From “faster is slower” to “faster is faster”

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Abstract
The “faster is slower” effect raises when crowded people push each other to escape through an exit during a panic situation. As individuals push harder, a statistical slowing down in the evacuation time can be achieved. The slowing down is caused by the presence of small groups of pedestrians (say, a small human cluster) that temporary block the way out when trying to leave the room. The pressure on the pedestrians belonging to this blocking cluster raises for increasing anxiety levels and/or larger number of individuals trying to leave the room through the same door. Our investigation shows, however, that very high pressures alters the dynamics in the blocking cluster, and thus, changes the statistics of the time delays along the escaping process. It can be acknowledged a reduction in the long lasting delays, while the overall evacuation performance improves. We present results on this novel phenomenon taking place beyond the “faster is slower” regime.

Key Words: Panic, evacuation, clogging, time delays

1. Introduction

The “faster is slower” (FIS) effect states that the faster they try to reach the exit during an evacuation, the slower they move due to clogging near the door. This effect has been observed in social systems [1] as well as on other physical systems, such as grains flowing out a 2D hopper or sheep entering a barn [2].

Statistical research on the clogging delays (in the context of the “social force model”) has shown that a small group of pedestrians close to the door is responsible for blocking the way to the rest of the crowd. This *blocking clusters* appear as an arch-like metastable structure around the exit. The tangential friction between pedestrians belonging to this blocking structure was shown to play a relevant role with respect to the whole evacuation delays [3, 4]. However, either the amount of blocking structures or its time life can vary according to the door width, the presence of obstacles or fallen individuals [5, 7]. Further studies on blocking structures appearing in granular media research can be found in Refs. [8]- [11] .

The relevance of the *blocking structures* on the time evacuation performance has alerted researchers that the analysis of “reduced” systems rather than the whole crowd is still a meaningful approach to the FIS effect. In this context, the authors of Ref. [12] introduced a simplified breakup model for a small arch-like blocking structure (in a SFM setting). They examined theoretically the breakup of the arch due to a single moving particle, and observed a FIS-like behavior. Thus, they concluded that the essentials of the FIS phenomenon could

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be described with a system of only a few degrees of freedom.

To our knowledge, neither the theoretical approach nor the computational simulations have been pushed to extreme scenarios. That is, no special attention has been paid to those situations where the pedestrians experience very high anxiety levels (see Section 4) while the crowd becomes increasingly large.

In the current investigation we explore the pedestrians anxiety levels from a relaxed situation to very high pushing pressures (equivalent to anxiety levels close to 20 m/s). Our work is organized as follows: a brief review of the basic SFM can be found in Section 2. Section 3 details the simulation procedures used to studying the room evacuation of a crowd under panic. The corresponding results are presented in Section 4. Finally, the conclusions are summarized in Section 5.

2. Background

2.1 The Social Force Model

The "social force model" (SFM) states that our tendency to avoid overcrowded environments acts as a repulsive force, changing our dynamics, although our desire to reach some target point. Both effects (repulsion and desire) operate as *social forces* in pedestrian dynamics. Additionally, friction between people (and walls) is also a very important issue in crowd dynamics. Thus, the three forces (repulsion, desire and friction) are present in the equation of motion for any individual

$$m_i \frac{d\mathbf{v}^{(i)}}{dt}(t) = \mathbf{f}_d^{(i)}(t) + \sum_j \mathbf{f}_s^{(ij)}(t) + \sum_j \mathbf{f}_g^{(ij)}(t) \qquad (1)$$

where m_i is the mass of the pedestrian i, and $\mathbf{v}_i$ is its corresponding velocity. The subscript j represents all other pedestrians (excluding i) and the walls. $\mathbf{f}_d$, $\mathbf{f}_s$ and $\mathbf{f}_g$ are the desire force, the social (repulsion) force and the friction (or granular) force, respectively. See Refs. [1]-[6] for details.

The expression for each kind of forces are as follows

$$\begin{cases} \mathbf{f}_d^{(i)}(t) & = & m_i \dfrac{\mathbf{v}_d^{(i)}(t) - \mathbf{v}_i(t)}{\tau} \\ \mathbf{f}_s^{(ij)} & = & A_i \, e^{(r_{ij}-d_{ij})/B_i} \mathbf{n}_{ij} \\ \mathbf{f}_g^{(ij)} & = & \kappa \, g(r_{ij}-d_{ij}) \, \Delta\mathbf{v}_{ij} \cdot \mathbf{t}_{ij} \end{cases} \qquad (2)$$

where $\mathbf{v}_d^{(i)}$ is the desired velocity for pedestrian i, $\mathbf{v}^{(i)}$ is the current velocity, and τ, A_i, B_i and κ are fixed parameters. The magnitude $r_{ij} = r_i + r_j$ is the sum of the pedestrian's radius, while d_{ij} corresponds to the inter-pedestrian distance. Further details on each parameter can be found in Refs. [3]-[6].

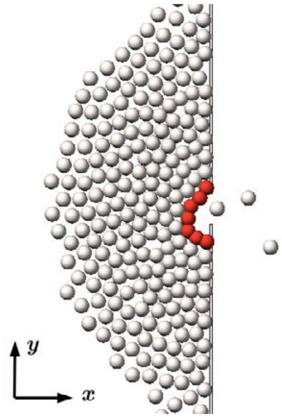


Figure 1: (Color on-line only) Snapshot of an evacuation process from a $20\,\mathrm{m} \times 20\,\mathrm{m}$ room, with a single door of 1.2 m width. The blocking structure is identified in red color. The rest of the crowd is represented by white circles. It can be seen three individuals that have already left the room. The desired velocity for the individuals inside the room was $v_d = 6\,$m/s.

2.2 Clustering structures

The time delays during an evacuation process are related to clogged people, as explained in Refs. [3, 4]. Groups of pedestrians can be defined as the set of individuals that for any member of the group (say, i) there exists at least another member belonging to the same group (j) in contact with the former. That is, the distance between them (d_{ij}) is less than the sum of their radius ($d_{ij} < r_i + r_j$).

During an evacuation process, different humans clusters may appear inside the room. But, some of them are able to *block* the way out. We are interested in the minimum set of human clusters that connects both sides of the exit. Thus, we will call *blocking clusters* or *blocking structures* to those human structures that block the exit (with the minimum number of individuals). Two blocking clusters are different if they differs at least in one pedestrian. That is, if they differs in the number of members or in pedestrians themselves. Fig. 1 shows (in highlighted color) a *blocking structure* near the door.

We define the *blocking time* as the cumulative period of time when the evacuation process is stopped due to any blocking cluster. That is, the sum of the “life time” of each blocking cluster (*blocking delays*).

3. Simulations

Most of the simulation processes were performed on a 20 m $\times$ 20 m room with 225 pedestrians inside. The room had a single exit on one side, as shown in Fig. 1. The door was placed in the middle of the side wall to avoid corner effects.

A few simulation processes were performed on 30 m $\times$ 30 m and 40 m $\times$ 40 m rooms with 529 and 961 pedestrians inside, respectively. The door was also placed in the middle of the side wall.

Statistics were taken over at least 30 evacuation processes. The recorded magnitudes

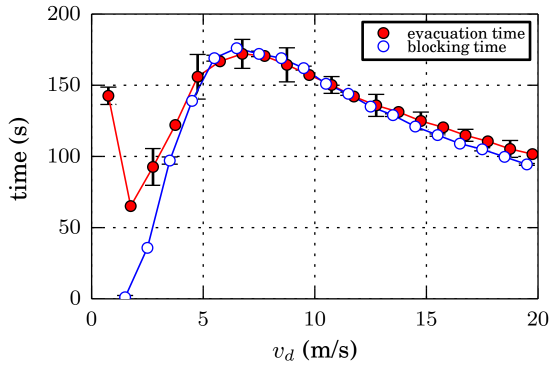


Figure 2: (Color on-line only) Evacuation time and blocking time as a function of the desired velocity v_d. Both data sets represent the mean values from 60 evacuation processes. The simulated room was 20×20 m with a single door of 1.2 m width on one side. The number of individuals inside the room was 225 (no re-entering mechanism was allowed). The simulation lasted until 160 individuals left the room.

were the pedestrian's positions and velocities for each evacuation process. We also recorded the corresponding social force f_s and granular force f_g actuating on each individual.

The simulating process lasted until 70% of individuals left the room. If this condition could not be fulfilled, the process was stopped after 1000 s. Data recording was done at time intervals of $0.05\ \tau$.

The explored anxiety levels ranged from relaxed situations ($v_d < 2$ m/s) to extremely stressing ones ($v_d = 20$ m/s). This upper limit may hardly be reached in real life situations. However, we have already shown in Ref. [13] that very high desired velocities may resemble the same "social pressure" of an increasingly large number of pedestrians acknowledging a moderate anxiety levels.

The simulations were supported by LAMMPS molecular dynamics simulator with parallel computing capabilities [14]. The time integration algorithm followed the velocity Verlet scheme with a time step of 10^{-4} s. All the necessary parameters were set to the same values as in previous works (see Refs. [5, 6]).

4. Results

4.1 Evacuation time versus the desired velocity

Fig. 2 shows the evacuation time (filled symbols and red line) for a wide range of desired velocities v_d, many of them beyond the interval analyzed by Helbing and co-workers (see Ref. [1]). The *faster is slower* regime can be observed for desired velocities between 2 m/s and 8 m/s (approximately). However, the evacuation time improves beyond this interval, meaning that the greater the pedestrian's anxiety level, the better with respect to the overall time saving.

Therefore, we actually attain a *faster is faster* regime for desired velocities larger than 8 m/s, instead of the expected *faster is slower* regime. This is a novel behavior that has not been reported before (to our knowledge) in the literature. This effect holds even if we include the elastic force introduced by Helbing et al. in Ref. [1] (not shown in Fig. 2).

The overall time performance has been reported to be related to the *clogging delays*, understood as the period of time between two outgoing pedestrians (see Refs. [3, 4, 5] for details). But, since most of the time delays correspond to the presence of *blocking structures* near the door, we examined closely those delays due to blockings for increasing anxiety levels (*i.e.* desired velocities v_d).

Fig. 2 exhibits (in hollow symbols and blue line) the computed blocking time for a wide range of desired velocities. That is, the cumulative "life time" of all the *blocking clusters* occurring during an evacuation process. Notice that the blocking delays become non-vanishing for $v_d > 2\,$m/s. This threshold corresponds to those situations where the granular forces become relevant, according to Refs. [3, 4]. It is, indeed, the lower threshold for the *faster is slower* effect.

No complete matching between the mean evacuation time and the blocking time can be observed along the interval $2\,\mathrm{m/s} < v_d < 4\,\mathrm{m/s}$. We traced back all the time delays experienced by the pedestrian, and noticed that the time lapse between the breakup of the blocking structure and the leaving time of the pedestrians (belonging to this blocking structure) was actually a relevant magnitude. This *transit time* explained the difference between the evacuation time and the blocking time.

According to Fig. 2, the evacuation time appears to be highly correlated to the blocking delays above $v_d = 4\,$m/s. Thus, the noticeable enhancement in the evacuation performance taking place between $8\,$m/s and $20\,$m/s (*i.e.* the "faster is faster" effect) is somehow related to the enhancement in the blocking time. In other words, the delays associated to the blocking clusters appear to explain the entire *faster is faster* effect.

We next measured the evacuation time for three different crowd sizes. We chose a relatively small crowd (225 pedestrians), a moderate one (529 pedestrians) and a large one (961 pedestrians). The corresponding room sizes were 20×20 m, 30×30 m and 40×40 m, respectively. The results are shown in Fig. 3.

The three situations exhibited in Fig. 3 achieve a *faster is faster* phenomenon, since the slope of each evacuation curve changes sign above a certain desired velocity. As the number of individuals in the crowd becomes larger, the v_d interval attaining a negative slope increases. That is, only a moderate anxiety level is required to achieve the *faster is faster* phenomenon if the crowd is large enough.

Notice that the larger crowd (*i.e.* 961 individuals) attains the steepest negative slope. Thus, as more people push to get out (for any fixed desired velocity v_d), the faster they will evacuate.

For a better insight of the "faster is faster" phenomenon, we binned the blocking delays into four categories. This allowed a close examination of the qualitative changes in the delays when moving from the "faster is slower" regime to the "faster is faster" regime. Fig. 4 shows the mean number of blocking delays (for each category) as a function of v_d.

The four categories represented in Fig. 4 have positive slopes for small anxiety levels

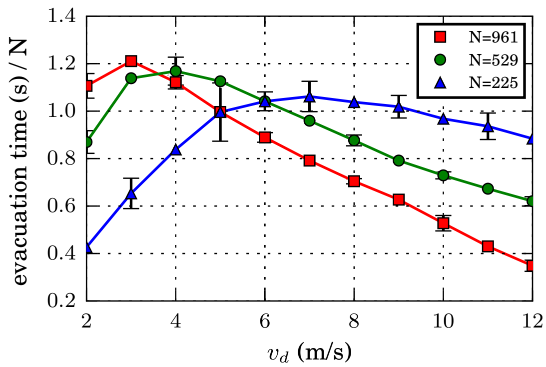


Figure 3: (Color on-line only) Evacuation time per individual vs. desired velocity for N=225, N=529 and N=961 (no re-entering mechanism was allowed). The rooms sizes were 20×20 m, 30×30 m and 40×40 m, respectively, with a single door of 1.2 m width on one side. Mean values were computed from 30 evacuation processes until 70% of pedestrians left the room.

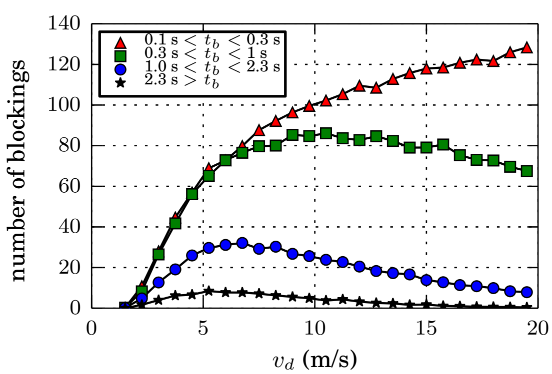


Figure 4: (Color on-line only) Mean number of blocking delays for four different time categories (see legend for the corresponding blocking times t_b) as a function of the desired velocity v_d. The simulated room was 20×20 m with a single door of 1.2 m width on one side. The number of individuals inside the room was 225 (no re-entering mechanism was allowed). Mean values were computed from 60 realizations. The simulation lasted until 160 individuals left the room.

(say, $v_d \lesssim 8$ m/s). This is in agreement with the "faster is slower" regime, since the faster the pedestrians try to evacuate, more time they spend stuck in the blocking structure.

A qualitative change, however, becomes noticeable beyond 8 m/s. The slopes corresponding to the three most long lasting categories change sign. Thus, the individuals spend less time stuck in the blocking structure for increasing anxiety levels.

It is true that the short lasting delays increase for high anxiety levels. But a quick inspection of Fig. 4 shows that the increase in the short lasting category (red triangular symbols) is not enough to balance the decrease in the other categories. Consequently, the overall evacuation time follows the same qualitative behavior as the long lasting categories (say, the *faster is faster* behavior).

The above investigation may be summarized as follows. The scenario for high anxiety levels (say, $v_d > 4$ m/s) corresponds to a "nearly always" blocking scenario. However, two different blocking instances can be noticed. The "faster is slower" corresponds to the first instance. The "faster is faster" is the second instance appearing after either high values of v_d or increasing number of pedestrians. Many long lasting blockings seem to break down into shorter blockings, or even disappear (see Fig. 4).

We examined closely the mean number of breakup processes taking place during the evacuation. We bin these breakups along the parallel direction with respect to the door (*i.e.* y-axis, according to Fig. 1), in order to associate the breakups to a meaningful position. For any desired velocity, the breakup position was likely to occur straight in front of the exit (roughly, at the door middle position). Therefore, this region became of special interest with respect to the breakup process.

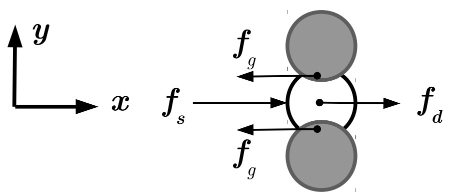
From our current simulations and previous work (see Ref. [6]), we realized that the mid-position corresponds to the crowd area of highest pressure (for an exit width of 1.2 m). This is in agreement with the maximum amount of breakups, since higher pushing forces may help forward the blocking pedestrians.

4.2 Stationary blocking model

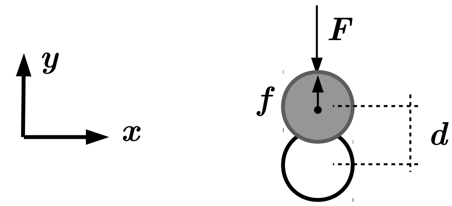
For a better understanding of the relation between the crowd pushing forces and the breakup process, we decided to focus on the behavior of a single pedestrian who tries to get released from the blocking structure.

We mimicked a small piece of the blocking structure (*i.e.* red individuals in Fig. 1) as two individuals standing still, but separated a distance smaller than the pedestrian's diameter. A third pedestrian was set in between the former, mimicking the pedestrian who tries to get released from the blocking structure. Fig. 5a represents this set of three pedestrians. Notice that Fig. 5a may represent any piece of the blocking structure, but according to the previous result, it will usually correspond to the middle piece of the blocking structure.

The crowd was assumed to push the pedestrian in the middle from behind (f_s force along the x-axis in Fig. 5a). Besides, the crowd actuated on the still pedestrian (*i.e.* F force), in order to counterbalance the social repulsion in the y-direction, as shown in Fig. 5b.



(a) force balance for the x-axis.



(b) force balance for the y-axis.

Figure 5: Force balance for a moving pedestrian between two still individuals. The moving pedestrian is represented by the white circle, while the gray circles correspond to the still individuals. The movement is in the $+x$ direction. f_s represents the (mean) force due to other pedestrians pushing from behind. f_d is the moving pedestrian's own desire. f_g corresponds to the tangential friction (*i.e.* granular force) between the moving pedestrian and his (her) neighbors. F and f are the forces actuating on the upper (still) pedestrian. f corresponds to the social repulsive force due to the moving pedestrian, while F represents the counter force for keeping the pedestrian still.

The center of mass of the three pedestrians were initially aligned and the velocity of the individual in the middle was initially set to zero.

According to Section 2 and the balance condition for the still pedestrian, the crowd force F is related to the compression distance between the two pedestrians as follows

$$2r - d = B\ \ln(F/A) \tag{3}$$

for the known values A and B.

The crowd pushing force depends linearly on the anxiety level of the pedestrians v_d for a fixed number of individuals at equilibrium, according to Ref. [13]. Thus, it seems reasonable, as a first approach, that f_s and F are related as

$$f_s = F = \beta v_d \tag{4}$$

where β is a fixed coefficient, thats depends linearly on the number of individuals in the crowd. Notice that f_s and F are assumed to be equal because of the geometry of the mimicking model.

We computed the period of time required for the moving pedestrian to get released from the other two (still ones). This time is supposed to mimic the blocking time of the blocking structure, since the three pedestrians represent a small piece of this structure. Fig. 6 shows the blocking time as a function of the desired velocity v_d.

A comparison between Fig. 2 and Fig. 6 shows the same qualitative behavior for the blocking time, although the scale along the v_d axis is somehow different. The blocking time slope changes sign at 7 m/s in Fig. 2, while Fig. 6 shows a similar change at 3.75 m/s. This discrepancy can be explained because of the chosen value of β.

The chosen value for β in Fig. 6 was 2000 (see caption). This value is roughly the expected force due to the crowd of 225 pedestrians (and $v_d = 2$ m/s). However, as the pedestrians evacuate from the room, the crowd force diminishes. The "effective" force

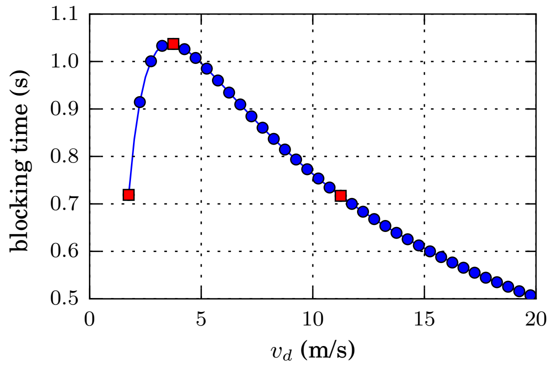


Figure 6: (Color on-line only) Blocking time of the three pedestrians model (one moving pedestrian between two still ones) as a function of the desired velocity v_d. The initial velocity of the moving pedestrian was set to zero. The crowd pressure was set to $F = f_s = 2000\,v_d$. Each blocking time was recorded when the moving pedestrian lost contact with the other individuals. Desired velocities of $v_d = 1.75$ m/s, $v_d = 3.5$ m/s and $v_d = 11.25$ m/s are indicated in red color (and squared symbols). The blocking time for $v_d = 1.75$ m/s and $v_d = 11.25$ m/s are the same. Only one realization was done for each v_d value.

along the whole process is actually smaller, and so is the β value. Thus, the "effective" maximum blocking time is expected to lie at a larger v_d value than 3.75 m/s.

The above reasoning is also in agreement with the evacuation time shown in Fig. 3 for an increasing number of pedestrians. The maximum evacuation time takes place at lower anxiety levels (*i.e.* v_d values) as the crowd size becomes larger. Therefore, the pushing force βv_d downscales the *faster is faster* threshold, as expected from our simple model.

So far, the mimicking model for a small piece of the blocking structure exhibits a *faster is slower* instance for low crowd's pushing forces, and a *faster is faster* instance for large pushing forces.

4.3 Non-stationary blocking model

We were able to establish a connection between the breakup process and the crowd pushing force in Section 4.2. Now, we will examine the force balance on the moving pedestrian along this process. As already mentioned, our attention is placed on initially aligned pedestrians with null velocity.

Fig. 7 shows the force balance on the moving pedestrian (in the mimicking model) along the breakup process. The balance is expressed as the ratio between the *positive forces* and the *negative forces*. The former corresponds to the sum of all the forces that push the moving pedestrian towards the exit (*i.e.* the own desired force, and the social force from all the neighbors). The latter corresponds to the force in the opposite direction to the movement (*i.e* the granular force). According to Section 2 and Fig. 5

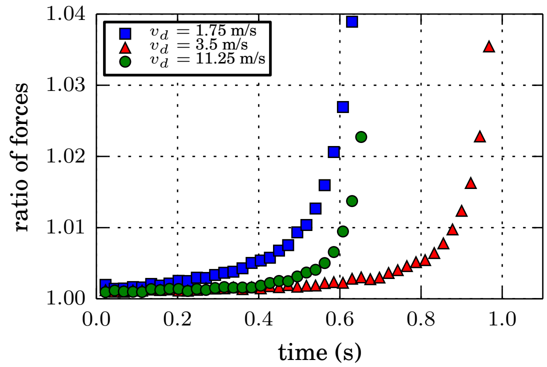


Figure 7: (Color on-line only) Ratio of *positive forces* (desire force and social repulsion) and *negative force* (granular) on the moving pedestrian as a function of time for three desired velocities (see text for details). The initially velocity of the moving pedestrian was zero. The simulation finished when he looses contact with the other individuals. One realization is done.

$$\text{ratio} = \frac{f_s + F + f_d}{2f_g} \tag{5}$$

where f_s and F correspond to the pushing forces from the crowd and the still neighboring pedestrian, respectively. Both are social forces in nature. Notice, however, that only the contribution on the x-axis is relevant in the mimicking model (see Fig. 5).

Fig. 7 presents three different situations, corresponding to those desired velocities highlighted in squared symbols (and red color) in Fig. 6. The three situations stand for any *faster is slower* instance, the maximum blocking time instance and any *faster is faster* instance, respectively. But care was taken in choosing similar blocking times for the first and the third situation, in order to achieve a fair comparison.

The three situations shown in Fig. 7 exhibit a ratio close to unity during the first stage of the process. This means that all the forces actuating on the moving pedestrian are approximately balanced.

Notice that this quasi-stationary stage lasts until the very end of the breakup process (say, 1% above unity). However, a striking positive slope can be seen during the last stage of each process. The slopes are quite similar on each process (although shifted in time), and thus, this last stage seems not to be relevant in the overall blocking time. We can envisage the last stage as an expelling process before the blocking structure breaks into two pieces.

An important conclusion can be derived from the inspection of Fig. 7: although the breakup process is actually non-stationary, the balance constrain (ratio $\simeq$ 1) is quite accurate for the early stage of the breakup process.

4.4 Remarks

From our point of view, the balance constrain (that is, ratio $\simeq 1$) is actually the main reason for the *faster is faster* phenomenon to take place.

Recall that the *positive* forces $f_s + F + f_d$ correspond to the cumulative pushing forces of the crowd (f_s and F) and the moving pedestrian (f_d). The latter, however, is not relevant with respect to the former because most of the positive force is done by the crowd (for example, f_d is approximately 10% of f_s for 225 individuals). Thus, the *positive* forces are roughly $f_s + F = 2\beta v_d$, according to Section 4.2. The balance constrain becomes approximately

$$\frac{\beta v_d}{f_g} \simeq 1 \tag{6}$$

Eq. (6) is meaningful since it expresses the fact that the *negative* force f_g balances the pushing force, in order to keep the pedestrian moving forward (at an almost constant velocity). However, the granular force is currently $f_g = \kappa\, v\, B \ln(\beta v_d/A)$. The $B \ln(\beta v_d/A)$ factor corresponds to the compression between the pedestrian and his (her) neighbor in the blocking structure. It can easily be shown from Eq. (6) that

$$v^{-1} \sim \frac{\ln(\beta v_d/A)}{\beta v_d/A} \tag{7}$$

The slope of v^{-1} is positive for low anxiety levels (*i.e.* v_d values), but changes sign as the anxiety level becomes increasingly large. Since the blocking time varies as v^{-1}, we may conclude that Eq. (7) mimics the *faster is slower* and the *faster is faster* instances.

The logarithm in Eq. (7) is the key feature for the slope change. Recall from Eq. (3) that $\ln(\beta v_d/A)$ stands for the compression in the blocking structure. But, although compression increases for an increasing pushing force of the crowd, it seems not enough to diminish the pedestrian velocity in order to hold the *faster is slower* phenomenon at high anxiety levels. Consequently, the blocking time decreases, achieving a *faster is faster* instance.

5. Conclusions

Our investigation focused on the evacuation of extremely anxious pedestrians through a single emergency door, in the context of the “social force model”. No previous research has been done, to our knowledge, for such extreme scenarios.

Unexpectedly, we acknowledged an improvement in the overall evacuation time for desired velocities above 8 m/s (and a crowd size of 225 individuals). That is, the *faster is slower* effect came to an end at this anxiety level, while a novel *faster is faster* phenomenon raised (at least) until a desired velocity of 20 m/s. This unforeseen phenomenon was also achieved for increasingly large crowds and lower desired velocities.

A detailed examination of the pedestrian’s blocking clusters showed that the *faster is faster* instance is related to shorter “life times” of the blocking structures near the exit. The long lasting structures taking place at the *faster is slower* instance now breakup into short lasting ones. The breakup is most likely to occur at the middle position of the exit.

We mimicked the breakup process of a small piece of the blocking structure through a minimalistic model. The most simple model that we could image was a moving pedestrian between two still individuals. Although its simplicity, it was found to be useful for understanding the connection between the crowd's pushing forces and the breakup process.

The mimicking model for the blocking structure showed that a force balance between the crowd's pushing force and the friction with respect to the neighboring individuals held along the breakup. Only at the very end of the process, the pedestrian was expelled out of the blocking structure.

We concluded from the force balance condition that friction was the key feature for the *faster is faster* instance to take place. As the crowd pushing force increases, the compression between individuals in the blocking structure seems not enough to provide a slowing down in the moving pedestrian. Thus, the *faster is slower* instance switches to a *faster is faster* instance. The latter can be envisaged as brake failure mechanism.

We want to stress the fact that, although we investigated extremely high anxiety situations, *faster is faster* instance may be present at lower desired velocities if the crowd size is large enough. We were able to acknowledge the *faster is faster* phenomenon for desired velocities as low as $v_d = 4$ m/s when the crowd included 1000 individuals approximately.

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