Regime Switching Asymmetric-GARCH Models for Estimating Financial Risk in the Nigerian Stock Index

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Abstract

This paper applies various Markov switching asymmetric GARCH models in estimating value-at-risk (VaR) and its coherent complement Expected shortfall(ES) on returns of the Nigerian stock index. This was done by considering a mixture of Student's-t distributions with varying variances over different time and regimes. Single regime asymmetric GARCH models were compared with their Markov switching counterparts. We found that although the Markov switching models were able to adjust for spurious high persistence found in the single regime asymmetric GARCH models. Under relative performance and hypothesis-testing evaluations, the VaR forecasts derived from the Markov-switching GARCH models were not necessarily preferred to their single regime counterparts.

Key Words: Markov-switching GARCH ; Financial Risk ; Expected shortfall(ES) ; Asymmetric-GARCH ; value-at-risk (VaR)

1. Introduction

A major consequence of the recent global financial crisis is the improvement of the regulatory process of the Basel Accords (currently the Basel III Accords)(on Banking Supervision, 1996);(on Banking Supervision, 2011). Financial institutions of leading nations are obliged to meet stringent capital requirements and rely on state-of-the-art risk management systems ((bas,)). It is undoubtful that better risk management practices should lead to a higher stability of the economy and ultimately translate to social benefits for the critical mass. It would also translate to more confidence for potential startups, encouraging the inception of small and medium scale enterprises. Nigeria as a nation has not been exempt from the current global crisis. Being a major oil producing state, a lot of economic development in the country is hinged largely on proceedings from trading crude. Hence it is not surprising that staggering prices of crude would significantly rock the Nigerian economy. The economy is currently struggling heavily under the recent drop in crude oil prices resulting in major job losses and a rise in the birth of small business as families seek other sources of livelihood. The need for seamless functioning banking and insurance systems and of course credit system cannot be over emphasised at this time. Modeling the volatility of financial markets is central in risk management. Furthermore, back testing existing risk models and comparing the estimation techniques used to calibrate these models is even more critical. Research on modeling volatility dynamics using time series models has been active since the creation of the original ARCH model by (Engle, 1982) and its generalization by (Bollerslev, 1986). Many variants that sprung from GARCH models have been proposed to capture additional stylized facts observed in financial markets. However, the choice of an optimal model still remain evasive as different market dynamics play out in different markets and different time periods. GARCH-type models have been shown to recognize that there may be important nonlinearities, asymmetries, and long-memory properties in the volatility process; see (Bollerslev et al., 1986), (Engel and Keen, 1994) for a

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review. One important finding is that estimates based on GARCH-type models can be biased by structural breaks or regime switches in the volatility dynamics (see, e.g.,(Bauwens and Sucarrat, 2010); (Bauwens et al., 2015). These breaks typically occur during periods of financial upheaval in the said markets. Estimating a GARCH model on data displaying a structural break yields a non-stationary estimated model which could lead to poor risk predictions.(Ardia, 2009);(Ardia et al., 2017) suggested markov-switching GARCH models (MSGARCH) whose parameters can change over time according to a discrete latent (i.e., unobservable) variable as a way to handle the problem. These models can quickly adapt to variations in the unconditional volatility level, which should improve risk predictions (see, e.g.,(Marcucci, 2005);(Ardia, 2009)).

This paper compares the forecast performance of selected single regime asymmetric models and 2-regime asymmetric GARCH models in providing risk forecasts for the Nigerian stock index. The forecasting performance of each of the models is tested for the financial time series based on 250 out-of-sample monthly (percentage) log-returns of the Nigerian stock index. The period considered ranges from 1985 to 2015. We assess the performance of the various models in forecasting the left-tail (i.e., losses) of the conditional distribution of the assets' returns. Single regime GARCH and 2-regime MSGARCH models are estimated by the Maximum Likelihood (ML) method (see (Haas et al., 2013), (Marcucci, 2005). Risk forecasts are backtested using the Unconditional Coverage(UC) and Conditional Coverage (CC) tests of (Christoffersen, 1998);(Kupiec, 1995) and the DQ test of (Engle and Manganelli, 2004).

The rest of this paper is structured as follows: In Section 2, we discuss the single regime GARCH models and 2-regime MSGARCH models, we also outline the details of the single and 2-regime versions of the each of the models considered. In Section 2.5 we discuss the risk estimation methods considered and give an overview of the risk forecast backetesting procedures. Section 3 describes the data used. The results are presented in Section 4. Finally, Section 5 concludes.

2. Methods

2.1 GARCH models

Consider a stock index of price y_t, with corresponding continuously compounded rate of return r_t defined as $r_t = 100[log(y_t) - log(y_t - 1)]$, where the t indicates the time index under consideration. The basic GARCH(1,1) model for the returns series is given as follows:

$$r_t = \delta + \eta\sqrt{h_t} \tag{2.1.1}$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta h_{t-1} \tag{2.1.2}$$

Where $\alpha_0 > 0$, $\alpha_1 > 0$ and $\beta_1 \geq 1$

2.2 Markov Switching GARCH Models

Consider $r_t \in \mathbb{R}$, the (percentage) log-return of a stock index of price y_t at time t. The general Markov- switching GARCH specification assumes that:

$$r_t \mid (s_t = k, I_{t-1}) \sim \mathcal{D}(0, h_{k,t}, \epsilon_k) \tag{2.2.1}$$

where $\mathcal{D}(0, h_{k,t}, e_k)$ is a continuous distribution with zero mean, time-varying variance $h_{k,t}$, and additional shape parameters gathered in the vector ϵ_k. Furthermore, it is assumed that the integer- valued stochastic variable s_t, defined on the discrete space $\{1, \ldots, K\}$, evolves according to an unobserved first order ergodic homogeneous Markov chain with transition probability matrix $P \equiv \{p_{i,j}\}_{i,j=1}^{K}$, with $p_{i,j} \equiv \mathbb{P}[s_t = j \mid s_{t-1} = i]$. The information set up to time $t-1$ is denoted by I_{t-1},i.e. $I_{t-1} \equiv \{r_{t-1}, i > 0\}$. Given the parametrization of $D(\dot{)}$,$\mathbb{R}[r_t^2 \mid s_t = k, I_{t-1}] = h_{k,t}$, that is, $h_{k,t}$ is the variance of r_t conditional on the realization of s_t. Note that the conditional mean of the return is assumed to be zero across time and regimes. The conditional variance of r_t is assumed to follow a GARCH-type model. Hence, conditionally on regime $s_t = k$, $h_{k,t}$ is available as a function of past returns and the additional regime-dependent vector of parameters θ_k ((Ardia et al., 2017)) such that, $h_{k,t} \equiv \lambda(r_{t-1}, h_{k,t-1}, \theta_k)$ where $\lambda(\dot{)}$ is an I_{t-1} measurable function which defines the filter for the conditional variance and also ensures its positiveness. It is also assumed that $h_{k,1} \equiv \bar{h}_k(k = 1, \dots, K)$, where $\bar{h}_k$ is a fixed initial volatility level for regime k, that we set equal to the long-run unconditional volatility in regime k. Depending on the shape of $\lambda(\dot{)}$. Flexible definitions of the filter $\lambda(\dot{)}$ can be defined, e.g.to account for the asymmetric reaction of volatility to the sign of past returns (i.e. leverage effect) a Markovswitching GJR model with K regimes can be defined (see (Ardia et al., 2017) for details)In the next subsection, we give details of the variants of Equation 2.1.1 considered in this paper.

2.3 The Models

Here we give brief details of the GARCH models considered in this work. For each of the models described, we consider both single and 2-regime versions. The models are as follows

- Standard GARCH model (sGARCH): For the sGARCH model, the conditional variance of is modelled as (Bollerslev, 1986):

$$h_t = k + \delta_1 h_{t-1} + \theta_1 e_{t-1}^{\alpha} \tag{2.3.1}$$

- Glosten, Jagannathan, and Runkle GARCH (GJR-GARCH): (Glosten et al., 1993) proposed an asymmetric GARCH model that accounts for leverage effect. The model allows the conditional variance of to respond differently to shocks of either sign. It is defined as follows:

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 [1 - I_{e_{1-1>0}}] + \varepsilon e_{t-1}^2 [1 - I_{e_{1-1>0}}] + \beta_1 h_{t-1} \tag{2.3.2}$$

  where $I.$ is an indicator function.

- Exponential GARCH (EGARCH): (Nelson, 1991) introduced the EGARCH model, here there are no parameter constraints and logarithm of the conditional variance is modelled as follows:

$$log(h_t) = \alpha_0 + \alpha_1 \left| \frac{e_{t-1}}{h_{t-1}} \right| + \varepsilon \left| \frac{e_{t-1}}{h_{t-1}} \right| + \beta_1 log(h_1) \tag{2.3.3}$$

$$\tag{2.3.4}$$

- Threshold GARCH (TGARCH): (Zakoian, 1994) introduced a GARCH variation similar to the GJR-GARCH. The specification is one on conditional standard deviation instead of conditional variance such that:

$$h_t = K + \delta h_{t-1} + \alpha_1^+ e_{t-1}^+ + \alpha_1^- e_1^- \tag{2.3.5}$$

  where $e_{t-1}^+ = e_{t-1}$ if $e_{t-1} > 0$, and $e_{t-1}^+ = 0$ if $e_{t-1} \leq 0$. Likewise, $e_{t-1}^- = e_{t-1}$ if $e_{t-1} \leq 0$, and $e_{t-1}^- = 0$ if $e_{t-1} > 0$.

- Generalised autoregressive score model (GAS) :(Blasques et al., 2016) proposed the GAS model. The main feature of GAS models is that the evolution in the timevarying parameter vector θ_t is driven by the score of the conditional distribution $y_t \mid y_{1:t-1} \sim p(y_t; \theta_t)$ together with an autoregressive component:$\theta_{t+1} = +As_t + B\theta_t$. where,, A and B are matrices of coefficients with proper dimensions collected in e, and s_t is a vector which is proportional to the score of the conditional distribution.

2.4 Model evaluation

We compare model performances by employing the following penalised measures:

1. Akaike Information Criterion (AIC): It penalizes the loglikelihood for additional model parameters. AIC provides an asymptotically unbiased estimator of the expected Kullback discrepancy between the generating model and the fitted approximating model. it is computed as follows:
$$AIC = -2 \ln f(y \mid \hat{\theta}_k) + 2k$$
(Akaike, 1974).

2. Bayesian Information Criterion (BIC): It also penalizes the loglikelihood for additional model parameters, however this penalty increases as the number of records in the dataset increases. BIC provides a large-sample estimator of a transformation of the Bayesian posterior probability associated with the approximating model. it is computed as
$$BIC = -2 \ln f(y \mid \hat{\theta}_k) + k \ln n$$
(Kass and Raftery, 1995).

Note 2.1. It is noteworthy that AIC and BIC feature the same goodness-of-fit term, however, the penalty term of BIC is more stringent than the penalty term of AIC. (For $n \geq 8$, $k \ln n$ exceeds $2k$.) Consequently, BIC can be too restrictive and tends to favor smaller models than AIC.

2.5 Risk estimation and backtest

One step ahead predictive distribution is then computed for the returns series. VaR is computed as the $100_\alpha\%$ quantile of the predictive distribution and ES is computed as, $E[r_t \mid r_t > VaR_\alpha]$.

In this paper, we focus on focus on the VaR forecasts at the 1% and 5% risk levels. The first test used is the conditional coverage (CC) test of (Christoffersen, 1998), the common extension of the unconditional coverage (UC) test by (Kupiec, 1995). This approach is based on the study of the hit sequence $I_t^\alpha = \mathbb{I}\{y_t \leq VaR_t^\alpha\}$ where VaR_t^α denotes the VaR prediction at time t for risk level α. $\mathbb{I}\{\cdot\}$ is an indicator function such that I_t^α equal to one if the condition holds, and zero otherwise. A sequence of VaR forecasts at risk level α has correct conditional coverage if $\{I_t^\alpha; t = 1, \ldots, H\}$ is an independent and identically distributed sequence of Bernoulli random variables with parameter α. This hypothesis can be verified by testing jointly the independence on the series and the unconditional coverage of the VaR forecasts.

The dynamic quantile (DQ) test of (Engle and Manganelli, 2004) was also considered. This method jointly tests for UC and CC and has more power than previous alternatives under some form of model misspecification. The series of interest is defined as $\{I_t^\alpha - \alpha; t = 1, \ldots, H\}$. Under correct model specification, we have the following moment conditions: $E(\{I_t^\alpha - \alpha)\} = 0$, $E(\{I_t^\alpha - \alpha \mid I_{t-1}\}) = 0$,$E(\{(I_t^\alpha - \alpha)(I_{t'}^\alpha - \alpha)I_{t-1})\} = 0$ for $t \neq t'$ (Engle and Manganelli, 2004).

3. Data

The data is composed of 366 adjusted monthly closing prices of the Nigerian Stock Exchange (NSE) between 1985-2015. Figure1 gives the times plot of the NSE stock prices and corresponding log returns over the period considered.

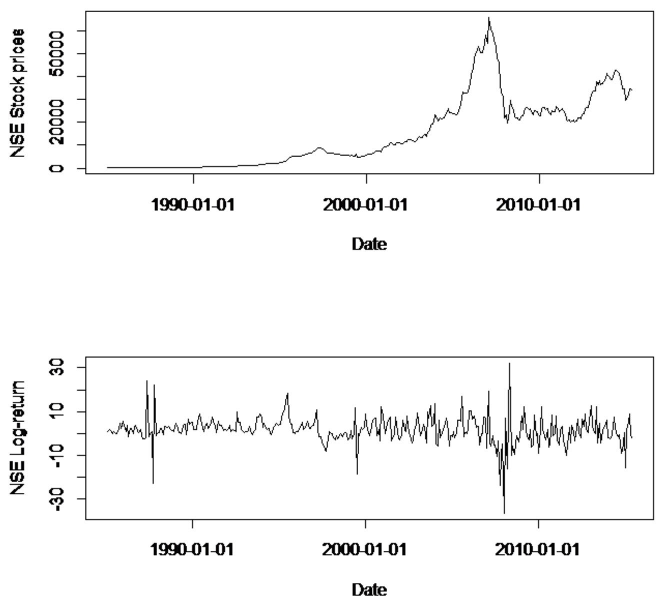

Figure 1: Time plot of NSE stocks (top panel) and NSE log returns (lower panel)

Figure 2 presents the Autocorrelation (ACF) and Parital Autocorrelation (PACF) plots of the returns series. Both plot do not record any periodicity in the returns series. This is further confirmed by the result of the Augmented Dickey-Fuller (ADF) Test, which returns a p-value less than 0.01 confirming that at 5% and 1% level of significace we are safe to say that there is no periodicity in the returns series.

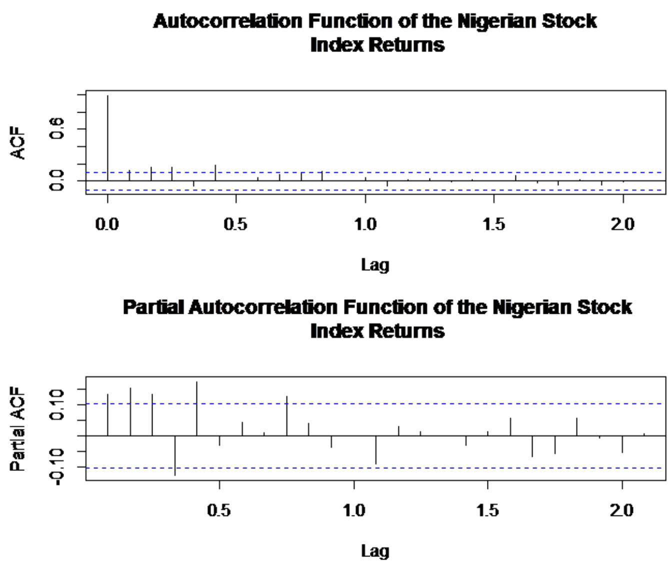

Figure 2: ACF of NSE log Returns (top panel) and PACF of NSE log returns (lower panel)

The summary statistics on the data is presented in Table 1

Table 1: Summary statistics of NSE log returns

Statistic	values
nobs	365.00
NAs	0.00
Minimum	-36.59
Maximum	32.35
1. Quartile	-1.05
3. Quartile	4.29
Mean	1.56
Median	1.61
Sum	570.58
SE Mean	0.32
LCL Mean	0.94
UCL Mean	2.19
Variance	36.67
Stdev	6.06
Skewness	-0.50
Kurtosis	7.81
JB test(p-value)	0.00

Skewness, kurtosis and Jarque-Bera test indicates that the returns are far from being unconditionally normally distributed,thus supporting our choice of more flexible distributional assumptions to improve model performance. This paper thus chooses the student's t distribution rather than the Gaussian distribution.

4. Results

4.1 Model evaluation (AIC and BIC)

Table 2 records the AIC and BIC scores for each of the models considered. The scores for both the single and 2-regime versions of the models are recorded.

Table 2: AIC and BIC results for each model

	Single regime model		2-regime model	
Model	AIC	BIC	AIC	BIC
sGARCH	2260.93	2280.43	2221.93	2268.73
GJR-GARCH	2259.84	2283.24	2236.90	2291.50
EGARCH	2242.38	2265.78	2194.37	2248.97
TGARCH	2241.02	2264.42	42793.00	42847.60
GAS	2249.52	2269.02	2226.85	2273.65

- Apart from the TGARCH model, the information criteria consistently selects the 2-regime model.
- The 2-regime EGARCH model was selected as the best model.

4.2 VaR Backtest results

1. Dynamic Quantile test results In Table 1, the results of the dynamic quantile (DQ) test are presented while the results of the Mean Absolute Deviation (ADmean) and the Maximum Absolute deviation (ADmax) are presented in Table 1.

	2-regime model		Single-regime model	
	DQ	p-value	DQ	p-value
sGARCH	32.41	0.00	23.46	0.00
GJR-GARCH	32.41	0.00	32.42	0.00
E-GARCH	0.77	0.99	32.42	0.00
TGARCH	0.76	0.99	-	-
GAS	0.76	0.99	0.39	1.00

- DQ tests report slightly better forecasts with the 2-regime models than for the single regime models. Suggesting that the 2-regime models might be better suited to forecasting VaR for the Nigerian Stock Index (NSE). On the other hand the Apart from the GAS model for which the 2-regime models perform better, the single regime models perform better for all the other models.

	Single regime model		2-regime model	
Model	Ad mean	AD max	AD mean	AD max
sGARCH	3.60	6.38	6.81	15.06
GJR-GARCH	3.60	6.38	5.91	10.93
E-GARCH	2.27	3.27	8.03	11.08
TGARCH	5.04	5.99	-	-
GAS	5.04	5.99	2.53	5.55

2. Unconditional coverage test results The results of the Unconditional Coverage (UC) test are presented in Table 2 below.

	Single regime model		2-regime model	
Model	LRuc	p-value	LRuc	p-value
sGARCH	20.33	0.00	16.93	0.00
GJR-GARCH	20.33	0.00	20.33	0.00
E-GARCH	24.41	0.00	20.33	0.00
TGARCH	24.41	0.00	-	-
GAS	24.41	0.00	20.33	0.00

- The UC tests report that similar performance for both single and 2-regime models.

3. Conditional coverage test results The results of the Conditional Coverage (CC) test are presented in Table 3 below

	Single regime model		2-regime model	
Model	LRcc	p-value	LRcc	p-value
sGARCH	20.38	0.00	17.02	0.00
GJR-GARCH	20.38	0.00	20.38	0.00
E-GARCH	24.43	0.00	20.38	0.00
TGARCH	24.43	0.00	-	-
GAS	24.43	0.00	20.38	0.00

- The CC tests report that similar performance for both single and 2-regime models.

4. Actual over Expected exceedance ratio

Model	2-regime	1-regime
sGARCH	0.16	0.22
GJR-GARCH	0.16	0.16
E-GARCH	0.11	0.16
TGARCH	0.11	-
GAS	0.11	0.16

- Similar to the UC and CC tests, the Actual over expected exceedance ratio for both single and 2-regime models are not too far apart.

5. Summary

In this paper we applied selected Markov switching asymmetric GARCH models in estimating value-at-risk (VaR) and its coherent complement Expected shortfall(ES) on returns of the Nigerian stock index. This was done by considering a mixture of Student's-t distributions over different time and regimes. Single regime asymmetric GARCH models were compared with their Markov switching counterparts. The forecasted risk measures were back tested using the unconditional coverage (UC)test of (Kupiec, 1995), the conditional coverage (CC) test of (Christoffersen, 1998) and Dynamic Quantile (DQ) test of (Engle and Manganelli, 2004). We found that while the Markov switching models were able to adjust for spurious high persistence found in the single regime asymmetric GARCH models, forecast performance is not too different between single and 2-regime models.

The results of this work has raised some questions concerning the behaviour of the Nigerian stock index.

- For Nigerian stock index, do monthly dynamics differ significantly from daily dynamics? The authors hope to investigate by running the same analysis on adjusted daily closing prices.
- Is sample size a critical determinant in the choice of single or multiple regime models? The authors hope to investigate this by extending the data to cover more years.

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