Estimating Country Indexes Returns Based on Earnings Forecasts

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Abstract

In this article, we present statistical models showing that expected earnings are an appropriate forecast measure for future returns. Since earnings estimates data for countries are available since 2003 and these are not available for companies, this article focuses on country indexes. The hypothesis is that future equity indexes prices for different countries are influenced by earnings estimates as well as by risk and currency levels. Models have been tested for six Countries selected among 47 Developed and Emerging countries covered by MSCI.

For this purpose, SUTSE structure - Seemingly Unrelated Time Series Equations - introduced by Harvey in 1989 is adopted as well as Durbin and Koopman's structural model disclosed in 2001. In structural models, time series are interpreted as the sum of the components. These are estimated recursively through smoothing algorithms. Results will be shown including four error measures – MAPE, R^2, Mean Squared Error and Forecast Standard Error - calculated for the models and its' variables.

Key Words: Equity Indexes, Earnings Estimates, SUTSE, Structural Models

1. Introduction

This essay combines earnings' estimates with quantitative tools to predict investment returns.

Practitioners believe asset class selection has been the major determinant of success in portfolio management (Brinson 1995). Ibbotson (2010) explains that the original intent of mentioned Brinson's paper was, de facto, not about returns but rather for return variations. Considering the traditional asset classes (Wikipedia 2017), Equity is the one with the highest standard deviation (Brinson 1990). Therefore, it is the most critical from the point of view of returns, risk and performance attribution evaluation.

The increased integration of economies and markets is causing some institutional investors to view the global equity markets as the relevant Market Portfolio. International diversification is playing a crescent role in portfolio management. Active tactical regional allocations might be an alternative (Subramanian 2009). In addition, covering a very large number of companies presents practical difficulties and the challenge of data availability. In the whole world, there are around 2,500 large and mid-cap companies, and circa 6,200 small caps companies, that are participants of world indexes. Usually, these are covered by capital markets major players' analysts (MSCI, 2017).

Company discounted cash flows are probably the most accurate way to determine a company's value. But to elaborate a discounted cash flow for a single company is a quite complex task and for 2,500 or 8,700 companies quite impossible to cover in a short article. An alternative is using the Country Price/Earnings estimates ratio, nicknamed P/E ratio, a very popular measure amongst equity market analysts and portfolio managers. Price stands for the price of the share or market value of a company, and earnings represent the total profits or losses of a company or a per share version. MSCI provides these figures for 47 countries and 9 Regional Indexes. The latter includes The World, Developed Markets, Emerging Markets and the combination of three regions – 1.Americas, 2.Europe, Middle East and Africa - EMEA, and 3.Asia Pacific, with their developed and emerging countries subsets. The availability of data and the simplification of calculations directed our choice to focus on a regional and country approach rather than on companies. Expected consensus data concerning forward 12 months Price/Earnings ratios allows implying future earnings since the variable P in the ratio is known.

The first simulations tested if earnings expectations variations would anticipate market returns variations. Market returns were represented by country and region Indexes. To improve the analysis, Risk and Currency Exchange Rates were also tested. The risk was included in the tests because it is a dimension usually combined with returns. Since the data are denominated in US Dollars, the exchange rate was included in the tests because global investors mostly think in dollar terms and an exchange rate fluctuation would alter prices in dollars. Currency Exchange Rates and Risk data are also available (Sources: Bloomberg and Thomson Eikon).

5-year CDS (Credit Default Swaps) were used as the measure for Country Risk. They express a faster adjustment to information, despite a more volatile nature than other measures such as ratings and sovereign interest rates (Damodaran, 2017*a*). As future earnings synthesize returns, country risk expresses risk. A 5% basic interest was added to the CDS figures to express a discount rate. The sum of 5% and the CDS is very similar to a country sovereign bond interest rate. The reason for this simplification is that since 2008, interest rates have been very low. This is due to Central Banks economic stimuli and 5% is a very close number to the U.S. Bond mean rate for previous 10 years, from 1998 to 2008.

Several time lags were tested including zero, one, three, and 12 months. The best results were for the one-month time lag. Thus the other ones were not mentioned in this article.

One aspect that was not taken into consideration in this article is the equity risk premium. This is due to the range of values that might be chosen (Damodaran 2017*b*). Another aspect not considered is that the companies' risk is more connected to where they obtain their revenues than to where they are headquartered.

2. Data

For each country and regional aggregate 7 models could be calculated. These include Index as a function of Expected Earnings, Risk, and Currency, each one separately, two of the last three combined, and all three combined. So, considering the 47 countries and 9 regional aggregates there are 392 models possible.

All results would not fit this article so a few countries and one regional aggregate were chosen to show the results: one developed and one emerging market for each of the three regions - Americas, EMEA, and Asia Pacific. The criteria were the largest country GDP. GDP size was an important factor as well as the diverse drivers of growth in these countries. So, USA, Germany, Japan, Brazil, Russia, and China were the chosen sample. The All-Country World index model was also built. If the criteria would have been Market Capitalization, instead of GDP, United Kingdom, and South Africa would have replaced Germany and Russia.

Only the Large and Mid-Capitalization Companies – representing around 85% of the total market cap - had their data analyzed for simplicity purposes. MSCI was the data source for Indexes and Forward Earnings, and Thomson Reuters and Bloomberg were the sources for Country Risk and Currency Exchange Rates.

3. Methodology: A Multivariate State Space Model for Estimating Country Indexes Returns

The model class called SUTSE (Seemingly Unrelated Time Series Equation) was introduced by Harvey (1989) and later used by Fernandez and Harvey (1990), Moauro and Savio (2005), Jules (2009) and others.

The SUTSE model with the level (μ), the trend (υ), the seasonal (γ) and the cyclic (ψ) stochastic components for the series in the SUTSE formulation is described in the equations below:

$$\begin{bmatrix} y_{1,t+1} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,t+1} \\ \mu_{2,t} \\ \mu_{3,t} \\ \mu_{4,t} \end{bmatrix} + \begin{bmatrix} \gamma_{1,t+1} \\ \gamma_{2,t} \\ \gamma_{3,t} \\ \gamma_{4,t} \end{bmatrix} + \begin{bmatrix} \psi_{1,t+1} \\ \psi_{2,t} \\ \psi_{3,t} \\ \psi\gamma_{4,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{bmatrix} \quad \text{where } \varepsilon_t \backsim N(0, \Sigma_\varepsilon) \quad (1)$$

The evolution of the components – the level (μ), the trend (υ), the seasonal (γ) and the cyclical (ψ) stochastic - presented in equation (1) are in Appendix 1.

In this section, a general SUTSE is presented. It includes four stochastic components. The generic model was tested for the six countries as well as for the All Country World data. Then, a few evaluation criteria and residual analysis were employed to check if this model would be suitable for the database.

The bivariate SUTSE model previously presented can be cast into the state space form, and once this is done, estimation of the unobserved components is accomplished by use of the Kalman filtering and related algorithms, such as smoothing and prediction error decomposition of the likelihood function. A Gaussian linear state space form consists of two equations. The first is the observations equation, which describes the evolution of a p-variate time series y_t , t=1,2,3,4 in terms of the components that are encapsulated in the vector α_t. The second is the state equation specifying the way each of the components contained in α_t (μ, υ, γ e ψ) evolves stochastically. More specifically:
Observation equation:

$$y_t = Z_t + \alpha_t + d_t + \varepsilon_t \quad t = 1,2, \dots n \quad (2)$$

State equation:

$$\alpha = T_t\alpha_t + c_t + R_t\eta_t \quad (3)$$

Where:

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \backsim NID \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix} , \begin{bmatrix} H_t & 0 \\ 0 & Q_t \end{bmatrix} \right] (4)$$

$$E[\varepsilon'_t\alpha_1] = E[\eta'_t\alpha_1] = 0, \forall t \quad \alpha_1 \backsim N(a_{1\,,}\, p_1) \; (5)$$

The system matrixes, Z_t, d_t, T_t and c_t are deterministic, and the errors are considered independent of each other and independent of the initial state. H_t and Q_t contains the variance and covariance matrices (Σ_ε, Σ_η, Σ_ϵ e Σ_k)[1]. The fixed unknown elements contained of the system matrixes are estimated together with the state vector. This is accomplished by use of the Kalman Filter, a set of recursive equations which gives estimated values for the mean and covariance matrices of the state vector at any period. Full details on the Kalman filtering deduction and related algorithms can be found in Durbin & Koopman (2001).

Initially, the EM (Expectation Maximization) algorithm was used to estimate the variances and covariances of the observations (H_t) state (Q_t) equations. These estimates were used as starting values for the diffuse initialization because the EM algorithm moves in direction of the highest value. The details of the algorithm are described in the revised book by Durbin and Koopman (2001) published in 2014. STAMP and R language were used to model

[1] See Appendix 1

the data. STAMP is a statistical econometric software system for time series models with unobserved components such as trend, seasonal, cycle, and irregular.

To investigate the existence of long-term causality between the variables (future returns, expected profits, currency, and risk), the Toda and Yamamoto (1995) causality test was used. According to Kannebley Júnior (2002), "this methodology allows testing restrictions on the coefficients of a level VAR model, using standard Wald asymptotic distribution, without considering restrictions on the order of integration or co-integration between the variables of the model."

The usual Granger causality thesis was not used because, according to the results of the unit root tests, by Zivot and Andrews (1992), the selected series present at least one structural break, which evidences the non-stationary characteristic of the series. The results of the unit root test and the causality tests following the methodology of Toda and Yamamoto (1995) are presented in Table 1.

Table 1: Unit Root Test and Toda & Yamamoto (1995) Causality Test

Country / Region	Index as Function of:	Unit Root Test	Toda & Yamamoto Causality Test
Brazil	Earnings	H0 Rejected	H0 Rejected
Brazil	Currency	H0 Rejected	H0 Not Rejected
Brazil	Risk	H0 Rejected	H0 Rejected
Germany	Earnings	H0 Rejected	H0 Rejected
Germany	Currency	H0 Rejected	H0 Not Rejected
Germany	Risk	H0 Rejected	H0 Rejected
Russia	Earnings	H0 Rejected	H0 Rejected
Russia	Currency	H0 Rejected	H0 Rejected
Russia	Risk	H0 Rejected	H0 Rejected
Japan	Earnings	H0 Rejected	H0 Not Rejected
Japan	Currency	H0 Rejected	H0 Not Rejected
Japan	Risk	H0 Rejected	H0 Rejected
China	Earnings	H0 Rejected	H0 Rejected
China	Currency	H0 Rejected	H0 Rejected
China	Risk	H0 Rejected	H0 Rejected
USA	Earnings	H0 Rejected	H0 Rejected
USA	Risk	H0 Rejected	H0 Not Rejected
World	Earnings	H0 Rejected	H0 Not Rejected

Table 1 shows that the null hypothesis can be rejected. This means that the series does not have structural breaks. The results of the Zivot and Andrews test (1992) confirmed our expectations that in analyzing the recent history of the selected regions, it is believed that there may be structural breaks due to macroeconomic instability throughout the period.

According to the Toda & Yamamoto (1995) causality test, the hypothesis of causality between the series of future index returns, expected profits, currency and risk are not rejected. From then on, the generalized and non-generalized SUTSE models were applied. To select Level, Trend, Seasonality and Cycle components, the residual analysis was used. The residual analysis includes Normality Test (Bowman-Shenton) and Residual Autocorrelation Durbin-Watson homoscedasticity test. In addition, likelihood and R2 were calculated for the Index, Earnings, Risk and Currency variables.

Table 2 shows the models' results. The Blue colored line shows the model chosen for each country.

Table 2: SUTSE Models per Selected Country or Region

Country / Region	Index as funcion of:	Components				R^2				Evaluation Criteria: Residual Normality Test	Residuals Autocorrelation	Homoscedasticity Test
		Level	Trend	Seasonality	Cycle	Index	Earnings	Risk	Currency	Bowman-Shenton Test	Durbin-Watson Test	
Brazil	earnings; risk	yes	yes	yes	yes	57%	52%		57%	H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings; risk	yes	yes	yes		52%	51%		52%	H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings; risk	yes		yes		52%	48%		47%	H0 rejected for Risk	H0 Accepted	H0 Accepted
Brazil	earnings	yes	yes	yes	yes	58%	69%			H0 rejected for Earnings	H0 Accepted	H0 Accepted
Brazil	earnings	yes	yes	yes		61%	68%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings	yes		yes		60%	70%			H0 rejected for Earnings	H0 Accepted	H0 Accepted
Brazil	earnings; risk	yes	yes	yes	yes	50%	55%	51%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings; risk	yes	yes	yes		53%	64%	64%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings; risk	yes		yes		52%	67%	63%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings	yes	yes	yes	yes	50%	55%			H0 accepted for all residuals	H0 Rejected	H0 Accepted
Brazil	earnings	yes	yes	yes		50%	50%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Brazil	earnings	yes		yes		55%	56%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Russia	earnings; risk; currency	yes	yes	yes	yes	59%	73%	70%	58%	H0 rejected for Risk & Currency	H0 Accepted	H0 Accepted
Russia	earnings; risk; currency	yes	yes	yes		54%	72%	65%	54%	H0 rejected for Risk & Currency	H0 Accepted	H0 Accepted
Russia	earnings; risk; currency	yes		yes		55%	73%	67%	59%	H0 rejected for Risk	H0 Accepted	H0 Accepted
Russia	earnings; risk	yes	yes	yes	yes	55%	79%	73%		H0 rejected for Risk	H0 Accepted	H0 Accepted
Russia	earnings; risk	yes	yes	yes		58%	82%	74%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
Russia	earnings; risk	yes		yes		50%	76%	71%		H0 rejected for Risk	H0 Accepted	H0 Accepted
Russia	earnings; currency	yes	yes	yes	yes	59%	75%		71%	H0 rejected for Currency	H0 Accepted	H0 Accepted
Russia	earnings; currency	yes	yes	yes		51%	80%		49%	H0 rejected for Currency	H0 Accepted	H0 Accepted
Russia	earnings; currency	yes		yes		51%	74%		43%	H0 rejected for Currency	H0 Accepted	H0 Accepted
Russia	earnings	yes	yes	yes	yes	61%	76%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Russia	earnings	yes	yes	yes		59%	77%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Russia	earnings	yes		yes		54%	74%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings; risk; currency	yes	yes	yes	yes	62%	59%	65%	70%	H0 rejected for Risk	H0 Rejected	H0 Accepted
China	earnings; risk; currency	yes	yes	yes		65%	61%	72%	88%	H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings; risk; currency	yes		yes		63%	56%	61%	83%	H0 rejected for Risk	H0 Accepted	H0 Accepted
China	earnings; risk	yes	yes	yes	yes	64%	57%	82%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings; risk	yes	yes	yes		67%	65%	82%		H0 rejected for Currency	H0 Accepted	H0 Accepted
China	earnings; risk	yes		yes		69%	63%	81%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings; currency	yes	yes	yes	yes	51%	39%		39%	H0 rejected for Earnings and Currency	H0 Rejected	H0 Accepted
China	earnings; currency	yes	yes	yes		49%	25%		36%	H0 rejected for Index, Earnings & Currency	H0 Rejected	H0 Accepted
China	earnings; currency	yes		yes		67%	61%		66%	H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings	yes	yes	yes	yes	62%	67%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings	yes	yes	yes		63%	72%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
China	earnings	yes		yes		59%	75%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Germany	earnings; risk	yes	yes	yes	yes	61%	60%	83%		H0 rejected for Risk	H0 Rejected	H0 Accepted
Germany	earnings; risk	yes	yes	yes		60%	60%	90%		H0 rejected for Earnings and Risk	H0 Rejected	H0 Accepted
Germany	earnings; risk	yes		yes		62%	62%	92%		H0 rejected for Earnings and Risk	H0 Rejected	H0 Accepted
Germany	earnings	yes	yes	yes	yes	59%	57%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Germany	earnings	yes	yes	yes		54%	57%			H0 rejected for Index	H0 Accepted	H0 Accepted
Germany	earnings	yes		yes		57%	53%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
Japan	risk	yes	yes	yes	yes	48%		61%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
Japan	risk	yes	yes	yes		46%		73%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
Japan	risk	yes		yes		58%		75%		H0 accepted for all residuals	H0 Accepted	H0 Accepted
USA	earnings	yes	yes	yes	yes	50%	28%			H0 rejected for Index & Earnings	H0 Accepted	H0 Accepted
USA	earnings	yes	yes	yes		59%	68%			H0 rejected for Earnings	H0 Accepted	H0 Accepted
USA	earnings	yes		yes		59%	73%			H0 rejected for Earnings	H0 Accepted	H0 Accepted
World	earnings	yes	yes	yes	yes	60%	51%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
World	earnings	yes	yes	yes		64%	63%			H0 accepted for all residuals	H0 Accepted	H0 Accepted
World	earnings	yes		yes		64%	55%			H0 accepted for all residuals	H0 Accepted	H0 Accepted

In the models for Brazil, Germany, USA and the World the selected variables were Indexes and Earnings. Index and Risk worked better for the Japanese model. Index, Earnings, and Risk were selected for the Russian model and, finally, Index, Earnings, Risk, and Currency were chosen for the Chinese model. When the three tests shown in Table 2 indicate residual normality, non-auto correlation and homoscedasticity were examined. If all passed, the model with the highest R2 was picked.

4. Results

12-month Forward Earnings estimates data are available in MSCI starting in June 2003. Indexes and Currency data are also available for this period. Risk data for Japan were available as of August 2008. All models were calculated with data until June 2015. The forecasts from July 2015 to March 2017 were compared to the effective data.

Table 3 shows a summary of error measures for the chosen models per country. It specifies the variables used, and four measures: Mean Absolute Percentage Error – MAPE, R^2, Mean Squared Error and Forecast Standard Error. For the Index, Earnings, and Risk, MAPE was lower for the selected Developed Markets countries than for the Emerging Markets. Forecast Standard Errors were also lower for developed market countries. On the other hand, R^2 was higher for Emerging Markets countries. Besides the error measures, the volatility for the Indexes was included in the Table´s last line. Brazil and Russia have a quite higher pattern than the other countries.

Table 3: Error Measures per Model

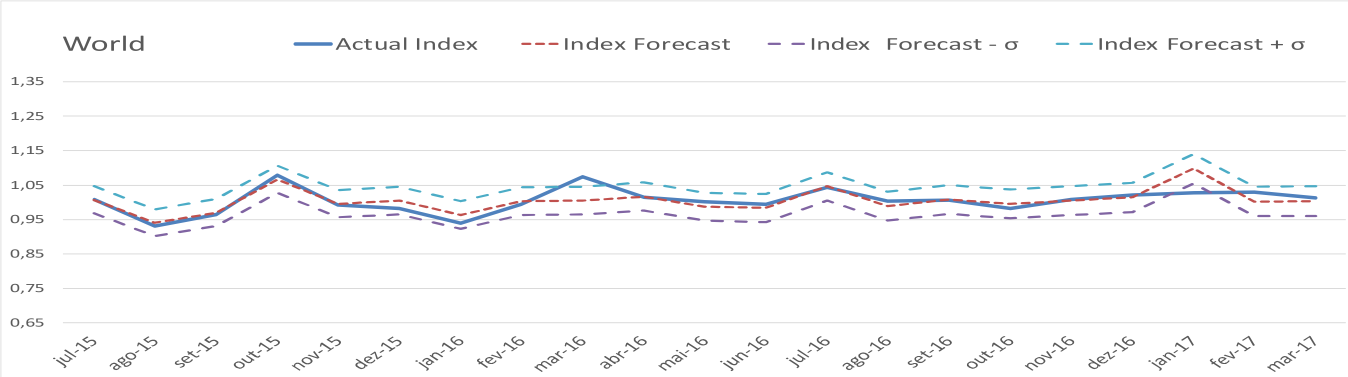
	Region, Country	World	USA	Germany	Japan	Brazil	Russia	China
	Model Variables	Index , Earnings	Index , Earnings	Index , Earnings	Index, Risk	Index , Earnings	Index , Earnings , Risk	Index, Earnings, Risk, Currency
Index	Mean Absolute Percentage Error	1,3%	1,4%	3,1%	2,1%	5,3%	3,1%	3,3%
	R^2	54,2%	51,9%	61,5%	52,2%	72,8%	60,6%	59,8%
	Mean Squared Error	0,00034	0,00030	0,00116	0,00088	0,00491	0,00137	0,00127
	Forecast Std Error	0,03873	0,03658	0,06788	0,04190	0,09088	0,09676	0,07186
Earnings	Mean Absolute Percentage Error	0,5%	0,5%	0,9%		3,6%	4,6%	1,2%
	R^2	43,2%	27,3%	71,7%		74,0%	64,5%	74,7%
	Mean Squared Error	0,00003	0,00003	0,00010		0,00201	0,00268	0,00014
	Forecast Std Error	0,01622	0,00921	0,03577		0,04181	0,04109	0,01416
Risk	Mean Absolute Percentage Error				0,6%		2,6%	4,0%
	R^2				46,8%		61,9%	35,9%
	Mean Squared Error				0,00005		0,00072	0,00220
	Forecast Std Error				0,01913		0,04835	0,01984
Currency	Mean Absolute Percentage Error							1,8%
	R^2							38,6%
	Mean Squared Error							0,00094
	Forecast Std Error							0,00398
Volatility [1]		12,3%	12,5%	16,8%	16,6%	35,3%	34,6%	20,3%

1- Anualized 156 weeks volatility ao of December 2016.

If the models would have been re-calculated every month including the previous month indexes and the new market forecasts, the results might be better and the standard error smaller.

Charts for the chosen sample are presented below.

Chart 1: World Forecasted and Actual Indexes



All charts use the same range for the vertical axis. It can be noted that Brazil and Russia have higher range variations. Volatility and political crisis might be the explanation.

Chart 2: USA Forecasted and Actual Indexes

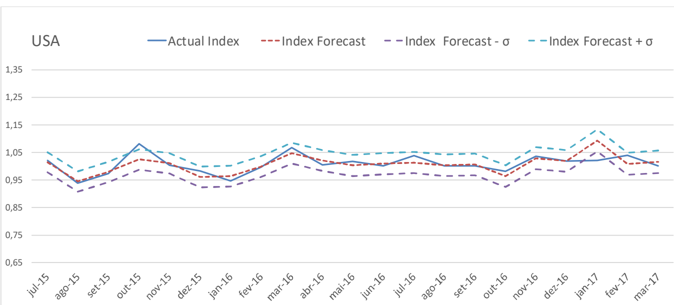


Chart 3: Brazil Forecasted and Actual Indexes

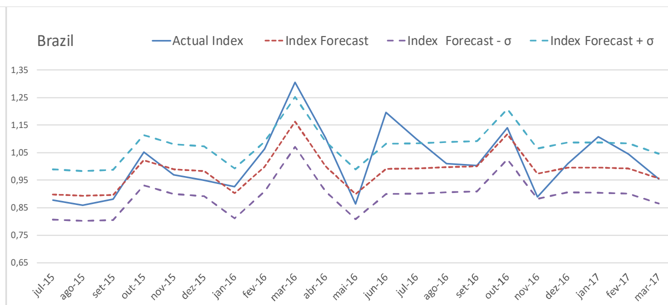


Chart 4: Germany Forecasted and Actual Indexes

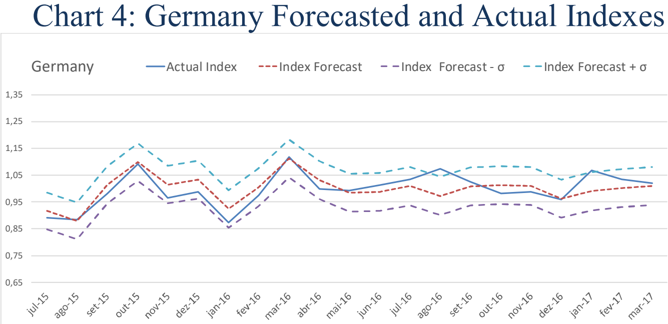


Chart 5: Russia Forecasted and Actual Indexes

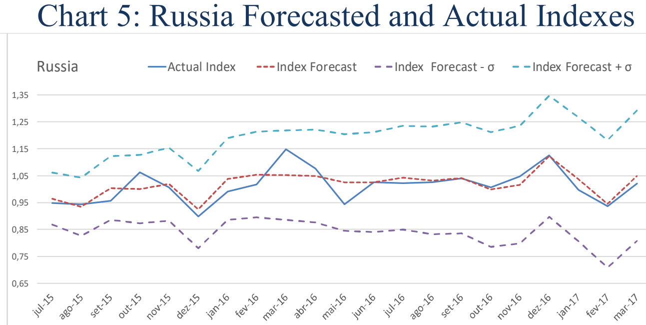


Chart 6: Japan Forecasted and Actual Indexes

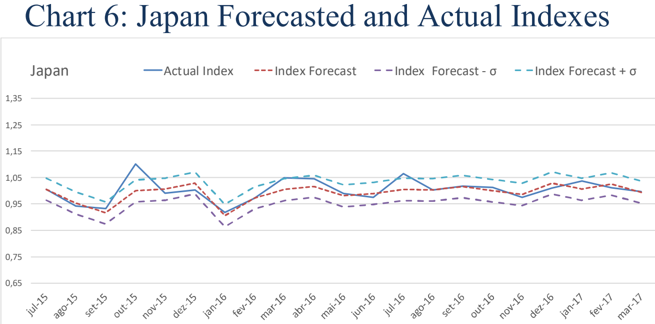
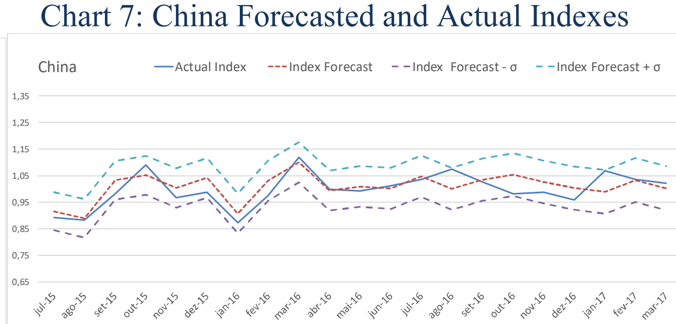


Chart 7: China Forecasted and Actual Indexes



5. Conclusions

The increased integration of economies and capital markets is causing investors to view the Global Market as the relevant Portfolio to be taken into consideration. Among the more relevant Asset Classes usually invested, Equity is the one with higher standard deviation. Therefore, it is the most critical from the point of view of Returns, Risk and Performance Attribution evaluation.

Country Data are available for Indexes, Country Risk and Currency Exchange Rates and Estimated Returns. This allows the calculation of the impact of earnings estimates variations, complemented by risk and currency variations in the Index returns for countries and regions. These factors were tested separately and combined to choose the most appropriate model for each country of the sample.

Models with time-varying coefficients (for example SUTSE models) are more suitable for time series, compared to the usual models with Box and Jenkins static parameters. The model parameters in this article are re-estimated as new data appear (every period t). Since they are dynamic, they come closer to reality. Other important facts are: (i) the non-mandatory stationarity of the series of models and (ii) the acceptance of non-normality of the residues. For this reason, a larger range of time series can be predicted by the models used here., The models obtained satisfactory results.

They have shown that expected earnings are an appropriate forecast measure for future index returns. The estimates, in most periods, were within the calculated confidence interval, as can be observed in the Charts in section 4. When on a few occasions, this did not occur, a specific macroeconomic effect or political crisis, may have been the reason.

Earnings estimates for the USA should be viewed with caution since the residuals of the model with variant coefficients in the selected time are not normal (see table 2). And that was one of the premises adopted for the series estimation.

The models presented here are easy to understand, easy to interpret economically because the series can be divided into components with economic interpretation such as level, seasonality, and cycle. and they also and easily applied, because the STAMP program is extremely didactic and self-explanatory. Replications of this work can be expanded to other databases.

Models like the ones shown could be used by investors to improve investment strategies. Computational means and estimates availability would determine the quality and detail of the results.

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Appendix I – Details on the Methodology

The equations 6, 7, 8 e 9 present the evolution of the stochastic components described in equation (1): the level (μ), the trend (υ), the seasonality (γ) and the cyclic (ψ).

$$\begin{bmatrix}\mu_{1,t+1}\\ \mu_{2,t}\\ \mu_{3,t}\\ \mu_{4,t}\end{bmatrix} = \begin{bmatrix}\mu_{1,t}\\ \mu_{2,t-1}\\ \mu_{3,t-1}\\ \mu_{4,t-1}\end{bmatrix} + \begin{bmatrix}\upsilon_{1,t+1}\\ \upsilon_{2,t}\\ \upsilon_{3,t}\\ \upsilon_{4,t}\end{bmatrix} + \begin{bmatrix}\eta_{1,t+1}\\ \eta_{2,t}\\ \eta_{3,t}\\ \eta_{4,t}\end{bmatrix} \text{ where } \eta_t \backsim N(0, \Sigma_\eta) \quad (6)$$

$$\begin{bmatrix}\upsilon_{1,t+1}\\ \upsilon_{2,t}\\ \upsilon_{3,t}\\ \upsilon_{4,t}\end{bmatrix} = \begin{bmatrix}\upsilon_{1,t}\\ \upsilon_{2,t-1}\\ \upsilon_{3,t-1}\\ \upsilon_{4,t-1}\end{bmatrix} + \begin{bmatrix}\zeta_{1,t+1}\\ \zeta_{2,t}\\ \zeta_{3,t}\\ \zeta_{4,t}\end{bmatrix} \text{ where } \zeta_t \backsim N(0, \Sigma_\zeta) \quad (7)$$

$$\begin{bmatrix}\gamma_{1,t+1}\\ \gamma_{2,t}\\ \gamma_{3,t}\\ \gamma_{4,t}\end{bmatrix} = \begin{bmatrix}-\sum_{j=1}^{n-1}\gamma_{1,t+1-j}\\ -\sum_{j=1}^{n-1}\gamma_{2,t-j}\\ -\sum_{j=1}^{n-1}\gamma_{3,t-j}\\ -\sum_{j=1}^{n-1}\gamma_{4,t+1-j}\end{bmatrix} + \begin{bmatrix}\epsilon_{1,t+1}\\ \epsilon_{2,t}\\ \epsilon_{3,t}\\ \epsilon_{4,t}\end{bmatrix} \text{ where } \epsilon_t \backsim N(0, \Sigma_\epsilon) \quad (8)$$

$$\begin{bmatrix}\psi_{1,t+1}\\ \psi *_{1,t+1}\\ \psi_{2,t}\\ \psi *_{2,t}\\ \psi_{3,t}\\ \psi_{*3,t}\\ \psi_{4,t}\\ \psi *_{4,t}\end{bmatrix} = \left[\rho \quad \begin{bmatrix}cos\lambda_c & sen\lambda_c\\ -sen\lambda_c & cos\lambda_c\end{bmatrix} \quad \otimes I_2\right] \begin{bmatrix}\psi_{1,t}\\ \psi *_{1,t}\\ \psi_{2,t-1}\\ \psi *_{2,t-1}\\ \psi_{3,t-1}\\ \psi_{*3,t-1}\\ \psi_{4,t-1}\\ \psi *_{4,t-1}\end{bmatrix} + \begin{bmatrix}k_{1,t+1}\\ k_{2,t}\\ k_{3,t}\\ k_{4,t}\end{bmatrix} \text{ where } k_t \backsim N(0, \Sigma_k) \quad (9)$$

Where $(\lambda_c) = \frac{2\pi}{T_c}$ and $0 < \rho < 1$. λ_c or T_c are variables that need to be estimated.

In SUTSE models, series relate themselves through correlations presented in $\Sigma_\eta, \Sigma_\zeta, \Sigma_\epsilon$ e Σ_k matrixes. For example, the level of series 1, (index future returns), is related to the level of series 2 (expected earnings) through the co-variants included the variance-covariance matrix Σ_η.

According to Koopman, Andrew and Harvey (2006), cycles might be introduced in multivariate models. As the cycle in each series is boosted by two disturbances, there are two sets of disturbances and these are assumed as having the same variance matrix: $(k_t k_t') = E(k_t^* k_t^{*'}) = \Sigma_k,\ \ E(k_t\ k_t^{*'}) = 0,\ \ t = 1, \dots, T$, where Σ_k is a matrix of é N x N variance.

Cycles in different series have the same properties, that is, they have the same autocorrelation function. Koopman (2006) calls them similar cycles. The strength of a cycle in a series depends on the variance of its disturbance.