A comparison of new developments of the Henderson filters for real time trend-cycle estimation

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Abstract

The Henderson filters (1916) jointly with the Musgrave filters (1964) have been used for trend-cycle estimation in officially published data by statistical agencies around the world. The Henderson filters are symmetric and the Musgrave ones are asymmetric and we shall refer to both as symmetric and asymmetric Henderson filters. These filters have the good property of fast detection of true turning points, but the limitations of large revisions when data are added, and a large amount of unwanted ripples (9 and 10 month cycles) for short filter lengths.

The purpose of this paper is to present a brief description of three estimators developed to reduce the Henderson filter limitations. We do so from a theoretical viewpoint by looking at the respective gain and phase shift functions and empirically by application to a sample of economic indicators. We calculate the mean square revision error as new observations are added to the series and the time delay to estimate a turning point that for illustrative purposes we have chosen to be December 2007.

Key Words: Cascade linear filters, kernel trend-cycle filters, turning point detection, revisions.

1. Introduction

The linear filter developed by Henderson (HF) (Henderson, 1916) is the most widely applied to estimate the trend-cycle component in seasonal adjustment software such as the US Bureau of Census II - X11 method (Shiskin et al., 1967) and all its ARIMA variants. Major studies have been done on trend-cycle estimation during the last twenty years by making changes to the Henderson filters. The emphasis has been on determining the direction of the short-term trend for an early detection of a true turning point and on reducing the size of the revisons when new observations are added to the series.

In 1996, Dagum developed a nonlinear trend-cycle estimator to improve on the classical 13-term Henderson filter. The NonLinear Dagum filter (NLDF) results from applying the 13-term symmetric Henderson filter (H13) to seasonally adjusted series where outliers and extreme observations have been replaced and which have been extended with extrapolations from an ARIMA model (Dagum, 1996).

Later on, in 2009, Dagum and Luati developed a Cascade Linear filter (CLF) which is an approximation to the NLDF to facilitate a larger application, since the new linear approximation does not need an ARIMA model identification. From another perspective, in 2008 and 2015, Dagum and Bianconcini developed a Reproducing Kernel filter (RKF) that was shown to produce better results for real time trend-cycle estimation. It should be noticed that following the tradition of statistical agencies, we shall call asymmetric Henderson filters the ones that correspond to the asymmetric weights developed by Musgrave (1964).

The main purpose of this study is to present a brief description of each Henderson modification, and to perform a theoretical comparison of CLF, RKF and the Henderson filter via their gain and phase shift functions. Furthermore, we perform an empirical comparison with leading, coincident and lagging socio-economic indicators calculating the mean

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square revision as new data is entered to the series as well as the time delay to pick up a true turning point.

This paper is presented as follows. Section 2 briefly introduce the main features of the three Henderson modifications, and compare the three linear filters, H13, CLF and RKF looking at their gain functions and phase shifts. Section 3 calculate the mean square revisions and the time delay (in months) to detect a true turning point. Since the CLF was developed only for 13 term length span we want also to see what happens when applied to series that require different length spans. We have chosen December 2007 to assess the capability of the CLF and RKF relative to the Henderson filter to calculate the time delay to detect the turning point.

2. Main assumptions of the nonlinear Dagum filter, the cascade linear filter and the reproducing kernel trend-cycle filter

2.1 The nonlinear Dagum filter

The modified Henderson filter developed by Dagum (1996) is nonlinear and a brief description is given here. We refer the reader to the Dagum reference for more details. It basically consists of: (a) extending the seasonally adjusted series with ARIMA extrapolated values, and (b) applying the 13-term Henderson filter to the extended series where extreme values have been modified using very strict sigma limits.

To facilitate the identification and fitting of simple ARIMA models, Dagum (1996) recommends, at step (a), to modify the input series for the presence of extreme values using very strict σ limits, such as $\pm 0.7\sigma$ and $\pm 1.0\sigma$. In this way, a simple and very parsimonious ARIMA model, the ARIMA(0,1,1), is often found to fit a large number of seasonally adjusted series. The main purpose of the ARIMA extrapolations is to reduce the size of the revisions of the most recent estimates, whereas that of extreme values replacement is to reduce the number of unwanted ripples produced by H13. An unwanted ripple is a 10-month cycle (identified by the presence of high power at $\omega = 0.10$ in the frequency domain) which, due to its periodicity, often leads to the wrong identification of a true turning point. In fact, it falls in the neighborhood between the fundamental seasonal frequency and its first harmonic. On the other hand, a high frequency cycle is generally assumed to be part of the noise pertaining to the frequency band $0.10 \leq \omega < 0.50$. The problem of the unwanted ripples is observed when H13 is applied to seasonally adjusted series, and also present when shorter Henderson filters are used.

The NLDF can be formally described in matrix notation as follows. Let $\mathbf{y} \in \mathbb{R}^n$ be the n-dimensional seasonally adjusted time series to be smoothed, which consists of a trend-cycle $\mathbf{TC}$ plus an erratic component $\mathbf{e}$, that is

$$\mathbf{y} = \mathbf{TC} + \mathbf{e} \tag{1}$$

It is assumed that the trend-cycle is smooth and can be well estimated by means of the 13-term Henderson filter applied to $\mathbf{y}$. Hence,

$$\widehat{\mathbf{TC}} = \mathbf{H}\mathbf{y} \tag{2}$$

where $\mathbf{H}$ is the $n \times n$ matrix (canonically) associated to the 13-term Henderson filter. Replacing $\widehat{\mathbf{TC}}$ in eq. (1) by eq. (2), we have

$$\mathbf{y} = \mathbf{H}\mathbf{y} + \mathbf{e} \tag{3}$$

or,

$$(\mathbf{I}_n - \mathbf{H})\mathbf{y} = \mathbf{e} \tag{4}$$

where $\mathbf{I}_n$ is the $n \times n$ identity operator on $\mathbb{R}^n$. Assign now a weight to the residuals in such a way that if the observation $y_t, t = 1, \ldots, n$, is recognized to be an extreme value (with respect to $\pm 2.5\sigma$ limits, where σ is a 5-year moving standard deviation) then, the corresponding residual e_t is zero weighted (*i.e.* the extreme value is replaced by $\hat{TC}_t$ which is a preliminary estimate of the trend). If y_t is not an extreme value then the weight for e_t is one (*i.e.* the value y_t is not modified). In symbols,

$$\mathbf{W}_0\mathbf{e} = \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})\mathbf{y}, \tag{5}$$

where $\mathbf{W}_0$ is a zero-one diagonal matrix, being the diagonal element w_{tt} equal to zero when the corresponding element y_t of the vector $\mathbf{y}$ is identified as an outlier. For instance, if in the series $\mathbf{y}$ the only extreme value is y_2, then the weight matrix for the residuals will be

$$\mathbf{W}_0 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{6}$$

Denoting by

$$\mathbf{e}_0 = \mathbf{W}_0\mathbf{e} \tag{7}$$

the vector of the modified residuals, then the series modified by extreme values with zero weights becomes

$$\mathbf{y}_0 = \mathbf{TC} + \mathbf{e}_0 \tag{8}$$

which can be written as

$$\mathbf{y}_0 = \mathbf{H}\mathbf{y} + \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})\mathbf{y} = [\mathbf{H} + \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})]\,\mathbf{y}. \tag{9}$$

Using (9), one year of ARIMA extrapolations are obtained in order to extend the series modified by extreme values. Denoting with $\mathbf{y}_0^E$ the extended series, that is the $n + 12$ vector whose first n elements are given by $\mathbf{y}_0$ while the last 12 are the extrapolated ones, in block-matrix notation we have

$$\mathbf{y}_0^E = \begin{bmatrix} [\mathbf{H} + \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})]\,\mathbf{y} \\ \mathbf{y}^{12} \end{bmatrix} \tag{10}$$

where $\mathbf{y}^{12}$ is the 12×1 block of extrapolated values. Setting

$$[\mathbf{H} + \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})]^{12} = \begin{bmatrix} [\mathbf{H} + \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})] & \mathbf{O}_{n\times 12} \\ \mathbf{O}_{12\times n} & \mathbf{I}_{12} \end{bmatrix} \tag{11}$$

and

$$\mathbf{y}^{+,12} = \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^{12} \end{bmatrix}, \tag{12}$$

$\mathbf{y}_0^E$ becomes

$$\mathbf{y}_0^E = [\mathbf{H} + \mathbf{W}_0(\mathbf{I}_n - \mathbf{H})]^{12}\,\mathbf{y}^{+,12}. \tag{13}$$

This concludes the operations involved in step (a) of the NLDF.

Step (b) follows. The procedure for obtaining $\mathbf{y}_0$ on the series $\mathbf{y}_0^E$ is repeated, but with stricter sigma limits (such as $\pm 0.7\sigma$ and $\pm 1.0\sigma$) and with different weights assigned to the residuals for the replacement of the extreme values. The estimates $\mathbf{y}^E$ computed over the series $\mathbf{y}_0^E$ are

$$\mathbf{y}^E = [\mathbf{H} + \mathbf{W}(\mathbf{I}_n - \mathbf{H})]^E\,\mathbf{y}_0^E. \tag{14}$$

The $(n+12)\times(n+12)$ matrix $[\mathbf{H}+\mathbf{W}(\mathbf{I}_n-\mathbf{H})]^E$ is analogue to $[\mathbf{H}+\mathbf{W}_0(\mathbf{I}_n-\mathbf{H})]^{12}$ except for the matrix $\mathbf{W}$ that is also diagonal, but with generic diagonal element w_{tt}, such that $w_{tt}=0$ if the corresponding value y_t falls out of the upper bound selected limits, say, $\pm1.0\sigma$, and $w_{tt}=1$ if the corresponding y_t falls within the lower bound selected limits, say, $\pm0.7\sigma$ and w_{tt} decreases linearly (angular coefficient equal to -1) from 1 to 0 in the range from $\pm0.7\sigma$ to $\pm1.0\sigma$. Under the assumption of normality, these sigma limits imply that 48% of the values will be modified (replaced by the preliminary smoothed trend), 32% will be zero weighted while the remaining 16% will get increasing weights from zero to one. Notice that $\mathbf{y}^E$ can also be written as

$$\mathbf{y}^E=[\mathbf{H}+\mathbf{W}(\mathbf{I}_n-\mathbf{H})]^E\,[\mathbf{H}+\mathbf{W}_0(\mathbf{I}_n-\mathbf{H})]^{12}\,\mathbf{y}^{+,12}. \tag{15}$$

Finally, the NLDF estimates are given by applying a 13-term Henderson filter to eq.(15), that is

$$\begin{aligned}\mathbf{H}\mathbf{y}^E &= \mathbf{H}\,[\mathbf{H}+\mathbf{W}(\mathbf{I}_n-\mathbf{H})]^E\,[\mathbf{H}+\mathbf{W}_0(\mathbf{I}_n-\mathbf{H})]^{12}\,\mathbf{y}^{+,12} \qquad (16)\\ &= \begin{bmatrix}\widehat{\mathbf{TC}}\\ \widehat{\mathbf{TC}}^{12}\end{bmatrix}\end{aligned}$$

where $\widehat{\mathbf{TC}}$ is the n-dimensional vector of smooth estimates of $\mathbf{y}$.
It is apparent that the NLDF method reduces drastically the effects of extreme values by repeatedly smoothing the input data via down weighting points with large residuals. Furthermore, the ARIMA extension enables the use of the symmetric weights of the 13-term Henderson filter for the last six observations and, thus, reduces the size of the revisions of the last estimates.

2.2 The cascade linear trend-cycle filter

A brief description of the Cascade linear symmetric and asymmetric filters follow. We refer the readers to Dagum and Luati (2009) for a full description.

The cascade filter is a linear approximation of the nonlinear Dagum filter for 13 term spans. The cascading is done via the convolution of several filters chosen for noise suppression, trend estimation and extrapolation. A linear filter offers many advantages over a nonlinear one. first, its application is direct and hence, does not require knowledge of ARIMA model identification. Furthermore, linear filtering preserves the crucial additive constraint by which the trend of an aggregated variable should be equal to the algebraic addition of its component trends, thus avoiding the selection problem of direct versus indirect seasonal adjustments. Also, the theoretical properties of a linear filter concerning signal passing and noise suppression can always be compared to those of other linear filters by means of spectral analysis.

The symmetric filter is the one applied to all central observations. In this case, the purpose is to offer a linear solution to the unwanted ripples problem. To avoid the latter, the NLDF largely suppresses the noise in the frequency band between the fundamental seasonal and first harmonic. In this regard, a cascade linear filter is derived by double smoothing the residuals obtained from a sequential application of H to the input data. The residuals smoothing is done by the convolution of two short smoothers, a weighted 5-term and a simple 7-term linear filters. The linear approximation for the symmetric part of the NLDF is truncated with weights normalized to add to one.

The smoothing matrix of the symmetric cascade linear filter is

$$\mathbf{H}\left[\mathbf{H}+\mathbf{M}_{7(0.14)}\left(\mathbf{I}_n-\mathbf{H}\right)\right]\left[\mathbf{H}+\mathbf{M}_{5(0.25)}\left(\mathbf{I}_n-\mathbf{H}\right)\right], \tag{17}$$

where $\mathbf{H}$ stands for the 13-term Henderson filter, $M_{5(0.25)}$ is the matrix representative of a 5-term moving average with weights $(0.250, 0.250, 0.000, 0.250, 0.250)$, and $M_{7(0.14)}$ is the matrix representative of a 7-term filter with all weights equal to 0.143.

On the other hand, the asymmetric filter is applied to the last six data points which are crucial for real time analysis. It is obtained by means of the convolution between the symmetric filter and the linear extrapolation filters for the last six data points. These asymmetric filters for the last six data points results from the convolution of: (1) the asymmetric weights of an ARIMA extrapolation model, (2) the weights of $M_{7(0.14)}$ and $M_{5(0.25)}$) filters repeatedly used for noise suppression, and (3) weights of the final linear symmetric filter.

The extrapolation filters are linearized by fixing both the ARIMA model and its parameter values. The latter are chosen such as to minimize the size of revisions and phase shifts. The ARIMA model was selected among some parsimonious processes found to fit and extrapolate well a large number of seasonally adjusted series. Such model is the ARIMA(0,1,1) with $\theta = 0.40$. A simple linear transformation allows to apply the asymmetric filter to the first six observations. Hence, the smoothing matrix associated to the asymmetric linear filter for the last six data points is obtained in two steps:

(1) a linear extrapolation filter for six data points is applied to the input series. This filter is represented by a $(n+6) \times n$ matrix $\mathbf{A}^*$

$$\mathbf{A}^* = \begin{bmatrix} & \mathbf{I}_n & \\ \mathbf{O}_{6\times n-12} & & \mathbf{\Pi}^*_{6\times 12} \end{bmatrix}$$

where $\mathbf{\Pi}^*_{6\times 12}$ is the submatrix containing the weights for the $n-5, n-4, ..., n$ data points. $\mathbf{\Pi}^*_{6\times 12}$ results from the convolution

$$\mathbf{H}\left[\mathbf{H} + \mathbf{M}_{7(0.14)}\left(\mathbf{I}_{n+12} - \mathbf{H}\right)\right]^E \mathbf{A}\left[\mathbf{H} + \mathbf{M}_{5(0.25)}\left(\mathbf{I}_n - \mathbf{H}\right)\right], \quad (18)$$

where $\left[\mathbf{H} + \mathbf{M}_{5(0.25)}\left(\mathbf{I}_n - \mathbf{H}\right)\right]$ is the $n \times n$ matrix representative of trend filter plus a first suppression of extreme values, $\left[\mathbf{H} + \mathbf{M}_{7(0.14)}\left(\mathbf{I}_{n+12} - \mathbf{H}\right)\right]^E$ is the $n \times n+12$ matrix for the second suppression of the irregulars applied to the input series plus 12 extrapolated values, generated by

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_n \\ \mathbf{\Pi}_{12\times n} \end{bmatrix}.$$

This $(n+12) \times n$ matrix $\mathbf{A}$ is associated to an ARIMA(0,1,1) linear extrapolations filter with parameter value $\theta = 0.40$.

(2) The symmetric filter is applied to the series extrapolated by $\mathbf{A}^*$, that is

$$\widehat{\mathbf{y}} = \mathbf{S}\mathbf{A}^*\mathbf{y}$$

where $\mathbf{S}$ is the $n \times (n+6)$ matrix given by

$$\mathbf{H}\left[\mathbf{H} + \mathbf{M}_{7(0.14)}\left(\mathbf{I}_n - \mathbf{H}\right)\right]\left[\mathbf{H} + \mathbf{M}_{5(0.25)}\left(\mathbf{I}_n - \mathbf{H}\right)\right]$$

The convolution $\mathbf{S}\mathbf{A}^*$ produces 12-term asymmetric filters for the last six observations, that are truncated and uniformly normalized in order to obtain the final asymmetric linear filters for the last observations.

The new filter is called the Cascade Linear filter (CLF), and there is a distinction between the Symmetric (SCLF) and the Asymmetric Linear Filters (ACLF). The CFL matrix for the 13-term filter is given in Table 1.

2.3 The reproducing kernel trend-cycle filter

Dagum and Bianconcini (2008, 2015) presented a reproducing kernel approach to modify the Henderson filters. The new set of filters are based on the reproducing kernel Hilbert space methodology. A Hilbert space is characterized by a kernel that reproduces, via an inner product, every function of the space.

Dagum and Bianconcini used the Berlinet (1993) theorem according to which a kernel estimator of order p can always be decomposed into the product of a reproducing kernel R_{p-1}, belonging to the space of polynomials of degree at most $p-1$, times a probability density function f_0 with finite moments up to order $2p$.

In this context, the equivalent kernel representation of the Henderson filter is given by

$$K_4(t) = \sum_{i=0}^{3} P_i(t)P_i(0)f_0(t), \qquad t \in [-1,1], \tag{19}$$

where f_0 is the density function, defined on $[-1,1]$, obtained through normalization of W_j and the P_i are the corresponding orthonormal polynomials. The $W_j \propto \{(m+1)^2 - j^2\}\{(m+2)^2 - j^2\}\{(m+3)^2 - j^2\}$ are chosen to minimize the sum of squares of the third differences of the wj weights to be applied to the input data.

Dagum and Bianconcini (2008) found that the biweight function $f_{0B}(t) = \left(\frac{15}{16}\right)(1-t^2)^2, t \in [-1,1]$, provides a good approximation for Henderson filters of short length, say, between 5 to 23 terms which are those used by statistical agencies (see also Bianconcini and Quenneville, 2010).

When applied to real data, the symmetric filter weights are derived as follows

$$w_j = \frac{K_4(j/b)}{\sum_{j=-m}^{m} K_4(j/b)}, \qquad j = -m, \ldots, m, \tag{20}$$

where b is a time-invariant global bandwidth parameter (same for all $t = m+1, \ldots, N-m$) selected to ensure a symmetric filter of length $2m+1$. The bandwidth parameter relates the discrete domain of the filter , that is $\{-m, \ldots, m\}$, with the continuous domain of the kernel function, that is, $[-1,1]$.

The derivation of the symmetric Henderson filter assumes the availability of $2m+1$ input values centered at t. However, at the end of the sample period, that is, $t = N-(m+1), \ldots, N$, only $2m, \ldots, m+1$ observations are available, and asymmetric filters have to be considered. Hence, at the boundary, the effective domain of the kernel function K_4 is $[-1, q^*]$, with $q^* << 1$, instead of $[-1,1]$ as for any interior point. This implies that the symmetry of the kernel is lost, and it does not integrate to unity on the asymmetric support ($\int_{-1}^{q^*} K_4(t)dt = 1$). Furthermore, the moment conditions are no longer satisfied, that is $\int_{-1}^{q^*} t^i K_4(t)dt = 0$, for $i = 1,2,3$. To overcome these limitations, several boundary kernels have been proposed in the literature. In the context of real time trend-cycle estimation, the condition that the kernel function integrates to unity is essential, whereas the unbiasedness property can only be satisfied with a great increase of the estimates variance. This is a consequence of the well-known trade-off between bias and variance. This latter becomes very large because most of the contribution to the real time trend-cycle estimates comes from the current observation which gets the largest weight. Based on these considerations, Dagum and Bianconcini (2008, 2013) have followed the so-called 'cut and normalize" method (Kyung-Joon and Schucany, 1998), according to which the boundary kernels K_4^* are obtained by cutting the symmetric kernel K_4 to omit that part of the function lying between q^* and 1, and by normalizing it on $[-1, q^*]$. That is,

$$K_4^{q^*}(t) = \frac{K_4(t)}{\int_{-1}^{q^*} K_4(t)\mathrm{d}t} = \frac{\det(\mathbf{H}_4^0[1,\mathbf{t}])f_{0B}(t)}{\det(\mathbf{H}_4^0[1,\boldsymbol{\mu}^{q*}])}, \qquad t \in [-1, q^*], \tag{21}$$

Applied to real data, the "cut and normalize" method yields the following formula for the asymmetric weights:

$$w_{q,j} = \frac{K_4^{q^*}(j/b_q)}{\sum_{j=-m}^{q} K_4^{q^*}(j/b_q)} = \frac{\det(\mathbf{H}_4^0[1, \mathbf{j}/\mathbf{b_q}])(1/b_q) f_{0B}(j/b_q)}{\det(\mathbf{H}_a)}, \tag{22}$$

for $j = -m, \ldots, q$, and $q = 0, \ldots, m-1$, where $b_q, q = 0, \ldots, m-1$, is the local bandwidth, one for each asymmetric filter. Dagum and Bianconcini (2015) show that given the length of the filter and the density function, the properties of the asymmetric filter depend on the bandwidth parameter b_q.

A filter is defined as optimal if it minimizes revisions and time lag to detect a true turning point and the optimal b_q is the one that minimizes the distance between the gains function of the symmetric and asymmetric filters. Since the m asymmetric filters corresponding to a $2m+1$ symmetric filter are time-varying, the local bandwidth parameters are also time varying. The weights of the RKF matrix for the 13-term filter are given in Table 2.

2.4 Theoretical comparison of HF, CLF and RKF

Next, we show superimposed the gain functions of the three linear symmetric filters for 13 terms. It can be seen that, whereas the HF and RKF are practically the same, this is not the case for the CLF.

Figure 1: Gain functions of the 13-term symmetric filters

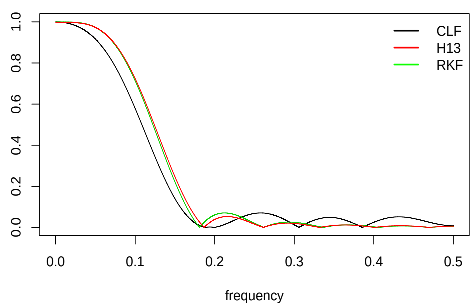


Figure 2 shows the gains of the three linear last point filters superimposed, whereas Figure 3 shows their phase shifts functions.

Figure 2: Gain functions of the last point filters associated with the 13-term symmetric CLF, HF, and RKF.

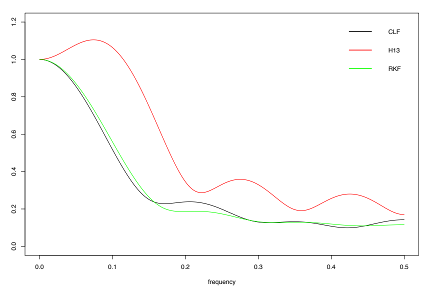


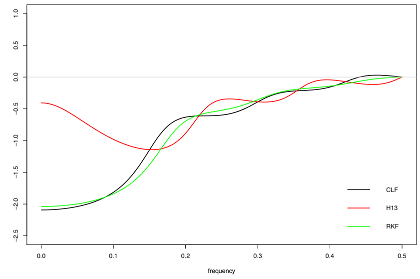
Table 1: Weight system of the 13-tern CLF

0.22994356	0.21807687	0.18236778	0.13420192	0.11420000	0.07648294	0.04472693	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.18174310	0.22861344	0.20512398	0.16697939	0.10825490	0.07510604	0.03269660	0.00148256	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.12963359	0.18398191	0.22628900	0.19648469	0.14898637	0.08264389	0.04642842	0.00507274	-0.01952060	0.00000000	0.00000000	0.00000000	0.00000000
0.06377485	0.13145712	0.18512908	0.22237015	0.18728138	0.13687215	0.06917620	0.03378736	-0.00447445	-0.02537383	0.00000000	0.00000000	0.00000000
0.03072539	0.06456224	0.13239634	0.18278836	0.21750020	0.18159631	0.13070808	0.06364420	0.02951682	-0.00709931	-0.02633862	0.00000000	0.00000000
-0.00635989	0.03043143	0.06509409	0.13248429	0.18313975	0.21861339	0.18294929	0.13215216	0.06466463	0.03009318	-0.00688184	-0.02638048	0.00000000
-0.02700000	-0.00700000	0.03100000	0.06700000	0.13600000	0.18800000	0.22400000	0.18800000	0.13600000	0.06700000	0.03100000	-0.00700000	-0.02700000

Table 2: Weight system of the 13-tern RKF

0.22362	0.21564	0.19266	0.157478	0.11444	0.06902	0.02714	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.21065	0.22352	0.21065	0.174523	0.12230	0.06460	0.01357	-0.01982	0.00000	0.00000	0.00000	0.00000	0.00000
0.15391	0.21013	0.23100	0.21013	0.15391	0.08000	0.01250	-0.02600	-0.025570	0.00000	0.00000	0.00000	0.00000
0.06338	0.14212	0.20452	0.22808	0.20452	0.14212	0.06338	-0.00217	-0.02978	-0.01617	0.00000	0.00000	0.00000
-0.00258	0.06319	0.14245	0.20533	0.22909	0.20533	0.14245	0.06319	-0.00258	-0.02996	-0.01593	0.00000	0.00000
-0.02983	0.00060	0.06762	0.14651	0.20848	0.23179	0.20848	0.14651	0.06762	0.00060	-0.02983	-0.01855	0.00000
-0.01986	-0.02982	0.00217	0.07010	0.14921	0.21106	0.23429	0.21106	0.149208	0.07010	0.00217	-0.02982	-0.01986
0.00000	-0.01855	-0.02983	0.00066	0.06762	0.14651	0.20848	0.23179	0.20848	0.14651	0.06762	0.00060	-0.02983
0.00000	0.00000	-0.01593	-0.02996	-0.00258	0.06319	0.14245	0.20533	0.22909	0.20533	0.14245	0.06319	-0.00258
0.00000	0.00000	0.00000	-0.01617	-0.02978	-0.00217	0.06338	0.14212	0.20452	0.22808	0.20452	0.14212	0.06338
0.00000	0.00000	0.00000	0.00000	-0.02557	-0.02600	0.01250	0.08000	0.15391	0.21013	0.23100	0.21013	0.15391
0.00000	0.00000	0.00000	0.00000	0.00000	-0.01982	0.01357	0.06460	0.12230	0.17452	0.21065	0.22352	0.21065
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.02714	0.06902	0.11444	0.15748	0.19267	0.21564	0.22362

Figure 3: Phaseshift functions of the last point filters associated with the 13-term symmetric CLF, HF, and RKF.



3. Empirical Evaluation

We evaluated empirically the performance of the CLF and RKF with a sample of socio-economic series calculating the Mean Square Percentage revision Error (MSPE), and the time delay (in months) to detect a true turning point.

3.1 Reduction of revision size in real time short-term trend estimates

The reduction of revisions in real time trend-cycle estimates is very important because the most recent estimates are preliminary but often used to assess the current stage of the economy. Statistical agencies and major users of these indicators are reluctant to large revisions because these can lead to wrong decision taking and policy making concerning the current economic situation.

The series considered are all seasonally adjusted, where also trading day variations and extreme values have been removed if present. The series are of different length, but the periods selected sufficiently cover the various lengths published for these series. For each series, the length of the filters is selected according to the I/C (noise to signal) ratio, as classically done in the X11/X12ARIMA procedure (Ladiray and Quenneville, 2001). In the sample, the ratio ranges from 0.20 to 1.98, hence filters of 9 and 13 terms are applied. The comparisons are based on the relative filter revisions between the final symmetric filter S and the last point asymmetric filter A, that is,

$$R_t = \frac{S_t - A_t}{A_t}, \qquad t = 1, \ldots, N.$$

For each series and for each estimator, we calculate the ratio between the Mean Square Percentage Error (MSPE) of the revisions corresponding to the filters derived following the CLF and RKF methodologies and those corresponding to the last point of the HF. For all the estimators, the results illustrated in Table 3 show that the ratio is always smaller than one, indicating that either the cascade linear or the kernel last point predictors introduce smaller revisions than the HF. But, it is also apparent from Table 3 that the CLF reduces more than the RKF the revisions of almost 10%. However, this is only true when the correct filter length selected using the noise to signal ratio is 13-term. Indeed, as shown in Table 4, when the filter length is 9-term, being the cascade linear filter only developed for 13-term, it produces largest revisions than the 9-term RKF.

3.2 Turning point detection

It is important that the reduction of revisions in real time trend-cycle estimates is not achieved at the expense of increasing the time lag to detect the upcoming of a true turning

Table 3: Revisions when filters of 13 terms are appropriate

Macro-area	Series	$\frac{RKF}{H13}$	$\frac{CLF}{H13}$
Leading	Average weekly overtime hours: manufacturing	0.492	0.455
	New orders for durable goods	0.493	0.451
	New orders for nondefense capital goods	0.493	0.452
	New private housing units authorized by building permits	0.475	0.430
	University of Michigan: consumer sentiment	0.480	0.411
Lagging	Average (mean) duration of unemployment	0.509	0.484
	Inventory to sales ratio	0.483	0.438
	Index of total labor cost per unit of output	0.515	0.474

Table 4: Revisions when filters of 9 terms are appropriate

Macro-area	Series	$\frac{RKF}{H13}$	$\frac{CLF}{H13}$
Leading	Composite index of ten leading indicators	0.466	0.813
	S&P 500 stock price index	0.454	0.700
	M2 money stock	0.508	0.997
	10-year treasury constant maturity rate	0.446	0.565
Coincident	Composite index of four coincident indicators	0.472	0.822
	All employees: total nonfarm	0.517	1.054
	Industrial production index	0.477	0.908
	Manufacturing and trade sales	0.471	0.842
Lagging	Composite index of seven lagging indicators	0.480	0.929
	Commercial and industrial loans at all commercial banks	0.473	0.951

point. A turning point is generally defined to occur at time t if (downturn):

$$y_{t-k} \leq \cdots \leq y_{t-1} > y_t \geq y_{t+1} \geq \cdots \geq y_{t+m}$$

or (upturn)

$$y_{t-k} \geq \cdots \geq y_{t-1} < y_t \leq y_{t+1} \leq \cdots \leq y_{t+m}.$$

Following Zellner et al. (1991), it is selected $k = 3$ and $m = 1$ given the smoothness of the trend cycle data. To determine the time lag needed by an indicator to detect a true turning point it is calculated the number of months it takes for the real time trend-cycle estimate to signal a turning point in the same position as in the final trend-cycle series. The time delays for each estimator are shown in Tables 5 and 6, when filters of 13 and 9 terms are respectively appropriate. It shows that the filters based on the reproducing kernel methodology take one month (on average), whereas those based on the cascade linear filter take two months if the 13-term filter is used correctly, but three months when a 9-term filter should have been used.

The fastest the upcoming of a turning point is detected the fastest new policies can be applied to counteract the impact of the business-cycle stage. Failure to recognize the downturn in the cycle or taking a long time delay to detect it may lead to the adoption of policies to curb expansion when in fact, a recession is already underway.

Table 5: Time lag when filters of 13 terms are appropriate

Macro-area	Series	RKF	CLF
Leading	Average weekly overtime hours: manufacturing	1	2
	New orders for durable goods	1	2
	New orders for nondefense capital goods	1	1
	New private housing units authorized by building permits	1	1
	University of Michigan: consumer sentiment	1	5
Lagging	Average (mean) duration of unemployment	1	3
	Inventory to sales ratio	1	1
	Index of total labor cost per unit of output	1	1
	Average time lag in months	**1**	**2**

Table 6: Time lag when filters of 9 terms are appropriate

Macro-area	Series	RKF	CLF
Leading	Composite index of ten leading indicators	1	1
	S&P 500 stock price index	2	3
	10-year treasury constant maturity rate	1	5
Coincident	Composite index of four coincident indicators	1	5
	All employees: total nonfarm	1	5
	Industrial production index	1	5
	Manufacturing and trade sales	1	1
Lagging	Composite index of seven lagging indicators	1	1
	Commercial and industrial loans at all commercial banks	1	1
	Average time lag in months	**1.1**	**3**

4. Conclusions and further research

For spans of 13 terms, relative to the Henderson filters the CLF and RKF reduce the size of the revisions as new data are entered into the series. Moreover, the CLF decreases the size of the revisions relative to the RKF by nearly 10 %. Similarly, the phase shifts functions take a reduced number of months to detect a true turning point relative to the Henderson, but here the RKF is to be preferred because it reduces the time delay to close to one month whereas the CLF is of two months. When the CLF, that was developed for 13 terms, is applied to series where different spans are more appropriate, it systematically produces poorest values than the RKF.

It would be interesting to extend the research on the CLF using reproducing kernel methodology such that it can cover series for which different spans are suitable. One of our projects in the near future will be on this direction.

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