Extending the Applications of Simulation-based Approaches in the Teaching of Elementary Statistics

Sherry L. Hix, Dianna J. Spence
University of North Georgia, 82 College Circle, Dahlonega, GA 30597

Abstract
Much has been written during the last decade about using simulations in Elementary Statistics courses. Although growing in popularity, the use of simulations is not yet considered a universal standard for these courses. This paper seeks to review the most commonly reported and recommended practices for incorporating simulations into a statistics course, and then to suggest possible extensions for the roles that such simulations can play in the course. Specifically, we briefly review how simulations have been used to develop conceptual understanding of statistical inference. We then describe multiple ways to position simulations within student projects, and we explore ways to leverage simulations to illustrate the underlying assumptions and resulting limitations of more traditional statistical tests.

Key Words: Simulation, elementary statistics, statistics education

1. Historical Context

Facilitating student learning in the development of statistical reasoning has historically posed many challenges, but significant modifications have evolved in the way we teach an introductory statistics course (Cobb, 2007; Rossman & Chance, 2014). These changes continue to provide a more informed perspective for researchers and instructors so that the experiences designed for students are more relevant and designed to better launch and promote student learning (Tintle, et al., 2011). Decades ago, students were given convenient, contrived data sets to analyze. These were eventually replaced with real data sets-- some that the students were given, others that students collected. Another important shift moves students from working problem sets at the end of the chapter, with each exercise assessing some piece of data analysis or inference, to having students collect, describe, analyze data and make inferences from that work. Along with these changes, simulations are used to illustrate a closer approximation of a described population. Students conduct simulations by hand for several repetitions, and then use technology to simulate thousands of repetitions. The call for a shift in pedagogy for introductory statistics courses is supported by the Guidelines for Assessment and Instruction in Statistics Education (GAISE, Aliaga et al., 2005); however, the shift required, which includes the use of simulations, is not yet considered a universal standard for these courses. Major introductory statistics textbooks that leverage simulations are in circulation (e.g., Lock, et al., 2013; Tintle, et al., 2015). An open source book employing these strategies is also available online (see Diez, Barr, & Çetinkaya-Rundel, 2014). This paper seeks to review the most commonly reported and recommended practices for incorporating simulations into a statistics course, and then to suggest possible extensions for the roles that such simulations can play in the course.

2. Uses of Simulations

The online applets accompanying Tintle et al.'s (2015) textbook were used to generate the figures associated with the examples given in this paper (see Chance & Rossman, 2015). Simulations are frequently used to help students understand the distribution of a particular statistic and the likelihood of a range of sample behaviors under certain conditions, i.e., the null hypothesis. Suppose an event is known to happen 80% of the time and, in a particular sample of size n = 6, only occurs 65% of the time. A simulation can be used to investigate how often a proportion that low may occur if 80% is the true likelihood of occurrence. Students can understand that either the event is no longer likely to be happening 80% of the time, or the sample was just an occurrence that could easily have happened by chance. Every student in a classroom can simulate this situation by placing four chips of one color and one chip of another color in a bag. When the bag is mixed thoroughly, students can draw (with replacement) a sample of six chips. Drawing one of the four same-colored chips represents the event happening; drawing the one different-colored chip represents the event not happening. Students can perform this simulation several times in a short amount of time and record the number of successes in each sample on a class dot plot in order to show the distribution of successes. After students have the experience of actually observing the population and selecting samples from the population, then they can use computer-based simulations to quickly draw thousands of random samples.

Another powerful way to use simulations is to show the limitations of a theoretical hypothesis test. For example, suppose a Google pop-up advertisement has a link that is known to be followed 8% of the time. But recent data from a sample of n = 20 reveal the link was followed 0% of the time. Are these data so different from the historical information due to chance, or has the trend in following these advertisement links changed? Students often want to use the normal approximation for this distribution, as shown in Figure 1. In this distribution, the proportion of values at or below 0 is 0.0936.

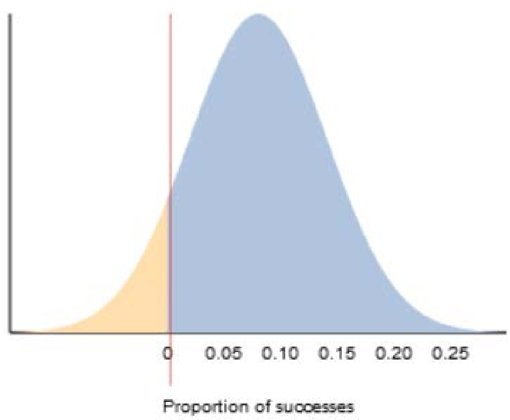

Figure 1: Students' use of a normal approximation for inference on a proportion

Yet if we use a computer simulation of 1000 samples of size n = 20 with $\pi = 0.08$, we see that the proportion of sample statistics as extreme as 0 is 0.181. This distribution is shown in Figure 2. Students will agree that the distribution observed in Figure 2 is not approximately normal and would best be described as skewed, rendering the normal approximation inappropriate. After seeing this comparison, students may more readily understand the nature of the conditions that must be met before using a theoretical model.

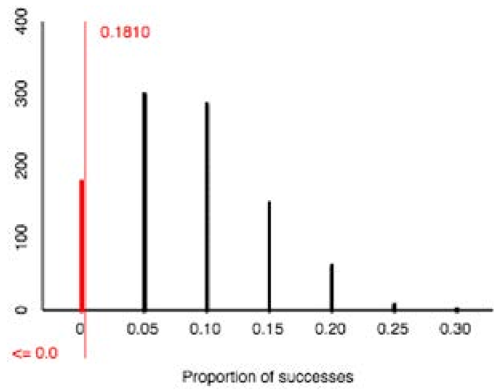

Figure 2: Distribution of students' simulation of 1000 sample proportions

Overlaying the normal approximation with the 1000 samples of size n = 20 when $\pi = 0.08$ as in Figure 3 shows the poor fit of the normal distribution for this scenario, including the large portion of area under the left tail of the normal curve that actually has no observations.

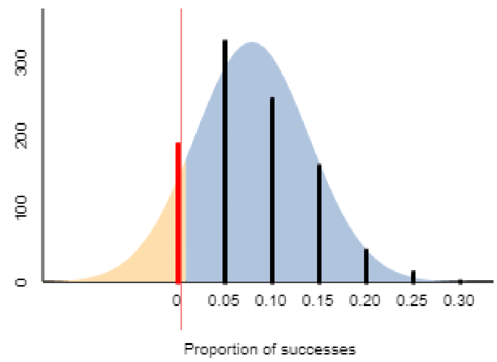

Figure 3: Normal approximation for inference on a proportion overlaying distribution of 1000 simulated sample proportions

Finally, simulations can be used to verify the results of theoretical hypothesis tests. One scenario where this practice can be useful is in the context of a student-led statistical project. In such projects, students plan, collect and describe data, then analyze their data and interpret the results of their analysis. The students can describe the null hypothesis with a concrete model and simulate it using technology to make a determination about what values of the statistic would seem extra-ordinary. It can be useful to have students make this determination prior to conducting their analysis and interpretation. The empirical simulation-based results can be compared to the theoretical results.

For instance, one group of students investigated whether the Atlanta Hawks basketball team scored more points per game at home or away, a problem inspired by similar examples in a textbook by Tabor and Franklin (2013). The students gathered data from a particular season as shown in Figure 4.

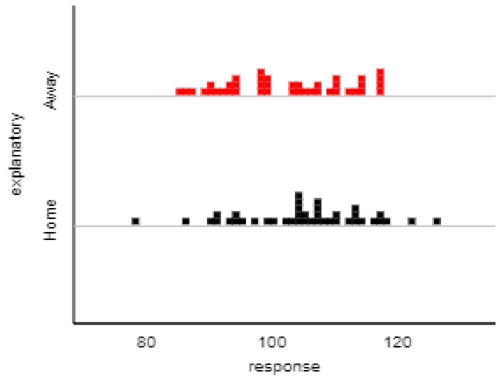

Figure 4: Atlanta Hawks basketball team total points scored per game during one season

Using the online applet, students performed one randomization of the above data; the result is shown in Figure 5.

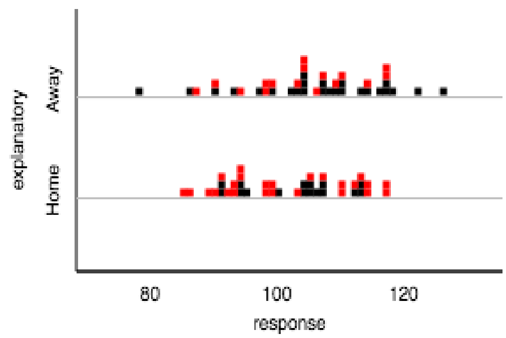

Figure 5: Students' randomization of team points scored per game, retaining colors of original data set (home or away)

Calculating the difference of the means of the away scores and the home scores for many randomizations produces the distribution shown in Figure 6. The proportion of samples in the distribution with a difference greater than the one observed in the sample is 0.09. The corresponding area under the t-curve (the theoretical model) is 0.1014.

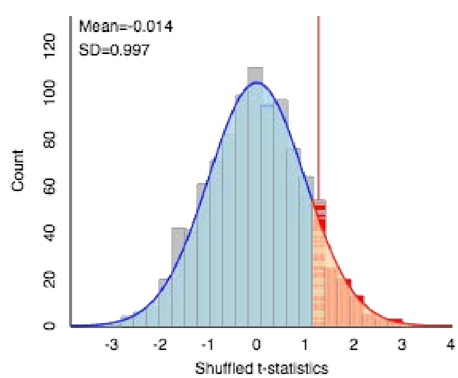

Figure 6: t-distribution approximation for inference on difference of means overlaying many randomizations of sample data

A similar example can be seen in the often-used student project comparing prices at competing stores. This project compares items of the same brand and size sold at Kroger and Walmart. Figure 7 shows the original sample of prices for corresponding items from both stores.

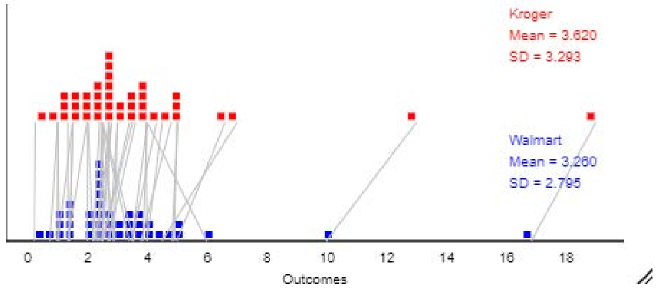

Figure 7: Students' sample of prices for the same items at two different stores

Figure 8 shows how students used the applet to produce one randomization of the data from the above sample.

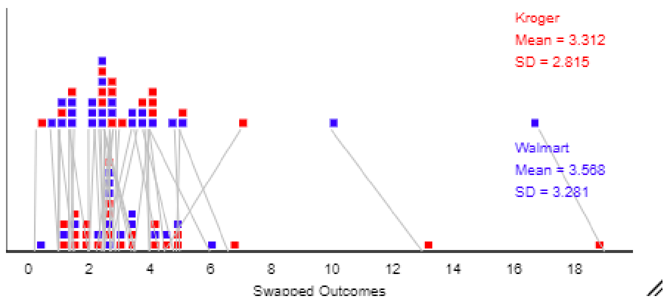

Figure 8: Students' randomization of price data for the same item at two different stores

Performing this randomization many times, the distribution of the mean difference can be seen in Figure 9. The proportion of samples in this distribution with a mean difference greater than the one observed is 0.003, and the corresponding area under the t-curve 0.0064.

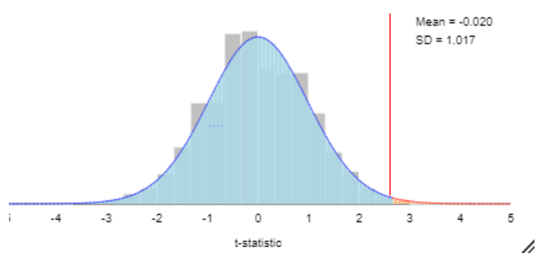

Figure 9: t-curve approximation for inference on mean difference overlaying many randomizations

3. Summary

Simulations in an introductory statistics course hold many possibilities for student learning. Students can initially use a concrete model to generate samples that help them understand what values a statistic might have under the null hypothesis. They can then transition from that concrete model to an applet to simulate many more samples and observe the distribution of that statistic. The distribution they generate then serves as their basis for inference. This interaction between the student and the representations of the null hypothesis and analysis of a sample and its "extreme-ness" can be very powerful for students as they are developing statistical reasoning.

Simulations can also be used to show the limitations of a theoretical test when the data do not satisfy assumptions necessary for the theoretical model. Juxtaposing the simulated distribution with the theoretical distribution can reveal the role and importance of those assumptions. The simulated distribution also provides students a mechanism for inference that avoids problems associated with an inappropriate theoretical model. Further, simulations can verify the theoretical model when it is appropriate. This extension of the use of simulations can engage students in more authentic learning about the conditions necessary for inference with theoretical distributions.

References

Aliaga, M., Cuff, C., Garfield, J., Lock, R., Utts, J. & Witmer, J. (2005). *Guidelines for Assessment and Instruction in Statistics Education (GAISE): College Report*. American Statistical Association. Available at http://www.amstat.org/education/gaise/

Chance, B., & Rossman, A. (2015). *Applets for Introduction to Statistical Investigations*, www.rossmanchance.com/ISIapplets.html.

Cobb, G. (2007). The introductory statistic course: A Ptolemaic curriculum? *Technology Innovations in Statistics Education, 1*(1).

Diez, D. M., Barr, C. D., & Çetinkaya-Rundel, M. (2014). *Introductory Statistics with Randomization and Simulation*. OpenIntro. Available at http://openintro.org

Lock, R., Frazer Lock, P., Lock Morgan, K., Lock, E., & Lock, D. (2013). *Statistics: Unlocking the Power of Data*. New York: Wiley.

Rossman, A., & Chance, B. (2014). Using simulation-based inference for learning introductory statistics. *WIREs Computational Statistics, 6*, 211-221.

Tabor, J., & Franklin, C. (2013). *Statistical Reasoning in Sports.* New York: W.H. Freeman and Company.

Tintle, N.L., Chance, B., Cobb, G., Rossman, A., Roy, S., Swanson, T., & VanderStoep, J. (2015). *Introduction to Statistical Investigations*. New York: Wiley.

Tintle, N., VanderStoep, J., Holmes, V-L., Quisenberry, B., & Swanson, T. (2011). Development and assessment of a preliminary randomization-based introductory statistics curriculum. *Statistics Education Research Journal, 11*, 21-40.