The Lights of Mathile 212

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Abstract

The Mathile Center for the Natural Sciences (named for Clayton L. Mathile, an alumnus and benefactor of Ohio Northern University) houses the Department of Mathematics and Statistics at ONU. As Associate Professor of Statistics, I have lectured for many courses in room Mathile 212. Like other faculty and students using this room, I have found the lighting system to be very confusing, as getting the desired lighting in the room often requires several seconds of fidgeting with numerous light switches. After six and one-half years of frustration, I decided it was time to figure out how it all works.

Key Words: Lighting system, Fidgeting, Frustration

1. The Configuration of Lights and Light Switches in Mathile 212

Mathile 212 is a classroom with dimensions of roughly thirty-three feet (south to north) by twenty-four feet (front to back). It contains thirty-eight desks and a table for the instructor at the front of the room – it is crowded, and I would prefer having only thirty desks.

One can enter Mathile 212 at the front of the room, from either a south door or a north door. Between the two doors are two sets of light switches (south and north), separated by the blackboard at the front of the room.

There are four rows of lights, which we shall label as R1, R2, R3, and R4 (from front to back).

There are a total of nine light switches, five by the south door, four by the north door. We shall label these (from south to north) as {S1, S2, S3, S4, S5} (south) and {S6, S7, S8, S9} (north).

2. Easily Obtained First Results

It was relatively easy, with a bit of trial-and-error fidgeting with the light switches, to obtain the following results:

- S5 controls R1. When the switch is up, the lights are on. When the switch is down, the lights are off. Thus, these first row lights, located immediately above the front blackboard, have only two “brightness levels”: “off” and “on”. It should be noted that R1 is thus controlled by just one light switch, S5, located on the south side.

- Lights in R2, R3, and R4 all have three "brightness levels": "off", "dim", and "bright". Figuring out how the light switches control the brightness levels was the greatest challenge of this endeavor.
- S3 & S4 control R2 on the south side. S6 & S7 control R2 on the north side. Having switches on both sides of the room controlling R2 allows one to control these lights upon entering from either side, south or north. Having pairs of switches on each side allows one to control the "brightness level" at three levels.
- The lights in R3 & R4 are always at the same brightness level as each other, and are controlled by the same switches. We shall henceforth refer to these two rows of lights together as "R3&4".
- S1 & S2 control R3&4 on the south side. S8 & S9 control R3&4 on the north side. Thus, in the same manner as with R2, we have two pairs of switches (one pair on the south, the other pair on the north) controlling the lights in R3&4,

3. The Control of Brightness Levels

3.1 Initial thoughts and observations

Arbitrarily I chose to initially focus my attention on how light switches {S1, S2, S8, S9} control the brightness level of the R3&4 lights – naturally anticipating that {S3, S4, S6, S7} would control the brightness level of R2 in a similar manner.

Not being the brightest bulb in the socket, it took me longer than I care to admit to recognize that the key to figuring out how the light switches control the brightness levels was to put together the following table:

S1	S2	S8	S9	Brightness Level
U	U	U	U	bright
U	U	U	D	dim
U	U	D	U	dim
U	D	U	U	dim
D	U	U	U	dim
U	U	D	D	off
U	D	U	D	off
U	D	D	U	bright
D	U	U	D	bright
D	U	D	U	off
D	D	U	U	off
U	D	D	D	dim
D	U	D	D	dim
D	D	U	D	dim
D	D	D	U	dim
D	D	D	D	bright

Working entirely on my own, constructing this table required walking back and forth (from south to north to south to north, *etc.*), switching switches up ("U") and down ("D") – many more times than would have been needed if I had organized my trip itinerary more efficiently. It should be noted that there are $2^4 = 16$ lines in the table, one for each possible combination of four dichotomous variables.

Examination of the table led me to the following thinking. The variable "Brightness Level", which can be viewed as the "response variable" for this situation, is an "ordered categorical variable", with the three brightness levels having a natural ordering, but not representing actual quantities. I wondered whether or not "Brightness Level" could alternatively be interpreted as a "discrete quantitative variable", perhaps taking on the values 0 for "off", 1 for "dim", and 2 for "bright".

The four switches, {S1, S2, S8, S9}, can be viewed as dichotomous ("U" or "D") "predictor variables". I recognized (it's a point I emphasize to all of my students, even in first semester introductory courses) that these could be viewed alternatively as "quantitative", with (arbitrarily) value "1" representing "U" and "0" representing "D".

From examination of the table, it didn't take long to realize that the effects of the switches (*i.e.* the dichotomous predictors) are not "simply additive" (*i.e.* the brightness level is not simply a function of how many switches are up or down), and that some kind of an interaction among the effects is occurring.

3.2 Cutting to the chase

At this point I will explain what my conclusions are regarding how the light switches determine the (quantitative) "Brightness Levels". It is not possible for me to recall the exact (or even approximate) sequence of questions and ideas and thoughts that led me to these conclusions.

The key realization is that the brightness level which one perceives is determined by two "light tubes", each of which can be turned on or off. If neither "light tube" is turned on, then $Y = 0$ and the lights appear "off". If only 1 "light tube" is turned on, then $Y = 1$ and the lights appear "dim". If both "light tubes" are turned on, then $Y = 2$ and the lights appear "bright".

It should be noted that the lights in Mathile 212 are covered by opaque plastic, so that the individual light tubes cannot be seen. Thus, I inferred their existence based on my analysis of the brightness level patterns, rather than through direct observation or any prior knowledge about how lights work. (Note: There are lights in various locations of my building, such as my office, where the actual light tubes can be seen – but I never really looked at or thought about them much, until long after I had figured out how things worked mathematically! Again, not the brightest bulb in the socket.)

The determination of whether a particular light tube is turned on or off is based on the positions of the light switches. Let us now refer to the two light tubes as "LT #1" and "LT #2". The status of a particular light tube ("on" or "off") is determined by two of the four light switches {S1, S2, S8, S9}), one on the south side, one on the north side.

Arbitrarily, we can assume that S1 (south) and S9 (north) determine the status of LT #1 and that S2 (south) and S8 (north) determine the status of LT #2. In particular, when S1 and S9 are both up or both down, LT #1 is turned on. When one is up and the other down, LT #1 is turned off. Similarly, when S2 and S8 are both up or both down, LT #2 is turned on, and when one is up and the other down, LT #2 is turned off.

3.3 A summary of the results

We let Y denote the brightness level of R3&4, taking on the values of 0 (for "off"), 1 (for "dim"), and 2 (for "bright").

We may now express Y as the sum of two dichotomous variables, X_1 and X_2, each taking on the values 1 and 0:

$$Y = X_1 + X_2, \text{ with } X_1 = 1 \text{ or } 0 \quad \& \quad X_2 = 1 \text{ or } 0.$$

Here X_1 denotes the status of LT #1 and X_2 denotes the status of LT #2.

The brightness level Y is thus determined as in the following table:

X_1	X_2	Y	**Brightness**
0	0	0	off
0	1	1	dim
1	0	1	dim
1	1	2	bright

Determination of the values of X_1 and X_2

$X_1 = 1$ if (S1 → U & S9 → U) or (S1 → D & S9 → D)

0 if (S1 → U & S9 → D) or (S1 → D & S9 → U)

$X_2 = 1$ if (S2 → U & S8 → U) or (S2 → D & S8 → D)

0 if (S2 → U & S8 → D) or (S2 → D & S8 → U)

3.4 Table summarizing the process for R3&4 lights

S1	S9	X_1	S2	S8	X_2	Y	Brightness Level
U	U	1	U	U	1	2	bright
U	D	0	U	U	1	1	dim
U	U	1	U	D	0	1	dim
U	U	1	D	U	0	1	dim
D	U	0	U	U	1	1	dim
U	D	0	U	D	0	0	off
U	D	0	D	U	0	0	off
U	U	1	D	D	1	2	bright
D	D	1	U	U	1	2	bright
D	U	0	U	D	0	0	off
D	U	0	D	U	0	0	off
U	D	0	D	D	1	1	dim
D	D	1	U	D	0	1	dim
D	D	1	D	U	0	1	dim
D	U	0	D	D	1	1	dim
D	D	1	D	D	1	2	bright

3.5 Results for R2 lights

As stated earlier, I naturally expected that the determination of the brightness levels of the R2 lights by {S3, S4, S6, S7} would work in the exact same manner as the determination of the brightness levels of the R3&4 lights by {S1, S2, S8, S9}. This is, in fact, the case. In fact, the table given immediately above can be repeated entirely, only with the column labels "S1, S2, S8, S9" replaced by "S3, S4, S6, S7". Virtually all other results can also be given by a simple replacement of appropriate numbers.

4. The Next Day (Surprise!)

When I came back to Mathile 212 the next day I was astonished and dismayed to find that the results which I had obtained previously no longer fully applied! In particular, although S1 and S9 determined whether LT #1 was turned on or off in the "same manner" as was observed the previous day, S2 and S8 now determined whether LT #2 was turned on or off in what can only be described as an "opposite manner".

Specifically, LT #2 was now turned off when the S2 and S8 switches were set in the same direction (UU or DD), and were turned on when set in the opposite directions (UD or DU). How could this possibly happen, I asked? Why me?

With some additional thought, and based on my years of experience teaching in this room, I figured out what was going on! The lights in Mathile 212 (and other classrooms) have an energy saving feature which causes them to turn off when the room is not being used (which is determined by a motion detector). If someone (or their dog) enters the room shortly after a row of lights (R2 or R3&4 – R1 works slightly differently) have gone out,

the motion detector senses this and the row of lights go back on. However, if a longer period of inactivity goes by, the lighting system resets itself, so that when someone now enters the room, the lights in the row will not automatically come back on – they will only come on when one or more switches are flipped. Thus, the orientation of paired light switches at the time the lighting system resets itself determines whether {UU or DD} or {UD or DU} turns the associated light tube on.

For example, when the lighting system last reset itself before I returned the next day, the S1 and S9 switches must have been in an opposite orientation, *i.e.* UD or DU, so that this combined orientation caused LT #1 to be turned off – and therefore UU or DD would cause LT #1 to be turned on. In contrast, the S2 and S8 switches must have had the same orientation when the system reset itself, *i.e.* UU or DD, so that this combined orientation caused LT #2 to be turned off – and therefore UD or DU would cause LT #2 to be turned on. Get it?

5. Final Comments

A colleague of mine at Ohio Northern University questioned whether this topic was really appropriate for a presentation at the 2016 Joint Statistical Meetings, asking "Is this statistics"? I think it is. Milo Schield (Augsburg College), who attended my poster presentation, agrees.

References

I ain't got no stinkin' references.