

Relative and Trend Revision Methods for Benchmarking Two Sets of Time Series

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Abstract

Data from the same target population collected at different spans of time can be inconsistent. For example, estimates collected monthly do not necessarily sum to measures from an annual survey. The adjustment process referred to as benchmarking solves inconsistency problems such as this. Economic programs at the Census Bureau use the relative and trend revision methods to benchmark quarterly and monthly time series. Fagan (1999) generalized the methods to benchmark a series broken out in two ways. This paper describes the methods presented by Fagan and use of them on economic data.

Keywords: Growth rate, objective function, steepest feasible descent

Disclaimer

Any views expressed are those of the author and not necessarily those of the U.S. Census Bureau.

1. Introduction

In addition to conducting the Economic Census every five years, the U.S. Census Bureau runs monthly, quarterly and annual programs that provide measures of our economy. The statistics are from over 100 separate surveys and census programs. Many of the programs gather data from the same target population. Data series from programs that target the same population at different spans of time may have inconsistencies. For example, sales estimates collected from a quarterly survey do not necessarily sum to sales measured from an annual survey. Benchmarking is the data adjustment method used to solve such inconsistencies.

The data series combined by benchmarking occur at time spans referred to as low (repeat after a long span of time; such as an annual survey or census) and high (repeat after a short span of time; such as a quarterly or monthly survey) frequency. Usually, measures from low frequency programs are detailed and reliable. The details and precision are due to many reasons such as large sample sizes and additional processing time. Programs that occur at high frequencies provide timely measures that are less detailed and better estimates of period-to-period (for example, month-to-month or quarter-to-quarter) change.

When we benchmark at the Census Bureau, our goal is to generate a “revised” time series that preserves high frequency data’s period-to-period changes (growth rates). Causey and Trager (1981; see also Trager, 1982; Bozik and Otto, 1988) developed the methodology we use to benchmark a series. In general, the methods (relative and trend revision) obtain solutions by minimizing objective functions subject to constraints. The first revision method, relative, solves a system of linear equations derived from partial derivatives of a Lagrange equation and the second, trend (referred to as the Causey-Trager method), uses an iterative process (steepest feasible descent) to minimize a nonlinear objective function.

We use these methods to solve 1-dimensional (1-D) benchmarking problems (adjusting *only* one high frequency series) and a generalized form of the methods (Fagan, 1999) to solve 2-dimensional (2-D) benchmarking problems (two sets of series; each set consists of series that are adjusted simultaneously). In this paper, we present the 2-D benchmarking problem, solutions and empirical results from the Building Permits Survey (BPS). The discussion includes survey results for both relative and trend revision methods.

2. 2-Dimensional Benchmarking

When a high frequency series is broken down in two ways such as housing by number of units (2, 3-4, 5 or more) and region (Northeast (NE), Midwest (MW), South (SO), West (WE)), we refer to the sets of series as group 1 (units) and 2 (region). To benchmark each series in groups 1 and 2, we adjust them at the same time through a process referred to as 2-D benchmarking. The benchmarking methods presented in Brown (2012) that use constrained optimizing techniques to solve for revised series are the foundation of 2-D benchmarking methods: relative and trend revision. The 2-D relative revision method minimizes a linear objective function (extension of Denton's (1971) proportional first difference objective function) and the 2-D trend revision method uses an iterative algorithm to minimize a growth rate objective function. The data used by both methods and revised series generated by the processes are described in section 2.1. Details about the revision methods are in sections 2.2 and 2.3.

2.1 Benchmarking Data

The data¹ processed through 2-D benchmarking are two sets of unadjusted high frequency series (original series) and corresponding low frequency series (benchmarks). In addition, the process can use link points, which are estimates from periods just before the revision span (for example, using link point December 2012 for a monthly series with a revision span that begins in January of 2013). We refer to the adjusted series created by benchmarking as the revised series and define 2-D benchmarking data as follows:

2-D Benchmarking Data: For both original and revised series, n equals the number of time periods (for example, $n=24$ when a monthly original series has two annual benchmarks), i and j represent breakdown time series within each group (1 & 2) and t represents the time period.

- Original Series – High Frequency

$$\text{Group 1- time series: } \{x_t^{(i)}, t = 1, \dots, n\} \text{ for } i=1, \dots, G1 \text{ breakdown series} \quad (1)$$

$$\text{Group 2- time series: } \{z_t^{(j)}, t = 1, \dots, n\} \text{ for } j=1, \dots, G2 \text{ breakdown series} \quad (2)$$

$$\text{where, } \sum_{i=1}^{G1} x_t^{(i)} = \sum_{j=1}^{G2} z_t^{(j)} \text{ for } t=1, \dots, n \quad (3)$$

¹ Economic programs at the Census Bureau benchmark two types of original series, flow and stock. Currently, the 2-D benchmarking solution can process only flow series, which measure accumulating activity over time.

- Benchmarks – Low Frequency

For groups 1 and 2, m = number of benchmark periods² and $k = 1, \dots, m$ denotes the benchmark period. Each benchmark (T_k and S_k) covers time periods from the original series that span from b_k (beginning) to e_k (ending) such that $t = 1 \leq b_1 \leq e_1 < b_2 \leq e_2 < \dots < b_m \leq e_m \leq n$.

Group 1- time series: $\{T_k^{(i)}, k = 1, \dots, m\}$ for $i=1, \dots, G1$ breakdown series (4)

Group 2- time series: $\{S_k^{(j)}, k = 1, \dots, m\}$ for $j=1, \dots, G2$ breakdown series (5)

where,
$$\sum_{i=1}^{G1} T_k^{(i)} = \sum_{j=1}^{G2} S_k^{(j)} \quad \text{for } k=1, \dots, m \quad (6)$$

- Revised Series

Group 1- time series: $\{y_t^{(i)}, t = 1, \dots, n\}$ for $i=1, \dots, G1$ revised series (7)

Group 2- time series: $\{w_t^{(j)}, t = 1, \dots, n\}$ for $j=1, \dots, G2$ revised series (8)

Revised Series meet the following constraints:

$$\sum_{t=b_k}^{e_k} y_t^{(i)} = T_k^{(i)} \quad \text{for } i=1, \dots, G1 \quad (9)$$

$$\sum_{t=b_k}^{e_k} w_t^{(j)} = S_k^{(j)} \quad \text{for } j=1, \dots, G2 \quad (10)$$

$$\sum_{i=1}^{G1} y_t^{(i)} = \sum_{j=1}^{G2} w_t^{(j)} \quad \text{for } t=1, \dots, n \quad (11)$$

2.2 Relative Revision

The relative revision method (Fagan, 1999) minimizes an objective function based on Denton’s proportional first difference:

$$F_R(y, w) = \sum_{i=1}^{G1} \sum_{t=1}^{n-1} \left(\frac{y_{t+1}^{(i)}}{x_{t+1}^{(i)}} - \frac{y_t^{(i)}}{x_t^{(i)}} \right)^2 + \sum_{j=1}^{G2} \sum_{t=1}^{n-1} \left(\frac{w_{t+1}^{(j)}}{z_{t+1}^{(j)}} - \frac{w_t^{(j)}}{z_t^{(j)}} \right)^2 \quad (12)$$

given the revised series constraints (9-11).

² Each benchmark period refers to a nonoverlapping span of time from the original series. For example, the number of time periods (n) = 2x4 when the number of benchmark periods (m) = 2 and the series frequency (f) = 4 (quarterly).

The revised series (y_t & w_t) are obtained by solving a system of linear equations from partial derivatives of this Lagrange Equation:

$$\begin{aligned}
 L_R(y, w, \lambda, \mu, \alpha) = & F_R(y, w) \\
 & + 2 \sum_{i=1}^{G1} \sum_{k=1}^m \lambda_k^{(i)} \left(\sum_{t=b_k}^{e_k} y_t^{(i)} - T_k^{(i)} \right) \\
 & + 2 \sum_{j=1}^{G2} \sum_{k=1}^m \mu_k^{(j)} \left(\sum_{t=b_k}^{e_k} w_t^{(j)} - S_k^{(j)} \right) \\
 & + 2 \sum_{t=1}^n \alpha_t \left(\sum_{i=1}^{G1} y_t^{(i)} - \sum_{j=1}^{G2} w_t^{(j)} \right)
 \end{aligned} \tag{13}$$

The partial derivatives set equal to 0 form a system of $(G1+G2)(n+m)+n-m$ equations (see appendix A for the system of equations with a link point). The approach, LU decomposition, to solving the system of equations in matrix form is as follows

$$\mathbf{A}\tilde{\boldsymbol{\theta}} = \mathbf{B}$$

where, $\mathbf{A} = \begin{bmatrix} \mathbf{D} & \mathbf{E} \\ \mathbf{E}' & \mathbf{0} \end{bmatrix}$

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}'\mathbf{d} & 0 & \dots & 0 \\ 0 & \mathbf{d}'\mathbf{d} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{d}'\mathbf{d} \end{bmatrix}, \text{ if link point } \mathbf{d} = \begin{bmatrix} 1 & 0 & 0 & \dots \\ -1 & 1 & 0 & \dots \\ 0 & -1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$\mathbf{E} = [\mathbf{E}_1 \quad \mathbf{E}_2]$$

$\mathbf{E}_1 = \mathbf{XZ} * \mathbf{H}$, \mathbf{XZ} is a diagonal matrix of original values and \mathbf{H} is a diagonal matrix of $f \times 1$ matrices with the value one.

\mathbf{E}_2 is a matrix of diagonal matrices \mathbf{g}_1^i and $-\mathbf{g}_2^j$. \mathbf{g}_1^i is a diagonal matrix of series group 1 original values (x^i) and \mathbf{g}_2^j is a diagonal matrix of series group 2 original values (z^j). Each matrix excludes columns where $t=e_k$.

$\mathbf{0}$ is a matrix of the value zero.

$$\tilde{\theta} = \begin{bmatrix} \tilde{Y} \\ \tilde{W} \\ \lambda \\ \mu \\ \alpha \end{bmatrix}; \text{ unknowns}$$

Revised series are derived from \tilde{Y} and \tilde{W} and the Lagrange multipliers are in λ , μ , and α .

$$\mathbf{B} = \begin{bmatrix} \mathbf{L} \\ \mathbf{T} \\ \mathbf{S} \\ \mathbf{0} \end{bmatrix}; = \begin{bmatrix} \mathbf{lr}^i \\ \vdots \\ \mathbf{lc}^j \\ \vdots \end{bmatrix}, \text{ where } \mathbf{lr}^i \text{ and } \mathbf{lc}^j \text{ are } n \times 1 \text{ matrices with link point values}$$

$$\mathbf{lr}^i = \begin{bmatrix} y_0^i / x_0^i \\ 0 \\ \vdots \end{bmatrix} \text{ and } \mathbf{lc}^j = \begin{bmatrix} w_0^j / z_0^j \\ 0 \\ \vdots \end{bmatrix}$$

\mathbf{T} and \mathbf{S} are matrices of the benchmarks for series group 1 and 2.

$\mathbf{0}$ is a matrix of the value zero.

Solving for the unknowns ($\tilde{\theta}$) is programmed in SAS®³. The solution to $\tilde{\theta} = \mathbf{A}^{-1}\mathbf{B}$ is obtained by using the matrix inverse function (INV) built into PROC IML. The function uses “LU decomposition followed by back substitution to solve for the inverse” (SAS, 2011). For more on the relative revision method, see Fagan (1999).

2.3 Trend Revision

The trend revision method (Fagan, 1999) minimizes the following growth rate objective function with the revised series constraints (9-11):

$$F_T(y, w) = \sum_{i=1}^{G1} \sum_{t=1}^{n-1} \left(\frac{y_{t+1}^{(i)}}{y_t^{(i)}} - \frac{x_{t+1}^{(i)}}{x_t^{(i)}} \right)^2 + \sum_{j=1}^{G2} \sum_{t=1}^{n-1} \left(\frac{w_{t+1}^{(j)}}{w_t^{(j)}} - \frac{z_{t+1}^{(j)}}{z_t^{(j)}} \right)^2 \quad (14)$$

The approach to finding a minimum uses an iterative nonlinear programming method, steepest feasible descent. In general, the method consists of the following steps for each p^{th} iteration until stopping criteria are met:

- 1) Find feasible direction from $(y, w)_{p-1}$.
- 2) Find the point that minimizes F_T along the line through $(y, w)_{p-1}$ and in the direction of step 1. Use it for the next iteration.

³ SAS and all other SAS institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc. in the USA and other countries. ® indicates USA registration.

The method uses the auxiliary function

$$F_T^*(s) = \sum_{i=1}^{G1} \sum_{t=0}^{n-1} \left(\frac{y_{t+1}^{(i)} + s d_{t+1}^{(i)}}{y_t^{(i)} + s d_t^{(i)}} - \frac{x_{t+1}^{(i)}}{x_t^{(i)}} \right)^2 + \sum_{j=1}^{G2} \sum_{t=0}^{n-1} \left(\frac{w_{t+1}^{(j)} + s e_{t+1}^{(j)}}{w_t^{(j)} + s e_t^{(j)}} - \frac{z_{t+1}^{(j)}}{z_t^{(j)}} \right)^2 \quad (15)$$

where d, e is the direction vector and the step size (scalar) is s .

In practice, we implement this approach by first assigning the relative revision solutions that meet the revised series constraints (9-11) as starting values $(y, w)_{p=0}$. Next, we process the following steps repeatedly until they meet convergence criteria or maximum number of iterations:

- Find direction vector, $\mathbf{D} = \begin{bmatrix} \mathbf{d} \\ \mathbf{e} \end{bmatrix}$
 - Solve $\tilde{\mathbf{D}} = \mathbf{A}^{-1} \mathbf{B}$

$$\tilde{\mathbf{D}} = \begin{bmatrix} \tilde{\mathbf{d}} \\ \tilde{\mathbf{e}} \\ \boldsymbol{\lambda} \end{bmatrix}; \text{ unknowns}$$

The direction vector is derived from $\tilde{\mathbf{d}}$ and $\tilde{\mathbf{e}}$ and the Lagrange multipliers are in $\boldsymbol{\lambda}$.

$$\mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{E} \\ \mathbf{E}' & \mathbf{0} \end{bmatrix}$$

\mathbf{I} is an identity matrix.

$$\mathbf{E} = [\mathbf{E}_{11} \quad \mathbf{E}_{12}]$$

\mathbf{E}_{11} is a diagonal matrix of $f \times 1$ matrices with the value one.

\mathbf{E}_{12} is a matrix of diagonal matrices \mathbf{g}_{11}^i and \mathbf{g}_{12}^j . \mathbf{g}_{11}^i is a diagonal matrix of the value one and \mathbf{g}_{12}^j is a diagonal matrix of the value negative one. Each matrix excludes columns where $t=e_k$.

$\mathbf{0}$ is a matrix of the value zero.

$$\mathbf{B} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{0} \end{bmatrix}$$

\mathbf{a} is series group 1's matrix of terms from the auxiliary function's (F_T^*) directional derivative.

\mathbf{b} is series group 2's matrix of terms from the auxiliary function's (F_T^*) directional derivative.

For more on \mathbf{a} and \mathbf{b} see Appendix B.

$\mathbf{0}$ is a matrix of the value zero.

- Derive $\mathbf{D} = \begin{bmatrix} \mathbf{d} \\ \mathbf{e} \end{bmatrix}$ from $\tilde{\mathbf{D}} = \begin{bmatrix} \tilde{\mathbf{d}} \\ \tilde{\mathbf{e}} \\ \lambda \end{bmatrix}$

$$d_t^{(i)} = \frac{\tilde{d}_t^{(i)}}{\Delta}; \quad e_t^{(j)} = \frac{\tilde{e}_t^{(j)}}{\Delta}$$

$$\text{where, } \Delta = \sqrt{\sum_{i=1}^{G1} \sum_{t=1}^n (\tilde{d}_t^{(i)})^2 + \sum_{j=1}^{G2} \sum_{t=1}^n (\tilde{e}_t^{(j)})^2}$$

- Test if direction vector is sufficiently close to zero. If true, stop processing and output iteration $p-1$ values.

$$\left| \sum_{i=1}^{G1} \sum_{t=1}^n a_t^{(i)} d_t^{(i)} + \sum_{j=1}^{G2} \sum_{t=1}^n b_t^{(j)} e_t^{(j)} \right| < \text{set tolerance}$$

- Find step size, s .
 - Find points that bracket the minimum.⁴
 - Find minimum in the direction \mathbf{D} using the parabolic interpolation procedure, Brent's Method.⁵
- Derive revised value for p^{th} iteration:

$$y_t^{(p),(i)} = y_t^{(p-1),(i)} + s d_t^{(i)} \quad \text{and} \quad w_t^{(p),(j)} = w_t^{(p-1),(j)} + s e_t^{(j)}$$

- Test stopping criteria

$$\left| F_T^{(p)}(y, w) - F_T^{(p-1)}(y, w) \right| \leq \text{set tolerance}$$

or

$$p^{\text{th}} \text{ iteration} = \text{maximum \# of iterations}$$

3. Building Permits Survey

The BPS “provides data on the number of new housing units authorized by building permits” (U.S. Census Bureau | Building Permits, “Overview”). The survey generates

⁴ The algorithm used is described in section 10.1 (Initially Bracketing a Minimum) of *Numerical Recipes 3rd Edition: The Art of Scientific Computing* (Press, Teukolsky, Vetterling & Flannery, 2007).

⁵ The algorithm used is described in section 10.3 (Parabolic Interpolation and Brent's Method in One Dimension) of *Numerical Recipes 3rd Edition: The Art of Scientific Computing* (Press, Teukolsky, Vetterling & Flannery, 2007).

national, state and local level estimates from a monthly survey (original series) and annual census (benchmarks) of permit issuing places. Annually, the survey benchmarks monthly single-family (1) and 2+ unit series to annual totals. The 2+ series are in the data structure for 2-D benchmarking. They are broken down by unit type and region (see Tables 1a and 1b). Each group of series is benchmarked at once to annual totals, so for each month the sum of units equals the region total (sum of rows). For example, $U.S. 2^+_{Jan} = 2_{Jan} + 3-4_{Jan} + 5^+_{Jan} = NE_{Jan} + MW_{Jan} + SO_{Jan} + WE_{Jan}$. In addition, the monthly benchmarked values of each breakdown sum to the annual totals (sum of columns).

Table 1a: Series 1 - Units

Unit Type			U.S. 2+	
2	3-4	5+		
			Jan	
			...	
			Dec	
<i>Annual</i>	2	3-4	5+	U.S. 2+

Table 1b: Series 2 - Region

Region				U.S. 2+	
NE	MW	SO	WE		
				Jan	
				...	
				Dec	
<i>Annual</i>	NE	MW	SO	WE	U.S. 2+

For comparison of the 2-D benchmarking methods, relative and trend revision, we combined BPS 2+ benchmarking results from 2007–2013. Similar to production, we processed each calendar year through the relative and trend revision methods with a link point (December of the previous year). The research data does not match published results because they go through additional processing (for example, rounding and raking).

First, we reviewed the objective functions described in sections 2.2 and 2.3. For each benchmarking method, we derived both objective functions. Table 2 shows averages of the objective functions. As expected, the smallest objective function averages for each revision are those minimized by their respective methods. The expected pattern is the same for almost every benchmark period. One set of relative revision results has an unexpected pattern, trend objective function slightly smaller than relative. For that benchmark period, there is one series with a month-to-month spike in permits. The growth rates around the spike range from approximately 248% to -83%.

Table 2: Average Objective Functions (2007-2013) by Revision Method

Objective Function*	Revision Method	
	Relative	Trend
OBJ-R	0.00614	0.01059
OBJ-T	0.00751	0.00374

*OBJ-R is the relative revision’s objective function (12) and OBJ-T is the trend revision’s objective function (14).

The relative revision method minimizes differences between ratios of revised to original series values. For each benchmarking method, we calculated these ratios and reviewed each series. In general, the relative revision and trend lines of each series were very similar but the trend line had a few more changes in movement that we refer to as “kinks.” For most series, both lines show “kinks” at each benchmark. For an example, see Appendix C, Figure 1.

Next, we calculated absolute revisions of the period-to-period change (ut_t^m)

$$ut_t^m = \left| \frac{y_t^m}{y_{t-1}^m} - \frac{x_t}{x_{t-1}} \right| \quad \text{for } t = 2, \dots, n^* \quad (16)$$

where, m (benchmarking method) = R (Relative) or T (Trend)
 n^* is the last time period

We graphed box plots of the absolute revisions. Most of the plots that combine benchmark periods show smaller, less variable trend absolute revisions (see Appendix C, Figure 2a). When the revisions were broken down by each benchmark period, we generally saw similar patterns as seen in the graphs that combined results (see Appendix C, Figure 2b).

In addition, we calculated the difference between absolute revisions

$$dt_t = ut_t^R - ut_t^T; t = 2, \dots, n^*. \quad (17)$$

Tables 3a and 3b show series results for the differences. The positive sign and magnitude of each maximum value show the relative revision method generated each of the biggest differences largest absolute revision. Overall, region series SO has the smallest average difference in absolute revisions.

Table 3a: Difference in Absolute Revisions (Units)

Series	Difference in Absolute Revision			
	Mean	Median	Minimum	Maximum
2 Units	.0004	.0000	-.0109	0.0138
3-4 Units	.0010	.0004	-.0086	0.0192
5 Units or More	.0007	.0003	-.0069	0.0102

Table 3b: Difference in Absolute Revisions (Region)

Series	Difference in Absolute Revision			
	Mean	Median	Minimum	Maximum
NE	.0049	.0011	-0.0343	0.0843
MW	.0018	.0005	-0.0116	0.0219
SO	.0002	.0001	-0.0042	0.0066
WE	.0005	.0003	-0.0035	0.0079

To review the results over time we graphed differences in absolute revisions. Series graphs show that on average, the relative revision method has higher absolute revisions. In addition, they show for most series, minimum differences occur at the beginning of a benchmark period. This might be due to processing each benchmark period separately. For an example, see Appendix C, Figure 3.

4. Summary

At the Census Bureau, a 2-D benchmarking method, which is a generalized form of the Causey-Trager approach, simultaneously adjusts two sets of series from the BPS. In general, the method minimizes a growth rate objective function subject to constraints. The solution comes from an iterative nonlinear programming method (steepest feasible descent) that uses relative revision results as starting points. For the survey data reviewed, most of

the results were as expected. The smallest average objective functions for each revision are those minimized by their respective methods and on average, the Causey-Trager approach has smaller absolute revisions of month-to-month change.

Future work will focus on investigating the robustness of the 2-D benchmarking method. Research topics include investigating alternative nonlinear programming techniques and the method's performance and consistency. The review will include processing additional types of economic time series such as quarterly and processing more than one benchmark period. In addition to evaluating the current method, we continue to look into alternative approaches that will meet the Census Bureau's benchmarking goal, which is to generate "new" time series that preserve high frequency data's growth rates.

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Relative Revision – System of Linear Equations with a Link Point ($t=0$)

$$\text{Let } \tilde{y}_t^{(i)} = \frac{y_t^{(i)}}{x_t^{(i)}}, t \in \{1, \dots, n\}; i \in \{1, \dots, G1\}$$

$$\tilde{w}_t^{(j)} = \frac{w_t^{(j)}}{z_t^{(j)}}, t \in \{1, \dots, n\}; j \in \{1, \dots, G2\}$$

$$2\tilde{y}_1^{(i)} - \tilde{y}_2^{(i)} + \lambda_1^{(i)} x_1^{(i)} + \alpha_1 x_1^{(i)} = \frac{y_0^{(i)}}{x_0^{(i)}} \quad i \in \{1, \dots, G1\}$$

$$\left(-\tilde{y}_{t+1}^{(i)} + 2\tilde{y}_t^{(i)} - \tilde{y}_{t-1}^{(i)}\right) + \lambda_{k(t \in (b_k, e_k))}^{(i)} x_t^{(i)} + \alpha_t x_t^{(i)} = 0 \quad \begin{array}{l} t \in \{2, \dots, n-1\}; \\ i \in \{1, \dots, G1\} \end{array}$$

$$\left(-\tilde{y}_{n-1}^{(i)} + \tilde{y}_n^{(i)}\right) + \lambda_{m(n \in (b_m, e_m))}^{(i)} x_n^{(i)} = 0 \quad i \in \{1, \dots, G1\}$$

$$2\tilde{w}_1^{(j)} - \tilde{w}_2^{(j)} + \mu_1^{(j)} z_1^{(j)} - \alpha_1 z_1^{(j)} = \frac{w_0^{(j)}}{z_0^{(j)}} \quad j \in \{1, \dots, G2\}$$

$$\left(-\tilde{w}_{t+1}^{(j)} + 2\tilde{w}_t^{(j)} - \tilde{w}_{t-1}^{(j)}\right) + \mu_{k(t \in (b_k, e_k))}^{(j)} z_t^{(j)} - \alpha_t z_t^{(j)} = 0 \quad \begin{array}{l} t \in \{2, \dots, n-1\}; \\ j \in \{1, \dots, G2\} \end{array}$$

$$\left(-\tilde{w}_{n-1}^{(j)} + \tilde{w}_n^{(j)}\right) + \mu_{m(n \in (b_m, e_m))}^{(j)} z_n^{(j)} = 0 \quad j \in \{1, \dots, G2\}$$

$$\sum_{t=b_k}^{e_k} x_t^{(i)} \tilde{y}_t^{(i)} = T_k^{(i)} \quad \begin{array}{l} k \in \{1, \dots, m\}; \\ i \in \{1, \dots, G1\} \end{array}$$

$$\sum_{t=b_k}^{e_k} z_t^{(j)} \tilde{w}_t^{(j)} = S_k^{(j)} \quad \begin{array}{l} k \in \{1, \dots, m\}; \\ j \in \{1, \dots, G2\} \end{array}$$

$$\sum_{i=1}^{G1} x_t^{(i)} \tilde{y}_t^{(i)} - \sum_{j=1}^{G2} z_t^{(j)} \tilde{w}_t^{(j)} = 0 \quad \begin{array}{l} t^* \in \{1, \dots, n\} \\ \text{*except} \\ \text{where } t=e_k \end{array}$$

Trend Revision- 'a' and 'b' Terms of Directional Derivative

$$a = \begin{bmatrix} a_1^{(1)} \\ \vdots \\ a_n^{(G1)} \end{bmatrix}; \text{ with link point}$$

$$a_1^{(i)} = \frac{\left(\frac{y_1^{(p),(i)}}{y_0^{(i)}} - \frac{x_1^{(i)}}{x_0^{(i)}}\right) - \left(\frac{y_2^{(p),(i)}}{y_1^{(p),(i)}} - \frac{x_2^{(i)}}{x_1^{(i)}}\right) (y_2^{(p),(i)})}{(y_1^{(p),(i)})^2} \quad i \in \{1, \dots, G1\}$$

$$a_t^{(i)} = \frac{\left(\frac{y_t^{(p),(i)}}{y_{t-1}^{(p),(i)}} - \frac{x_t^{(i)}}{x_{t-1}^{(i)}}\right) - \left(\frac{y_{t+1}^{(p),(i)}}{y_t^{(p),(i)}} - \frac{x_{t+1}^{(i)}}{x_t^{(i)}}\right) (y_{t+1}^{(p),(i)})}{(y_t^{(p),(i)})^2} \quad \begin{array}{l} t \in \{2, \dots, n-1\}; \\ i \in \{1, \dots, G1\} \end{array}$$

$$a_n^{(i)} = \frac{\left(\frac{y_n^{(p),(i)}}{y_{n-1}^{(p),(i)}} - \frac{x_n^{(i)}}{x_{n-1}^{(i)}}\right)}{y_{n-1}^{(p),(i)}} \quad i \in \{1, \dots, G1\}$$

$$b = \begin{bmatrix} b_1^{(1)} \\ \vdots \\ b_n^{(G2)} \end{bmatrix}; \text{ with link point}$$

$$b_1^{(j)} = \frac{\left(\frac{w_1^{(p),(j)}}{w_0^{(j)}} - \frac{z_1^{(j)}}{z_0^{(j)}}\right) - \left(\frac{w_2^{(p),(j)}}{w_1^{(p),(j)}} - \frac{z_2^{(j)}}{z_1^{(j)}}\right) (w_2^{(p),(j)})}{(w_1^{(p),(j)})^2} \quad j \in \{1, \dots, G2\}$$

Trend Revision- 'a' and 'b' Terms of Directional Derivative - continued

$$b_t^{(j)} = \frac{\left(\frac{w_t^{(p),(j)}}{w_{t-1}^{(p),(j)}} - \frac{z_t^{(j)}}{z_{t-1}^{(j)}} \right)}{w_{t-1}^{(p),(j)}} - \frac{\left(\frac{w_{t+1}^{(p),(j)}}{w_t^{(p),(j)}} - \frac{z_{t+1}^{(j)}}{z_t^{(j)}} \right) \left(w_{t+1}^{(p),(j)} \right)}{\left(w_t^{(p),(j)} \right)^2} \quad \begin{array}{l} t \in \{2, \dots, n-1\}; \\ j \in \{1, \dots, G2\} \end{array}$$

$$b_n^{(j)} = \frac{\left(\frac{w_n^{(p),(j)}}{w_{n-1}^{(p),(j)}} - \frac{z_n^{(j)}}{z_{n-1}^{(j)}} \right)}{w_{n-1}^{(p),(j)}} \quad j \in \{1, \dots, G2\}$$

Building Permits Survey 2-D Benchmarking Results – Figures 1 and 2a

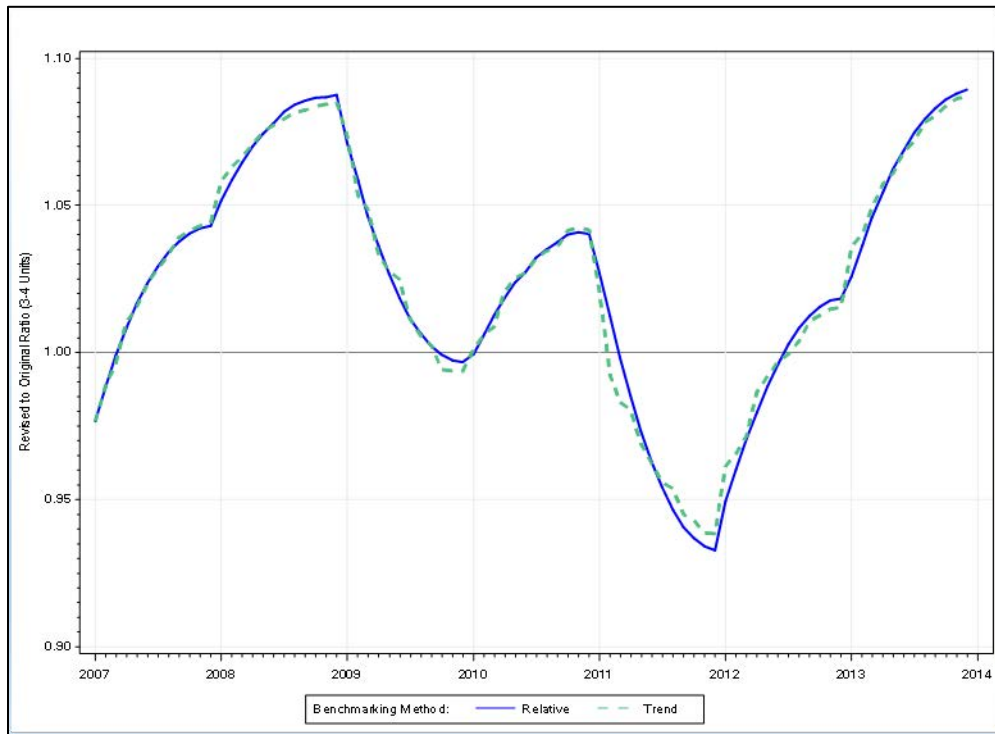


Figure 1: Ratio of Revised to Original - 3-4 Units

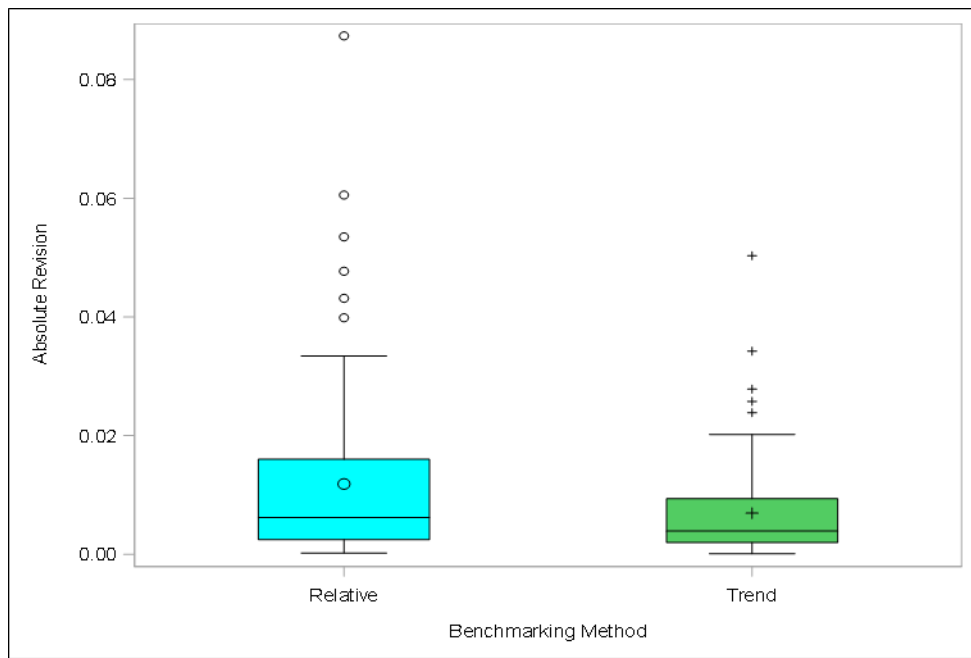


Figure 2a: Absolute Revisions (Period-to-Period Change) - Northeast

Building Permits Survey 2-D Benchmarking Results – Figures 2b & 3

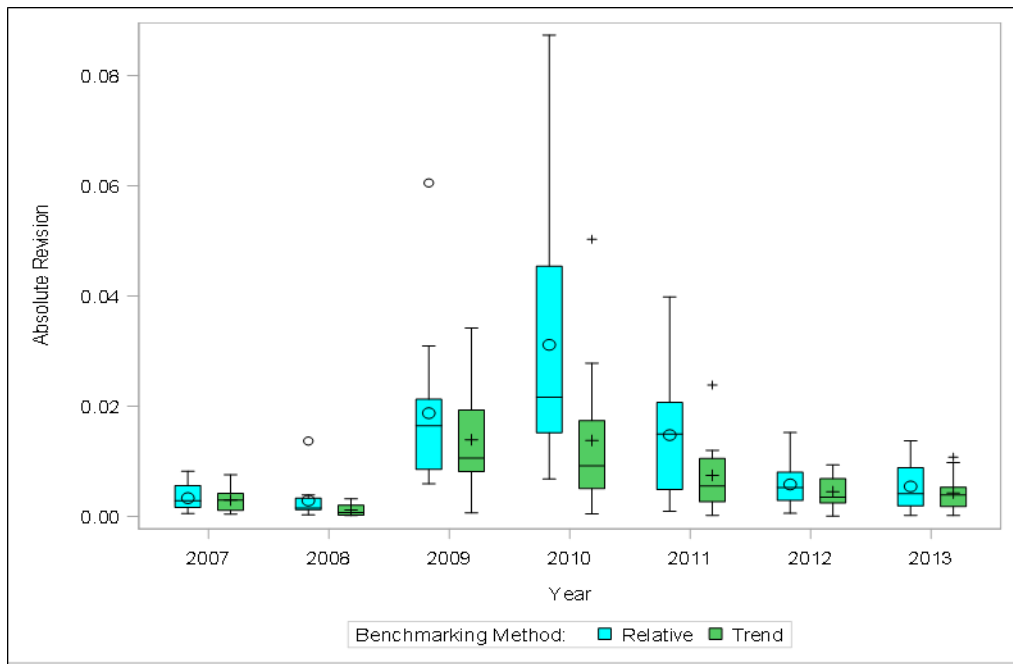


Figure 2b: Absolute Revisions (Period-to-Period Change by Year) - Northeast

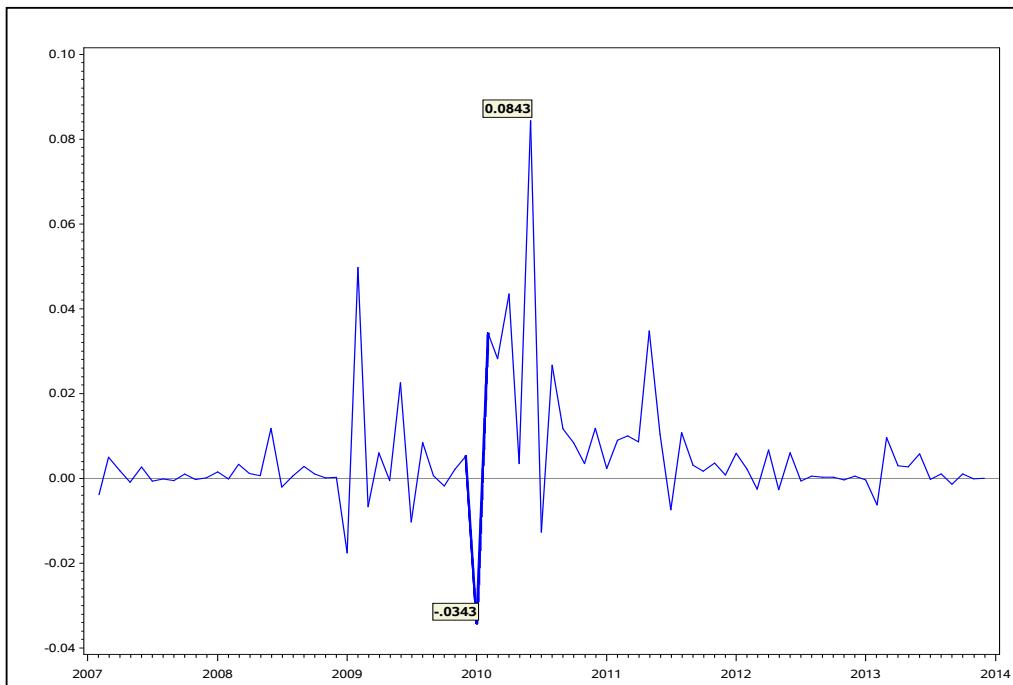


Figure 3: Difference in Absolute Revisions - Northeast