The panic contagion probability during an evacuation process

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Abstract

Panic spreads like a contagious disease among people in a crowd. Any individual closely exposed to a panic source may express his (her) feelings, alerting others of imminent danger. The more intense these feelings are, the more likely that a neighboring individual move to panic. Thus, a "contagion probability" exists for the panic spreading. The contagion probability plays a major role in the overall evacuation process. We examined the evacuation dynamics in the context of the "Social Force Model". Our investigation shows that two possible evacuation patterns may appear, according to the contagion probability level. Both patterns are in agreement with real life recordings from crowded events. We were able to determine the probability threshold for the occurrence of each evacuation pattern. We further investigated how the panic spreading gradually stops if the source of danger ceases.

Key Words: Panic, Contagion, Calm Down.

1. Introduction

Many authors called the attention on the fact that panic is a contagious phenomenon [1, 2, 3, 4]. Panic may spread over any simple "social group" if some kind of coupling mechanism exists between agents [1]. This coupling mechanism corresponds to social communication appearing in the group. As a consequence, the individuals (agents) may change their anxiety state from relaxed to a panic one (and back again) [1].

Panic contagion over the crowd can be attained if the coupling mechanism between individuals is strong enough and affects many neighboring pedestrians [1]. Research on random lattices shows that the coupling stress becomes relevant whenever the number on neighbors is small (*i.e.* less than four). That is, a small connectivity number between agents (pedestrians) requires really moving gestures [1].

Recent investigation suggests that other psychological mechanisms than social communication can play an important role during the panic spreading over the crowd [2, 3, 5]. Susceptibility appear as relevant attributes that control the panic propagation [2]. Consequently, diseases contagion models are usually introduced when studying the panic spreading. The Susceptible-Infected-Recovered-Susceptible (SIRS) model raises as a suitable research tool for examining the panic dynamics. The spreading model is, therefore, represented as a system of first order equations [2, 5].

According to the SIRS model implemented in Ref. [2], a dramatic contagion of panic can be expected in those crowded situations where the individuals are not able to calm down quickly. However, these individuals may relax after some time due to "stress decay"

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(if no emotions of fear are received by the corresponding neighbors) [3, 4]. That is, there is actually not a probability to switch from the anxious (infected) state to the relaxed (recovered) state as in the SIRS model, though a natural decay occurs along time. Thus, the increase in the "inner stress" and the "stress decay" are actually the two main phenomena attaining for the pedestrians behavior.

Our investigation focuses on a real life situation. Our aim is to develop a model for describing a striking situation, where many individuals may suddenly switch to an anxious state. We will focus on the video analysis in order to obtain reliable parameters from a real panic-contagion events, and further test these parameters on computing simulations.

2. The Social Force Model context

The Social Force Model (SFM) exploits the idea that human motion depends on the people's own desire to reach a certain destination, as well as other environmental factors [6]. The former is modeled by a force called the "desire force", while the latter is represented by social forces and "granular forces". These forces enter the motion equation as follows

$$m_i \,\frac{d\mathbf{v}^{(i)}}{dt} = \mathbf{f}_d^{(i)} + \sum_{j=1}^{N} \mathbf{f}_s^{(ij)} + \sum_{j=1}^{N} \mathbf{f}_g^{(ij)} \tag{1}$$

where the i, j subscripts correspond to any two pedestrians in the crowd. $\mathbf{v}^{(i)}(t)$ means the current velocity of the pedestrian (i), while $\mathbf{f}_d$ and $\mathbf{f}_s$ correspond to the "desired force" and the "social force", respectively. $\mathbf{f}_g$ is the friction or granular force.

The $\mathbf{f}_d$ attains the pedestrians own desire to reach a specific target position at the desired velocity v_d. But, due to environmental factors (*i.e.* obstacles, visibility), he (she) actually moves at the current velocity $\mathbf{v}^{(i)}(t)$. Thus, the acceleration (or deceleration) required to reach the desired velocity v_d corresponds to the aforementioned "desire force" as follows [7]

$$\mathbf{f}_d^{(i)}(t) = m_i \,\frac{v_d^{(i)}\,\mathbf{e}_d^{(i)}(t) - \mathbf{v}^{(i)}(t)}{\tau} \tag{2}$$

where m_i is the mass of the pedestrian i and τ represents the relaxation time needed to reach the desired velocity. $\mathbf{e}_d$ is the unit vector pointing to the target position. Detailed values for m_i and τ can be found in Refs. [7, 8].

Besides, the "social force" $\mathbf{f}_s(t)$ represents the socio-psychological tendency of the pedestrians to preserve their *private sphere*. The spatial preservation means that a repulsive feeling exists between two neighboring pedestrians, or, between the pedestrian and the walls [7, 6]. This repulsive feeling becomes stronger as people get closer to each other (or to the walls). Thus, in the context of the social force model, this tendency is expressed as

$$\mathbf{f}_s^{(ij)} = A_i \, e^{(r_{ij}-d_{ij})/B_i}\mathbf{n}_{ij} \tag{3}$$

where (ij) corresponds to any two pedestrians, or to the pedestrian-wall interaction. A_i and B_i are two fixed parameters (see Ref. [9]). The distance $r_{ij} = r_i + r_j$ is the sum of the pedestrians radius, while d_{ij} is the distance between the center of mass of the pedestrians i and j. $\mathbf{n}_{ij}$ means the unit vector in the $\vec{ji}$ direction. For the case of repulsive feelings with the walls, d_{ij} corresponds to the shortest distance between the pedestrian and the wall, while $r_{ij} = r_i$ [7, 6].

It is worth mentioning that the Eq. (3) is also valid if two pedestrians are in contact (*i.e.* $r_{ij} > d_{ij}$), but its meaning is somehow different. In this case, $\mathbf{f}_s$ represents a body repulsion, as explained in Ref. [10].

The granular force $\mathbf{f}_g$ included in Eq. (1) corresponds to the sliding friction between pedestrians in contact, or, between pedestrians in contact with the walls. The expression for this force is

$$\mathbf{f}_g^{(ij)} = \kappa\,(r_{ij} - d_{ij})\,\Theta(r_{ij} - d_{ij})\,\Delta\mathbf{v}^{(ij)} \cdot \mathbf{t}_{ij} \tag{4}$$

where κ is a fixed parameter. The function $\Theta(r_{ij} - d_{ij})$ is zero when its argument is negative (that is, $r_{ij} < d_{ij}$) and equals unity for any other case (Heaviside function). $\Delta\mathbf{v}^{(ij)} \cdot \mathbf{t}_{ij}$ represents the difference between the tangential velocities of the sliding bodies (or between the individual and the walls).

3. The inner stress model context

Complementary to the SFM, the "inner stress" stands for the cumulative emotions that the pedestrian receives from his (her) neighbors. This magnitude may change the pedestrian's behavior from a relaxed state to panic, and consequently, we propose that his (her) desired velocity v_d increases as follows [4]

$$v_d(t) = [1 - M(t)]\,v_d^{\min} + M(t)\,v_d^{\max} \tag{5}$$

for $M(t)$ representing the "inner stress" as a function of time. The minimum desired velocity $v_d^{\min}$ corresponds to the (completely) relaxed state, while the maximum desired velocity $v_d^{\max}$ corresponds to the (completely) panic state.

The inner stress $M(t)$ in Eq. (5) is assumed to be bounded between zero and unity. Vanishing values of $M(t)$ mean that the pedestrian is relaxed, while values approaching unity correspond to a very anxious pedestrian (*i.e.* panic state).

The emotions received from the pedestrian's surrounding are responsible for the increase in his (her) inner stress $M(t)$. But, in the absence of stressful situations, some kind of relaxation occurs (say, the "stress decay"), attaining a decrease in $M(t)$. Following Ref. [11], a first order differential equation for the time evolution of $M(t)$ can be assumed

$$\frac{dM}{dt} = -\frac{M}{\tau_M} + \mathcal{P} \tag{6}$$

The differential ratio on the left of Eq. (6) expresses the change in the "inner stress" with respect to time. Whenever the pedestrian receives alerting emotions from his (her) neighbors (expresses by the contagion efficiency $\mathcal{P}$), the "inner stress" is expected to increase. But, if no alerting emotions are received, his (her) stress is expected to decay according to a fixed relaxation time τ_M. Thus, the first term on the right of Eq. (6) handles the settle down process towards the relaxed state. The second term on the right, on the contrary, increases his (her) stress towards an anxious state.

We assume that the parameter $\mathcal{P}$ attains the emotions received from alerting (anxious) neighbors within a certain radius, called the *contagion radius*. As described in Appendix A, if k pedestrians among n neighbors are expressing fear, then the actual value of $\mathcal{P}$ is

$$\mathcal{P} = J\left\langle \frac{k}{n} \right\rangle \tag{7}$$

where the parameter J represents an *effective contagion stress* (see Appendix A for details). This parameter resembles the pedestrian susceptibility to enter in panic. For simplicity we further assume that this parameter is the same for all the pedestrians.

The symbol $\langle\cdot\rangle$ represents the mean value for any short time interval (see Appendix A for details). However, for practical reasons, we will replace this mean value with the sample value k/n at each time-step.

3.1 The stress decay model

The pedestrian "stress decay" corresponds to the individual's natural relaxation process in the absence of stimuli (*i.e.* emotions), until he (she) settles to relaxed. This behavior is mathematically expressed through the relaxation term in Eq. (6). Thus, in the absence of stimuli (that is, vanishing values of $\mathcal{P}$), it follows from Eq. (5) and Eq. (6) that

$$v_d(t) = v_d^{\min} + (v_d^{\max} - v_d^{\min})\, M(0)\, e^{-t/\tau_M} \tag{8}$$

for any fixed value $M(0)$ at $t = 0$, and a vanishing value of $M(t)$ long time after ($t \gg \tau_M$). The characteristic time τ is different from τ_M. Ref. [12] suggests that $\tau_M \simeq 50$ seconds.

The characteristic time τ_M may be different from the suggested value according to specific environmental factors. Eq. (8) proposes the way to handle an estimation of τ whenever the composure desired velocity $v_d(t_c)$ is known (t_c being the time required to arrive to composure). Assuming $M(0) = 1$, it is straight forward that

$$\tau_M^{-1} = \frac{1}{t_c}\,\ln\left(\frac{v_d^{\max} - v_d^{\min}}{v_d(t_c) - v_d^{\min}}\right) \tag{9}$$

We computed an experimental value for τ_M within the context of the panic spreading analyzed in Section 4. According to Eq. (9) and setting $v_d^{\min}$ = 0 m/s, $v_d^{\max}$ = 4 m/s and $v_d(t_c)$ = 0.5 m/s ($t_c = 20$ s), this value equals approximately 10 seconds.

4. The video analysis

On June 3rd 2017, many Juventus fans were watching the Champions League final between Juventus and Real Madrid on huge screens at Piazza San Carlo. During the second half of the match, a stampede occurred when one (or more) individuals shouted that there was a bomb. More than 1000 individuals were injured during the stampede, although it was a false alarm. Fig. 1 captures two moments of the panic spreading (see caption for details). The arrow in Fig. 1b points to the individual that caused the panic spreading. He will be called the *fake bomber* throughout this investigation.

(a) Before panic. (b) Pedestrians in panic. (c) Panic spreading.

Figure 1: (Color on-line only) (a) Snapshot of the crowd watching the football match. The screen is on the left (out of the scene). The pedestrians on the right are actually in fear due to the *fake bomber*. The (b) snapshot corresponds to the same scene as (a) but shifted to the right (actually, the camera appearing in this image is the one that captured the (a) image). The *fake bomber* appears in the scene and is indicated with a green arrow. (c) Analysis of the panic spreading among the crowd. The blue and orange profiles represent the relaxed and anxious pedestrians, respectively, associated to the (a) image (see text for details). In fact, in the (b) image we can observe (on the right of the image) the camera that captured the (a) image. The total number of contour bodies is $N = 131$.

The recordings from Piazza San Carlo show how the pedestrians escape away from the "panic source", that is, from the *fake bomber*. It can be seen in Fig. 1b the opening around the panic source a few seconds after the shout. The opening exhibits a circular pattern around the *fake bomber*. This pattern gradually slows down as the pedestrians realize the alarm being false. Approximately 20 seconds after the shout, the pedestrians calm down to the relaxed state while the opening closes.

In order to quantify the panic contagion among the crowd, we split the video into 14 images. The frame rate was 2 frames per second. Thus, the time interval between successive images was 0.5 seconds. This time interval corresponds to the acceleration time τ in the SFM.

Fig. 1c shows the profile corresponding to the first image. Any (distinguishable) pedestrian in Fig. 1a is outlined in Fig. 1c as a body contour. The contour colors represent relaxed pedestrians (*i.e.* blue in the on-line version) or pedestrians in panic (*i.e.* orange in the on-line version). The latter correspond to the individuals that suddenly changed their motion pattern. That is, individuals that turned back to see what happened or pedestrians that were pushed towards the screen (on the left) due to the movement of his (her) neighbors.

The panic spreading shown in Fig. 1c occurs from right to left, until nearly all the contour bodies switch to the panic state (*i.e.* orange in the on-line version). Notice, however, that a few pedestrians may remain relaxed for a while, even though his (her) neighbors have already switched to the panic state. Or, on the contrary, pedestrians in panic may be completely surrounded by relaxed pedestrians, as appearing on the left of Fig. 1c. Both instances are in agreement with the hypothesis that pedestrians may switch to a panic state according to an *contagion efficiency* $\mathcal{P}$. See Appendix A for details on the $\mathcal{P}$ computation within the contagion radius.

The inspection of successive images provides information on the new anxious or panicking pedestrians and the state of their current neighbors. Appendix B summarizes this information, while detailed values for the contagion efficiency $\mathcal{P}$ and the contagion stress J are reported in Table 1. Notice that the data sampling is strongly limited by the total

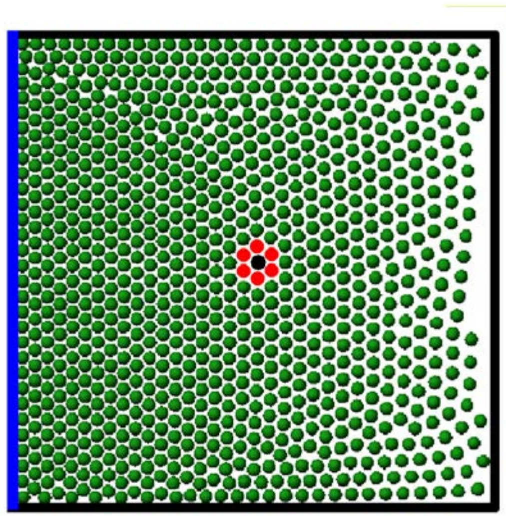


Figure 2: Snapshots of the initial configuration for the Turin (Italy) simulation. The relaxed pedestrians are represented in green circles on both images. The *fake bomber* is represented in black, while his first neighbors are represented in red (see text for details). The blue line on the left represents the wide screen.

number of outlined pedestrians (that is, 131 individuals). Thus, the reported values for $t > 4\,$s are not really suitable as parameter estimates because of the finite size effects. In order to minimize the size effects, we focused on the early stage of the contagion were the contagion stress J seems to be (almost) stationary (see Fig. 5).

The (mean) contagion stress for the Turin incident was found to be $J = 0.1 \pm 0.055$ (within the standard deviation). This value appears to be surprisingly low according to explored values in the literature (see Ref. [11]). However, we shall see in Section 6 that this stress is enough to reproduce real life incidents.

5. Simulations

We mimicked the Turin incident (see Section 4) by first placing 925 pedestrians inside a 21 m $\times$ 21 m square region. The pedestrians were placed in a regular square arrangement, meaning that the occupancy density was approximately 2 people/m^2. After their desire force was set (see below), the crowd was allowed to move freely until the pedestrian's velocity vanished. This balance situation can be seen in Fig. 2 and corresponds to the initial configuration for the panic spreading simulation.

We assumed that the pedestrians are attracted to a wide screen on the left (blue line in Fig. 4a) in order to have a better view of the football match. Thus, a (small) desire force pointing towards the screen was included at the beginning of the simulation. This force equaled $m\,v_d/\tau$ for the standing still individuals ($v(0) = 0$), according to Eq. (2). We further assumed that the pedestrians were in a relaxed state at the beginning of the simulation, and therefore, we set $v_d = 0.5\,$m/s [9]. This value accomplished a local density that did not exceed the maximum expected for outdoor events, say, 3-4 people/m^2.

The pedestrian in black in the middle of the crowd in Fig. 4a represents the *fake bomber* appearing in the video. He is responsible for triggering the panic contagion at the beginning of the simulations. For simplicity, we assumed that he remained still during the panic spreading process.

Recall that the event takes place outdoor. Piazza San Carlo, however, is surrounded by walls (as can be seen in Fig. 4). We considered along the simulations that the crowd always remained inside the *piazza* and no other pedestrian were allowed to get inside during the process.

The video that captures the panic spreading over the crowd let us classify the pedestrians into those moving relaxed or those moving anxiously. These are qualitative categories that can be easily recognized through the pedestrian's behavioral patterns. An accurate value for the inner stress M seems not to be possible from the videos. Thus, we assume that the pedestrians may be in one of two possible states: relaxed or in panic. The former means that his (her) desired velocity does not exceed a fix threshold v_d^{lim}, or M_{lim}, according to Eq. (5). The latter means that the individual surpassed this threshold.

We already mentioned that the desired velocity $v_d = 0.5$ m/s is in correspondence with either accepted literature values for relaxed individuals and the expected local density for approximately 900 individuals. Hence, we set $v_d^{\mathrm{lim}} = 0.5$ m/s as a reasonable limit for the pedestrian to be considered relaxed.

For simplicity, v_d^{min} and v_d^{max} (Eq. (5)) were set to zero and 4 m/s, respectively, in all the simulations. The maximum velocity $v_d^{\mathrm{max}} = 4$ m/s corresponds to reasonable anxiety situations appearing in the literature [6, 8, 13, 14].

The panic contagion process was implemented as follows. First, we associated an effective contagion stress $\mathcal{P}^{(i)}$ to each relaxed individual, according to Eq. (7). That is, we computed the fraction of neighbors in the panic state k to the total number of neighbors n within a fix contagion radius of 2 m (from the center of mass of the corresponding relaxed pedestrian i). Second, we randomly switched the relaxed pedestrians to the panic state, according to the associated effective contagion stress $\mathcal{P}^{(i)}$. The $\mathcal{P}^{(i)}$ values were updated at each time step (say, 0.05 s).

Notice that this contagion process may be envisaged as a Susceptible-Infected-Susceptible (SIS) process. The Susceptible-to-Infected transit corresponds to the (immediate) increase of v_d from 0.5 m/s to 4 m/s (with effective contagion stress $\mathcal{P}^{(i)}$). The Infected-to-Susceptible transit corresponds to the stress decay from 4 m/s back to 0.5 m/s.

We want to remark the fact that the emotions received by an individual in the panic state were neglected, and thus, did not affect the stress decay process. This should be considered a first order approach to the panic contagion process.

The simulations were implemented on the LAMMPS molecular dynamics simulator [15]. LAMMPS was set to run on multiple processors. The chosen time integration scheme was the velocity Verlet algorithm with a time step of 10^{-4} s. Any other parameter was the same as in previous works (see Refs. [13, 8]).

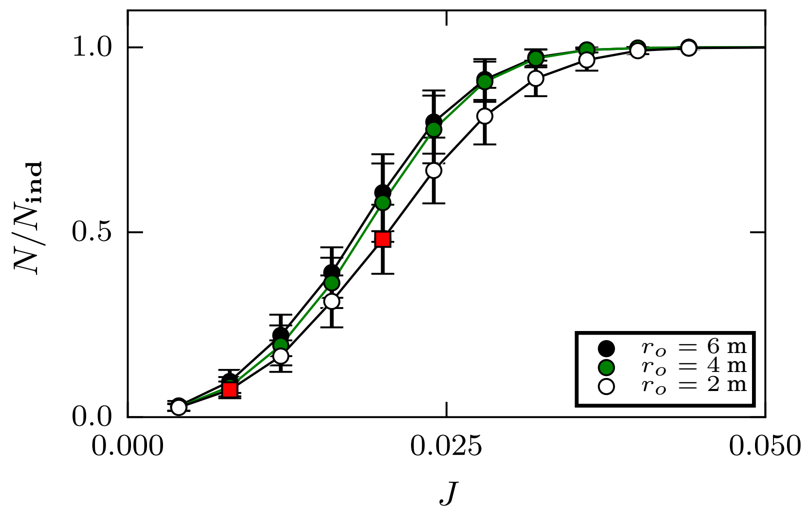


Figure 3: Normalized number of anxious pedestrians during the first 20 s of the escaping process as a function of the contagion stress J for r_o = 2 m, 4 m and 6 m. N is the number of anxious pedestrians. The plot is normalized with respect to the total number of individuals (N_{ind} = 925). J-values of 0.01 and 0.02 are indicated in red color (and squared symbols). Mean values were computed from 60 realizations. The error bars corresponds to $\pm\sigma$ (one standard deviation).

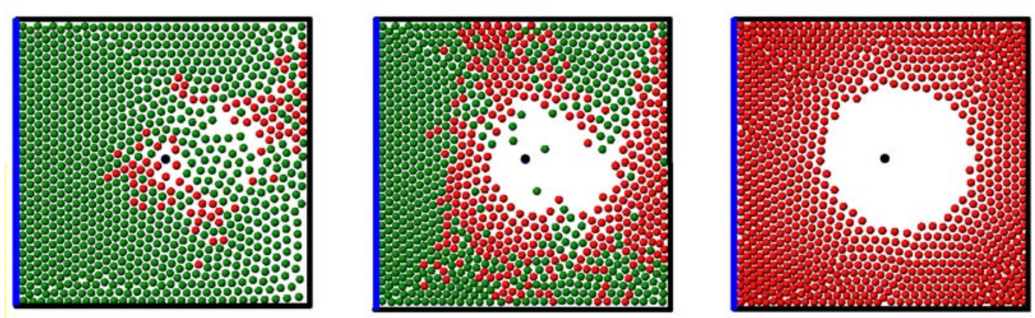
6. Results

As a first step, we measured the mean number of anxious pedestrians during the first 20 s of the escaping process for a wide range of contagion stresses (J). This is shown in Fig. 3. As can be seen, the number of anxious pedestrians increases for increasing contagion stresses. That is, as pedestrians become more susceptible to the fear emotions from his (her) neighbors, panic is allowed to spreads easily among the crowd.

The fraction of pedestrians that switch to the anxious state exhibits three qualitative categories as shown in Fig. 3. For J ranging between 0 to 0.01, no significant spreading appears. But this scenario changes rapidly for the J (intermediate) range between 0.01 and 0.03. The slope in Fig. 3 experiences a maximum throughout this interval. However, if the stress becomes stronger (say, above 0.03), the majority enters into panic regardless of the precise value of J. A seemingly threshold for this is around $J = 0.04$.

Notice that Fig. 3 is in agreement with the experimental Turin value for the mean contagion stress ($J = 0.100 \pm 0.055$, see Section 4). The panic situation at Piazza San Carlo, as observed from the videos, shows that all the pedestrians moved to the panic state. The snapshot in Fig. 1b illustrates the situation a while after the (fake) bomber called for attention.

The panic contagion shown in Fig. 3 does not appear to change significantly for increasing contagion radii. We explored situations enclosing only first neighbors (2 m) to situations enclosing as far as 6 m. The number of pedestrians in panic always attained a maximum slope at almost the same J value for all the investigated situations. This value (close to 0.025) seems to be an upper limit for any weak panic spreading situation, or the



(a) $J = 0.01$ (b) $J = 0.02$ (c) $J = 0.09$

Figure 4: Snapshots of different escaping processes for three values of contagion stress in the first 15 seconds. The different colors of the circles represents the anxiety state of each pedestrian. Relaxed and panic pedestrians are represents in green and red circles, respectively. The *fake bomber* is place at the center of the region and is represented in black circle. Relaxed pedestrians desire to reach the screen located on the left (blue line).

lower limit for any widely spreading situation. We may hypothesize that two qualitative regimes may occur for the panic propagation in the crowd.

Following the above working hypothesis, we turned to study any morphological patterns for both regimes. Fig. 4 represents three possible situations after $15\,\text{s}$ since the (fake) bomber shout (see caption for details).

Fig. 4a corresponds to the lowest contagion stress ($J = 0.01$). We can see an amorphous “branching” pattern for those pedestrians in panic (red circles). That is, a branch-like configuration is present around the (fake) bomber. From the inspection of the whole process through an animation, we further noticed that these branches could be classified into two types (see below). The “branching” profile is also present in Fig. 4b for $J = 0.02$, although this category exhibits an extended number of pedestrians in panic. The highest contagion stress category ($J = 0.09$), instead, adopts a circular profile (see Fig. 4c).

In summary, low contagion stresses correspond to the (qualitative) branch-like regime, while high contagion stress correspond to the (qualitative) circular-like regime. The snapshot in Fig. 1b clearly shows a circular-like regime, as expected for the obtained experimental value of J.

The branching-like profile in Piazza San Carlo is not completely symmetric since the pedestrian’s density is higher near the screen area (on the left of Fig. 4) than in the opposite area. The pedestrians near the screen can not move away as easily as those in the opposite direction. Thus, the panic contagion near the screen occurs among almost static pedestrians, while the contagion on the opposite area occurs among moving pedestrians. Both situations, although similar in nature, produce an asymmetric branching. We labeled as *passive* branching the one near the screen, and *active* branching the one in the opposite direction.

7. Conclusions

The contagion of panic offered a challenge to the emotional mechanism operating on the pedestrians. We included the “inner stress” and “stress decay” as the main processes triggered during a panic situation. Although the simplicity of this model, we attained fairly good agreement with a real panic-contagion event.

We handled the coupling mechanism between individuals through the contagion stress parameter J. This parameter appears to be responsible for increasing the “inner stress” of the individuals. Our first achievement was getting a real (experimental) value for J. The value for the Piazza San Carlo event was 0.1 ± 0.055.

We further noticed through computer simulations that J controls the contagion dynamics. The Piazza San Carlo event illustrates the dynamic arising for high values of J, where everyone moves away from the source of stress. However, this might not be the case for low values of J. Our simulations, show that panic propagates weakly for low values of J. This produces a branch-like, slow panic spreading around the source of danger (for a simple rectangular geometry). If J exceeds (approximately) 0.025 the panic contagion spreads freely in a circular-like profile (for a simple rectangular geometry).

We want to remark that different contagion radii (between 2 m and 6 m) did not produce significant changes on our simulations. This was unexpected, and thus, we may speculate that “spontaneous” contagion out of the usual contagion range may not produce dramatic changes, if the probability of “spontaneous” contagion is small.

Recall that the increase in the “inner stress” is the underneath mechanism allowing the panic to spread among the crowd. The “emotional decay”, however, seems not to play a relevant role in Piazza San Carlo (and in our simulations). This is because the experimental characteristic time for the “emotional decay” is $\tau = 10\,\text{s}$, allowing anxious pedestrians to settle back to the relaxed state after $20\,\text{s}$.

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A. The contagion efficiency

Any individual among the crowd may increase his (her) anxiety level if his (her) neighbors are in panic. This is actually the propagation mechanism for panic: one or more pedestrians express their fear, alerting the others of imminent danger. The latter may get into panic and thus, a “probability” exists for getting into panic.

We hypothesize that the “probability to danger” (*contagion efficiency*) is the cumulative effect of the alerting neighbors. That is, if k pedestrians among n neighbors are expressing fear, then the contagion efficiency $\mathcal{P}_n$ of an individual is

$$\mathcal{P}_n = p_n(1) + p_n(2) + ... + p_n(n) \tag{10}$$

where $p_n(k)$ represents the contagion efficiency of $k = 1, 2, ..., n$ pedestrians (among n neighbors) expressing fear. The distribution for $p_n(k)$ is a Binomial-like distribution if any neighbor expresses panic with fixed contagion efficiency p, regardless of the feelings of other neighbors. If the feelings of any neighbor (among n pedestrians) is not completely independent of the other neighbors, $p_n(k)$ should be assessed as a Hypergeometric-like distribution.

For the purpose of simplicity we assume that the Binomial-like distribution is a valid approximation for the $p_n(k)$ computation. Consequently,

$$\mathcal{P}_n = \sum_{k=1}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) p^k (1-p)^{n-k} = 1 - (1-p)^n \tag{11}$$

The mean value of neighbors expressing fear $\langle k \rangle$ is np. Thus,

$$\mathcal{P}_n = 1 - \left(1 - \frac{\langle k \rangle}{n}\right)^n \tag{12}$$

It is worth noting that this expression holds for a fix value of n. That is, the contagion efficiency $\mathcal{P}_n$ is conditional to the amount of neighboring individuals n. The contagion efficiency for any number of neighbors $n = 1, 2, ..., M$ is

$$\mathcal{P} = \sum_{n=1}^{M} \mathcal{P}_n \, \pi_n \tag{13}$$

where π_n means the contagion efficiency that there are n neighbors surrounding the anxious pedestrian. Notice that the expression (13) neither includes the term for $n = 0$, nor the terms above M. The situation $n = 0$ is not considered here since it corresponds to a "spontaneous" contagion to danger. The situation $n > M$ corresponds to far away individuals, and thus, not really capable of alerting of danger. The limiting value M, however, is supposed to be related to a pertaining distance and the the crowd packing density.

There is no available information on the values of π_n, although it may be written as the ratio $\pi_n = z_n/M$ (number of current neighbors with respect to the maximum number of neighbors).

Recalling Eq. (12), the contagion efficiency $\mathcal{P}_n$ may be expanded as

$$\mathcal{P}_n = 1 - (1 - np + ... + p^n) = p \, f_n(p) \tag{14}$$

The function $f_n(p)$ stands for the summation

$$f_n(p) = n - \frac{n(n-1)}{2} \, p + ... + p^{n-1} \tag{15}$$

Each contributing terms in $f_n(p)$ may be envisage as the alert to danger due to groups of individuals of increasing size (for a fix number of neighbors n). Notice, however, that the expression (15) holds if the feelings between neighboring pedestrians are completely independent. Otherwise, the function $f_n(p)$ should be considered unknown.

The overall contagion efficiency reads

$$\mathcal{P} = \sum_{n=1}^{M} \frac{z_n}{M} \frac{\langle k \rangle}{n} f_n(p) \simeq J \left\langle \frac{k}{n} \right\rangle \tag{16}$$

where J represents an *effective stress* for the propagation, since it expresses in some way the efficiency of the alerting process. That is, no panic propagation will occur for vanishing values of J, while the pedestrian susceptibility to fear emotions will become more likely as J increases. The stress J may depend, however, on the probability p. Appendix B shows that this dependency is weak enough to be omitted in a first order approach.

The fraction $\langle k/n \rangle$ corresponds to the mean fraction of neighbors expressing fear with respect to the total number of neighbors. This mean fraction is computed over all the possible number of neighbors, according to Eq. (16).

B. The sampling procedure for Turin

The effective stress J may be evaluated from any real life situation. Details on the sampling procedure for the Turin incident at Piazza San Carlo are given in Section 4.

As a first step, we identified those individuals that switched to the panic state along the image sequence. We also identified the surrounding pedestrians for each anxious individual, and labeled them as neighboring individuals (regardless of their current anxiety state). For simplicity, we used the same profile (shown in Fig. 1c) throughout the image sequence.

The mean fraction $\langle k/n \rangle$ was obtained straight forward from this data. Table 1 exhibits the corresponding results (see second column).

Notice that the surrounding pedestrians actually correspond to the most inner ring of pedestrians enclosing the anxious individual, but not the ones within a certain radius. This radius, however, can be estimated from the (mean) packing density of the crowd.

The anxious pedestrians at the border of the examined area of Piazza San Carlo (see Fig. 1) are not included in Table 1 since it was not possible to identify *all* of their surrounding pedestrians.

The fraction of the anxious pedestrians n_p to the total number of individuals N is a suitable estimate for the overall contagion efficiency $\mathcal{P}$. However, as panic propagates, the acknowledged anxious pedestrians n_p diminish because the number of previously relaxed individuals reduces inside the analyzed area. Thus, the estimate of $\mathcal{P}$ follows a sampling "without replacement" procedure. That is, the fraction estimate is $n_p/(N - N_p)$, where N_p corresponds to the number of individuals in panic until the previous time step.

Fig. 5 shows the effective stress J computed as the ratio between $\mathcal{P}$ and $\langle k/n \rangle$. The contagion efficiency $\mathcal{P}$ was estimated either as $n_p/(N - N_p)$ (*i.e.* without replacement) or n_p/N (*i.e.* with replacement). It can be seen that the sampling effects can be neglected for $t \leq 4\,\mathrm{s}$.

The J estimates exhibited in Fig. 5 are not completely stationary along the interval $0.5\,\mathrm{s} \leq t \leq 4\,\mathrm{s}$. However, the increasing slope is not relevant for a first order approach.

Table 1: Data provided from the Turin video (see Section 4 for details). Samples were taken at 0.5 s time intervals. The second column shows the number of pedestrians n_p that switched to the panic state at the corresponding time stamp. The third column exhibits the (mean) ratio between neighbors in panic with respect to the surrounding neighbors. The fourth column corresponds to the contagion efficiency $\mathcal{P}$ computed as a "no-replacement" procedure (see text). The last column corresponds to the *contagion stress* J computed from the third and fourth columns. The total number of individuals was $N = 131$.

t	n_p	$\langle k/n \rangle$	$n_p/(N - N_p)$	J
0.5	1	0.17	0.0077	0.0453
1.0	1	0.20	0.0077	0.0385
1.5	5	0.43	0.0391	0.0909
2.0	5	0.42	0.0406	0.0967
2.5	2	0.13	0.0169	0.1300
3.0	4	0.55	0.0345	0.0627
3.5	6	0.36	0.0536	0.1489
4.0	13	0.64	0.1226	0.1916
4.5	11	0.68	0.1183	0.1740
5.0	10	0.52	0.1219	0.2344
5.5	22	0.63	0.3055	0.4849
6.0	29	0.90	0.5800	0.6444
6.5	15	0.88	0.7143	0.8117

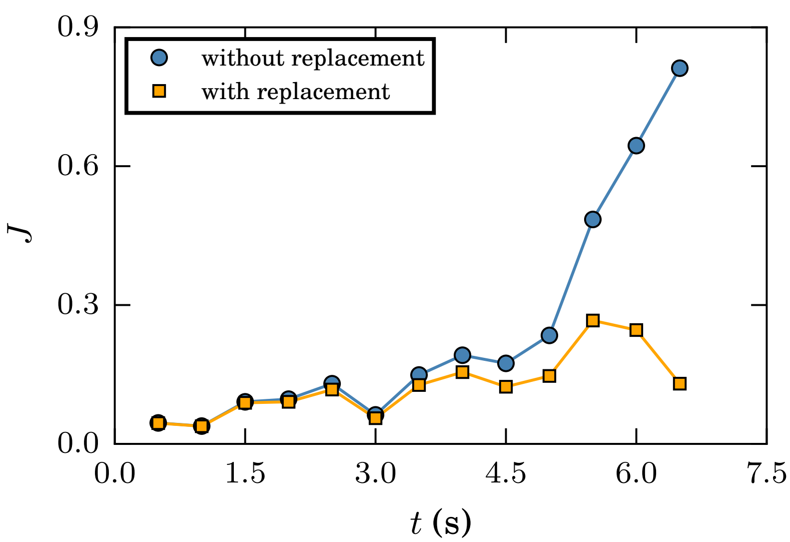

Figure 5: (Color on-line only) The contagion stress J as a function of time t in seconds (see text for details). The rounded symbols (in blue color) correspond to the J values computed from a crowd of $N = 131$ individuals and a sampling procedure "without replacement" (see Table 1 for details). The squared symbols correspond to the J values computed from the same crowd, but following a sampling procedure "with replacement". The mean stress for $0.5\,\mathrm{s} \leq t \leq 4\,\mathrm{s}$ is $J = 0.1 \pm 0.055$.

The mean value for the effective stress along this interval is $J = 0.1 \pm 0.055$.

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