Measurement of Chain Drift in the Chained CPI-U

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Abstract

Chained price indexes may drift from fixed base indexes as price change occurs over time. Fixed base indexes may not exhibit drift, but the weights are susceptible to being less representative than the weights of chained indexes. This paper introduces basic price index concepts including chain drift and weight representivity and uses Bureau of Labor Statistics (BLS) Consumer Price Index (CPI) data to demonstrate that high levels of weight representivity and low levels of index chain drift can co-exist for the superlative Tornqvist formula.

Key Words: BLS, Chained Consumer Price Index, C-CPI, chain drift, weight representativity

1. Background

The US CPI measures the average change in prices paid by urban consumers for a fixed market basket of goods and services in the United States.[1] The CPI is revised every other year to update the market basket item structure and weights. Therefore, the "fixed market basket" is limited to a fixed item structure and weights from January of an even year to December of an odd year.

The goal of the CPI is to approximate a Cost of Living Index (COLI), which measures the minimum change in expenditures needed to attain a base period standard of living at current period prices. Given that measurement of a standard of living is not directly observable, the BLS produces several indexes that approximate a conditional COLI.

The CPI-U is the "headline" CPI index. It is calculated using a Lowe formula, also referred to as a modified Laspeyres formula, where the weights are biennial expenditures. Fixed quantity weights overestimate a COLI because they fail to capture when consumers respond to price increases by substituting to relatively lower priced comparable goods or services. Although the CPI-U is constrained by fixed quantity weights within a 24 month time frame, biennial revisions to weights and resulting indexes are chained together which allows for longer term substitution in response to price change, and therefore diminishes the impact of overestimating a COLI.

In contrast, the Chained-CPI-U (C-CPI-U) is calculated using a superlative Tornqvist formula, where the prices and weights are from a current month and previous month and then are chained together across months. Monthly weights reflect consumer substitution in response to price change, and therefore provide a closer approximation to a COLI. The

[1] See Bureau of Labor Statistics Handbook of Methods Chapter 17.

final C-CPI-U is not produced in real time concurrent with the CPI-U due to a lag necessary for surveying and processing monthly weights. To fill this 10-12 month gap, preliminary C-CPI-U indexes are estimated using a Constant Elasticity of Substitution (CES) formula. An initial C-CPI-U index is estimated monthly concurrently with the CPI-U and chained onto the C-CPI-U historical series. The series is updated quarterly until the final C-CPI-U index is published about one-year after the initial. The proceeding analysis is limited to the final C-CPI-U.

2. Overview of Index Chain Drift

Index chain drift in the simplest form is defined as the difference between a fixed base index and a chained index. A price index can be constructed using a direct or chained approach. The direct approach measures price change from a fixed base period index to the current period, t. The chained approach measures price change from the previous period, t-1, to the current period, t.[2] The fixed base index is a direct measure of long term price change requiring a fixed index structure, which is most appropriate for measuring inflation over a longer time frame. The fixed index structure of base period weights results in weights that are less representative over time. The chained index is a cumulative measure of long term price change, and is most appropriate for measuring inflation in real time due to revisions to the index structure as well as revisions to weights that are more representative of current period prices.

The choice between a fixed base and a chained index is a tradeoff between transitivity and representitity.[3] Transitivity is a desirable property of a price index, meaning a price index should register no price change when there are symmetric changes in its component prices in two time periods. This is violated when chain drift occurs. Drift is caused by the relationship between price change and weight change from the chained short term trend overstating or understating the long term trend of the direct index. Chained indexes are susceptible to drift when peaks and troughs occur, as exhibited by seasonality or index oscillation. If an index is chained at the peaks or at the troughs, then the chained series will be respectively high or low relative to a longer term fixed base index. Additionally, indexes chained more frequently will be at greater risk for drifting due the increased likelihood of catching a wave upwards on a peak, or sinking downwards on a trough. The upwards or downwards movement of a chained index relative to a fixed based is commonly referred to as a "bounce" effect.[4] When prices are negatively correlated with quantity weights, then the chained index will drift higher than the fixed base index as displayed in the fresh fruits example and results below.[5]

3. Aggregate Index Estimation to Measure Drift

A price index is a measure of price change from an originating period to a future period. For most government statistical agencies, a CPI is a measure of weighted price change from one period to a future period by the aggregation of elementary level price indexes and quantity weights. This paper focuses on upper level aggregation of elementary indexes and weights to estimate aggregate indexes. The following elementary price index data are based

[2] See ILO Consumer Price Index Manual: Theory and Practice: 15.6, 15.77.
[3] See Forsythe and Fowler (1981), and Lent (2000).
[4] See Szulc (1983), Kurtzon (2006), ILO: 15.82 Footnote 61
[5] See Aizcorbe and Jackman (1993), Kurtzon (2016), Australian Working Paper (1996)

on 1-month chained price relatives. The following elementary weight data are based a fixed constant quantity weight shares from the based period (P_0Q_0), base period shares updated to a more recent time period (P_VQ_B), and monthly weight shares (P_tQ_t). These three forms will be used for the corresponding fixed base formulas, chained biennial revision formulas, and chained monthly formulas respectively.

To process fixed base formulas, the item structure is static over the entire time span. For example, in January 2008, the elementary item for convalescent care moved CPI major group classifications from Other Goods and Services (item GD06) to Medical Care (item MD03). To create a fixed structure from December 1999 to December 2014 this item was recoded to GD06 throughout the entire time span, where the new indexes were rebased as needed according to the old structure and the weights were recalculated according to the old structure. Fixed base recoding and processing activities occurred for a subset of items due to the 2008 and 2010 CPI item structure changes. Additionally, extreme outlier index relatives were edited to mitigate the impact to aggregate index results.[6]

For this paper, formulas measuring drift are based on the Lowe CPI-U formula and Tornqvist C-CPI-U formulas. Table 1 shows the Laspeyres/Lowe formulas and Table 2 shows the Tornqvist formulas.[7] The Tornqvist formulas described in Table 2 are essentially geometric mean formulas weighted by an average share between the current and previous period for the chained monthly revisions. The fixed base and chained biennial revisions modify the previous period share and corresponding denominator of the index relative.[8]

Table 1. Laspeyres / Lowe Formulas

	Index	Weight	Index
Fixed Base	Fixed base Index = 12 month average 2001	Fixed 2001 annual weight share	$FB_{t,0}^{LA} = \sum_j \left(\frac{\hat{P}_{Ak}\hat{Q}_{Ak}}{\sum_j \hat{P}_{Aj}\hat{Q}_{Aj}}\right)\left(\frac{IX_{tk}}{IX_{Ak}}\right)$
Chained Biennial Revision	Pivot month revised & chained biennially	Aggrega-tion weight revised biennially	$CBR_{t,0}^{LO} = \sum_j \left(\frac{(\hat{P}_{\alpha k}\hat{Q}_{\beta k})(IX_{tk})}{(\hat{P}_{\alpha k}\hat{Q}_{\beta k})(IX_{vk})}\right)$

[6] Outlier index relatives were identified as greater than 20 or less than .05; index relatives greater than 20 were imputed to 20, and relatives less than .05 were imputed to .05. The index level was then recalculated based on the imputed relative.
[7] Since the weighted arithmetic average of the Lowe index is equivalent to a Laspeyres with lagged weights Table 1 references Laspeyres / Lowe where Laspeyres is applicable to the Fixed Base and Chained Monthly Revisions formulas.
[8] Additional formulas based on fixed quantity share were also evaluated, but the end results closely approximated the Fixed Based versions for the respective formulas.

Chained Monthly Revision	Monthly chained	Previous period month weight share	$CMR_{t,0}^{LC} = \prod_{0,t-1}\left[\sum_{j}\left(\frac{M_{t-1k}}{\sum_j M_{t-1j}}\right)\left(\frac{IX_{tk}}{IX_{t-1k}}\right)\right]$

1. IX_{tk} = Basic level index for period t
2. IX_{Ak} = Fixed base basic level 12 month average index for base period
3. $\hat{P}_{Ak}\hat{Q}_{Ak}$ = Fixed base basic level annual expenditure weight
4. $\hat{P}_{Aj}\hat{Q}_{Aj}$ = Fixed base aggregate level annual expenditure weight
5. IX_{vk} = Basic level index for pivot period
6. $\hat{P}_{\alpha k}\hat{Q}_{\beta k}$ = Basic level aggregation weight
7. IX_{t-1k} = Basic level index for period t-1
8. M_{t-1k} = Basic level monthly expenditure weight for period t-1
9. M_{t-1j} = Aggregate level monthly expenditure weight for period t-1

Table 2. Tornqvist Formulas

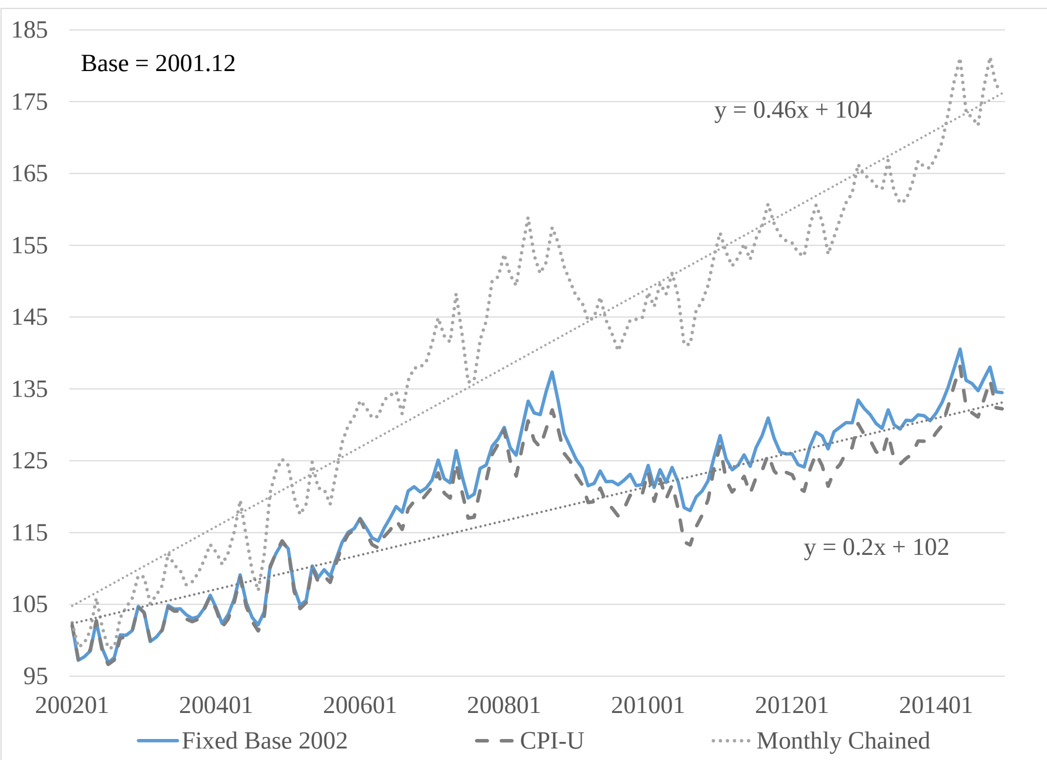
	Index	Weight	Index
Fixed Base Pivot	Fixed base Index = 12 month average 2001	Previous period fixed base 2001 annual share	$FB_{t,0}^{T} = \prod_{k}\left(\frac{IX_{tk}}{IX_{Ak}}\right)^{\left(\frac{\left(\frac{\hat{P}_{Ak}\hat{Q}_{Ak}}{\sum_j \hat{P}_{Vj}\hat{Q}_{Vj}}\right)+\left(\frac{M_{tk}}{\sum_j M_{tj}}\right)}{2}\right)}$
Chained Biennial Pivot Revision	Pivot month revised & chained biennially	Previous period cost weight share pivot revised biennially	$CBR_{t,0}^{T} = \prod_{k}\left(\frac{IX_{tk}}{IX_{vk}}\right)^{\left(\frac{\left(\frac{\hat{P}_{vk}\hat{Q}_{\beta k}}{\sum_j \hat{P}_{vj}\hat{Q}_{\beta j}}\right)+\left(\frac{M_{tk}}{\sum_j M_{tj}}\right)}{2}\right)}$
Chained Monthly Revision	Monthly chained	Previous period month share	$CMR_{t,0}^{T} = \prod_{0,t-1}\left[\prod_{k}\left(\frac{IX_{tk}}{IX_{t-1k}}\right)^{\frac{\left(\left(\frac{M_{t-1k}}{\sum_j M_{t-1j}}\right)+\left(\frac{M_{tk}}{\sum_j M_{tj}}\right)\right)}{2}}\right]$

1. $\hat{P}_{Vk}\hat{Q}_{\beta k}$ = Basic level fixed cost weight for pivot period
2. $\hat{P}_{Vj}\hat{Q}_{\beta j}$ = Aggregate level fixed cost weight for pivot period
3. M_{tk} = Basic level monthly expenditure weight for period t
4. M_{tj} = Aggregate level monthly expenditure weight for period t
5. $\hat{P}_{vk}\hat{Q}_{\beta k}$ = Basic level cost weight for pivot period
6. $\hat{P}_{vj}\hat{Q}_{\beta j}$ = Aggregate level cost weight for pivot period

3.1 Fresh Fruits Laspeyres/Lowe Formulas Example

Pronounced drift is displayed by the fresh fruits index in Graph 1 based on the Laspeyres / Lowe chained monthly revision compared to the fixed base formula. Fresh fruits contains four elementary items and 38 elementary areas for a total of 152 elementary cells each month. The fixed base 2002 and CPI-U indexes displays a slope increase of approximately 0.2 per month, however the monthly chained index displays a slope increase of approximately 0.46 per month. In December 2014, the monthly chained index drifts upward to an index level of about 175, compared to index levels of 135 and 133 for the fixed base and CPI-U formulas respectively. The frequency of monthly chaining overrides any benefit in updated weights, which results in approximately 40 points of drift for the chained monthly Laspeyres index.

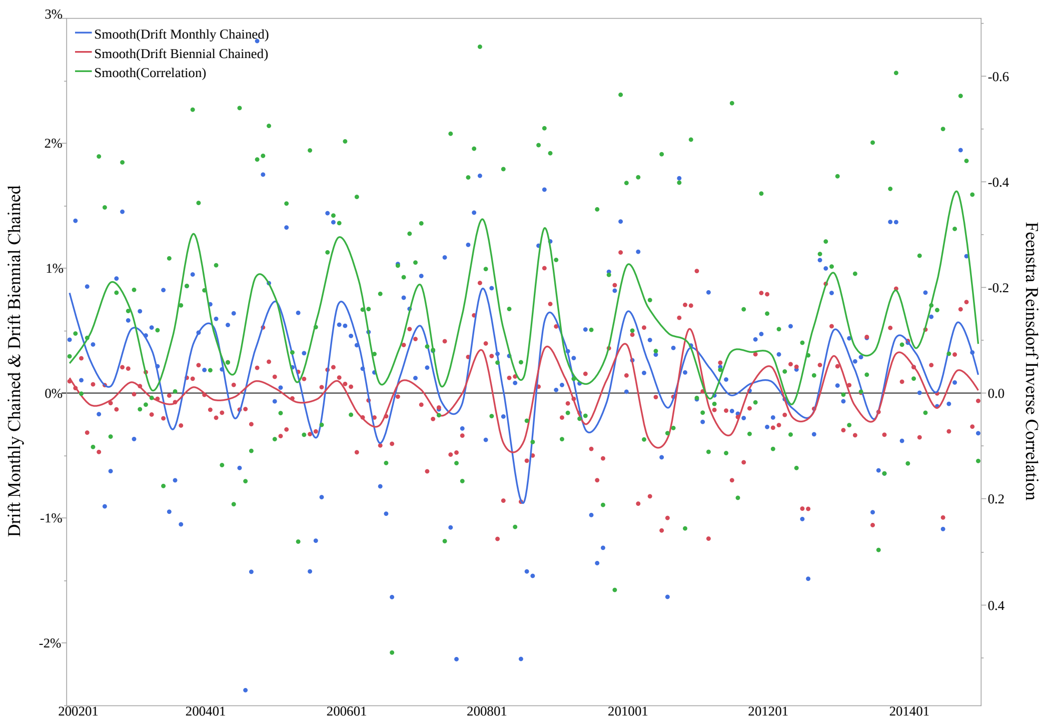
Graph 1. Fresh Fruits Laspeyres/Lowe Formulas



Drift analysis of 1-month percent changes for fresh fruits is performed to remove the cumulative effect of drift as the number of index months increases, and pinpoint months where the direct measure of drift peaks coincide with an inverse correlation measure of monthly weight vs. index logged relatives. In Graph 2, the 1-month drift of monthly chained scaled on the left axis is displayed as blue dots that range from -.02 to .03 with a mean of .002; the drift of biennial chained is displayed as red dots that range from -.01 to .01 with a mean of -.00007. The 1-month analysis of drift supports the cumulative index analysis of drift that drift for monthly chained series is larger than the biennial chained series. The Feenstra Reinsdorf model of the inverse correlation of 1 month weight vs. index logged relatives is a measure of a bivariate normal density ellipse with 90% coverage scaled on the right axis, and is displayed as green dots, which range from -.7 to .5 with a mean of -.13.[9]

To mitigate the noisy dot estimates of 1-month change, a cubic spline soother was applied to the three series with a lambda of .0001. The peaks for the inverse correlation appear to lead or coincide with the larger peaks of the smoothed drift monthly chained series and smaller peaks of the smoothed drift chained biennial series, which provides evidence of the diagnosis of the direct measure of 1-month percent change of drift as related to the weight vs. price logged relatives.

Graph 2. Fresh Fruits Smoothed 1 Month Percent Change Drift and Feenstra Reinsdorf Model Inverse Correlation



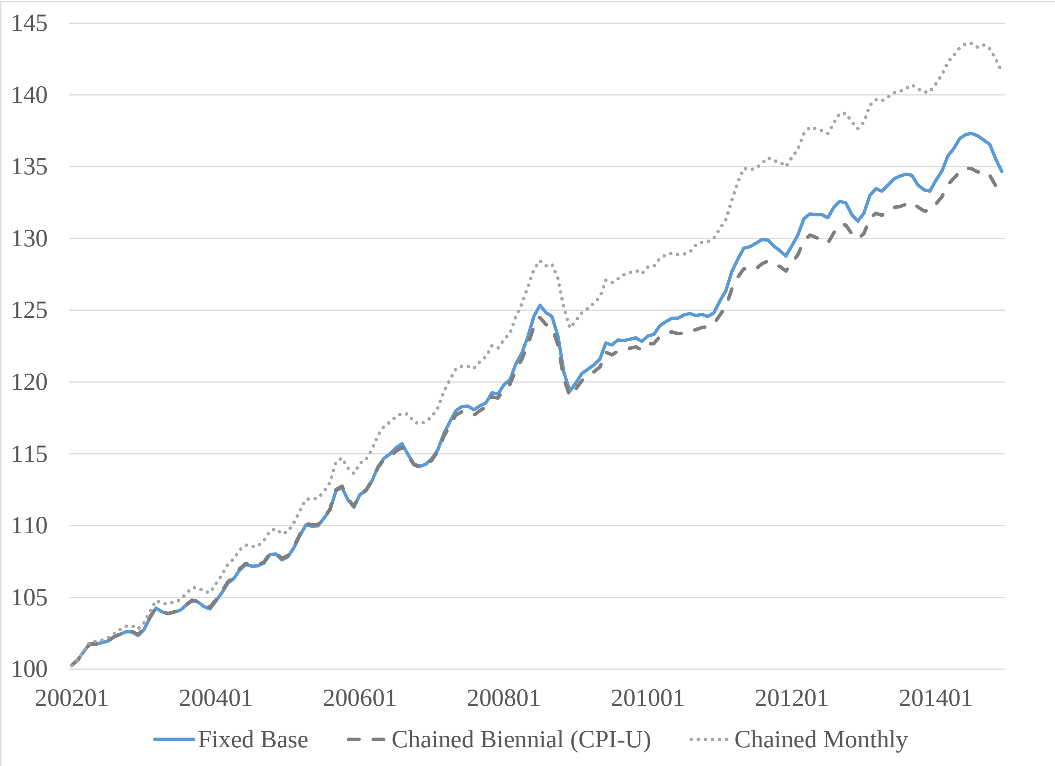
3.2 Results Measuring Drift

Does drift for the all-items aggregate index exist comparable to the fresh fruits index example? The following analysis evaluates drift within the Laspeyres/Lowe formula and

[9] See Feenstra and Reinsdorf (2007), Greenlees (2010).

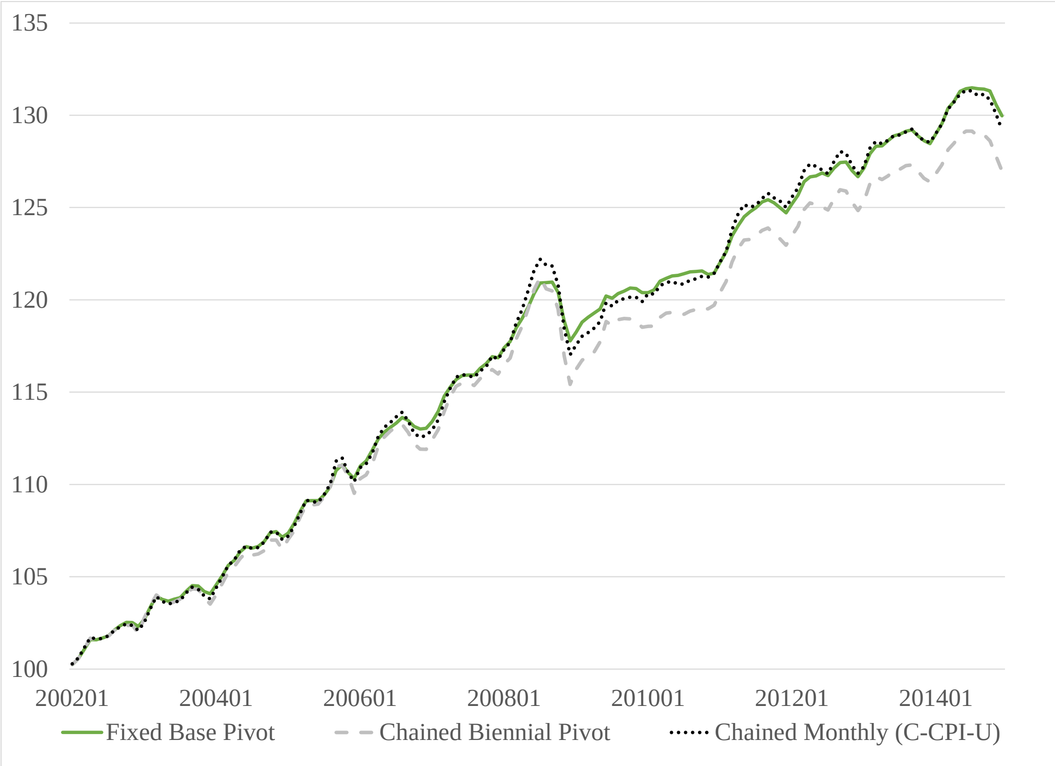
Tornqvist formula by comparing the fixed base version to the biennial chained and monthly chained versions. The December 2014 chained monthly terminal index terminal value is about 5.2% greater than the fixed base, noticeably drifting upward as early as January 2003. In contrast, the chained biennial index value is about 1.3% less than the fixed base. The more representative weights of the chained biennial index pull it lower than the fixed base index beginning in January 2009.

Graph 3. Laspeyres/Lowe Index Comparison



Graph 4 shows results for the Tornqvist formulas. The December 2014 chained monthly terminal index terminal value is about -0.2% less than the fixed base index, with a noticeable temporary bump above the fixed base index from April 2008 to November 2008, and temporary bump below the fixed base index from December 2008 to June of 2009. The chained biennial index value is about 0.6% less than the fixed base index, where the chained biennial index pulls lower than the fixed base index beginning December 2008. The results of Tornqvist formula are somewhat muted by limiting half of the expenditure share as either a fixed value for the fixed base index, or as a cost weight share constant over 24 months equivalent to the CPI published cost weights relative importance values.

Graph 4. Tornqvist Index Comparison



Graph 5 displays the index level difference between fixed base indexes and chained counterparts. The difference between the Laspeyres Fixed Base index and the Chained Monthly Laspeyres index increases steadily to a final index level drift value of 7, highlighting the cumulative effect of drift. There is generally no difference between the Laspeyres fixed base index and the official CPI-U index until about January 2006, when the fixed base index is consistently higher than the CPI-U. The difference peaks in July from 2007 to 2014, perhaps indicative of summer prices and weights becoming more important over time. In contrast, the difference between the Tornqvist fixed base index and both the official C-CPI-U and the Tornqvist chained biennial index exhibit a sharp spike upward in June 2008 and then downward until December 2008, with little movement otherwise. Table 3 summarizes results in terms of absolute value differences, where the C-CPI-U represents the smallest sum of absolute value difference of 40, monthly mean difference of 0.26, and root mean squared error of 0.02 indicative of a small amount of drift relative to the other formulas. Note that this analysis is limited to measuring drift as simply the chained less the fixed base, and therefore results contain both a representativity weight effect difference, combined with a formula effect difference.

Graph 5. Within Formula Drift: Chained Less Fixed Base

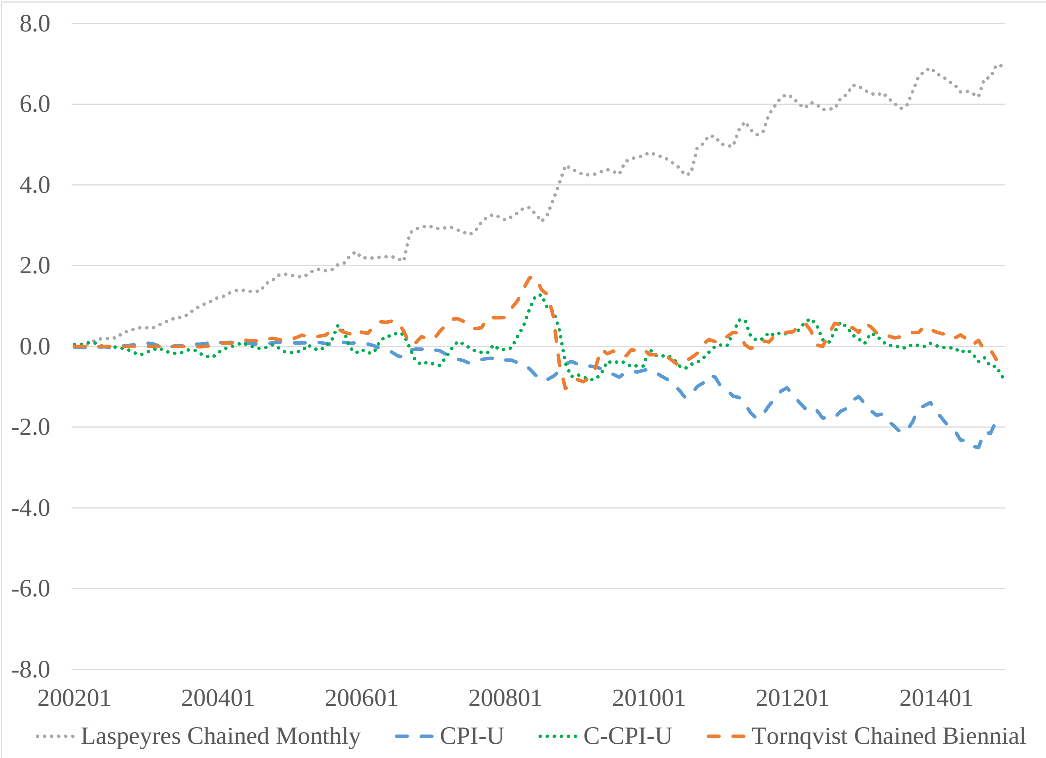


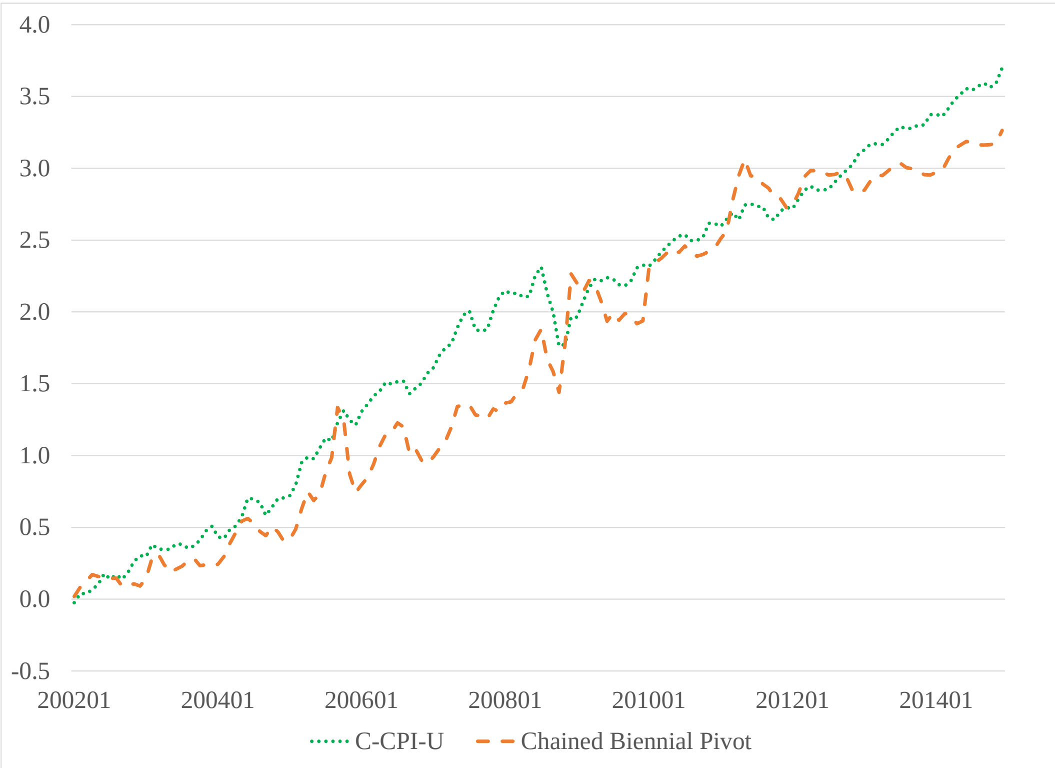
Table 3. Summary of Drift and Substitution Absolute Value Differences

	Drift- Within Formula Weight + Formula Effect				Substitution- Across Formula Weight + Formula Effect	
	Laspeyres Chained Monthly	CPI-U	C-CPI-U	Tornqvist Chained Biennially	CPI-U Less C-CPI-U	CPI-U Less Tornqvist Chained Biennially
SUM	563.22	111.63	40.21	50.21	295.28	261.49
MEAN	3.61	0.72	0.26	0.32	1.89	1.68
RMSE	0.17	0.06	0.02	0.03	0.09	0.09

Substitution is another measure that defines the level of representivity across formulas, which is more significant than measuring drift within formula alone due to publication and common questions associated with explaining the differences between these indexes. To highlight the effect of the Tornqvist formula and weights, substitution quantified as the

CPI-U less the C-CPI-U, and CPI-U less Tornqvist chained biennial index as displayed in Graph 6. The robust linear trend upward is indicative of monthly chaining of the Tornqvist indexes relative to the more fixed weights from the CPI-U. The C-CPI-U displays the greatest level of substitution in terms of a sum of absolute value differences of 295.3 and mean monthly level 1.9, slightly greater than the Tornqvist chained biennially where half of the weight is fixed each biennial period as displayed in Table 3. Similar to the drift results above, the substitution results contain both a representative weight effect, and a formula effect which includes drift.

Graph 6. Across Formula Substitution: CPI-U Less C-CPI-U, and CPI-U Less Tornqvist Chained Biennially



4. Conclusion and Future Work

The results above provide further evidence that the CPI-U and C-CPI-U exhibit a small amount of index chain drift, consistent with previous findings of Aizcorbe and Jackman, and Lent.[10] Although fixed base indexes are not practical for an index measuring price change, and numerous authors caution against their use, they do offer value in terms of measuring index chain drift.[11] As indicated above the measure of drift includes both a weight effect and formula effect for the C-CPI-U drift decomposition. While this paper does not attempt to isolate these effects, others have estimated that the majority of what this paper labels drift is due to representivity improvement of using more current weights. For example, Kurtzon approximates 90% of the drift is due to a weight effect which could be considered as representivity improvements, and 10% of the drift is due to formula effects

[10] See Aizcorbe and Jackman (1993) Table A1 pp. 32-33 and Lent (2000) Tables 1a and 1b p. 317.
[11] See Szulc (1983), and ILO Chapter 15.

that actually measure drift from the fixed base index over a January 2000 to December 2009 timeframe.[12] Additionally, substitution measures summarize the high level of across formula representivity, which includes both the weight and formula effect.

One of the original motivations for this research was to develop a better understanding of drift for the BLS CPI-U and C-CPI-U published products. The above analysis provides an initial direction, but significant value will be added once drift is decomposed into a weight effect and formula/other effect. Future analysis of drift will include analysis of additional formulas such as the Geometric Means and Constant Elasticity of Substitution, as well as continue to evaluate if drift should be measured as a cumulative value representative of the entire time frame of analysis reflective of a fixed market basket structure, or if it would be better to create consistent time spans to measure drift and therefore permit changes to the market basket structure. Future analysis should also evaluate drift of sub aggregate item groups relative to the correlation of weight change vs. index change to determine to what extent drift coincides with negative correlation of weights and indexes.

Acknowledgements:

Thanks to Anya Stockburger and Robert Cage for helpful comments and questions. Any opinions expressed in this paper are those of the author and do not constitute policy of the Bureau of the Labor Statistics.

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[12] See Kurtzon (2016) pp. 13-14.

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