Stochastic Seasonality, Contemporaneous Inference, and Forecasting in the Presence of Volatile Weather

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Abstract
Contemporaneous inference from economic data releases for policy and business decisions has become increasingly relevant in the high pace of the information age. The released data are typically filtered to eliminate seasonal patterns to reveal underlying trends and cycles. The nature of economic seasonal behavior is such that average seasonality, not actual seasonality, is filtered from the data. First, the paper suggests adjustments of the inference accounts for the stochastic seasonality. We formalize the issue and present a simple method to the informal inferential practice. Second, we provide a data-based method that allows for temperature adjustment to improve forecasting outcomes. With the assumption of climate change taking place, these methods are particularly important as weather patterns become more volatile.

Key Words: Seasonal adjustment, stochastic seasonality, forecasting, inference, temperature, weather

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1. Introduction

Contemporaneous inference and real-time forecasting based on economic data releases have become increasingly relevant in the information age where data come quickly, frequently, and in large quantities. The data are typically filtered to eliminate seasonal patterns, regular variations due to weather and holidays, etc., prior to their release in order to reveal underlying trends and cycles. Translation of these data into useful elements for business and policy decision making follows a standard (if not always formal) Bayesian updating process. While the trend component of the seasonal behavior can be detected and imperfectly adjusted by moving-average seasonal factors[3], the random component of seasonal behavior and how to incorporate that information in updating one's forecasts have not been well studied. Moreover, good rules of thumb as to how to interpret from data convoluted with random seasonal events, such as a greater than average snowfall in a season, are important but yet to well accepted as suggested by Hausman and Watson (1985), Wright (2013), and Stock (2013). An understanding and methodologies of the random component of seasonality could become more important in the presence of the more volatile climate pattern all over the world accompanied with the enlarged magnitude of the random component.

For example, we have heard the arguments over the past few winters as to how an extreme winter weather disrupted a normal seasonally-adjusted economy and a possible mean-reverting process could follow in the spring. The extremely warm winter[4] in 2012 boosted and kicked off some economic activities that was supposed to come later in the spring. The stronger 2012 winter performance was expected to, and indeed actually, be followed by a weaker spring performance[5]. In contrast, an extremely cold winter in 2014 rattled and/or delayed some economic activities, as pointed out by Bloesch and Gourio (2015). The question is: Beyond trends and cycles, will a weaker/stronger winter performance foresee a stronger/weaker spring one? This important question has just started to get attention from academies, private sectors, and governments but only few answers are provided. The paper presents some answers. If the random component of the winter weather just speeds up or postpones the economic activities across seasons, because of the predicted mean-reverting force, part of the spring's outcome will be predictable and therefore the seasonal adjustment method should incorporate the information. In contrast, if the random component of the winter has permanent impact on the economy, whether positive or negative, the conventional seasonal adjustment methods would still be valid.

Various recent newspapers pointed out that the U.S. GDP growth has averaged 1.3% in the first quarter (Q1) and 2.9% in all other quarters.[6] Since 2010, the disparity widened with GDP growth of 0.4% in Q1 and 2.9% in the other three quarters. In May 2015, the Bureau of Economic Analysis announced that they were working on a multi-pronged action plan to improve its estimates of GDP by identifying and mitigating potential sources of "residual" seasonality, which is when seasonal patterns remain in data even after they are

[3] This is the principle behind the Census Bureau's X-12 ARIMA seasonal adjustment method.
[4] The aggregate deviation of temperature in January, February, and March is 16.7 degree higher than its long-term average. See Figure 6 in details.
[5] See, for example, "Mild winter may have artificially inflated jobs data, economists fear," Washington Post, April 6, 2012.
[6] See, for example, "Will first-quarter GDP get better after it gets worse?" Wall Street Journal, May 29, 2015.

adjusted for seasonal variations.[7] Rudebusch, Wilson, and Mahedy (2015) find a strong positive residual seasonality of GDP growth in Q1 over the past two decades, especially after 2008. They propose a second seasonal adjustment approach and suggest that 2015:Q1 GDP growth would be 1.8% rather than 0.2%.

Seasonal adjustment is a common pre-analysis transformation of economic time series data. The benchmark for this practice is the path-breaking work on the Census X-11 methodology by Shiskin, Young and Musgrave (1967). As an illustration of the pervasiveness of the publication of mechanical seasonal adjustment, the U.S. Bureau of Labor Statistics (BLS) publishes 299 seasonally adjusted time series on consumer prices and 734 on employment, and the U.S. Bureau of Economic Research GDP release table contains 35 seasonally adjusted series. To accomplish the task of the seasonal adjustment of thousands of series by the U.S. Government as well as the many thousands published by other organizations, a high degree of automation is required[8]. Some specific annual events such as 4/5-week fixed effects and Easter/Labor Day adjustments are standard for all series and only require the development of a few common matrices for many time series, but deeper analysis of the causal seasonal events and their underlying economic dynamics are left to others.

Statistical issues with respect to seasonal adjustment are many[9], but the general overall process as practiced by statistical reporting agencies has changed only slowly over time since the debates of Pierce (1979), Bell and Hillmer (1984, 1985), Maravall (1984), and Sims (1985). Recent econometric research (*e.g.* Canova and Ghysels (1994), Canova and Hansen (1995), Franses and de Bruin (2000), and Koopman and Franses (2002)) have observed seasonal factor changes and seasonal variance changes associated with cyclical time series movements suggesting that seasonality in economic time series may be much more complex than previously thought. But these important avenues of research have not resolved the issues raised by Bell *et al.* (1984); that of economists and journalists using the thousands of more or less mechanically seasonally adjusted series for inference and decisions. Greenaway-McGrevy (2013) proposes a new semiparametric multivariate approach, as opposed to conventional univariate approach, to seasonal adjustment under the assumption that the trend component is driven by the common changes among a group of time series. Wright (2013) examines how the Great Recession, due to its timing in 2008:Q4 and 2009:Q1, distorts the following seasonally adjusted payroll employment. Boldin and Wright (2015) propose a seasonally and weather adjusted approach by incorporating temperature and snowfall data.

The contributions of this paper are twofold. First, we provide a simple model to adjust the time-varying variance of the seasonal factors and suggest an improved confidence interval and inference. Second, by incorporating observable seasonal factor, i.e. winter temperature, we produce an improved real-time forecast on economic variables in the spring. The article is organized as follows. Section 2 explains the average seasonal-adjustment model. Section 3 presents the scholastic seasonal-adjustment model. Section 4 shows the results of a Monte

[7] See Bureau of Economic Analysis' Blog Post: http://blog.bea.gov/2015/05/22/residual-seasonality-gdp/.

[8] Jaditz (1994) explores the rationale and exceptions to seasonal adjustment.

[9] See Blitzer, Case, Maitland, Shiller and Stiff (2010) for a discussion of some of the anomalous results from the application of Census X12 to the S&P/Case Shiller Home Price Indices. They do not critique the X12 program but state that it "works as intended" and point out that these results are data generated.

Carlo simulation. Sections 5 suggests a method to improve inference. Section 6 proposes a method to improve forecast. Section 7 concludes.

2. The Average Seasonal-Adjustment Model

The statistical theory that admits the widespread practice of seasonal adjustment is straightforward. Consider a time series Y_{it} where $i = 1,\ldots, r$ is the within-year period and $t = 1,\ldots, T$ is the annual indicator. The traditional component formulation of the time series is given by: $Y_{it} = C_{it}\ S_{it}\ E_{it},$ where C is the trend/cycle component, S is the seasonal component and E is the random disturbance component. The time series is then transformed by taking the natural logarithm to be:

$$y_{it} = c_{it} + s_{it} + \varepsilon_{it} \qquad \text{where } \varepsilon \sim \text{i.i.d. } (0, \sigma^2) \qquad (1)$$

By removing the seasonal component, s_{it}, the seasonally adjusted series, y^*_{it}, is comprised of the trend/cycle (underlying fundamentals) and a serially uncorrelated random shock. There are a number of mechanical ways to remove s_{it} from y_{it}. These include filtering (Shiskin, Young and Musgrave (1967), Burman (1965), Dagum and Quenneville (1993), Jain (2001), Koopman and Ooms (2006)), seasonal differencing (Box, Jenkins, and Reinsel (2008), Granger and Newbold (1986), Hillmer, and Tiao (1982)), fixed effects or dummy variables (Barsky and Miron (1989), Osborn (1990)), and spectral decompositions (Proietti (2000), Burman (1965)). While each have their merits and, depending on the nature of the time series, one may be indicated over the others[10], they all have the objective of finding a seasonal factor S_i, $i = 1, 2,\ldots, r$ which when subtracted from y_{it} in (1) yields a time series devoid of seasonal effects. In some variations $S_i = S_i(t)$ in order to capture the change in seasonal effects over time. The time variation is assumed to follow a smooth path as seasonality morphs from one generation to the next.

There is one key assumption imbedded in this process. All seasonal adjustment transformations (after suitable data transformations) are assumed to remove a component that is calendar dependent and, with the exception of temporal drift, is approximately constant year to year[11]. Thus the estimation of the seasonal factors, when done correctly, throws off i.i.d. errors subsumed in the error component of the seasonally adjusted series.

Seasonal components in many economic time series are derived from events such as weather. When it snows or is extremely cold, people do not generally go out and shop for cars. Thus automobile sales in February are generally below average for the year. The seasonal component of automobile sales is a behavioral reaction to external stimuli, snow and temperature. If it does not snow in February, the behavioral reaction is bound to be different. Therefore the seasonal component changes. Since the difference between the amount of snowfall and temperature in any given year and the long run average is properly viewed as a random variable, so then is a component of S_i, the behavioral reaction to it. Importantly, this shift in car purchases has an asymmetric effect on the data. The behavioral response in February does not change car sales in December or in January, but it does change them in March and April.

[10] See for example, Franses, Hylleberg and Lee (1994) for a discussion on differencing versus seasonal fixed effects and Ghysels, Lee and Siklos (1993) on a comparison of methods using U.S. macroeconomic data.
[11] Ghysels, Osborn and Rodrigues (2003) discuss violations of this assumption and methods to identify unit root and near unit root seasonal processes.

Removing the average seasonal response from the time series does not remove the entire seasonal response, and to the extent that the time series datum is treated as devoid of seasonal behavior, this induces an error to any inference derived from it. The more variable the stimulus (*i.e.* the more important seasonality is), the more seasonality remains.

3. The Stochastic Seasonal-Adjustment Model

Suppose there exists a singular random seasonal event that occurs in the first period of the year represented by W_{1t}, where t is the annual time index. W impacts the value of the time series y_{it} by an amount λW_{1t}. By definition, a seasonality is a temporary change from trend that is then compensated for in other periods within the year so as to preserve the annual average of the time series. In this example, we assume for simplicity's sake that the compensation is spread equally throughout the year. Formally that requires that fixed λ, $\Sigma W_{it} = 1$[12]. Let the seasonal pattern of W_{it} for a series of r sub-annual periods be described by:

$$W_{1t} = \{w^* + \Phi, w^*, w^* - \Phi\} \text{ with probability } \{1/3, 1/3, 1/3\}$$
$$W_{it} = W_{1t} / (r\text{-}1) \quad \text{for all } i \neq 1 \tag{2}$$

One can think of w^* as a winter storm which can either be severe, average, or mild. Almost all economic time series that are seasonally adjusted would have, given enough historical data, the estimated seasonal factors:

$$S_1 = -\lambda\, w^*$$
$$S_i = +\lambda\, w^* / (r\text{-}1) \qquad i = 2,\ldots, r \tag{3}$$

And the seasonally adjusted series would then be:

$$y_{it} = c_{it} + s_{it} + S_i + \varepsilon_{it} \tag{4}$$

If the seasonal influence were in fact removed ($\Phi = 0$) then:

$$s_{it} + S_i = 0$$
$$y^*_{it} = c_{it} + \varepsilon_{it} \tag{5}$$

Equation (5) would then be a seasonally adjusted series, which by construction would be a stationary time series. The purpose of this seasonal adjustment is to be able to draw inferences that would not be polluted by the presence of ephemeral seasonal phenomena. The inference is of the form:

$$H_0\colon |\Delta y^*_{it}| = |\Delta c_{it} + \Delta \varepsilon_{it}| < \delta \tag{6}$$

For large enough values of $|\Delta y^*_{it}|$ rejection of H_0 is an inference that the current observation on y is significantly different from the previous one.

Whether the analysis of (6) is a formal statistical hypothesis test or an implicit Bayesian posterior probability update, the size of δ matters. Since it is normally assumed that (5) is a close approximation to the time series cleaned of its seasonal component, δ is functionally

[12] It is important to note here that the parameter restriction is only on future seasonal values. Thus, the standard two-sided moving restriction on the sum of seasonal factors is, appropriately, relaxed. Ex post, it is not necessarily true that a sequence of twelve months of seasonal factors in monthly data sum to 1.

related to σ^2, the variance of ε. To be fair, it is generally recognized that W has some random component embedded in it, and that the random deviation from w^* ends up in the error term of (5), but this is generally not considered to be a problem[13]. This thinking is often not incorrect. First, the seasonal error in the economic data is not an i.i.d. process even if the seasonal event w^* is. That is because the behavior causing the seasonal blip in the data represents a temporal shifting in behavior across the year, as indicated in (2) and (3). Second, the way in which seasonal events translate into economic activity causes the variance of the deviation from average to be non-constant. Third, there is a temporal asymmetry to the impact of seasonal events.

Here we explore the implications of these in our example. For $i = 1$ there are 9 possible values for Δy^*_{1t}, each with a probability of 1/9:

1. $\Delta c_{1t} + \Delta\varepsilon_{1t} + \lambda\Phi(r\text{-}2)/(r\text{-}1)$
2. $\Delta c_{1t} + \Delta\varepsilon_{1t} + \lambda\Phi$
3. $\Delta c_{1t} + \Delta\varepsilon_{1t} + \lambda\Phi r/(r\text{-}1)$
4. $\Delta c_{1t} + \Delta\varepsilon_{1t} - \lambda\Phi/(r\text{-}1)$
5. $\Delta c_{1t} + \Delta\varepsilon_{1t}$
6. $\Delta c_{1t} + \Delta\varepsilon_{1t} + \lambda\Phi/(r\text{-}1)$
7. $\Delta c_{1t} + \Delta\varepsilon_{1t} - \lambda\Phi(r\text{-}2)/(r\text{-}1)$
8. $\Delta c_{1t} + \Delta\varepsilon_{1t} - \lambda\Phi$
9. $\Delta c_{1t} + \Delta\varepsilon_{1t} - \lambda\Phi r/(r\text{-}1)$ (7.a)

For $i = 2$ there are three possible values and they are not independent of the value of W at period 1:

1. $\Delta c_{2t} + \Delta\varepsilon_{2t} - \lambda\Phi r/(r\text{-}1)$ when $W_{1t} = w^* + \Phi$
2. $\Delta c_{2t} + \Delta\varepsilon_{2t}$ when $W_{1t} = w^*$
3. $\Delta c_{2t} + \Delta\varepsilon_{2t} + \lambda\Phi r/(r\text{-}1)$ when $W_{1t} = w^* - \Phi$ (7.b)

For $2 < i < r - 1$ there is one outcome:

1. $\Delta c_{it} + \Delta\varepsilon_{it}$ (7.c)

The variances of the three cases are:

$\text{Var}(\Delta y^*_{1t}) = 2\sigma^2 + (2/3)\lambda^2\Phi^2(1+(r\text{-}1)^{-2})$
(8.a)
$\text{Var}(\Delta y^*_{2t}) = 2\sigma^2 + (2/3)\lambda^2\Phi^2(r^2(r\text{-}1)^{-2})$
(8.b)
$\text{Var}(\Delta y^*_{it}) = 2\sigma^2$ *for all* $i \neq 1, 2$ (8.c)

And the co-variances of the time series are:

Lead/Lag	$\text{Cov}(\Delta y^*_{1t})$	$\text{Cov}(\Delta y^*_{2t})$	$\text{Cov}(\Delta y^*_{it})$ for all $i \neq 1, 2$

[13] Proietti (2004) and Tripodis and Penzer (2007) have studied the impact of monthly specific stochastic seasonal events by specifying the monthly variation in seasonality as a random walk. The asymmetry in the autocorrelation function as an outgrowth of the behavioral response to seasonal events was not incorporated in his study, nor has his analysis of the impact of monthly specific factors impacted the standard seasonal adjustment practice.

$-(r+1)$	0	0	0	
$-r$	$-(2/3)\lambda^2\Phi^2(r-1)^{-1}$	0	0	
$-(r-1)$	$-(2/3)\lambda^2\Phi^2(r-2)(r-1)^{-2}$	0	0	
$-(r-2)$	0	0	0	
.	.	.	.	
.	.	.	.	
.	.	.	.	
-2	0	0	0	
-1	$-\sigma^2 + (2/3)\lambda^2\Phi^2(r-1)^{-2}$	$-\sigma^2 + (2/3)\lambda^2\Phi^2 r(r-1)^{-1}$	$-\sigma^2$	
0	$2\sigma^2 + (2/3)\lambda^2\Phi^2(1+(r-1)^{-2})$	$2\sigma^2 + (2/3)\lambda^2\Phi^2(r^2(r-1)^{-2})$	$2\sigma^2$	
1	$\sigma^2 + (2/3)\lambda^2\Phi^2 r(r-1)^{-1}$	σ^2	σ^2	
2	0	0	0	
.	.	.	.	
.	.	.	.	
.	.	.	.	
$r-1$	0	0	0	
r	0	$-(2/3)\lambda^2\Phi^2 r(r-1)^{-2}$	0	
$r+1$	$(2/3)\lambda^2\Phi^2(r-1)^{-1}$	0	0	
$r+2$	0	0	0	(9)

In this simple example of seasonality we note three things. First, even though the non-seasonal components of the data series, Δy_{it}, might be stationary, the seasonally adjusted series, Δy^*_{it}, is not. The seasonally adjusted series violates the assumption that $\mathrm{Cov}(\Delta y^*_{kt}, \Delta y^*_{it}) = \rho(k-i)$ for all i and k. So the process of seasonal adjustment may smooth the original series somewhat by taking out the mean seasonality, but it does not generate a stationary time series from a series where nonstationarity is caused by the presence of a random seasonal component.

Second, the variances of the seasonally adjusted series are non-constant. If the analyst were to estimate the variance σ^2 by the standard methodology and then apply, say, a two standard deviation critical value to equation (6), then for some i, t, δ would be too small and for some δ would be too large. In our example, δ will be too small for Periods 1 and 2 and too large for the others. How much larger and how important this is becomes an empirical question[14]. However, we can say from our brief foray into the theory of seasonal adjustment that the larger λ and Φ are, the more likely there is to be inferential errors. Which is to say, the more important seasonal influences are in a series, the more likely the series is to be seasonally adjusted and the more likely current methods of adjustment by statistical bureaus will lead to inferential errors. The issue of how important this might be is explored via a Monte Carlo analysis in the next section.

4. Monte Carlo Simulation

Though the theory is quite clear—random seasonality creates nonstationary time series and seasonal adjustment does not correct for that nonstationarity—the question remains: is it an important or relatively inessential characteristic of seasonal patterns in time series? To be sure, if the random component of seasonal behavior were sufficiently small, then to the extent that there were errors of inference made by ignoring the random component, they

[14] Koopman, S. J., Ooms, M. and I. Hindrayanto (2009) calculated differing monthly seasonal frequencies in U.S. unemployment data.

would be infrequent and not terribly consequential. To answer the question, we ran a Monte Carlo simulation to bring out the factors that would make this an important characteristic of time series.

The structure of the simulation is a mean zero time series with four periods labeled Q1-Q4. There is no cyclical component, but there is a seasonal component that is modeled as a constant plus an MA process:

$$X_t = S_{it} + e_t + \beta_1 e_{it\text{-}1} + \beta_2 e_{it\text{-}2}, \quad i = 1, 2, 3, 4 \qquad (10)$$

S_{it} is fixed as $S_{1t} = 1$, $S_{2t} = 0$, $S_{3t} = -1$, $S_{4t} = 0$ for all t. The e_{it} are chosen from one of two distributions. The distributions are identical for $i = 1, 3$ and for $i = 2, 4$ and differ from each other by a scalar. The Monte Carlo simulation was conducted for alternative values of the scalar given β_1 and β_2, to ascertain how the size of the variance of the random component matters in inference with random seasonality and for alternative values of β_1 and β_2 given $S(t)$ to ascertain how the behavioral linkage between periods affects inference with random seasonality.

The experimental design was to construct 1,000 time series of length 88. Each series began with an initial condition of $e_{4,-1} = 0$ and $e_{3,-2} = 0$. The impact of the initial condition on the time series is washed out by $t = 4$, and the series employed in the analysis was $t = \{4,\cdots, 88\}$. An estimate of the seasonal factors was calculated using X-11 methodology, and the series were then seasonally adjusted. The seasonally adjusted series on $t = \{4,\cdots, 84\}$ were used to compute the standard deviation of the series with the entire sample, and individually for each quarter. The final step of the analysis was to take each observation $t = \{85,\cdots, 88\}$ and, using the total series standard deviation and the period specific standard deviation, calculate confidence intervals. Observations in which a decision was made that X_t, the mean zero time series, was significantly different from zero were recorded.

Tables 1 and 2 report the results from a variation in the variances. The not surprising result is that the greater the difference in variability between the random components of the periods, the more likely it is that using the series standard deviation will lead Type I errors in the high variability periods and Type II errors in the low variability periods. If the behavioral consequence of a seasonal event is to move 80% of the shock forward in time (that is, the shock in Q1 is mostly compensated for by a reduction in X_{t+1} and X_{t+2} akin to a pulling forward or backwards shopping due to weather considerations), then between 1 in 4 and 1 in 5 times inference using the total series standard deviation will be wrong. Moreover, between 1/3 and 1/2 the time incorrect inference will be made at least once during the year.

Tables 3 and 4 report the results from varying the linkage between random seasonal events in one period and another. The Monte Carlo simulation was conducted with β_1 varying between .8 and .1 and β_2 set to 0. The simulations show that the less a seasonal event affects other episodes, the more inferential errors are made. This result seems counter intuitive, but relates to the fact that the offsetting behavior in the linked periods often leads to incorrect inference. Moreover the total series is more volatile, and therefore the standard errors tend to be larger. Nevertheless, the frequency of at least one inferential error over a year in the simulations remains between 1/3 and 1/2.

Table 1: Frequency of Different Inferential Results from Replacing the Series Standard Deviations with Monthly Standard Deviation

	2.5% Significance (1 tailed test)				5% Significance (1 tailed test)			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Model Variance Ratio	2	1	2	1	2	1	2	1
Model MA Process		$e_t - 0.5e_{t-1} - 0.3e_{t-2}$				$e_t - 0.5e_{t-1} - 0.3e_{t-2}$		
Series Not Period	12%	0%	13%	0%	23%	0%	25%	0%
Period Not Series	0%	4%	0%	6%	0%	8%	0%	9%
Total	12%	4%	13%	6%	23%	8%	25%	9%
Model Variance Ratio	7	4	7	4	7	4	7	4
Model MA Process		$e_t - 0.5e_{t-1} - 0.3e_{t-2}$				$e_t - 0.5e_{t-1} - 0.3e_{t-2}$		
Series Not Period	11%	0%	11%	0%	22%	0%	24%	0%
Period Not Series	0%	4%	0%	5%	0%	7%	0%	7%
Total	11%	4%	11%	5%	22%	7%	24%	7%
Model Variance Ratio	3	2	3	2	3	2	3	2
Model MA Process		$e_t - 0.5e_{t-1} - 0.3e_{t-2}$				$e_t - 0.5e_{t-1} - 0.3e_{t-2}$		
Series Not Period	10%	0%	10%	0%	19%	0%	22%	0%
Period Not Series	0%	3%	0%	5%	0%	5%	0%	5%
Total	10%	3%	10%	5%	19%	5%	22%	5%
Model Variance Ratio	5	4	4	4	5	4	4	4
Model MA Process		$e_t - 0.5e_{t-1} - 0.3e_{t-2}$				$e_t - 0.5e_{t-1} - 0.3e_{t-2}$		
Series Not Period	8%	1%	8%	1%	17%	1%	18%	1%
Period Not Series	0%	3%	0%	3%	0%	3%	0%	3%
Total	8%	3%	8%	3%	17%	4%	18%	4%

Table 2: Frequency of Different Inferential Results from Replacing the Series Standard Deviations with Monthly Standard Deviation, At Least Once in Four Quarters Following Calculation of Seasonal Factors

		2.5% Significance	5% Significance
Model Variance Ratio	2:1		
Model MA Process	$e_t - 0.5e_{t-1} - 0.3e_{t-2}$	29%	49%
Model Variance Ratio	7:4		
Model MA Process	$e_t - 0.5e_{t-1} - 0.3e_{t-2}$	28%	46%
Model Variance Ratio	3:2		
Model MA Process	$e_t - 0.5e_{t-1} - 0.3e_{t-2}$	25%	42%
Model Variance Ratio	5:4		
Model MA Process	$e_t - 0.5e_{t-1} - 0.3e_{t-2}$	21%	36%

Table 3: Frequency of Different Inferential Results from Replacing the Series Standard Deviations with Monthly Standard Deviation

	2.5% Significance (1 tailed test)				5% Significance (1 tailed test)			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Model Variance Ratio	2	1	2	1	2	1	2	1
Model MA Process		$e_t - 0.8e_{t-1}$				$e_t - 0.8e_{t-1}$		
Series Not Period	12%	1%	8%	1%	16%	0%	18%	1%
Period Not Series	0%	2%	0%	2%	0%	8%	0%	4%
Total	12%	3%	8%	3%	16%	8%	18%	5%
Model Variance Ratio	2	1	2	1	2	1	2	1
Model MA Process		$e_t - 0.5e_{t-1}$				$e_t - 0.5e_{t-1}$		
Series Not Period	11%	0%	12%	0%	22%	0%	24%	0%
Period Not Series	0%	4%	0%	7%	0%	7%	0%	9%
Total	11%	4%	12%	7%	22%	7%	24%	9%
Model Variance Ratio	2	1	2	1	2	1	2	1
Model MA Process		$e_t - 0.25e_{t-1}$				$e_t - 0.25e_{t-1}$		
Series Not Period	15%	0%	14%	0%	29%	0%	28%	0%
Period Not Series	0%	4%	0%	6%	0%	10%	0%	12%
Total	15%	4%	14%	6%	29%	10%	28%	12%
Model Variance Ratio	2	1	2	1	2	1	2	1
Model MA Process		$e_t - 0.1e_{t-1}$				$e_t - 0.1e_{t-1}$		
Series Not Period	16%	0%	15%	0%	30%	0%	29%	0%
Period Not Series	0%	2%	0%	3%	0%	10%	0%	11%
Total	16%	2%	15%	3%	30%	10%	29%	11%

Table 4: Frequency of Different Inferential Results from Replacing the Series Standard Deviations with Monthly Standard Deviation, At Least Once in Four Quarters Following Calculation of Seasonal Factors

		2.5% Significance	5% Significance
Model Variance Ratio	2:1		
Model MA Process	$e_t - 0.8e_{t-1}$	19%	36%
Model Variance Ratio	2:1		
Model MA Process	$e_t - 0.5e_{t-1}$	27%	47%
Model Variance Ratio	2:1		
Model MA Process	$e_t - 0.25e_{t-1}$	31%	55%
Model Variance Ratio	2:1		
Model MA Process	$e_t - 0.1e_{t-1}$	32%	58%

5. A Method to Improve Inference: Univriate Series

5.1 Inference

As an example of the problem, consider the U.S. nonfarm payroll employment numbers. Policy makers and analysts closely watch these numbers as an indicator of labor market health. Depending on the change in employment as measured by this series, decisions are made including those related to contra-cyclical macroeconomic policy. Figure 1 displays the not-seasonally-adjusted and seasonally adjusted series from the BLS from 1984 through April of 2012. The date sequence was chosen to avoid two offsetting anomalous values relating to the 1982 recession and to eliminate the clear difference in the variance of the series before and after 1984. In addition to the regular seasonal pattern from non-adjusted series, the non-adjusted series has a much more volatile fluctuation than the adjusted series.

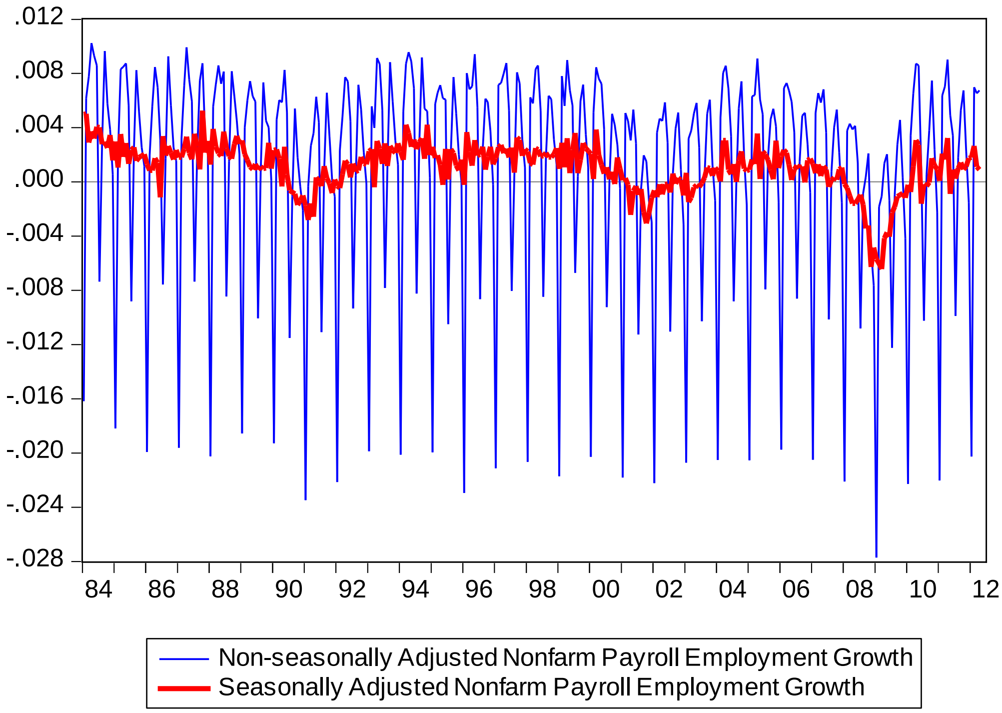


Figure 1: U.S. Nonfarm Payroll Employment, Not Seasonally Adjusted and Seasonally Adjusted, 1984-2012, First Difference of Log Value
Source: Bureau of Labor Statistics

As shown in Figure 2 of the Census X-12 method, there is a seasonal build up in employment in May and June, a seasonal correction or reduction in July and August, and another seasonal build up from September to December followed by a reduction the following January to March. Seasonal adjustment by the BLS seems to have corrected for these patterns and yielded a series that appears to be stationary with an imbedded cyclical component. Estimating the residual habitual behavior of the data with an AR(2) process yields a series of residuals with the expected white noise autocorrelation function.
The estimated time series model is:

$$\Delta y_t = 0.0001 + 0.4135\Delta y_{t-1} + 0.4516\Delta y_{t-2} + \varepsilon_t \qquad (11)$$

where Δy_t is the first difference of log nonfarm payroll employment. The adjusted R^2 = 0.68, all coefficients significant at 95%. To do inference with this data, analysts look at the size of the change in nonfarm employment and implicitly or explicitly compare it to some measure to see if the values indicate a turning point in the business cycle. Presumably, the δ employed relates to the standard deviation of the residuals from the series after adjusting for trend, cycle and seasonal components. In other words, δ is a function of the residual term[15].

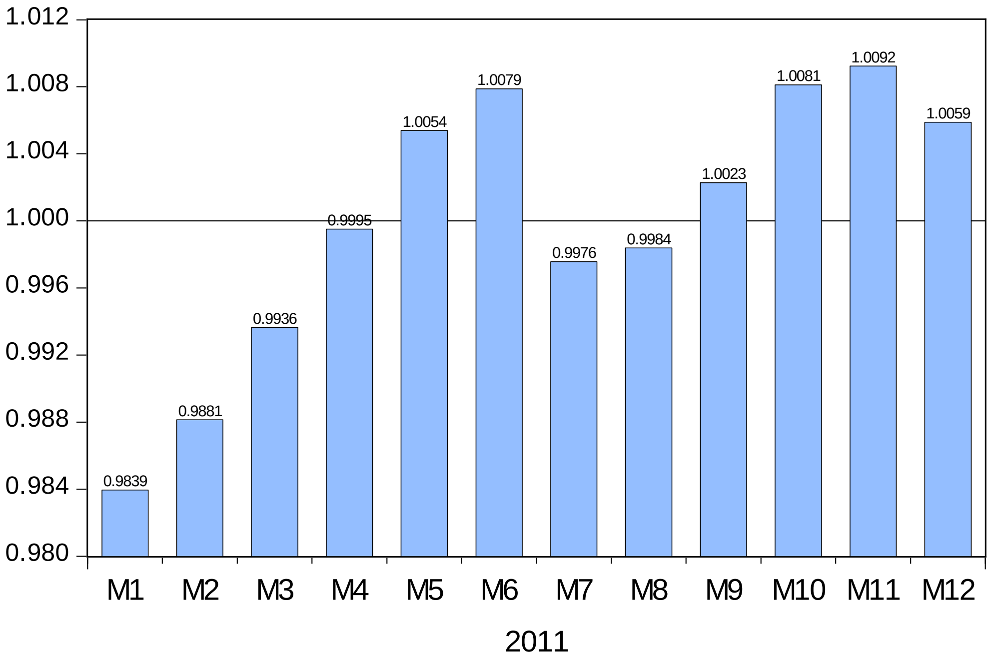


Figure 2: 2011 Monthly Seasonally Adjustment Factors for U.S. Nonfarm Payroll Employment, 1984-2012, Based on the X-12 Method from Census Bureau
Source: Data is from Bureau of Labor Statistics and factors are calculated by EViews' Census X-12 method

To see if the problem posed here exists, that of the nonstationarity of the seasonally adjusted time series when the behavioral response to seasonal factors varies with the season, inferences on this time series in the period since the end of the 2008 recession were compared. First, the sample variance of the residuals from (11) was computed for the entire sample and the monthly changes were normalized. Next the sample was segmented by month, and variances for each month were computed. A second set of normalized changes were computed using the monthly specific standard deviations.

Table 5 presents the results of the analysis. The first two columns show the changes in nonfarm payroll since the end of the 2008 recession using the two different normalizations. The following two columns answer the question: did the normalization matter for inference? Under the null hypothesis of stationarity, the answer should be, except for a small number of random instances, no. In this example, using a 90% critical value and assuming asymptotic normality, 14% of the months since June 2009 would have led to different inferences between statistics employing the total sample variance and those

[15] Tiller and Di Natale (2005) propose a model based variance to improve inference when the data is derived from sampling. See also Jaditz (2000).

employing the 12 monthly sample variances. This difference falls to 11% when a 95% critical value is used. The results from this example are in line with the Monte Carlo results above.

Table 5: Standardized Change in Log Seasonally Adjusted Nonfarm Payroll in the U.S.

Year	Month	Std. Deviation from Total Sample	Std. Deviation from Individual Months in Sample	Different Inference 90% Level	Different Inference 95% Level
2009	July	-2.627	-2.601	N	N
2009	August	-1.794	-2.201	N	Y
2009	September	-1.548	-1.594	N	N
2009	October	-1.547	-1.483	N	N
2009	November	-0.328	-0.325	N	N
2009	December	-1.335	-1.995	Y	Y
2010	January	-0.312	-0.371	N	N
2010	February	-0.274	-0.240	N	N
2010	March	1.476	1.265	N	N
2010	April	1.863	1.843	N	N
2010	May	4.012	4.157	N	N
2010	June	-1.297	-1.088	N	N
2010	July	-0.451	-0.446	N	N
2010	August	-0.396	-0.486	N	N
2010	September	-0.210	-0.216	N	N
2010	October	1.709	1.610	Y	N
2010	November	0.939	0.932	N	N
2010	December	0.930	1.390	N	N
2011	January	0.852	1.012	N	N
2011	February	1.702	1.496	Y	N
2011	March	1.900	1.628	Y	N
2011	April	1.935	1.914	N	N
2011	May	0.416	0.431	Y	N
2011	June	0.646	0.542	N	N
2011	July	0.738	0.731	N	N
2011	August	0.653	0.801	N	N
2011	September	1.551	1.597	N	N
2011	October	0.859	0.801	N	N
2011	November	1.202	1.193	N	N
2011	December	1.706	2.549	N	Y
2012	January	2.099	2.494	N	N
2012	February	1.973	1.735	N	Y
2012	March	1.088	0.932	N	N
2012	April	0.517	0.511	N	N
2012	May	0.585	0.606	N	N
2012	June	0.607	0.510	N	N

Consider the February 2012 employment change data. Using a normalization based on the entire series, the change in employment was 1.973 standard deviations, a level unlikely to be random at the 95% level. The Wall Street Journal’s report on the employment numbers was typical. “… (this is) the latest sign that the economy has gained momentum. … Still, most economists said Friday's report showed that the labor market is continuing to strengthen.” To be sure, the report contained a host of caveats, but the general tenor of this and other analyst reports was that February was a confirmation of a firming of the recovery. If the monthly variance estimates were used, one would have found that the change in non-farm employment fell from 2.49 standard deviations in January to 1.74 in February, and February’s change was not significantly different from random at the 95% level.

The reason for the disparity between the two sets of inferences lies in February having a higher variance in the change in non-farm employment than other months. The higher variance means that one ought to have discounted changes in February non-farm employment more heavily than in other months. Good analysts do this intuitively, but it is not common nor standard practice.

5.2 Model Residual Autocorrelation between Specific Months

In the previous example of the U.S. nonfarm employment time series, a seemingly stationary series was constructed. Though the series may technically be nonstationary, the extent to which it is might not matter. Under the null hypothesis of stationarity, autocorrelation functions estimated from a subset of the original data should not be systematically and significantly different from the full sample autocorrelation function. Figure 3 shows comparisons between the monthly sample autocorrelation functions, e.g. the January sample denotes Jan ’84, Jan ’85, Jan ’86, … Jan ’12, and the total sample. As expected, the total sample values, represented by triangles in the graphs, are close to zero. The monthly autocorrelation functions are asymmetric, and for each month there are estimated autocorrelation coefficients that are significantly different from the full sample model.

While the interactions between the months are quite complicated, March and May provide a good example of the nonstationary behavior derived from random seasonal events. In the March sample, March is negatively correlated with June (lead period 3), and the autocorrelation is significantly different from zero. The autocorrelation between March and January (lag 2) / February (lag 1) / April (lead 1), however, is not significantly different one from the other. April is negatively correlated with June, and the autocorrelation coefficient is significantly different from zero.

The explanation for this is straightforward. Seasonal hiring in March and April is associated with seasonal layoffs in June. When the amount of hiring is atypically large, the amount of firing will be as well. But prior to April, employers do not know that seasonal events are going to lead to more or less hiring. They are expecting the average. So the seasonal factors generated by standard methods fail to create a stationary series, even though the technical data, examined through our timeworn lens of time series analysis, seems to suggest they have.

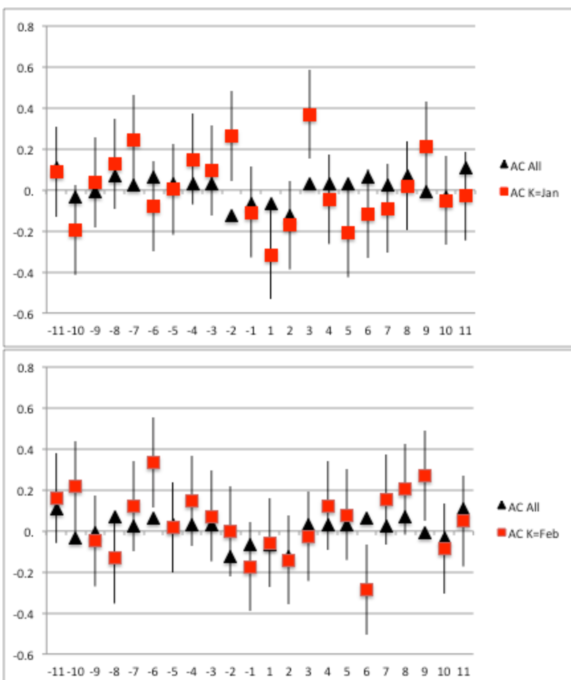
Note that the violation of stationarity derives, by definition, from month-over-month time-varying property as shown in (10). As the time horizon gets longer, say more than one year, the seasonal time variation becomes less important. This is the reason why the standard nonstationarity tests, such as Augmented Dickey-Fuller, Phillips-Perron (1988), Elliot, Rothenberg, and Stock (1996), fail to reject the null hypothesis of unit root for most seemingly-stationary seasonally-adjusted series empirically. This implies that the stochastic seasonal problem might not be crucial for the real-time, or out-of-sample, long-term forecast. Yet, for the real-time short-term forecast, it is imperative for forecasters to capture these seasonal shocks and systematically adjust their forward-looking correlation into forecasting. More importantly, the inference of seasonal abnormality is fundamentally different from cycle or trend changes.

5.3 Adjusting for Seasonal Serial Correlation

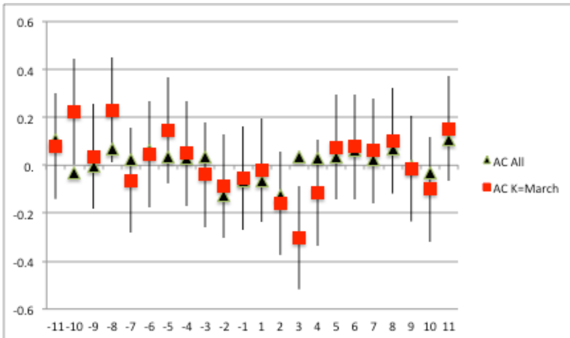
A solution to the above problems is to use estimated monthly autocorrelation functions, Figure 3, to estimate that part of the random seasonal that is projected forward from the seasonal event to future months. The new seasonally adjusted series is:

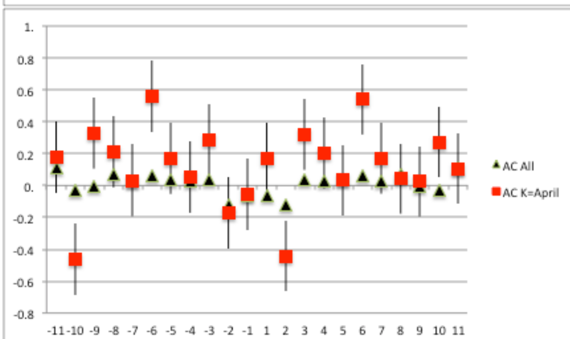
$$y^{**}_{it} = c_{it} + s_{it} + S_i - \zeta_{it} + \varepsilon^{*}_{it} \tag{12}$$

where ζ_{it} is the estimated seasonal from past deviations of the seasonal events from their average values, and ε^{*}_{it} is the new serially uncorrelated residual. It is still the case that S_i - ζ_{it} will differ from s_{it} by the amount of behavior induced by the deviation of the contemporaneous seasonal event from its average[16]. Therefore the monthly variances in y^{**}_{it} will not be constant over time, and an adjustment of the standardized changes using the new monthly variances is made to account for this.



[16] Tiao and Grupe (1980) analyze the structure of seasonal periodicity in ARMA models.


AC All
AC K=March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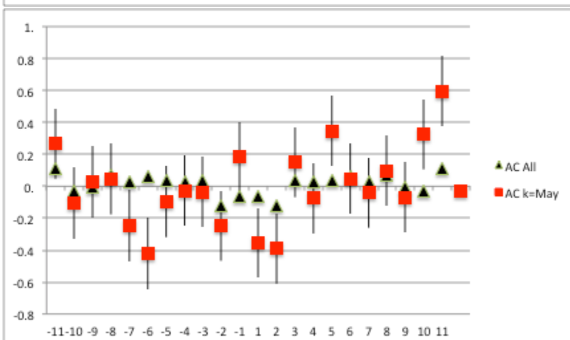
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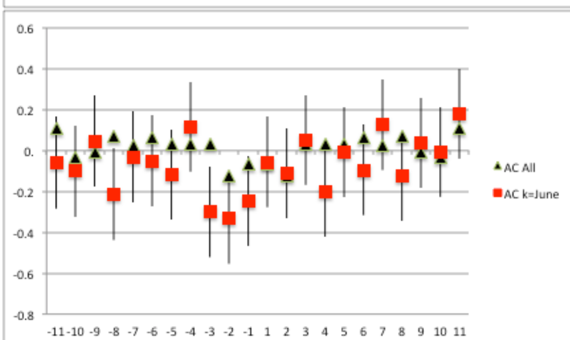
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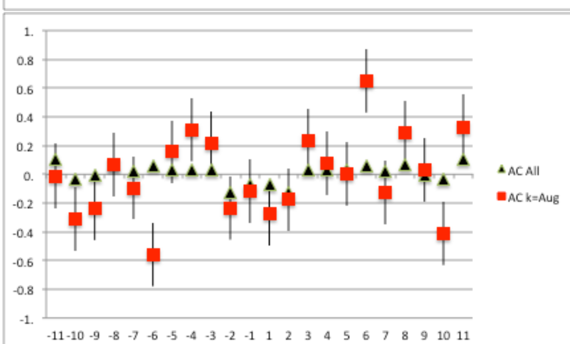
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AC k=Aug

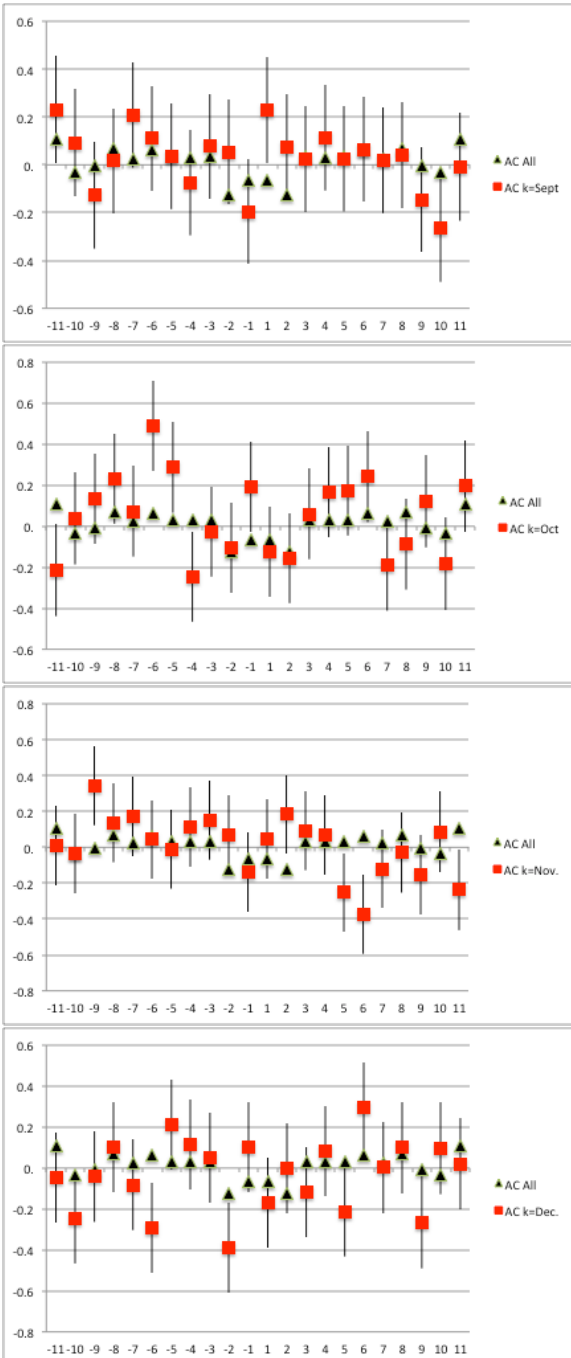

Figure 3: Sample Autocorrelation Functions Estimated for Each Month Compared to the Sample Autocorrelation Function for the Entire Sample (1984 – 2012). Seasonally Adjusted U.S. Nonfarm Payroll Residuals from the AR(2) Model
Source: Bureau of Labor Statistics

An estimate of the new, seasonal autocorrelation-corrected standardized differences is presented in Table 6. The first two columns represent the standardized changes in BLS seasonally adjusted nonfarm employment and the standardized changes using monthly autocorrelation corrections. There are two immediate observations. First, the number of inferential differences between the two series has increased from 14% to 23% for the 90% critical value and from 11% to 20% for the 95% critical value.

Table 6: Standardized Change in Log Seasonally Adjusted Nonfarm Payroll in the U.S.

Year	Month	Std. Deviation from Total Sample	Std. Deviation from Individual Months in Sample	Different Inference 90% Level	Different Inference 95% Level
2009	July	-2.627	-4.183	N	N
2009	August	-1.794	-0.821	Y	N
2009	September	-1.548	-3.156	Y	Y
2009	October	-1.547	-0.958	N	N
2009	November	-0.328	-1.626	N	N
2009	December	-1.335	-0.296	Y	Y
2010	January	-0.312	0.101	N	N
2010	February	-0.274	-0.708	N	N
2010	March	1.476	0.297	N	N
2010	April	1.863	1.010	Y	N
2010	May	4.012	1.278	Y	Y
2010	June	-1.297	1.177	N	N
2010	July	-0.451	1.468	N	N
2010	August	-0.396	1.327	N	N
2010	September	-0.210	-0.505	N	N
2010	October	1.709	2.709	N	Y
2010	November	0.939	1.163	N	N
2010	December	0.930	1.491	N	N
2011	January	0.852	0.549	N	N
2011	February	1.702	2.094	N	Y
2011	March	1.900	1.514	Y	N
2011	April	1.935	3.531	N	Y
2011	May	0.416	0.331	N	N
2011	June	0.646	1.055	N	N
2011	July	0.738	1.886	Y	N
2011	August	0.653	-0.447	N	N
2011	September	1.551	1.617	N	N
2011	October	0.859	1.622	N	N
2011	November	1.202	1.972	Y	Y
2011	December	1.706	1.666	N	N
2012	January	2.099	1.326	Y	Y
2012	February	1.973	2.061	N	N
2012	March	1.088	1.529	N	N
2012	April	0.517	1.388	N	N
2012	May	0.585	0.841	N	N
2012	June	0.607	1.274	N	N

Second, the pattern of changes using the BLS seasonally adjusted series since the end of the 2008 recession shows a sequence of two to three months of significant changes in nonfarm employment from time to time. These have, in fact, repeatedly given rise to inferences of a persistent move in the cyclical component of the series. In the autocorrelation adjusted series there are also some significant changes in nonfarm employment, but they are then followed by an insignificant change. The April, May 2011 sequence, thought to indicate a strengthening of labor markets, appears to be indistinguishable from random fluctuations, and January, February 2012, while perhaps encouraging, is hardly different from previous movements of the series.

The univariate series solution can be implemented with some changes in code and fully automated for the tens of thousands of seasonally adjusted series published each year. The filtered series used to construct seasonal factors in the X-11 and X-12 routines will generate residuals once the seasonal component is calculated. By calculating the standard deviation of those residuals by period and publishing them along with the series, all those who use the series will be able to observe the statistical difference in volatility and use it accordingly.

6. A Method to Improve Forecast: Multivariate Series

The second solution is to directly incorporate additional observable variables which are related to the real-time seasonality into the seasonal adjustment process. The multivariate solution focuses on real-time short-term forecast rather than inference as discussed in previous sections. It is similar to the practice of using Vector Autoregressive models instead of ARMA models to improve the forecasting outcomes.

6.1 The Temperature and Seasonal Factor

In the presence of extreme weather, an obvious candidate for many economic time series would be temperature. Figure 4 shows the mean and standard deviation of monthly temperature in the U.S. Figure 5 displays the seasonal factors of nonfarm payroll from 1984 to 2012 calculated by the Census X-12 method. It seems that the seasonal pattern for Q1 (Winter: January, February, and March) and Q2 (Spring: April, May, and June) is more dramatic in Snow Belt states, such as Minnesota, Illinois, and Ohio, than in Sun Belt states, such as California, Texas, and Florida. We run a pooled OLS regression in which the dependent variables are the calculated monthly seasonal factors for each state and the independent variables are the corresponding average monthly temperature for each state and summer dummy (July and August) as follows:

$$\text{Seasonal Factor}_{\textit{state, m}} = \alpha + \beta_1 \text{Temperature}_{\textit{state, m}} + \beta_2 \text{ Summer Dummy} \quad (13)$$

Our results were that $\alpha = 0.982$, $\beta_1 = 0.0004$ (t-stat = 9.21), $\beta_2 = -0.0108$ (t-stat= - 8.57), and the adjusted $R^2 = 0.22$. The statistical significant estimation of β_1 demonstrates that monthly temperature would influence the seasonal pattern instantaneously. The estimated model can then be employed to adjust the seasonal factors for the differential in actual temperature from the averages employed in the estimation.

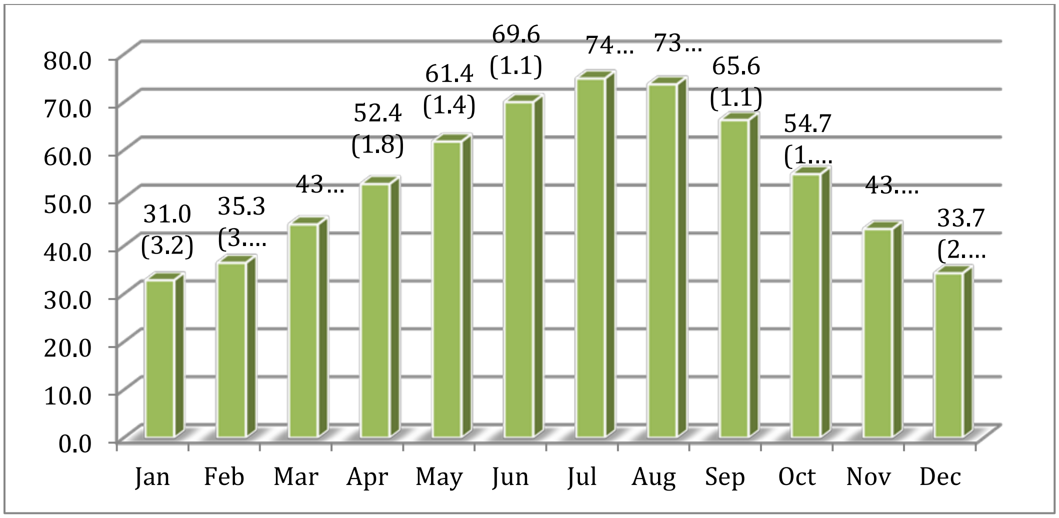

Figure 4: The Mean and Standard Deviation of Monthly Temperature in the U.S. (Contiguous 48 States, 1990-2011)
Source: National Climatic Data Center, standard deviations are in parenthesis.

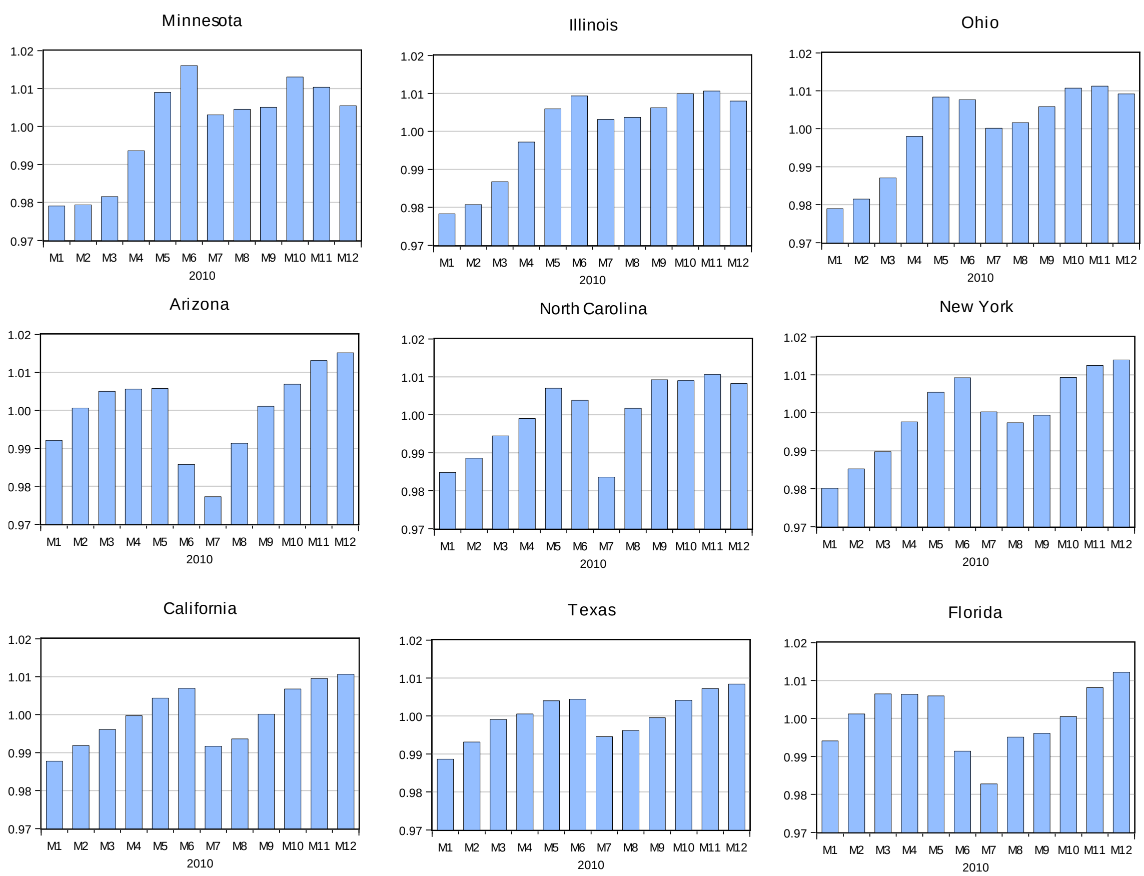

Figure 5: 2010 Monthly Seasonally Adjustment Factors for Nonfarm Payroll Employment in 9 States, 1984-2012, Based on X-12 Method from Census Bureau
Source: Data is from Bureau of Labor Statistics and factors are calculated by EViews' Census X-12 method.

6.2 A Temperature-Based Adjustment Method

The previous section proposes a simple way to adjust the seasonal factor in the presence of extreme weather so that the seasonally adjusted data would be less stochastically seasonal. The alternative way is to keep the estimated seasonal factor unchanged and to directly adjust and forecast non-seasonally adjusted economic data and then followed by the average seasonal adjustment. Here we suggest a simple data-based method with temperature adjustment on economic data. Using the nonfarm employment time series, we propose a hypothesis for the stochastic seasonality. Defining μ as the average employment growth in the data, the model—presumably based on additional knowledge of the relationship between the seasonal stimuli and the economic time series—is then:

(a) Normal winter: employment growth in Q1 is $\mu - s$, and Q2 is $\mu + ks$, while k and $s > 0$. Therefore, the seasonal difference of employment growth between Q2 and Q1 would be $(k+1)s$.
(b) Severe winter: employment growth in Q1 is $\mu - w_1 s$, and Q2 is $\mu + w_1 ks$, while k and $s > 0$ and $w_1 > 1$. Therefore, the difference of seasonal employment growth between Q2 and Q1 would be $w_1(k+1)s$. The severe winter discourages or delays many more economic activities and transactions, which will be realized in the spring. And this is why we have $w_1 > 1$.
(c) Warm winter: employment growth in Q1 is $\mu - w_2 s$, and Q2 is $\mu + w_2 ks$, while k and $s > 0$ and $w_2 < 1 < w_1$. Therefore, the difference of employment growth between Q2 and Q1 would be $w_2(k+1)s$.

Based on this hypothesis, we expect that the difference of employment growth from Q1 to Q2 will be larger in a severe winter while that in a warm winter will be smaller. Here we assume that summer and fall have no significant correlated seasonality. Equation (14) tests this hypothesis:

$$X_{it} = \alpha + \varphi X_{it\text{-}1} + \beta \text{Temdev}_{it} \tag{14}$$

where X is the difference between non-seasonal adjusted employment growth in Q1 (winter) and Q2 (spring) at state/sector i at year t from 1957 to 2011. Temdev is the winter temperature deviation from its historic winter temperature mean as partly shown in Figure 6. We suggest that φ, as an AR(1) model, would, by and large, capture the trend and cyclical components for an annual series[17]. If our hypothesis is correct, β would be significant and negative. As mentioned earlier, with a severe winter, *Temdev* would be negative and small, and X would be large. As a result, β would be negative. The results are displayed in Table 7, in which the upper panel shows the results with cross-state series and the lower panel shows the results with cross-sector series.

For the cross-state series, β is statistically significant (t-stat = -11.5) and negative (-0.0013) from the pooled OLS regression. If we decompose the coefficient for Q1 and Q2, respectively, β_1 is 0.0006 and β_2 is -0.0007. That said, with each degree higher than the average in winter temperature, we expect the employment growth in Q1/winter to go up by 0.0006%. However, this employment boost due to a warmer winter will face repercussions in Q2/spring, where employment growth will be expected to decline by 0.0007%. The state fixed effect exhibits similar estimations. For the cross-sector series, β is statistically significant (t-stat= -2.3) and negative (-0.0008) from the pooled OLS regression, although

[17] AR(2), AR(3), and AR(4) models are estimated and produced similar results at the national level. But at the individual state level, AR(1) is the most appropriate model, so we use the AR(1) model.

the magnitude is less than that of the cross-state series. And the sector fixed effect presents similar results.

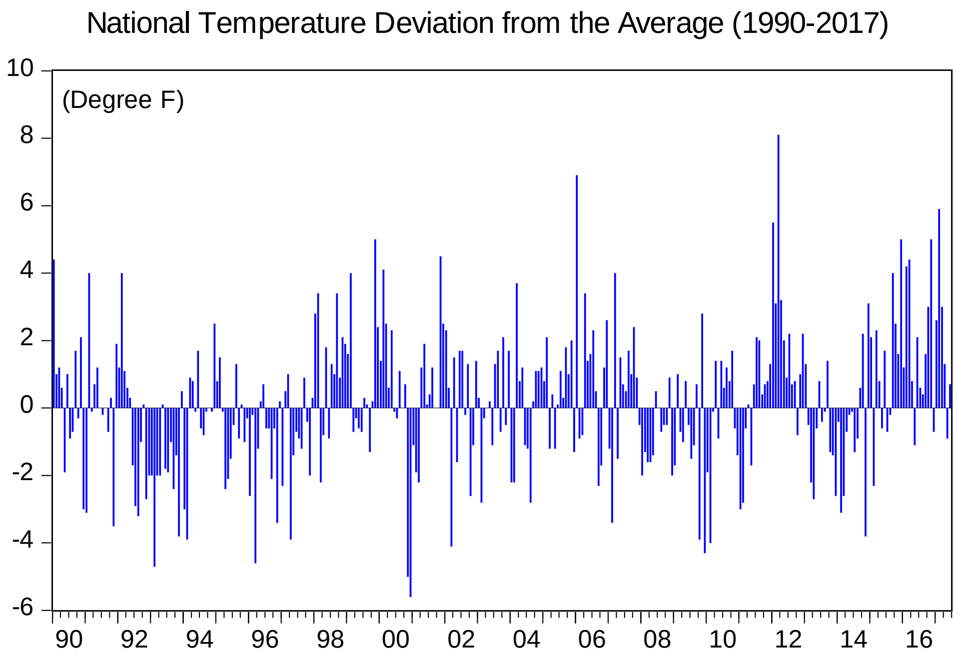

Figure 6: Monthly Temperature Deviation from the Average of the Month in the U.S. (Contiguous 48 States)
Source: National Climatic Data Center
Note: Average is based on the period of 1957 to 2011

If we run OLS for each state by (14), we could know which states are more sensitive in response to winter temperature abnormalities in terms of their winter and spring employment growth. The states with significant and negative β are shown in the shaded/yellow color in Figure 7. The significant states are mostly in the Midwest and the South. It is not surprising to see where in the West and in the Southeast the winter temperature aberration does not cause the winter/spring employment trade-off. The reason is because, for these relatively warmer-winter states, a big drop in winter temperature still produces a relatively mild winter with a smaller likelihood of snowstorms. Thus, we do not see a winter and spring employment growth tradeoff in these states. On the other hand, for states like Minnesota, Wisconsin, Michigan, and North Dakota with big positive or negative deviations in winter temperature, residents might still face more or less cold winters with snow and ice hindering economic activities. For instance, the average Q1 temperature in Minnesota from 1957 to 2012 is 16.5, and it was 27.2 in 2012. Even with an abnormally warm winter in 2012, it was still below freezing; therefore it will not affect economic behaviors. However, the average Q1 temperature in Iowa from 1957 to 2012 is 25.5, and it was 35.3 in 2012. The 10-degree higher winter, taking temperatures above freezing, will make a difference in economic behaviors.

If we run OLS for each sector by (14), three sectors present significant responses for their winter/spring employment tradeoff. They are construction, utilities, and information. For the construction and utilities sectors, the significant response is straightforward because these two sectors are directly affected by weather and temperature. For information, it is less clear. One possible reason might be that some sub-sectors, such us filming and other outdoor productions, are affected by weather as well.

Table 7: Estimations of Temperature-Adjusted AR(1) Model, 1957 to 2011

$X_{it} = \alpha + \varphi X_{it\text{-}1} + \beta \text{Temdev}_{it}$

	Cross States			
	Pooled OLS		State Fixed Effect	
	Coeff.	*t*-stat	Coeff.	*t*-stat
φ (AR(1))	0.88	103	0.72	53
β (Q2-Q1)	-0.0013	-11.5	-0.0014	-11.5
Observations	2592		2592	
Adjusted R^2	0.81		0.82	
φ (AR(1))	0.55	34.5	0.32	17.4
β (Q1)	0.0006	7.4	0.0006	7.5
Observations	2640		2640	
Adjusted R^2	0.32		0.41	
φ (AR(1))	0.86	87.8	0.67	47.4
β (Q2)	-0.0007	-5.9	-0.0008	-6.7
Observations	2592		2592	
Adjusted R^2	0.75		0.77	
	Cross Sectors			
	Pooled OLS		Sector Fixed Effect	
	Coeff.	*t*-stat	Coeff.	*t*-stat
φ (AR(1))	0.96	103	0.79	34.6
β (Q2-Q1)	-0.0008	-2.3	-0.0009	-2.5
Observations	734		734	
Adjusted R^2	0.93		0.94	
φ (AR(1))	0.85	47.4	0.55	18.5
β (Q1)	0.0005	2.1	0.0006	2.4
Observations	746		746	
Adjusted R^2	0.75		0.79	
φ (AR(1))	0.94	81.5	0.79	242
β (Q2)	-0.0003	-0.9	-0.0004	-1.1
Observations	734		734	
Adjusted R^2	0.90		0.91	

Note: X_{it} is the difference between non-seasonally-adjusted payroll employment growth in Q1 (winter) and Q2 (spring), where *i* denotes contiguous 48 states, and *t* denotes years.

6.3 In-sample and Out-of-sample Forecasting Performance

Using (14) and the estimated coefficients we obtained in Table 7, we calculate the in-sample mean squared errors (MSE: $(X\text{-}\hat{X})^2/N$, where X is the actual non-seasonally adjusted payroll growth difference between Q2 and Q1; $\hat{X}$ is the model projection) of the national payroll employment from 1957 to 2011 by two methods: (1) Equation (14)—AR(1) model with temperature adjustment, and (2) the AR(1) model. For cross-states pooled OLS model, MSE is 0.03932 for the temperature adjustment model (Equation (14)) while MSE is

0.04178 without the temperature adjustment model. The MSE is computed based on one-year-ahead rolling forecast.

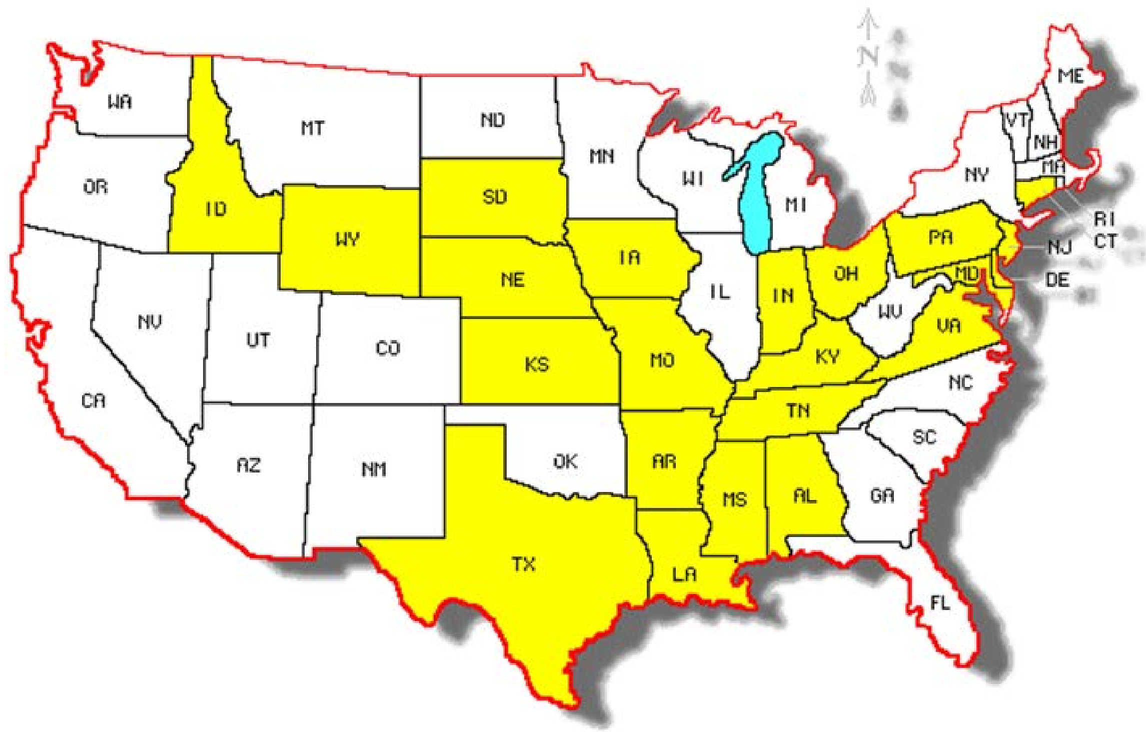

Figure 7: State’s Winter and Spring Employment that Responds Significantly with Winter Temperature Abnormality

Next, we use the in-sample estimates (1957-2011) from Table 7 to forecast out-of-sample estimates (2012-2017) by these two methods as shown in Table 8. The actual x is 8.5%, 10.1%, 11.1%, 11.1%, 10.2%, 10.2% from 2012 (Q2-Q1) to 2017 (Q2-Q1). The out-of-sample forecasts by the AR(1) model with temperature adjustment is 9.3%, 9.8%, 11.8%, 11.6%, 10.6%, and 9.7% with MSE of 0.00003, while forecasts by the AR(1) model are 11.5%, 9.6%, 11.0%, 11.9%, 11.9%, and 11.2% with MSE of 0.00022. Apparently, the forecasting performances, both in-sample and out-of-sample, from the model with temperature adjustment are better than the model without.

Table 8: Out-of-sample Forecast Mean Square Errors 2012 to 2017

	Actual X_t	Winter Temperature Deviation (Temdev_t)	Forecast Without Temperature Adjustment	Forecast With Temperature Adjustment
Forecast Model Based on the Estimators from 1957-2011			$X_t = 0.0219 + 0.879X_{t-1}$	$X_t = 0.0212 + 0.883X_{t-1} - 0.0013\times\text{Temdev}_t$
2012 Q2-Q1	8.5%	16.7	11.5%	9.3%
2013 Q2-Q1	10.1%	-1.5	9.6%	9.8%
2014 Q2-Q1	11.1%	-6.2	11.0%	11.8%
2015 Q2-Q1	11.1%	2.1	11.9%	11.6%
2016 Q2-Q1	10.2%	9.8	11.9%	10.6%
2017 Q2-Q1	10.2%	11.5	11.2%	9.7%
Out-of-sample Mean Square Errors (MSE)			0.0002296	0.0000343

Note: X_{it} is the difference between non-seasonally-adjusted payroll employment growth in Q1 (winter) and Q2 (spring), where *i* denotes contiguous 48 states, and *t* denotes years.

7. Conclusions

The purpose of this paper is to rekindle the debate on how one infers cyclical change from seasonally adjusted economic data. The past three years have generated ample evidence of inferences in the presence of extreme winters. There are three contributions of the paper. First, the autocorrelation correction suggested herein is an improvement to current practice. Second, the direct modeling of the behavioral responses is also suggested, and the improvement of the forecasting outcomes is presented. Third, the evidence from states and sectors validates our hypothesis that says that, given a certain range and industries, economic agents respond accordingly to the winter temperature.

What should be clear is that economic agents are not seasonal by themselves. Their seasonal behavior is a response to an external seasonal event or shock. If that event and the response are deterministic, then the answer for economists is to hit the Census X-12 button on the computer. But, more likely, the response to seasonal events will depend on the magnitude of the event. The more important the behavioral response, the more one has to worry about treating a non-stationary seasonally contaminated time series as a stationary de-seasonalized one. With regard to casual inference, the publication of the monthly standard deviation for each seasonally adjusted series would go a long way towards providing a standard measure by which contemporaneous analysis can be conducted. This does not require any new, sophisticated seasonal adjustment methodology, but simply a quantification of the period specific variances.

With regard to research employing seasonally adjusted time series, the solution to the problem comes from understanding the nature of the seasonal event as it affects individual behavior and why one is performing a seasonal adjustment transformation of the data in the first place[18]. If the seasonal event is a random stimulus with a fixed proportionate response function, then explicitly including seasonal data, such as the winter temperature deviation from its long-term mean, will solve the problem at least partly. If the seasonality cannot be modeled explicitly, then estimating the monthly variances and implicitly or explicitly incorporating them into inference with respect to the series is indicated. Finally, the question of what to do with series that are dominated by behavioral responses to random seasonal events when employing them in regression analysis is more complicated[19] and depends on how those seasonal events impact both the endogenous and exogenous variables.

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[18] This is an observation David Pierce made at the beginning of his (1979) article.
[19] See Sims (1974) and Wallis (1974) for a more complete discussion.

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