Validating a Time Series Model for Supply Chain Inventory Data

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Abstract

Supply chain inventory models quite often play a major role in the development of effective business strategies that impact profitability. These models are essential to effective business management and such models yield forecasts that are frequently needed throughout the organization. The focus of the current inquiry is operational and thus we are interested in identifying correlations among events occurring during a narrowly defined business cycle. In the present context, we will use appropriate Box and Jenkins methodologies to devise robust time series models that are parsimonious, reliable, and possess decidability.

Specifically, this paper revisits two business questions posed by a client regarding inventory data. In a recent paper, Morgan et al. identified interesting trends within that inventory data. Now, armed with additional out-year data, a new model will be developed and the results validated. In addition, the implications of the model results on real-time hiring and employment demands, as well aiding in the development of a potential training tool for new hires, will be outlined. As with the initial inquiry, the research will be directed at developing an adequate time series that represents monthly variation in inventory demand.

Key Words: Time Series, ARIMA, Supply Chain, Inventory

• Introduction

In a prior paper, Morgan et al. (2016), addressed two concerns related to effective business strategies and profitability. Our forecast modeling was the first step in providing management, firm data via supply chain modeling for making effective decisions. Clearly, the ability to successfully model is the starting point for enhancing management capabilities in an ever-changing business climate that is affected by many external economic factors. The limited application of supply chain models is perhaps understandable because of the large expense associated with building complex, comprehensive forecast models [Seliaman (2012)]. Such forecasting techniques are classified into three basic categories - causal models, smoothing techniques, or time series models. Causal models are derived using regression analysis, the smoothing techniques employ exponential fitting while the time series methods utilize Box et al. (1970) strategies. The focus of the current inquiry is operational and thus we are interested in establishing correlations among events occurring over a narrowly defined business cycle. In the present study, the appropriate Box-Jenkins methodologies will be used to devise a robust time series model that is parsimonious, reliable and possesses decidability.

This paper addresses two major business issues regarding our client's inventory data. First, we are interested in identifying apparent trends within the dataset and addressing management's goal of devising a model that can forecast future inventory demands. The client supporting this study is in the procurement business and there are several advantages to be gained if one can predict demand cycle requirements with reasonable certainty and lead time [Richards & Grinsted (2016)]. Typically, inventory must be stored which means there are costs associated with space rental and any reduction in inventory storage cost directly improves the bottom line. Additionally, given the cyclic nature of the business, management wants to reassess the present procurement strategy that has evolved over the life of the business enterprise.

• Approach

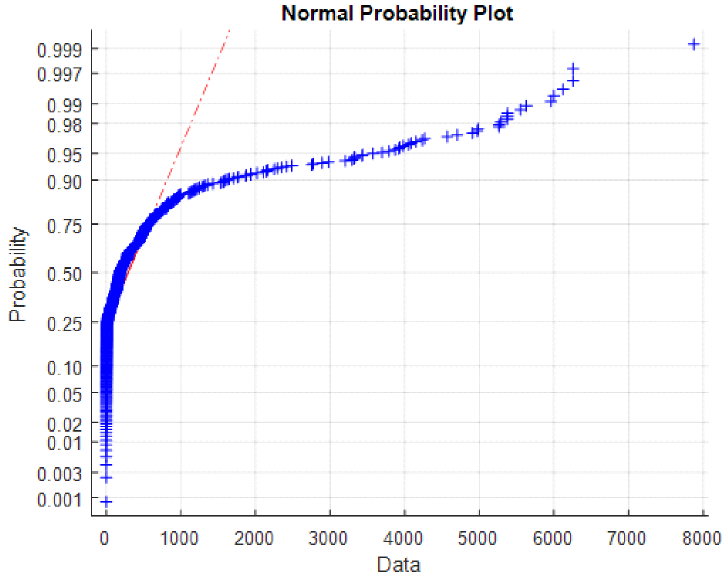
The technical approach involves using Box-Jenkins statistical time series techniques to gleam possible patterns from the supplied inventory transaction data that covers a year of business purchases. This initial inquiry was to focus on developing an adequate time series model that represented monthly variation in inventory demand. The model was developed using only the daily electronic transactions data for a fiscal year period provided by management. An expressed aim was to discern whether monthly variations in transactions dollars are random or serially correlated in time. Our initial findings would be instrumental in our decision to expand our investigation and construct an appropriate time series for the supplied transaction data.

Table 1 provides a brief statistical summary of procurement data for a single fiscal year. Results are provided for the first and second halves of the fiscal period. Even though the number of observations are roughly equal for the respective six-month periods there is a wide spread in the both the mean and variance between the two groups. The dollar values shown in that table have been rescaled to protect client identity. The data range is quite large and varies over five orders of magnitude as depicted by the normal probability plot of Figure 1.

Table 1. Transaction Data Summary

	First Half	Second Half
Count	328	281
Mean	359	1256
STD	416	3969
Range	0 - 2253	0 - 43157

Figure 1. Normal Probability Plot of Transaction Cost Data



A boxplot of the data is provided in Figure 2 and reveals a major outlier that clearly skews the distribution. Figure 3 where the variation in the quarterly mean transaction over the fiscal period is highlighted, suggest a strategy for detrending the data and isolating any underlying procurement practice.

Figure 2: Boxplot of Transaction Cost Data

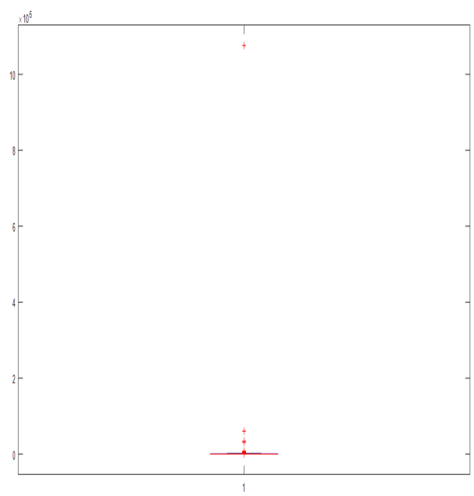
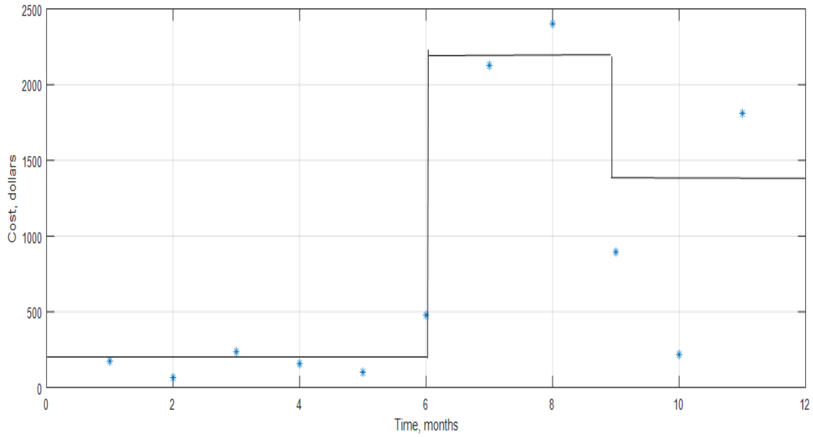
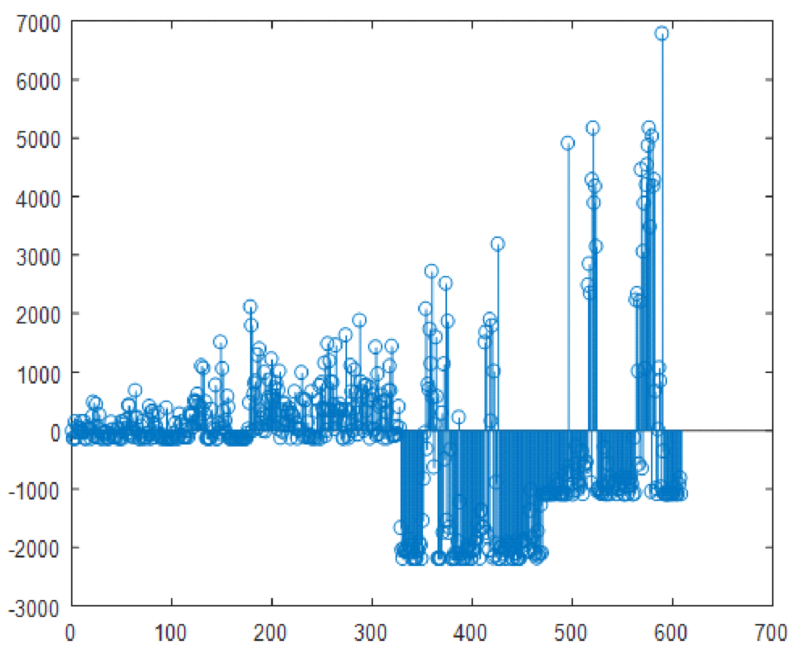


Figure 3: Quarterly Mean Transaction Cost



Hence, two modeling approaches were adopted. Both approaches involved removing the major outlier observed in the boxplot. Justification for eliminating this data point and its impact on the subsequent analysis will be revisited later in the discussion section. The first approach (Mean Filtered) will involve removing the quarterly means from the original data and then fitting the filtered data with an appropriate autoregressive integrated moving average (ARIMA) model. The second method (Full Model) employs standard differencing to obtain stationarity. The proposed supply chain modeling begins with a preprocessing step where the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots are used to estimate the ARIMA model order and differencing levels. After this initial processing step, outputs from the respective analyses are analyzed for goodness of fit. The MATLAB software was used to generate our results.

Figure 4: Mean Filtered Transaction Data



• Results and Discussion

As stated earlier, the autocorrelation and partial correlation (ACF and PACF) plots were used to identify the respective order of the autoregressive and moving average terms in our models. For the mean filtered model, a moving average of order 4 was found to be appropriate. Note that this model contained no difference term which meant that removing the quarterly means values was adequate. ARIMA statistics are summarized in Table 2 for this case. The two ARIMA models for the Full Model depicted in Tables 3 and 4 highlight the impact of appropriate differencing on the adequacy of models. These results like the mean filtered case do not contain an autoregressive term. The t-statistic was used to evaluate the fits of all the generated models.

From this analysis, it became apparent that the client had over time adopted a procurement strategy that deferred major purchases to the second half of the fiscal year in response to federal budget imposed constraints. That strategy involved processing all purchases less than $1000 immediately while shifting purchases between $1000 and $25000 to the second half of the fiscal year. It was also observed that management deferred the outlier purchase discussed earlier to the last quarter of the fiscal year. Removing a few of the other dominant outliers from the original set produce two distinct normally distributions respectively for the first and second halves of the fiscal year. This latter outcome is obviously a result induced by the deferred purchase inventory policy outlined above. A cursory examination of prior fiscal year data also reveals a similar operational pattern. A follow-up study will be used AIC to further evaluate time series model efficacy.

Table 2. Mean Filtered Model ARIMA (0,0,4)

Parameter	Value	Error	t-statistic
Constant	-222.9	115.4	-1.93153×10^{0}
MA (1)	4.38226×10^{-1}	2.30853×10^{-2}	1.89829×10^{0}
MA (2)	3.88955×10^{-1}	3.12535×10^{-2}	1.24452×10^{0}
MA (3)	2.58011×10^{-1}	2.777×10^{-2}	9.29098×10^{0}
MA (4)	2.34638×10^{-1}	2.69783×10^{-2}	8.69732×10^{0}
Variance	998419	33809.8	29.53

Table 3. Full Model: ARIMA (0,0,4)

Parameter	Value	Error	t-statistic
Constant	619.24	143.08	4.32792×10^{0}
MA (1)	3.80272 x10	2.22816×10^{-2}	17.0667×10^{0}
MA (2)	3.28714 x 10	2.82456×10^{-2}	11.6377×10^{0}
MA (3)	2.06007 x 10	2.261099×10^{-2}	7.89
MA (4)	2.0691 x 10	2.54373×10^{-2}	8.1369×10^{0}
Variance	924668	33343.6	27.7315×10^{0}

Table 4. Full Model: ARIMA (0,2,4)

Parameter	Value	Error	t-statistic
Constant	1.08195×10^{-1}	2.12591×10^{-1}	0.508935×10^{0}
MA (1)	-16.1885×10^{-1}	0.214211×10^{-1}	-75.573×10^{0}
MA (2)	$.6.333997 \times 10^{-1}$	0.355225×10^{-1}	17.8477×10^{0}
MA (3)	-0.445048×10^{-1}	0.412535×10^{-1}	-1.07881×10^{0}
MA (4)	0.331475×10^{-1}	0.237025×10^{-1}	1.39848×10^{0}
Variance	906092	29640.4	30.5695×10^{0}

• Conclusion

For the supplied data, most of transactions were relatively evenly distributed between the first and second halves of the fiscal year. Transactions less than $1000 were handled daily in a routine fashion. Those transaction between $1,000-$25,000 were shifted to the second half of the year in response to federal budget constraints. Any transaction exceeding the $25,000 threshold was shifted to the last quarter of the fiscal year. Removing major outliers from the original data set yields two distinct normally distributed populations. Reasonable model fits are obtained by the two approaches taken in this study.

References

Au, T. *Engineering Economics for Capital Investment Analysis*. Boston: Allyn & Bacon, 1983.

Biegler, L., I. Grossmann, and A. Westerberg. *Systematic Methods of Chemical Process Design*. Prentice Hall, New Jersey, 1997.

Box, G.E.P, G. M. Jenkins, and G. C. Reinsel. *Time Series Analysis: Forecasting and Control*. Wiley, New York, 1970.

Morgan III, M.H., C.B. Morgan, E.A. Morgan, and K.D. Morgan, 2016. *Time Series Models of Supply Chain Inventory Data*. In JSA Proceedings', Business, and Economic Statistic Section. Chicago, Ill: American Statistical Association, pp 2158-2164.

Park, C. *Contemporary Engineering Economics*. Reading, MA: Addison-Wesley, 1993.

Richards, G., and S. Grinsted. *The Logistics and Supply Chain Toolkit.* Amazon, 2016.

Seliaman, M.E. *Mathematical Models for Supply Chain Inventory Coordination*. LAMBERT Academic Publishing, 2012.

Sprott, J. C. "Chaos and Time Series Analysis," Oxford University Press, London, England, 2003.