Cumulative vs. Adjacent-category Logits: Readiness to Quit Smoking among Cancer Survivors

Zoran Bursac[1], Patricia J. Goedecke[1], Melissa A. Little[2], Robert C. Klesges[2]

[1]Division of Biostatistics, Department of Preventive Medicine, College of Medicine, University of Tennessee Health Science Center
[2] Center for Addiction and Prevention Research, Department of Public Health Sciences, College of Medicine, University of Virginia

Abstract

When modeling polytomous outcomes with more than two ordered response levels we can apply proportional odds or cumulative logit models, assuming a common set of slopes across the response functions. The resulting odds compare the directionality of higher order response levels to the lower ones. Depending on the data and the purpose of the analysis one might want to compare two neighboring response levels; this is possible with the application of the adjacent-category logit (ACL) model, which shares some similarities with the cumulative logit model. Furthermore, the ACL model allows us to relax the common slopes assumption while maintaining model validity, with predicted probabilities within the [0,1] interval. We illustrate the comparison of the two approaches while modeling a three-level outcome variable; readiness to quit (RTQ) smoking within the next month, one to six months, or longer than six months.

Key Words: Cumulative logit, adjacent-category logit

1. Introduction

Ordinal outcomes are common in health research. Depending on the question of interest, common analytical methods include the use of cumulative logit or proportional odds models; adjacent-category logit models; or in those instances where the proportionality assumption is not met, one might even consider generalized logit or stereotype logit models.

The example that motivated this paper is based on a pilot study of cancer survivors who are also current smokers. A cancer survivor is defined as any person with a cancer diagnosis, regardless of their treatment state. Current smokers are persons who have smoked at least 100 cigarettes in their lifetime, and currently smoke every day or at least some days. Not a lot is known about readiness to quit smoking in this population. Therefore the main outcome of interest was an ordinal variable, readiness to quit (RTQ) smoking. The ordinal categories were readiness (a) within the next month (<1mo), (b) one to six months (1–6mo), or (c) longer than six months (>6mo). An independent variable examined in association with the outcome was lung cancer diagnosis, as compared with diagnosis of any other cancer. The sample consisted of 110 cancer survivors who reported current smoking.

2. Background and Methods

2.1 Cumulative Logit

In the context of our example, y is an ordinal response, RTQ smoking, with k=3 categories. In addition, x is an explanatory variable comparing lung cancer versus other cancer diagnoses. We model

$P(y \le j)$; $j = 1, 2, \ldots, k-1$, using logits, such that

$\text{logit}[P(y \le j)] = \log[P(y \le j) / P(y > j)] = a + bx$ is a linear equation

for $j = 1, 2, \ldots, k-1$.

This is called the cumulative logit model. Effects are described by odds ratios, comparing the odds of being below versus above any point on the scale, hence termed cumulative odds ratios. The proportional odds assumption is that effect b is identical for every level of $j = 1, 2, \ldots, k-1$. Effect b is a cumulative log odds ratio found for every 1-unit increase in explanatory variable. This model uses the ordinality of y, without assigning specific category scores. [1]

2.2 Adjacent-category Logit (ACL)

In contrast to the above, the ACL model is given by

$\text{logit}[P(y = j) / P(y = j+1)] = a + bx$.

The odds ratio here uses adjacent categories. Interpretation makes use of a local odds ratio instead of a cumulative one. This model also has a proportional odds structure, in which effect b is the same for every level of $j = 1, 2, \ldots, k-1$. The ACL and cumulative logit models with proportional odds fit comparably in similar situations, and provide similar results. The ACL gives effects in terms of fixed categories, which is more suitable when one wants to provide interpretations for given categories rather than the observed continuum. ACL effects are also estimable for retrospective (case-control) studies. The ACL model with unequal slopes preserves a predicted probability range between [0,1], which is not the case for the cumulative logit model. [1, 2]

3. Results

Among those with lung cancer diagnoses, approximately 50% reported being RTQ smoking within 1 month; 30% within 1–6 months; and the remaining 20%, more than 6 months or not at all. The distribution of RTQ smoking was somewhat different among those with other than lung cancer diagnoses. (Figure 1)

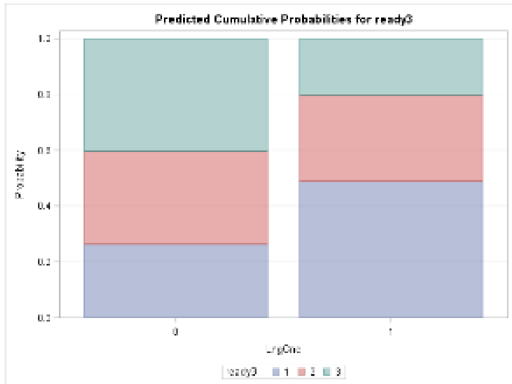

Figure 1. Distribution of RTQ smoking among those with lung cancer versus those with other cancer diagnoses.

3.1 Cumulative Logit with Equal Slopes

The cumulative logit model is expressed in terms of earlier RTQ smoking. Results show that those with lung cancer diagnoses are associated with 2.7 (95% CI 1.2–5.9; p=0.0135) times higher odds of RTQ sooner rather than later, compared with those having other cancer diagnoses. (Figure 2) Estimates from this model are larger than those from the adjacent-category logit model, because they refer to the entire outcome scale.

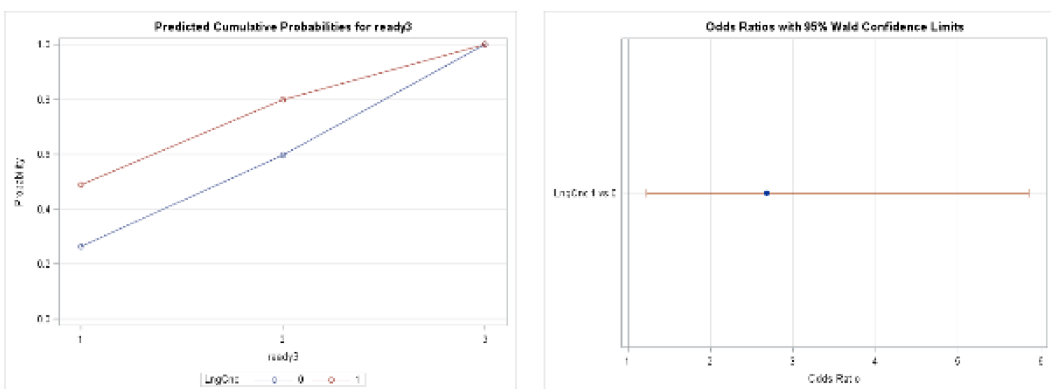

Figure 2. Predicted probabilities and point estimates for the cumulative logit model with equal slopes.

3.2 Adjacent-category Logit with Equal Slopes

For the ACL model with equal slopes, there are two response functions comparing readiness: <1mo to 1–6mo; and 1–6mo to >6mo. The common slope parameter is significant. Those with lung cancer diagnoses have 1.9 (95% CI 1.1–3.3; p=0.0165) times the odds of the next earlier level of RTQ smoking. (Figure 3) Standardized estimates from both models are similar; therefore neither model has greater power. The choice depends on interpretation.

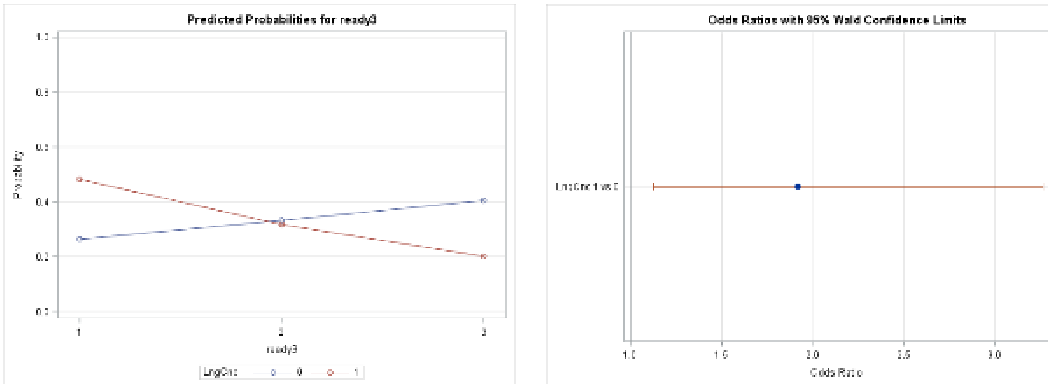

Figure 3. Predicted probabilities and point estimates for the adjacent-category logit model with equal slopes.

3.3 Adjacent-category Logit with Unequal Slopes

The adjacent-category logit with unequal slopes model has a relaxed common slopes assumption. The cumulative logit with unequal slopes could produce negative predicted probabilities, unlike ACL which maintains a [0,1] range. The OR is estimated for each adjacent logit for readiness. Those with lung cancer diagnoses have 2.4 (95% CI 0.9–6.5; p=0.0864) times the odds of RTQ <1mo vs 1–6mo, and 1.5 (95% CI 0.5–4.5; p=0.4933) times the odds of RTQ within 1–6mo vs >6mo. Neither comparison reached traditional significance at the alpha of 0.05. (Figure 4)

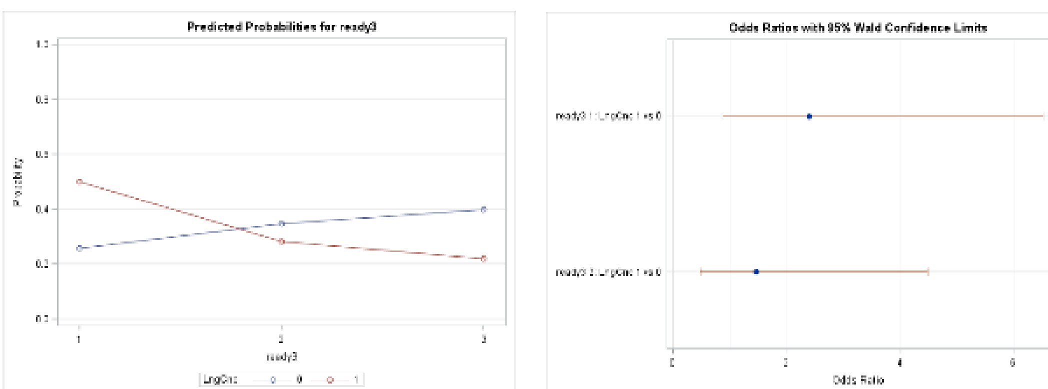

Figure 4. Predicted probabilities and point estimates for the adjacent-category logit model with unequal slopes.

4. Conclusions

We conclude that one's choice of method and underlying assumptions could affect the magnitude of association, significance, and inferences in a logit analysis. The cumulative logit and adjacent-category logit models with proportional odds fit comparably in similar situations. When standardized estimates from both models are similar, neither model has greater power. The cumulative logit model provides inference to the underlying continuum, a directional interpretation. The adjacent-category logit model gives effects in terms of fixed categories. Furthermore, the ACL model allows us to relax the common slopes assumption while maintaining model validity, with predicted probabilities within the [0,1] interval. Selection of the analytical model thus depends on the question of interest and on interpretational preferences.

References

1. Agresti, A. (2010). Analysis of Ordinal Categorical Data, Wiley, 2nd ed.
2. Stokes, M. (2015). SAS/STAT 14.1: Methods for Massive, Missing, or Multifaceted Data. SAS Global Forum Proceedings, Las Vegas, NV.