A Time Series-based Point Estimation of Stop Signal Reaction Times

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Abstract

The Stop Signal Reaction Time (SSRT) is a latency measurement for the unobservable human brain stopping process, and was formulated by Logan (1994) without consideration of the nature (go/stop) of trials that precede the stop trials. In 2017, the authors proposed asymptotically equivalent and larger indexes of mixture SSRT and weighted SSRT to address this issue from time in task longitudinal perspective, but estimation based on the time series perspective has still been missing in the literature. To test the hypothesis of no difference between time series based state space estimation of SSRT and Logan 1994 SSRT, two samples of SST data including real data and the simulated data were considered, and State-space missing data EM algorithm was applied for each subject's SST data, encompassing trial order. Using Logan's 1994 formulae on ordered SST data, the new state-space SSRT index was calculated. The results for both the real and the simulated data showed that state-space SSRT is significantly larger than Logan's 1994 SSRT, mixture SSRT, and weighted SSRT. As a conclusion, SSRT indexes based on the information of the preceding trial type are significantly larger than others.

Key Words: Stop Signal Reaction Times, State-Space Models, EM algorithm, Missing Data, Lognormal Distribution

1. Introduction

Inhibitory control has theoretical and empirical importance. While its theoretical importance is rooted in its nature as an internally-governed act of control, its empirical importance is due to the emergence of key results that support theories of development and inhibitory psychopathology (Logan, 1994). The Stop Signal Task (SST) paradigm is a useful tool by which inhibitory control can be studied (Verbruggen, Aron, Band, Beste,Bissett, Brockett, et al, 2019). The SST includes a go task and a stop task. In the go task, or "go trials", one of two symbols, such as X or O, is presented on a computer screen. Participants are instructed to choose between the X and O as fast as possible. In the stop task, or "stop trials", a short time after an X or O is presented, the participant hears an auditory "stop signal" through headphones; this is a Stop Signal Delay (SSD). The auditory signal indicates they must withhold their responses on that particular trial. In the most experiments, the stop trials constitute 25% of all SST trials (Figure 1).

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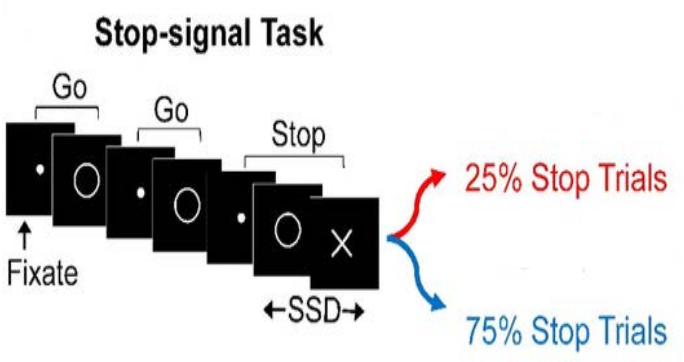

Figure 1: Stop-signal task (SST) design including two versions of the task: 25% stop trials and 75% stop trials, (image courtesy of (Manza, Hu, Chao, Zhang, Leung, and Li, 2016)).

The independent horse race model provides a theoretical framework in which the researchers can estimate the stop process' latency, or Stop Signal Reaction Times (SSRT), and factors associated with the probability of Successful Inhibition (SI) in the SST paradigm (Logan and Cowan, 1984). The independent horse race model assumes the finish times for the go reaction times (GORT) in the stop trials and the finish times for the stop process are stochastically independent. The SSRT is the time difference between the participant's internal response and the stop signal timing (Figure 2).

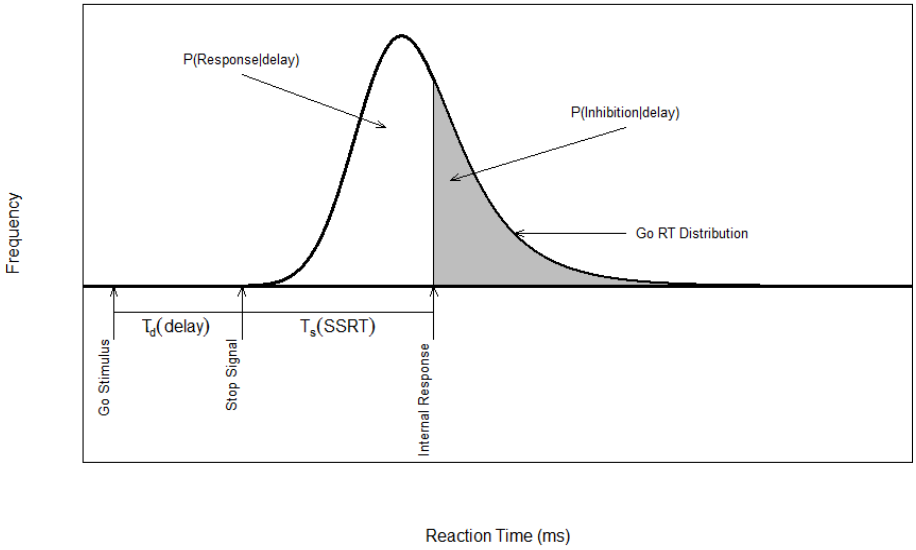

Figure 2: The independent horse race model: $SSD = T_d; SSRT = T_s$ (Logan, 1994- modified).

Two general frequentist and Bayesian approaches have been proposed in the literature to estimate SSRT (Verbruggen, Aron, Band, Beste, Bissett, Brockett, et al., 2019; Matzke, Dolan, Logan, Brown, Wagenmakers, 2013) The frequentist approach includes three methods: Colonius' method to calculate the entire distribution of the SSRT, the mean method to calculate the constant valued SSRT, and Logan's 1994 integration method to calculate

constant valued SSRT (Matzke, Dolan, Logan, Brown, and Wagenmakers, 2013). It has been shown that Colonius' method is impossible to use in the human experimental context, because it requires 250,000 trials for reliable estimates (Matzke, Dolan, Logan, Brown, and Wagenmakers, 2013). Furthermore, there is higher reliability and less bias in Logan's 1994 integration estimates versus the mean method estimates, particularly when the probability of SI is other than 50%. Given these observations, Logan's 1994 integration method has been recommended for the estimation of SSRT (Verbruggen, Aron, Band, Beste, Bissett, Brockett, et al., 2019). For a subject with go reaction times GORT in go trials, stop signal delay SSD of T_d and probability of successful inhibition SI in stop trials, Logan (1994) proposed the following frequentist point estimation of SSRT:

$$SSRT_{Logan1994} = Q_{GORT}(1 - P(SI|\overline{T_d})) - \overline{T_d} \tag{1}$$

where Q is the quantile function, and the average of T_d is taken over all stop trials. There are two assumptions for this method: The first implicit assumption is the equal impact of go trials and stop trials in SST data; that is, the impact of the preceding trial, either go or stop, on the current stop trial SSRT estimates is assumed to be the same. The second assumption is that there is no trigger failures or the trigger failures are randomized in the SST data(Matzke, Love and Heathcote, 2017). The authors have shown that the first assumption may be violated in the context of the tracking SST data[1](Soltanifar, Dupuis, Schachar, and Escobar, 2019). In order to address SSRT estimation given a violated assumption of equal impact of preceding trial type on SSRT, the authors partitioned SST data to type A cluster SST data, trials following a go trial, and type B cluster SST data, trials following a stop trial. Then, by considering cluster type $GORT_A, SSRT_A, GORT_B, SSRT_B$ and defining trial type weight $W_A = (\#\text{Type A stop trials})/(\#\text{Total stop trials}), W_B = 1 - W_A$, they proposed the following new frequentist indexes of SSRT (Soltanifar, Dupuis, Schachar, and Escobar, 2019):

$$SSRT_{Weighted} = W_A \times SSRT_A + W_B \times SSRT_B, \tag{2}$$

$$SSRT_{Mixture} = SSRT_{(W_A \times GORT_A + W_B \times GORT_B)}. \tag{3}$$

It has been shown than under specific, experimenter-pre-arranged conditions, the two indices are asymptotically equivalent, given increasing number of stop trial (Soltanifar, Dupuis, Schachar, and Escobar, 2019):

$$|SSRT_{Weighted} - SSRT_{Mixture}| \to 0, \ \ as \ \ m_{stop} \to \infty. \tag{4}$$

Moreover, the authors have shown that for both cases of the real SST data and the simulated SST data (Soltanifar, Dupuis, Schachar, and Escobar, 2019):

$$SSRT_{Weighted} - SSRT_{Logan1994} > 5.0 \ ms. \tag{5}$$

The proposed three indices of SSRT in equations (1) - (3) are calculated via longitudinal perspective on SST data. However, there is little information available to calculate SSRT when the SST data is considered as a time series data. The lack of literature becomes particularly important when the GORT time series has already been studied (Bosch, Ernestus, and Bores, 2014; Hyndman and Khandakar, 2008; Hyndman, Athnasopaubs, Bergmeir, Caceres, et al, 2018). Moreover, to the authors' best knowledge there is no study

[1]In the tracking SST data, the stop signal delay is dynamically increased by 50 ms after successful inhibition in the previous stop trial, or decreased by 50 ms after failed inhibition in the previous stop trial. In this way, the probability of successful inhibition $P(SI|\overline{T_d})$ approximates to 50% in overall stop trials; and, the estimations of SSRT are more reliable.

that assesses the impact of the preceding trial type (go/stop) on the current stop trial SSRT estimates in the time series context. Here, the linear relationship of the current time point outcome, in terms of the previous time point outcome in the time series, and the state-space feature can be simultaneously useful to address the impact of the preceding trial type on the current stop trial SSRT estimates (Diggle, 1990; Shumway and Stoffer, 2017).

The aim of the present paper is to estimate SSRT, given a violation of the assumption of equal impact of the preceding trial type (go/stop) on the current stop trial SSRT, using the missing state-space modelling on the entire SST data set. The outline of the paper is as follows. First, we consider the real SST data with given go reaction times (GORT), go reaction times on failed stop trials (SRRT), and stop signal delay time on stop trials (T_d) in the time series framework, and using a four-stage missing data state-space modelling, we compute state space SSRT with a lognormal distributional assumption. Second, we compare the new state space SSRT index with the three established indices in (1) - (3). Third, we repeat the previous explorations for the case of simulated data. Finally, we close with a discussion of the sensitivity of the distributional assumptions of the missing data state-space modelling, and the asymptotic behavior of the disparities between these indices.

2. Methods & Materials

2.1 Data

The study data included two sets of the real data and simulated data described below.

2.1.1 The Real Data

Data was collected at the Ontario Science Center in Toronto, Canada from 2009 to 2010. The sample includes 16,099 children, aged 6 to 17 years old (Crosbie, Arnold, Paterson, Swanson, Dupuis, Li, et al., 2013). Self or parent-reported demographic data were obtained. Each child completed a stop signal task (SST) comprised of 5 blocks of 24 trials (one practice block and four main blocks). Within the blocks, 25% of the trials contained a stop signal. The stop signal task algorithm was designed so that the probability of successful inhibition converged on 50%. The study's variables included stop-signal delay (centiseconds), previous trial type (stop/go), and ADHD status (yes/no). The ADHD variable was defined based on SWAN z-score (Brites, Salgado-Azoni, Ferrerira, Lima, and Ciasca, 2015), whether the score falls in the top 10% of the distribution (defined as ADHD case) or not (defined as control case).

2.1.2 The Simulated Data

The simulations are based on the assumption of the independent race model. Independent GORT and SSRT via the tracking method with initial $SSD = 200$ ms were simulated. On the stop trials, a successful inhibition was considered one for which $GORT > SSRT + SSD$; and, otherwise, it was considered a failed inhibition. An ex-Gaussian distribution was assumed for GORT, SSRT, and it was simulated by R package GAMLSS (Stasinopoulos and Rigby, 2016). This distribution has been extensively used in psychology, neuroscience, and as a time model for cognitive process in the study of reaction times(Palmer, Horowitz, Toralba and Wolfe, 2011; Roher and Wixted, 1994; Luce, 1991). We simulated SST data for each subject with type A GORT and SSRT ex-Gaussian distributions, type B GORT and SSRT ex-Gaussian distributions, and weights $W_A = 0.75$ and $W_B = 0.25$(see Appendix). We randomly merged the two cluster A and cluster B SST data for each subject. The baseline

ex-Gaussian distributional forms for type A cluster, type B cluster SST data, and subject SST sample are as follows in the Table 1:

Table 1: Structure of simulated stop task data by subject sample size (SST data sample size per subject: N=96,192,288,384,480,960)

SSRT(Δ mean, Δ STD)	n(subjects)	Cluster Type	n(subjects):cluster	GORT distribution	SSRT distribution
(increasing, increasing)	11	A	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(300,35,30)	ExG(130+a,70+a,60+a)
		B	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(450,50,30)	ExG(150+a,90+a,60+a)
(increasing, decreasing)	11	A	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(300,35,30)	ExG(130+a,70-a,60)
		B	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(450,50,30)	ExG(150+a,90-a,60)
(decreasing, increasing)	11	A	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(300,35,30)	ExG(130-a,70+a,60)
		B	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(450,50,30)	ExG(150-a,90+a,60)
(decreasing, decreasing)	11	A	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(300,35,30)	ExG(130-a,70-a,60-a)
		B	11 $(a = 2k : 1 \leq k \leq 11)$	ExG(450,50,30)	ExG(150-a,90-a,60-a)

2.2 Subjects

A random subsample of 44 participants (11 ADHD; 33 Control) age 6-17,with 96 SST trials, and a minimum of 10 stop trials preceded by a stop trial was selected from the real SST data in 2.1.1. For the simulated data, 44 subjects with a variety of increasing or decreasing mean and variance of their underlying SSRT distributions were simulated. Each simulated subject had 96, 192, 288, 384, 480, and 960 SST trials.

2.3 Statistical Inference

We considered the raw SST data, including GORT, SRRT and SSD, as a three-dimensional time series vector with missing values, and applied the missing data state-space models to it (Figure 3).

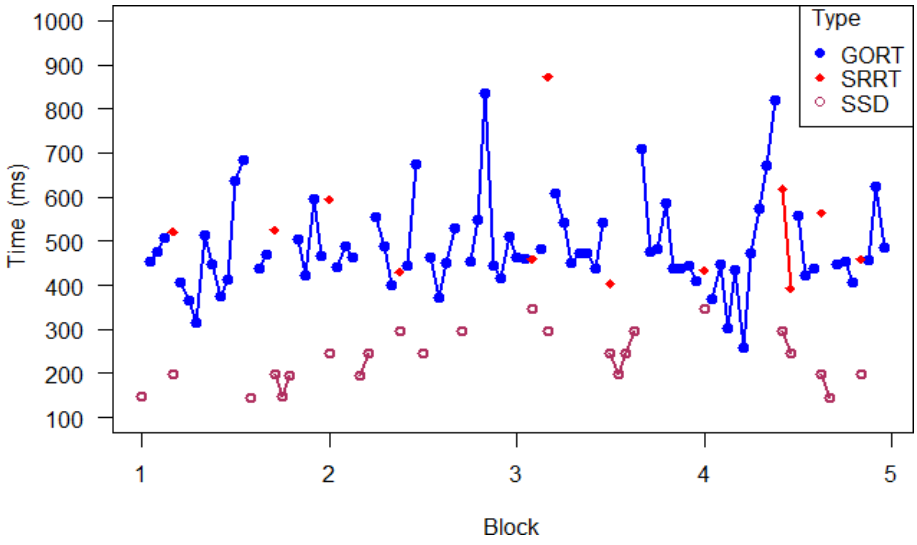

Figure 3: A 3-dimensional time series plot of Stop Signal Task (SST) data with 96 trials(72 go trials with GORT data points; 24 stop trials with 24 stop signal delay SSD data points and 12 signal respond reaction time SRRT data points).

Given that statistical technique, the state–space modelling of time series requires the input data to have normal distribution, we log transformed the data (Bland and Altman, 1996). In addition, the original lognormal distributional assumption for RT data is among the most accepted distributional forms in the RT literature, and it removes considerable

skewness, making the data's distribution near normal (Fengm, Hongyue, Lu, Chen, He, Lu, et al., 2014).

The details of the process are:

Step (i): We assumed a lognormal parametric distribution for GORT, SRRT, and SSD and, then we considered the following input data:

$$\begin{aligned} logGORT = log(GORT) \sim N(\mu, \sigma^2), \\ logSRRT = log(SRRT) \sim N(\mu^*, \sigma^{*2}) \\ logSSD = log(SSD) \sim N(\mu^{**}, \sigma^{**2}). \end{aligned} \tag{6}$$

Step (ii): We fit a Frequentist missing state-space model with relative tolerance of 1% to the incoming data(Shumway and Stoffer, 2017):

$$\begin{aligned} &\text{State Equation:}(x_t)_{3\times 1} = (\Phi)_{3\times 3}.(x_{t-1})_{3\times 1} + (w_t)_{3\times 1} : w_t \sim^{iid} N(0_{3\times 1}, Q_{3\times 3}) \\ &\text{Observation Equation:}(y_t^{(1)})_{q1t\times 1} = (A_t^{(1)})_{q1t\times 3}.(x_t)_{3\times 1} + (v_t^{(1)})_{q\times 1} : \\ &\qquad\qquad v_t^{(1)} \sim^{iid} N(0_{q1t\times 1}, R_{q1t\times q1t}). \end{aligned} \tag{7}$$

The observation matrix $A_t^{(1)}$ carries the trial type information from the previous trial to the current trial and either identity or identity with some diagonal values 0 whenever the preceding trial type is "go". The state equation can be written in the following format:

$$\begin{pmatrix} x_{t1} \\ x_{t2} \\ x_{t3} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} * \begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} w_{t1} \\ w_{t2} \\ w_{t3} \end{pmatrix} : \begin{cases} x_{t1} = logGORT_t \\ x_{t2} = logSRRT_t \\ x_{t3} = logSSD_t. \end{cases} \tag{8}$$

We applied the missing data EM method by R package ASTSA (Stoffer, 2017) to calculate the matrix $\Phi = (\phi_{ij})_{3\times 3}$ implying:

$$\begin{cases} \widehat{logGORT_t} = \phi_{11}.logGORT_{t-1} + \phi_{12}.logSRRT_{t-1} + \phi_{13}.logSSD_{t-1} \\ \widehat{logSRRT_t} = \phi_{21}.logGORT_{t-1} + \phi_{22}.logSRRT_{t-1} + \phi_{23}.logSSD_{t-1} \\ \widehat{logSSD_t} = \phi_{31}.logGORT_{t-1} + \phi_{32}.logSRRT_{t-1} + \phi_{33}.logSSD_{t-1} \ (1 \le t \le n) \end{cases} \tag{9}$$

where $n = 96$ for the real SST data and $n = 96k(k = 1,2,3,4,5,10)$ for the simulated SST data.

Step (iii). We used the estimated $\widehat{logGORT_t}(1 \le t \le m(m = 72k(k = 1,2,3,4,5,10))$ and used the frequentist MLE methods to fit normal distributions $N(\widehat{\mu}, \widehat{\sigma^2})$ to the corresponding state- space SST data (72k GORTs, 8k-14k SRRTs, 24k SSDs) matched to the original SST data.

Step (iv). The state-space estimation of SSRT for given probability of successful inhibition $P(SI)$ was computed as:

$$SSRT_{SS.Logan1994} = exp(\widehat{\mu} + \widehat{\sigma}.\Phi^{-1}(1 - P(SI))) - \overline{T_d}, \tag{10}$$

where Φ^{-1} is the quantile function of standard normal distribution, and the average of T_d is taken over all matched state-space stop trials.

We then repeat Steps (i) - (iv) with normal assumptions. Table 2 compares the overall

methodology applied in calculations of SSRT indices (1) - (3) with that of SSRT index in (10):

Table 2: Comparison of the current and State-Space method of estimations of SSRT

Method	Old
Used Data	Only GORT (72 trials) and SSD(24 trials)
Methodology	Does not consider impact of the preceding trial on the current trial
Distribution	Ex-Gaussian
Method	New
Used Data	Estimated state-space GORT (72 trials) and estimated state-space SSD (24 trials)
Methodology	Considers the impact of the preceding trial on the current trial via the observation matrix
Distribution	Lognormal/Normal

All formerly established SSRT indices in equations (1) - (3) were compared with the new index in equation (10) using the paired t-tests (PROC TTEST, (*SAS*/*STAT* software version9.4, 2012)). Given the distributional assumption for the SST data in the state space modelling (three variate lognormal or normal), independent sample t-tests were conducted between ADHD and control groups within each SSRT index.

3. Results

The results are divided into two subsections. In subsection 3.1, the new state space SSRT index is compared with $SSRT_{Weighted}$,$SSRT_{Mixture}$, and $SSRT_{Logan1994}$ in terms of size and differential impact between clinical groups. First, for the case of comparison of estimations, the following statistical hypothesis test is conducted:

$$\begin{aligned} H_0 &: SSRT_{SS.Logan1994} = SSRT_{Logan1994}, \\ H_1 &: SSRT_{SS.Logan1994} \neq SSRT_{Logan1994}, \end{aligned} \tag{11}$$

Second, for the case of differential impact between clinical groups, the following statistical hypothesis test is conducted:

$$\begin{aligned} H_0 &: SSRT^{ADHD}_{SS.Logan1994} = SSRT^{Control}_{SS.Logan1994}, \\ H_1 &: SSRT^{ADHD}_{SS.Logan1994} \neq SSRT^{Control}_{SS.Logan1994}, \end{aligned} \tag{12}$$

Similar hypothesis tests are conducted with replacing $SSRT_{Logan1994}$ in (11) with $SSRT_{Mixture}$ and $SSRT_{Weighted}$; and, with replacing $SSRT_{SS.Logan1994}$ in (12) with $SSRT_{Mixture}$ and $SSRT_{Weighted}$.

In subsection 3.2, the comparisons in terms of size of the estimates are repeated for the simulated data and the asymptotic behaviour of the sizes. Their sensitivity to the distributional assumptions is studied.

3.1 State-Space SSRT for the real SST data

Table 3 presents results for the new estimated state-space SSRT and compares it to the established SSRTs.

Table 3: Paired t-test and two sample t-test results of SSRT indices by distributional assumption (n = 44).

(a) Measurement Comparisons					
Measurement	Population	Distribution	Mean(95%CI)	t	Sig.(2-tailed)
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$	Overall	Lognormal	13.1(8.4,17.6)	5.7	< 0.0001
		Normal	21.9(17.4,26.3)	9.9	< 0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	Overall	Lognormal	-0.6(-9.8,8.7)	-0.1	0.9
		Normal	8.3(0.2,16.4)	2.1	0.04
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$	Overall	Lognormal	-1.2(-8.4,6.0)	-0.3	0.7
		Normal	7.7(1.3,14.1)	2.4	0.02
(b) Differential Impact					
Measurement	Population	SST Distribution	Mean(95%CI)	t	Sig.(2-tailed)
$SSRT_{SS.Logan}$	ADHD vs. Control	Lognormal	58.6(3.0,114.2)	2.3	0.04
		Normal	58.8(1.8,115.6)	2.3	0.04
$SSRT_{Logan1994}$		Ex-Gaussian	66.5(10.5,122.5)	2.6	0.02
$SSRT_{Mixture}$		Ex-Gaussian	65.6(5.4,125.9)	2.4	0.04
$SSRT_{Weightd}$		Ex-Gaussian	62.3(5.1,119.5)	2.4	0.04

There are five key results from Table 3. First, the $SSRT_{SS.Logan1994}$ was significantly larger than $SSRT_{Logan1994}$ under both distributional assumptions (Lognormal: 13.1: 95%CI =(8.1, 17.6); Normal: 21.4:95%CI =(17.4,26.3)); second, there were no significant differences between $SSRT_{SS.Logan1994}$ and two former indices $SSRT_{Mixture}$ and $SSRT_{Weighted}$ overall, under the lognormal distributional assumption. However, under normal distributional assumption, the former index was significantly larger than the latter two [8.3:95%CI = (0.2, 16.4); 7.7:95%CI = (1.3, 14.1), respectively]. Third, the ADHD participants had 58.6 ms $[95\%CI = (3.0, 114.2)]$ higher $SSRT_{SS.Logan1994}$ values than controls under the lognormal distributional assumption. A similar result was observed under the normal distributional assumption. Fourth, the differential impact of $SSRT_{SS.Logan1994}$ was slightly weaker than the two former new indices $SSRT_{Mixture}$ and $SSRT_{Weighted}$ under the lognormal distributional assumption (58.6 ms vs 65.6 ms, 62.3 ms). Finally, similar conclusions were found under the normal distributional assumption. Figure 4 depicts the comparison of the regular estimation of SSRT given by (1) and its state-space counterpart given by (10), given by the results in Table 3.

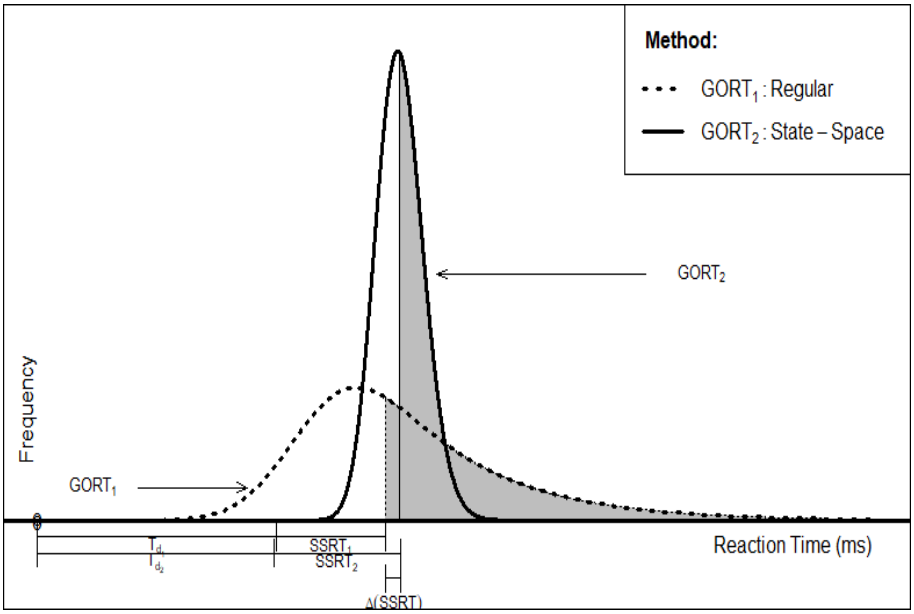

Figure 4: Regular and State-Space estimations of SSRT

From Table 3(a), it follows that under different underlying distributional assumptions for GORT, SRRT, and SSD, we obtain different estimates when comparing $SSRT_{SS.Logan1994}$ and the other three indices. To confirm the assumption of sensitivity of the state-space SSRT estimation to the underlying distributional assumption, we will conduct the comparisons for the simulations in the next section.

3.2 Simulations & Asymptotic Behaviour

In order to check the impact of the underlying distributional assumptions in the state-space models on the estimated SSRT indices, we simulate SST data as shown in subsection 2.1.2. Table 4 presents the results of pairwise t-tests for each given participant sample size m under the lognormal distributional assumption for the simulated ex-Gaussian GORT, SRRT, and SSRT. Such an assumption is justified, given that the shifted lognormal distribution provides a good fit for the ex-Gaussian distribution of the RT data (Ratcliff and Murdock, 1976).

Comparing the results from Table 3, Panel(a), and Table 4, we conclude:

Under the lognormal assumption for the real data:

Result (i): The difference between the state-space and the conventionally estimated $SSRT_{Logan1994}$ in the simulated data is the same as in the original real data. However, the size of differences in the former (8.1-11.8 ms) is smaller than the latter (13.1 ms), and with increasing simulated sample sizes, their gap diminishes.

Result (ii): The difference between the state-space estimated $SSRT_{Logan1994}$ and the conventional $SSRT_{Mixture}$ in the simulated data is in the range 8.5 – 11.7 ms and very different from the non-significant difference in the original real data.

Result (iii): The difference between the state-space estimated $SSRT_{Logan1994}$ and the conventional $SSRT_{Weighted}$ in the simulated data is in the range of 4.0 – 5.4 ms, and somewhat different than that of their non-significant difference in real data.

Table 4: Paired t-test results of simulated state-space SSRT indices by m(n = 264).

Pair	N(#SST)	m(#stop)	Mean(95%*CI*)	*t*	Sig.(2-tailed)
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$			8.1(6.5, 9.8)	9.9	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	96	24	8.5(6.8, 10.1)	10.3	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$			4.0(2.2, 5.8)	4.5	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$			10.2(9.3, 11.1)	23.1	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	192	48	10.4(9.5, 11.3)	23.1	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$			4.2(3.3, 5.2)	8.8	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$			10.4(9.5, 11.2)	22.8	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	288	72	10.6(9.6, 11.5)	22.8	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$			4.9(3.7, 5.4)	11.7	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$			11.4(10.4, 12.3)	24.5	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	384	96	11.1(10.0, 12.1)	12.3	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$			5.0(4.2, 5.8)	12.3	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$			11.4(10.5, 12.3)	26.6	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	480	120	11.3(10.4, 12.3)	24.1	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$			4.8(4.0, 5.6)	12.4	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Logan1994}$			11.8(11.2, 12.6)	34.0	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Mixture}$	960	240	11.7(11.0, 12.4)	33.3	<0.0001
$SSRT_{SS.Logan1994} - SSRT_{Weighted}$			5.4(4.8, 5.9)	20.0	<0.0001

Under the normal assumption for the real data:

Result (i): The difference between the state-space and conventionally estimated $SSRT_{Logan1994}$ in the simulated data (8.1 ms-11.8 ms) is significantly smaller than in the original real data (21.9).

Result (ii): The difference between the state-space estimated $SSRT_{Logan1994}$ and conventional $SSRT_{Mixture}$ in the simulated data (8.1 ms-11.8 ms) is similar to that of the real data (8.3 ms).

Result (iii): The difference between the state-space estimated $SSRT_{Logan1994}$ and conventional $SSRT_{Weighted}$ in the simulated data (8.1 ms-11.8 ms) is similar to that of real data (7.7 ms).

These two sets of results show that one needs to check underlying distributional assumptions for state-space models in calculating SSRT's state-space indices. Figure 5 presents the indices' differences in terms of simulated sample size. There are two main results: First, the main increment on the index difference occurs from sample size $m = 96$ to $m = 192$; Second, after simulated sample size $m = 480$, the trend has almost asymptotic constant behaviour.

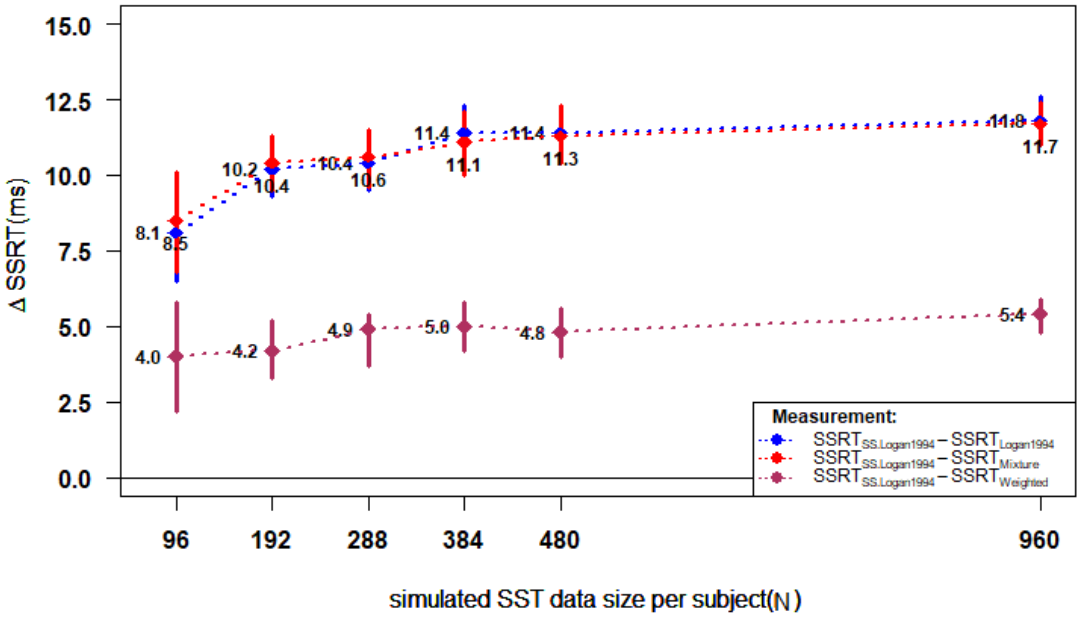

Figure 5: Disparities of simulated SSRT indices by sample size (n = 264).

4. Discussion

This study has presented a time series-based methodological approach for a more informed estimation of SSRT by considering state-space nature of SST time series data, and lognormal format of the involved distributions offering a new index of SSRT. It introduced time

series based state–space SSRT as the third index of SSRT, which considered trial order in the stop task data. It hypothesized that considering previous trial type (stop/go) in calculation of the new time series based index does impact estimations of SSRT, as it was shown in the case of the longitudinal approach (Soltanifar, Dupuis, Schachar, and Escobar, 2019).

The majority of findings of the study affirmed the hypothesis. State-space estimations of Logan's 1994 SSRT were 13.1 ms and 21.9 ms significantly larger than their regular estimations under a normal or lognormal distribution assumption for SST data, respectively. This result was confirmed by similar significantly higher estimations (8.1 ms - 11.8 ms) from simulated SST data under the ex-Gaussian distributional assumption. However, there were no significant differences between two indices of SSRT on their differential impact between clinical groups. There was consistency in comparing the results of the state-space SSRT and the Logan 1994 SSRT indices using both real and simulated SST data. While in the majority of cases, the new time series based index is different from established indices (as shown in this study), there are special cases where these four indices will be precisely equal. Two special cases include when each stop trial is preceded by a go trial, i.e. $(A_t^{(1)})_{q1t\times 3} = 0_3$ and when each stop trial is preceded by another stop trial, i.e. $(A_t^{(1)})_{q1t\times 3} = I_3$.

The study's results based on the time series method were consistent with those of longitudinal method (Soltanifar, Dupuis, Schachar, and Escobar, 2019), considering the impact of the preceding trial type on the current stop trial SSRT in the calculation of SSRT. The first consistency is that when the researcher considers the trial order when considering the SST data, they obtain significantly larger estimates versus when they ignore trial order. This is the common conclusion in both approaches. One explanation for this commonality is that a participant's stopping skills improves immediately after stop trials compared to go trials in the SST. This is in accord with examples in previous literature, such as a participant's optimized control skills in playing video games with dual task and task switching situation (Strobach, Frensch and Schubert, 2012), and a participant's improved visuomotor control in playing action video games (Li, Chen and Chen, 2016). In our case, once the order of SST trials is considered in the calculations, for the those preceded by a stop trial the participant's stopping improves by his or her on taking longer go reaction times (GORT), and hence, the latency of the stopping process SSRT increases. This yields to increase in the estimated SSRT. The second consistency is that there is no statistically significant difference between clinically differential impact(ADHD versus Control) using longitudinal perspective estimations of SSRT versus time series based estimation of SSRT. On defining the ADHD based on SWAN z-score we followed a trait based approach rather than emulating diagnosis.

The study's proposed time series based method is; however, less favourable than that of time in task longitudinal based method (Soltanifar, Dupuis, Schachar, and Escobar, 2019) given few considerations as follows: First, the calculations in the new method (in particular compared to the Weighted SSRT index in the old method) are more difficult than the old one. Second, the calculations in the new method are susceptible to satisfaction of underlying lognormal distributional assumption for the SST data. Third, the calculations in the new method are dependent to the size of the relative tolerance of the missing data EM algorithm. Finally, given that (i) ADHD as a trait is likely to reflect participants who make alot of errors in go trials and many signal responds in stop trials; (ii) the new estimation method considered preceding trial type into account; there was no progress in finding better differential impact (ADHD versus Control) in the new method.

This study's findings are restricted in a few aspects. The first is the assumption of a lognormal distribution for GORT, SRRT and SSRT in the state–space estimations of Logan 1994 SSRT. The optimum situation would be ex-Gaussian distributional assumption for GORT, SRRT, and SSRT in the new method such that the only remaining difference be-

tween the conventional method and the new method would be consideration of the nature (go/stop) preceding trials in the SST data. Thus, such assumption limited comparison of the results in their more customary assumption of ex-Gaussian distribution (Verbruggen, Aron, Band, Beste, Bissett, Brockett, et al., 2019). The second is the sensitivity of the state-state approach in calculating SSRT to a multivariate normal distributional assumption of the SST data, and upon violation of normality, inconsistent results may yield. This is evident from both real and the simulated SST data results, in simulated data, given good fit of the log-normal distribution to the simulated Ex-Gaussian SST data, the results for the calculated state-space SSRT versus the regular SSRT are consistent. The third one is that state-space calculation of SSRT depends on the relative tolerance of the missing data EM algorithm in the calculation of the state-space SST data. While we chose 1% for this purpose, other values may yield different state-space SST data and a different state-space SSRT estimate. Consequently, they may impact their comparison of non-state space SSRT estimates. Finally, for simplicity of the calculations, it was assumed that there were no trigger failures or randomized trigger failures in the SST data (Matzke, Love and Heathcote, 2017).

The approach outlined in this study should be replicated in the future research in two directions. The first is that the study should be replicated for in adult participants who can perform longer tasks and produce a higher number of stop trials (e.g., 200 SST trials with 50 stop trials, as recommended (Verbruggen, Aron, Band, Beste, Bissett, Brockett, et al., 2019), to confirm the current results at older ages and across a larger number of trials. The second is to consider non randomized trigger failures and their probabilities in the SST data in the calculations of the state-space Logan 1994 SSRT and to compare the results with the former established estimates.

Conclusion

The relative position of stop and go trials in the entire stop signal task data has been shown to be a key factor in the estimation of the associated SSRT based on the time in task longitudinal method(Soltanifar, Dupuis, Schachar, and Escobar, 2019). This study provided further evidence on this finding from time series perspective, paving the way for more refinement in the estimates. Given consistency of results in both methods and advantages of the first method, the researchers are recommended to consider Weighted SSRT ($SSRT_{Weighted}$) as the latest optimum option for estimation of the latency stopping process in the brain.

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Abbreviations

ADHD	Attention Deficit Hyperactivity Disorder
GORT	Reaction time in a go trial
GORTA	Reaction time in a type A go trial
GORTB	Reaction time in a type B go trial
SSD	Stop Signal Delay
SI	Successful Inhibition
SR	Signal Respond
SRRT	Reaction time in a failed stop trial
SSRT	Stop Signal Reaction Times in a stop trial
SSRTA	Stop Signal Reaction Times in type A stop trial
SSRTB	Stop Signal Reaction Times in type B stop trial
SST	Stop Signal Task
SWAN	Strengths and Weakness of ADHD-symptoms and Normal behavior rating scale

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Appendix

Cluster type Weight Calculations in the Simulation of SST data

Here we show that the weight $W_A = 0.75$ is the most natural weight given independence of assignment of stop or go process to the given trial. To see this, let $T_1, \cdots, T_{96}, S_1, \cdots, S_{24}$ and $G_1, \cdots, G_{72}$ denote all trials, stop trials and go trials. Then, given that 25% of all trials are go trials, it follows that:

$$\begin{aligned}
W_A &= \frac{\#\{\exists j(T_i = S_j), \exists k(T_{i-1} = G_k) | 1 \le i \le 96\}}{24} \\
&= \frac{\#\{\exists j(T_i = S_j), \exists k(T_{i-1} = G_k) | 1 \le i \le 96\}/96}{24/96} \\
&= \frac{P(\exists j(T_i = S_j), \exists k(T_{i-1} = G_k) | 1 \le i \le 96}{P(\exists j(T_i = S_j) | 1 \le i \le 96)} \\
&= \frac{P(\exists j(T_i = S_j) | 1 \le i \le 96) \times P(\exists k(T_{i-1} = G_k) | 1 \le i \le 96)}{P(\exists j(T_i = S_j) | 1 \le i \le 96)} \\
&= P(\exists k(T_{i-1} = G_k) | 1 \le i \le 96) \simeq 0.75.
\end{aligned}$$