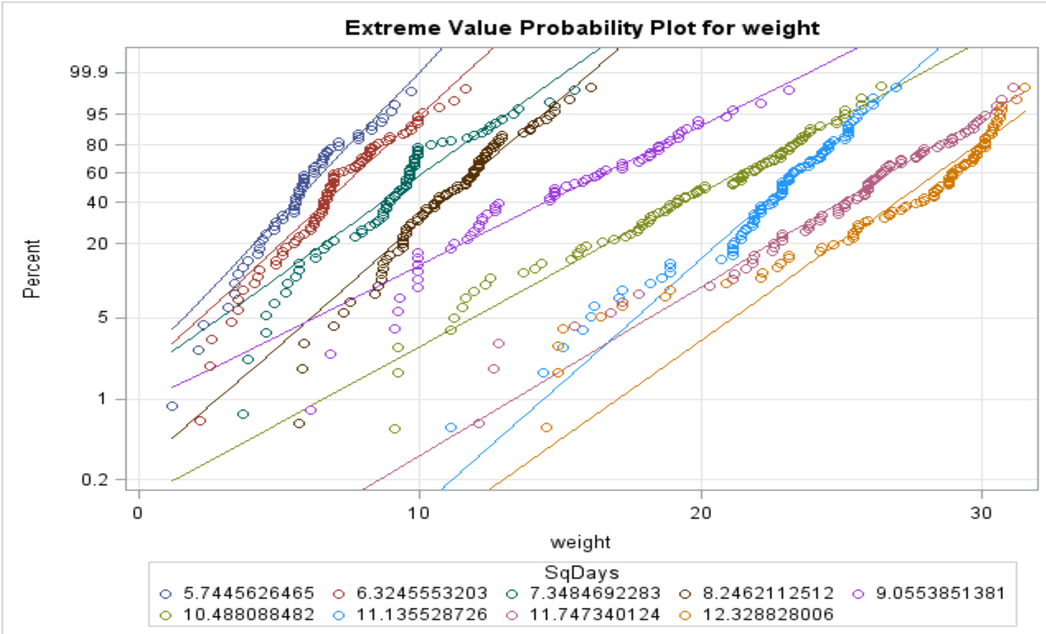

Figure 2: Identifying best distribution fit.

Finally, a joint model is estimated in which the scale parameter σ is limited to be equal for all levels of the explanatory variable and, at the same time, the locations parameters are estimated (Figure 3-left).

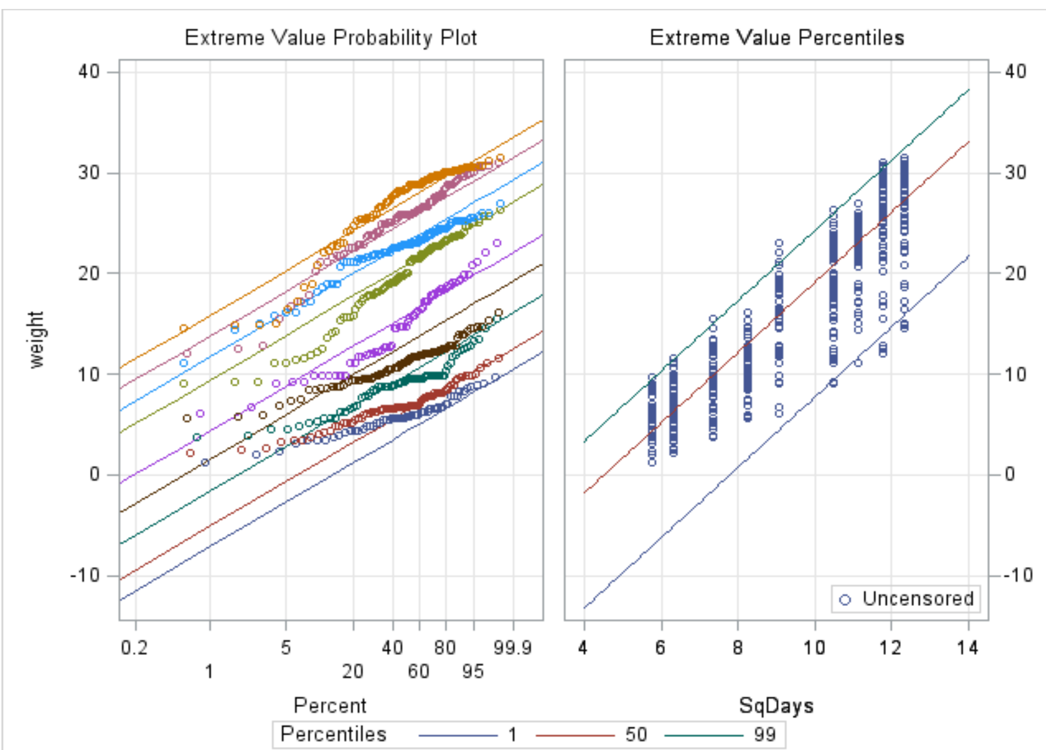

Figure 3: Left: Identifying the best distribution fit. Right: Real and estimated data with model

We subsequently assessed the assumptions of the model (Table 1 and Figure 4).

Goodness-of-Fit Tests for Extreme Value Distribution		
Test	Statistic	p-Value
Cramer-Von Mises	W-Sq 0.0886	Pr > W-Sq 0.166
Anderson-Darling	A-Sq 0.5513	Pr > A-Sq 0.175

Table 1: Extreme value Regression Model Y = sqrt(x), check for residuals assumptions

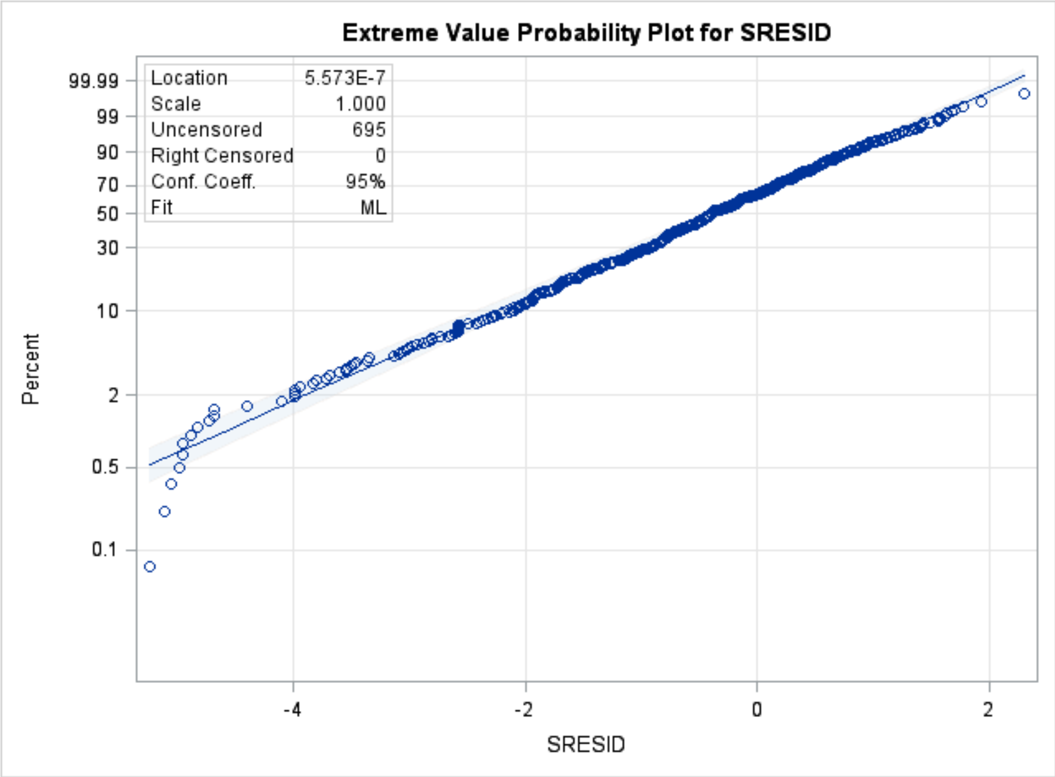

Figure 4: Left: Assessing Extreme Value Assumption

5. Results

Our model can be used to predict the distribution of sizes of shrimp reared inside submersible sea cages in similar conditions each season. This model does not only estimate the average weight but also the percentiles and the percentages of the sizes for a determined time. Further, the model is able to extrapolate information to predict the size distribution at a determined time in the future. This allows certainty for strategic planning purposes and decision making. The distribution of the shrimp weight can be better modeled using an extreme value distribution.

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