Dynamic Modeling of Factor Risks in Multi-strategy Hedge Fund Investment Portfolios

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Abstract
This paper proposes a Bayesian modeling framework to estimate the time-varying market factor risks of hedge fund investment portfolios–by studying hedge fund strategy indices. The explanatory variables are major market index and factor returns. Several dynamic models are proposed and compared. Time-varying model parameters, namely factor sensitivities and excess returns, are modeled as a "random walk plus noise" process. Investment industry experience provided guidance for selecting relevant factors in regressions models and setting prior (ex-ante) parameter values; Markov Chain Monte Carlo simulations generated posterior (ex-post) probability distributions to facilitate parameter (point and interval) estimates. Results from quantile regressions are also briefly discussed and compared with the results by dynamic regression models. The models presented here can be easily applied to individual hedge fund analysis and a diversified portfolio that incorporates hedge fund investments.

Key Words: Hedge Fund, Factor Risk, Bayesian Model, Dynamic Model, Markov Chain Monte Carlo

1. Introduction

Hedge funds (HFs) are privately placed investment vehicles that are free to invest in a wide range of securities and derivatives, are typically actively managed, take both long and short positions, use leverage to enhance return potential or asymmetry, and may participate in multiple regional markets. Hedge funds are only available to accredited investors and qualified purchasers.

Large pension funds, endowments, foundations, and financial institutions are the primary investors in hedge funds. Many investors have the ability to directly invest in hedge funds; while many others access HFs via fund-of-hedge-funds (FoHFs) to benefit from diversifications and lower volatilities. It's important to understand the underlying market factor risks of hedge funds in a multi-asset portfolio, or a multi-strategy hedge fund investment portfolio. The total investments in the hedge fund industry is estimated to be around $2.9 trillions in Q1 2016.

Return-based multi-factor modeling techniques have become very popular in the past two decades since Fama and French (1993) proposed a multi-factor regression framework to evaluate stock and bond risks. Fung and Hsieh (2004) populated return-based hedge fund style evaluation. Most papers on hedge fund analysis focused on longer term average

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factor risks and used data before the financial crisis of 2008. Since hedge funds are known to actively manage positioning, it would be useful to evaluate time-varying factor risks of hedge funds and compare the behaviors before and after 2008.

This paper attempts to estimate *dynamic* factor sensitivities and excess returns of major hedge fund strategies focusing on using the Bayesian method. The explanatory variables are major market index and factor returns. Several dynamic models are proposed and compared below. Investment experience provided guidance in selecting relevant factors in the regression models.

1.1 Hedge Fund Data

There are several hedge fund strategy classification systems, either asset weighted or equal weighted. The most popular asset-weighted HF indices are by Credit Suisse (CS). The major hedge fund strategies are listed below. Monthly CS index returns are available from www.hedgeindex.com.

- *Long/Short Equity (LSE), Equity Market Neutral (EMN), Dedicated Short (DS)*
- *Fixed Income Arbitrage (FIArb), Convertible Arbitrage (CVArb)*
- *Event Driven Multi-strategy (ED), Event Driven Distressed (Distr), Event Driven Risk Arbitrage (RiskArb)*
- *Global Macro (GM), Managed Futures (MF)*
- *Emerging Markets (EM)*
- *Multi-Strategy (MS)*

Another major HF index provider is Hedge Fund Research (HFR) Inc., which has four major HF strategy indices and more than ten sub-strategy indices. HFR publishes mostly equal weighted HFRI data, available by subscription only. The major strategies are Equity Hedged (EH), Even Driven (ED), Relative Value (RV), and Global Macro (GM). Asset-weighted HFRI major strategy indices were released in Q1 2016. Lipper/TASS is another vendor that provides equal weighted HF strategy indices.

Asset weighted strategy index returns are dominated by larger firms; while equal weighted index returns are dominated by small firms due to the sheer number of smaller firms. The actual returns of a multi-strategy hedge fund investment portfolio is likely to fall between those of asset-weighted and equal weighted indices.

This paper focuses on Credit Suisse HF strategy index data from Jan. 2006 to Mar. 2016, which includes several interesting sub-periods: (1) High volatility and financial crisis: 01/2008 to 02/2009. (2) European debt concerns: 04/2011 to 09/2011. (3) Growing equity markets: 01/2012 to 03/2014. (4) Federal Reserve "QE taper tantrum": 04/2013 to 12/2013. (5) Increased volatility: 08/2015 to 02/2016.

1.2 Market Factors

The market indices and factors used in this study are described below. Whenever possible, freely downloadable index and index ETF data from FRED (Federal Reserve Economic Research) and Yahoo websites are used. Monthly returns in excess of risk-free rates (of US 3-month Treasury Bills) are used in all regression models in this study.

1. MSCI World Equity: The primary risk factor of most directional hedge funds
2. MSCI EAFE Equity – S&P 500: International developed market equity bias
3. MSCI Emerging Market Equity – MSCI EAFE: Emerging market equity bias
4. Russell 2000 – Russell 1000: Small Cap – Large Cap (Small Cap bias)

5. Russell 1000 Growth – Russell 1000 Value: Growth – Value (Growth bias)
6. US 3-month Treasury Bill rates: Proxy of risk-free rates
7. US 10-year Treasury Notes: Proxy of sovereign note/bond total returns
8. Merrill Lynch Investment Grade (IG) Corporate Bond Index
9. Merrill Lynch High Yield Index – IG: High Yield (HY) credit bias
10. Merrill Lynch Distressed Index – IG: Distressed (DISTR) credit bias
11. Commodities: Average of import and export commodity prices
12. Trade weighted USD index (broad based)
13. Momentum: Multi-asset momentum indicator (developed by the author) consisted of four market segments: equities, sovereign, currency, and commodities

2. Modeling Dynamic Factor Risks

2.1 Rolling Window Classic Regressions

In the financial services industry, it is common to use rolling-window linear regressions to estimate the *time-varying* factor sensitivities (*betas*) and the excess return (*alpha*) of a return series vs. a set of benchmarks or factors. Typically, a window size of 18, 24, or 36 months is used. In the equation below, **Y** is the response (independent) variable; **X** is the explanatory (dependent) variable.

$$\mathbf{Y}(\ w(t)\ ) = \mathbf{X}(\ w(t)\ )^{T} \bullet \boldsymbol{\beta}(t) + \alpha(t), \text{ where } t = 1, ..., T$$
$$\text{and } w(t) = \{t-(m-1), t-(m-2), \ldots, t-1, t\}\ , m = \text{window size}$$

Rolling regressions are straightforward to implement, but the factor sensitivity shows a lagged response. To illustrate the point, the author used a single-factor 18-month rolling window regression model to estimate the equity beta of CS Long/Short Equity strategy index. Figure 1 shows the time-varying excess return and equity beta over time from 01/2006 to 03/2016. The first 18-month of data was used to run a static regression to estimate the initial values of beta and alpha. Note that the beta over time shows a dynamic effect, but the lowest point probably should have occurred in early 2009 instead of 2010 as shown in Figure 1 (right panel)–since Feb. 2009 marks the bottom of many equity markets during the financial crisis. Also, alpha should have been more negative in late 2008 instead of a small dip (left panel). Another issue with rolling regressions is the beta seems to jump around too much at times, e.g. from 2013 to 2014.

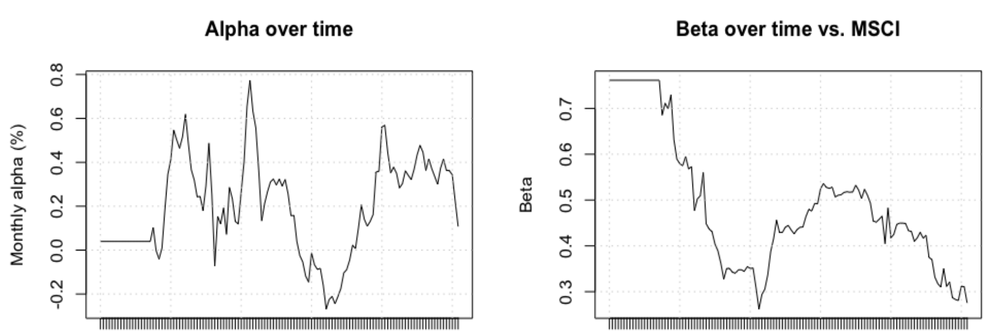

Figure 1: Alpha and beta over time by rolling window regressions: CS Long/Short Equity vs. MSCI World Equity

2.2 Dynamic Linear Model

The *dynamic linear model* (DLM), represented in the *state-space* form, is a powerful tool for modeling time-varying factor sensitivities. This approach originated in engineering research and applications in 1960s (West and Harrison, 1997). Our focus here is the *local level model*, also known as *"random walk plus noise"* model, shown below.

Observation equation: $Y(t) = \mathbf{X}(t)^{\mathbf{T}} \bullet \boldsymbol{\theta}(t) + v(t)$, where $\mathbf{X}(t)^{\mathbf{T}} = [1 \;\; x_1(t) \;\; x_2(t) \dots x_p(t)]$
State equation: $\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t\text{-}1) + \mathbf{w(t)}$, where $\boldsymbol{\theta}(t)^{\mathbf{T}} = [\alpha(t) \;\; \beta_1(t) \;\; \beta_2(t) \dots \beta_p(t)]$
and $v(t) \sim N(0, V)$, $\mathbf{w}(t) \sim N(0, \mathbf{W})$

Note that t is the time index; p is the total number of factors; **Y** is the response (independent) variable; **X** is the explanatory (dependent) variable; **θ** contains *unobserved* model parameters including alpha and beta(s); v and **w** are unknown errors modeled as Normal distributions with zero means. For simplicity, V and **W** are modeled as time invariant here; **W** is modeled as a diagonal matrix. Alternatively, **W** could be modeled as a full covariance matrix with an inverse-Wishart distribution.

The state equation, also called transition or evolution equation, for the model parameters **θ** has the form of a random walk. The observation equation, or called measurement equation, has the form of a linear regression. The state equation allows alpha and beta(s) to evolve over time while the **W** matrix controls the jump/drift size in each time iteration.

Initial values of model parameters are required in a DLM. Industry knowledge is used to provide guidance in setting ex-ante values. The very efficient Kalman filtering algorithm is used to estimate *online/real-time* posterior model parameters; Kalman smoothing is used to perform backward smoothing/correction. A popular R package called *dlm* is used by this study (Petris et al. 2009). A recent paper (Cai and Liang, 2010) also used the DLM approach to evaluate factor risks and excess returns of Lipper/TASS equal weighted HF strategy indices though with different factor sets and the data time period stopped at 12/2008.

2.2.1 Long/Short Equity

To contrast the model parameter estimates by the DLM with those by rolling regressions, the same CS Long/Short Equity strategy index vs. MSCI World Equity index are used. The (Kalman) filtered but un-smoothed mean alpha and beta values are shown in Figure 2. The major difference in the results is the beta curve is smoother and the lowest estimated beta occurred in early 2009, consistent with industry experience since Feb. 2009 marks the bottom of many equity markets during the financial crisis. The estimated alpha also had negative values in 2008, again consistent with experience. One shortcoming of the DLM is that posterior model parameter estimates are somewhat sensitive to initial (ex-ante) values. A static regression in the beginning may help provide better estimates of the initial parameter values.

2.2.2 Multi-Strategy

The author also applied the DLM method to evaluate CS Multi-Strategy HF index. The explanatory variables are MSCI World Equity index, Growth – Value, and Commodities. The (Kalman) filtered and smoothed posterior model parameters are shown in Figure 3. It's notable that this strategy has *positive average alpha* from 2009 to 2014. Many other HF strategies do not have this nice characteristic. The strategy index had moderate (0.2) equity beta but it greatly decreased after 2012; it also had higher growth bias leading to

2009 and it has been declining afterward; it also has a moderate and consistent sensitivity to commodities.

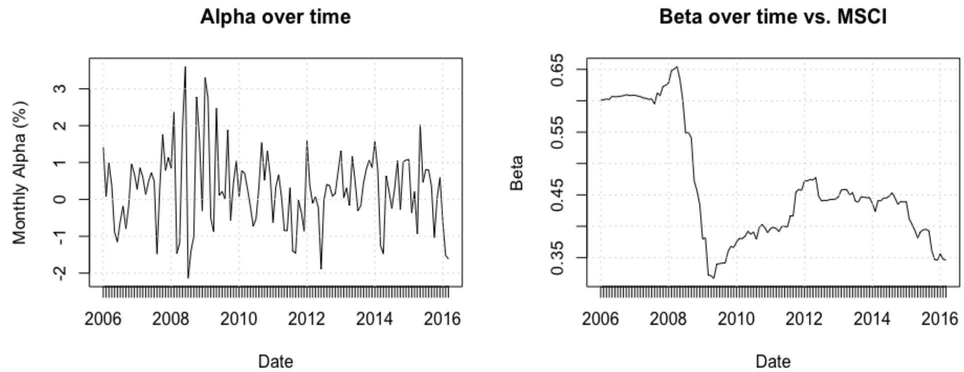

Figure 2: Alpha and beta over time by DLM: CS Long/Short Equity vs. MSCI World Equity

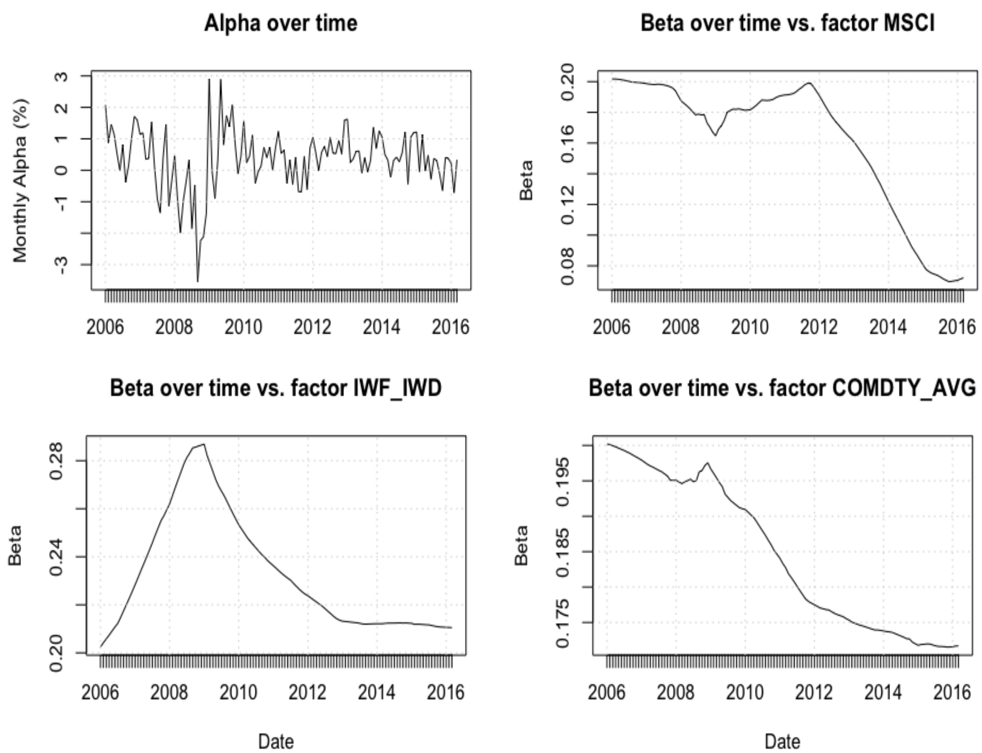

Figure 3: Alpha and betas over time by DLM: CS Multi-Strategy vs. factors

2.3 Dynamic Bayesian Regression Model

Though the DLM method is efficient, the Bayesian regression framework provides more flexibility in controlling model parameters. Essentially, the Bayesian paradigm considers all model parameters as random variables with probability distributions, not just unknown values. The author applied Bayesian regressions to study average long-term factor risks of hedge fund strategies and excess returns in (Chang 2014). This paper extends the study to time-varying factor risks over a 10-year period.

The dynamic Bayesian regression model is described as follows. (1) The first 18-month of data is used to run a static Bayesian regression to provide initial model parameter estimates. (2) Subsequently the model parameters follow the "random walk plus noise" model. All model parameter priors use diffuse (non-informative) distributions. For example, the prior of equity beta is a uniform distribution between -1 and 1; the prior of alpha is a Normal distribution with zero mean; all "precision" (inverse of variance) parameters use diffuse Gamma distributions. The *deviance information criterion* (DIC) is the indicator of how well the model fits the underlying *unknown* dynamic process, the lower the better. The posterior distributions are generated using Markov Chain Monte Carlo (MCMC) simulations, done by the JAGS (Just Another Gibbs Sampler) package and open-source R.

2.3.1 Long/Short Equity (LSE)
The CS LSE strategy is regressed vs. MSCI World Equity index. In Figure 4, the *median* model parameters (alpha and beta) and 75% Bayesian *credible intervals* are plotted. Similar to the results of the DLM method in Section 2.2.1, the equity beta is dynamic and hit the lowest point in early 2009, increased from 2009 until 2012, and continued declining until early 2016—consistent with industry experience. The beta curve is smoother than that by Kalman filtering but not so smooth as the result by Kalman smoothing. It's notable that the 75% credible region is pretty much away from zero, indicating the persistent equity beta of the LSE strategy. The Bayesian regression model for the LSE strategy has a (good) DIC value of -766.

The alpha values are negative in late 2008, consistent with experience. The alpha by this model is less jumpy compared to that of the DLM so is potentially be more useful in real-world applications such as investment risk evaluation and asset allocation decision making. The LSE strategy appears to provide positive alpha in risk-seeking market environments (e.g. 2009 and 2013) and negative or unreliable alpha in volatile market conditions (e.g. 2008 and late 2015).

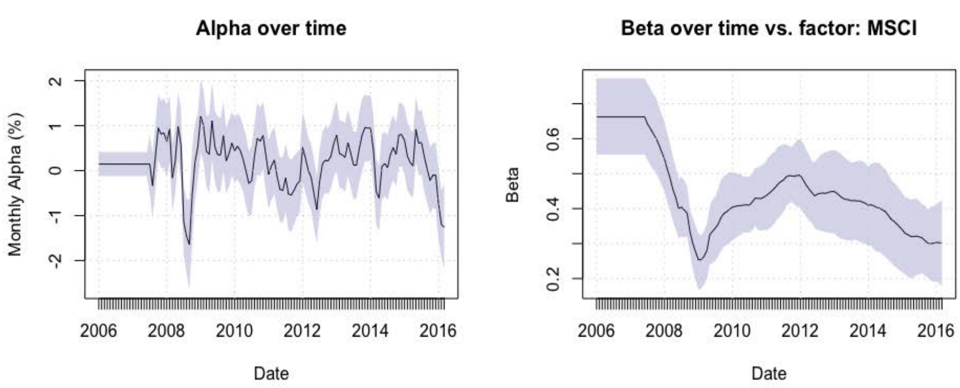

Figure 4: Alpha and beta over time by dynamic Bayesian regression: CS Long/Short Equity vs. MSCI World Equity

The drawback of the dynamic Bayesian regression method is that MCMC simulations are computationally intensive and do not allow online/real-time type of estimates offered by

Kalman filtering. Once new data points become available, MCMC simulations must be performed for the entire data period. Nonetheless, since Bayesian modeling is our focus in the study, more dynamic Bayesian regression results on other Credit Suisse HF strategies are presented in the next few subsections.

2.3.2 Fixed Income Arbitrage (FIArb)

Fixed Income Arbitrage hedge funds employ fundamental research to identify relative value opportunities in fixed income securities including corporate bonds, bank loans, securitized mortgages, asset-backed securities, and other structured products. Since these credit HFs are typically risk-seeking, the model uses Merrill Lynch HY index as the explanatory variable. Alternatively, the relative return of (HY—IG) could be used as the factor. The Bayesian regression model for the FIArb strategy has a (good) DIC value of around -779. The median alpha, beta, and 75% credible intervals are plotted in Figure 5.

The strategy has a consistent beta to lower credit-rating securities. The beta was high leading up to the financial crisis, dropped quickly in 2008, and hovered around 0.2 since 2009. The alpha was negative before 2009, stayed mostly positive until around 2013, but has been in decline since then.

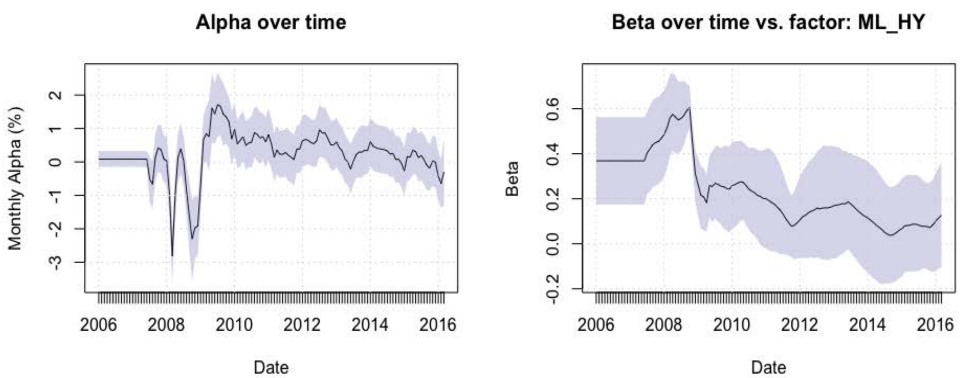

Figure 5: Alpha and beta over time by dynamic Bayesian regression: CS Fixed Income Arbitrage vs. factors

2.3.3 Managed Futures(MF)

Hedge funds classified as Managed Futures typically use trend-following systematic models to dynamically determine long and short positions in futures or forward contracts —spanning equity, sovereign rate/note/bond, currency, and commodity markets. Such a hedge fund usually constructs trading signals of various time horizons ranging from short term (1-week/1-month) to long term (12-month). Momentum and reversal indicators are often used together to determine aggregate position sizing. The explanatory variable for the MF strategy is a composite momentum indicator, developed by the author, composed of indices from the four above-mentioned market segments. The indicator is a weighted average of lagged 1-month, 3-month, 6-month, and 12-month momentum strengths. The Bayesian regression model for the MF strategy has a DIC value of around -534. As shown in Figure 6, the MF strategy index has a clear beta to momentum. The beta hit the bottom in early 2009 and has been increasing since then except a dip in the summer of 2001 when the European debt issue surfaced.

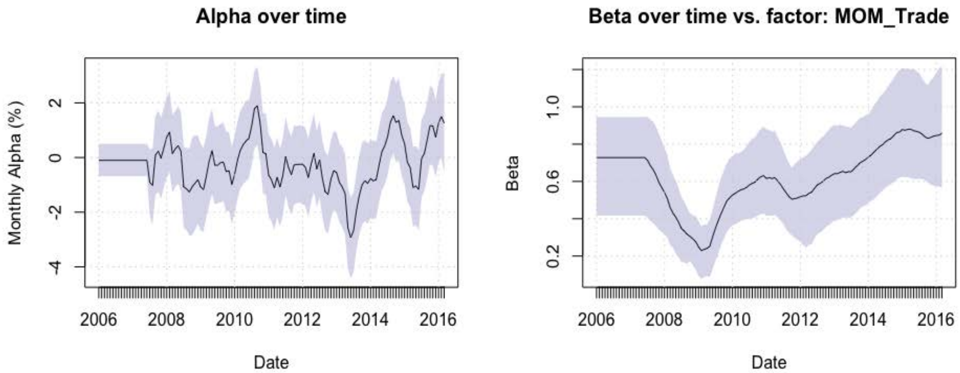

Figure 6: Alpha and beta over time by dynamic Bayesian regression: CS Managed Futures vs. factors

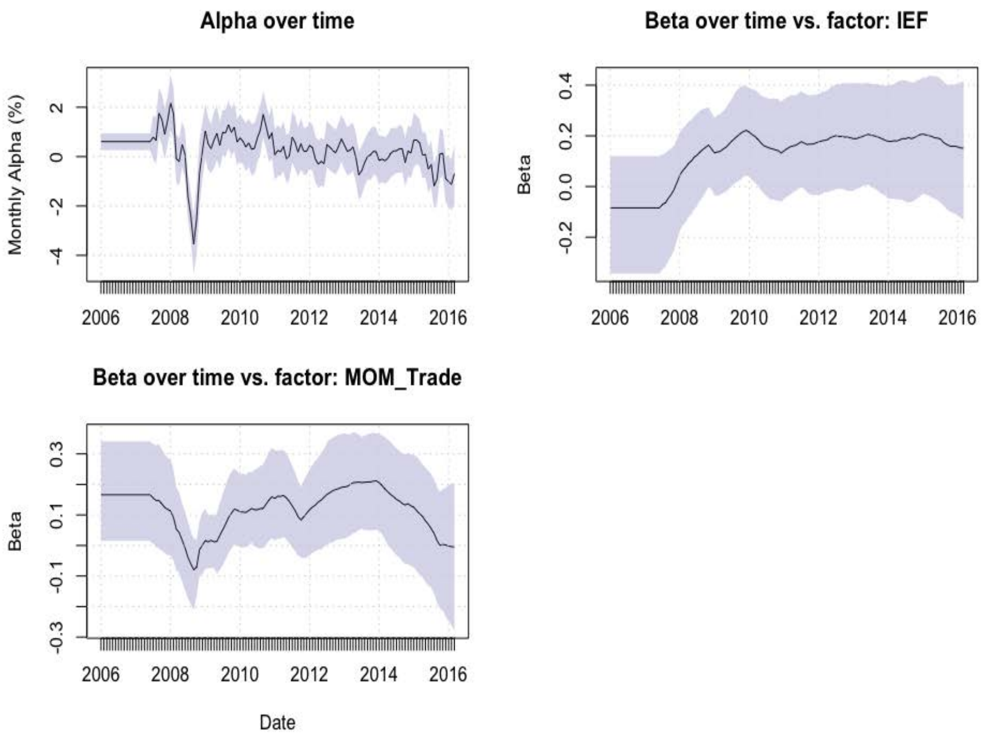

Figure 7: Alpha and betas over time by dynamic Bayesian regression: CS Global Macro vs. factors

2.3.4 Global Macro (GM)

Global Macro hedge funds rely on fundamental research to identify prevailing or forecast upcoming macro themes. Trades are usually constructed by discretionary portfolio managers. In addition to the momentum indicator, US 7-to-10-year Treasury Note total return ETF is added to the model as a proxy of sovereign bonds. The Bayesian regression model for the GM strategy has a DIC value of around -682. The median strategy model parameters are plotted in Figure 7. It has a modest but consistent factor beta to sovereign bonds. However, the beta to momentum is not as clear though slightly positive in most time periods until 2014, with the exception of 2008. The alpha shows a declining trend, reflecting the difficulties for discretionary macro traders to profit from their views in recent years.

2.3.5 Event Driven Distressed (DISTR)

Event Driven Distressed hedge funds strive to find undervalued credit instruments usually after a market dislocation, industry-wide credit downgrade, or firm re-structuring. These securities tend to be illiquid so are usually held for years. A good example is the Lehman bankruptcy in 2008 that led to many distressed securities that provided good returns or high intrinsic yields for several years since 2009.

Figure 8 shows the median alpha, beta, and 75% credible intervals of CS Distressed strategy. The strategy has positive beta to distressed securities in all time periods since 2006. On the other hand, the alpha is not reliable and appears cyclical: positive during risk-seeking years (2009 and 2013) and negative in volatile markets (2008, 2011, 2015).

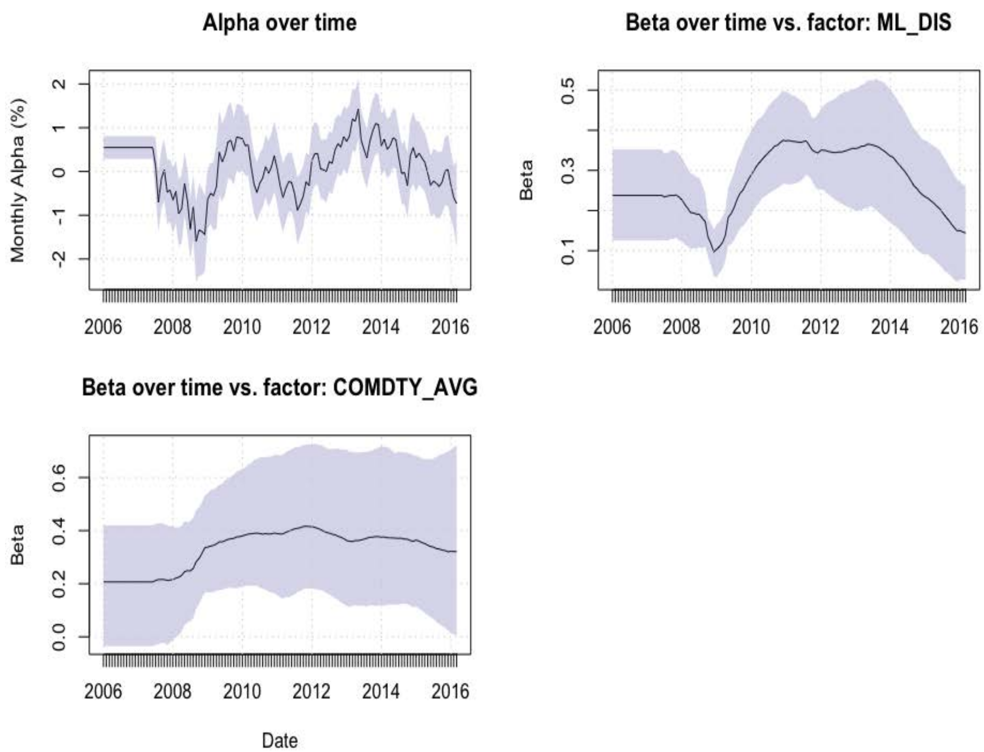

Figure 8: Alpha and betas over time by dynamic Bayesian regression: CS Event Driven Distressed vs. factors

2.3.6 Multi-Strategy (MS)

To estimate dynamic factor risks of the CS Multi-Strategy HF index, the author tested explanatory variables such as MSCI World Equity index, (Growth – Value), (Small – Big), (International Equity – Domestic Equity), (DM Equity – EM Equity), (HY – IG), Momentum, and Commodities. The strategy appears to have consistent factor betas to MSCI World Equity and commodities. In Figure 9, the median equity beta is moderate but consistent until 2014 when the 75% credible region covers zero. In Figure 10 (right panel), the median beta to commodities is also moderate and steady, but becomes unreliable after 2014 as the credible region is wide enough to include zero.

In Figure 10 (left panel), the median beta to "growth bias" (Growth – Value) is positive but becomes less reliable after 2013 due to the widening credible region.

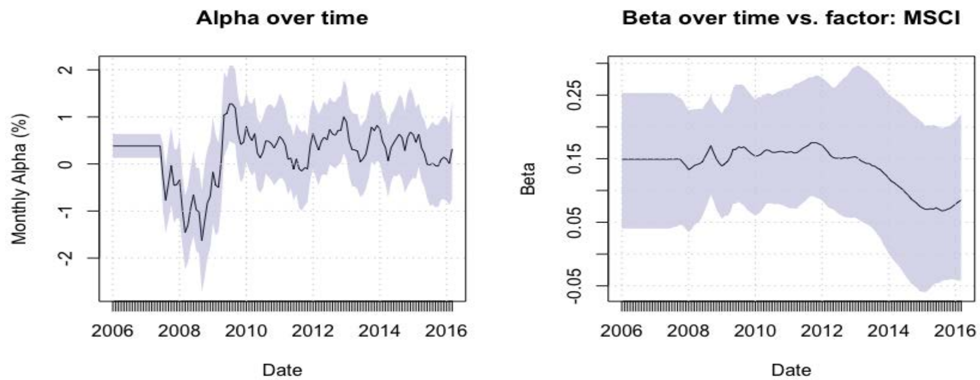

Figure 9: Alpha and beta over time by dynamic Bayesian regression: CS Multi-Strategy vs. MSCI World Equity

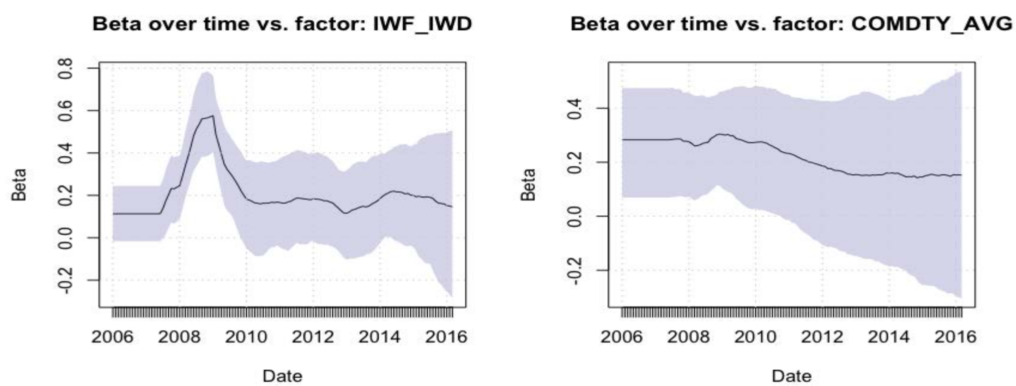

Figure 10: Betas over time by dynamic Bayesian regression: CS Multi-Strategy vs. Growth Bias and Commodities

2.3.7 Other HF Strategies

The author also ran dynamic Bayesian regressions on CS Convertible Arbitrage strategy index. The strategy shows consistent factor beta vs. (HY—IG) factor in the past 10 years with a dip in 2008. The graphs are not shown here to save space.

The most difficult strategy to model is Equity Market Neutral (EMN). The author included several "spread" factors such as (Growth – Value), (Small – Big), and relative sector returns. However, there appears no factor with a consistent 75% beta credible region away from zero. The alpha is also unreliable: for example, positive in 2013 and 2015; negative in 2008 and 2014. Obviously, more research is required in this area.

2.4 Bayesian Quantile Regression

The quantile regression method finds *conditional* relationships between response and explanatory variables by regressions at several data quantiles. The seminal QR method is proposed by Koenker and Bassett (1978). The recent Bayesian quantile regression (BQR) method was proposed by Yu and Moyeed (2001). In a paper by Meligkotsidou et al. (2009), quantile regressions were used to study long-term HF factor risks from 1990 to 2005 using the equal weighted HFRI indices. The author simplified the complex dynamic quantile regression analysis by using rolling-window Bayesian quantile regressions focusing on five quantiles: 10%, 25%, 50%, 75%, and 95%. The 10% and the 95% quantiles represent *tail* behaviors. The R package called BayesQR was used for analysis.

BQR results on CS Long/Short Equity strategy index during four 2-year periods are presented: (1) 2008 to 2009; (2) 2010 to 2011; (3) 2012 to 2013; (4) 04/2014 to 03/2016. The first two time periods represent market conditions more volatile than the latter two. The quantile alphas and betas are plotted in Figure 11 through Figure 14.

Figures 11 and 12 showed that quantile alphas and betas are negatively correlated during more volatile periods, implying worse alpha with higher beta—likely due to heightened factor correlations during stressed market conditions. By contrast, quantile alphas and betas in Figure 13 and Figure 14 are positively correlated across quantiles during more risk-seeking periods, which implies more alpha with more factor risk.

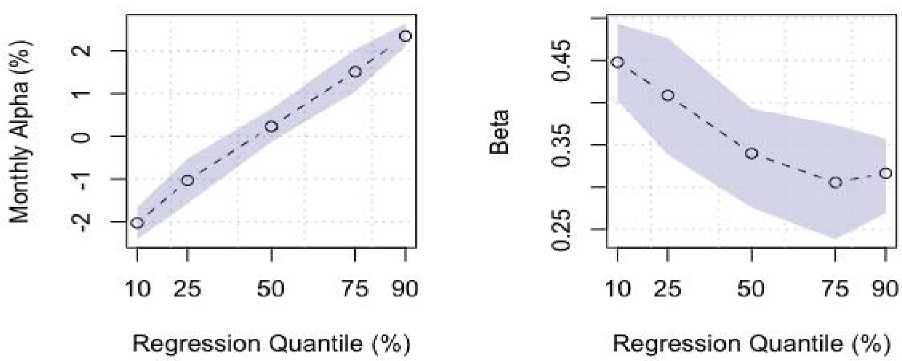

Figure 11: Alpha and beta using Bayesian quantile regressions: CS Long/Short Equity vs. MSCI World Equity from 01/2008 to 12/2009

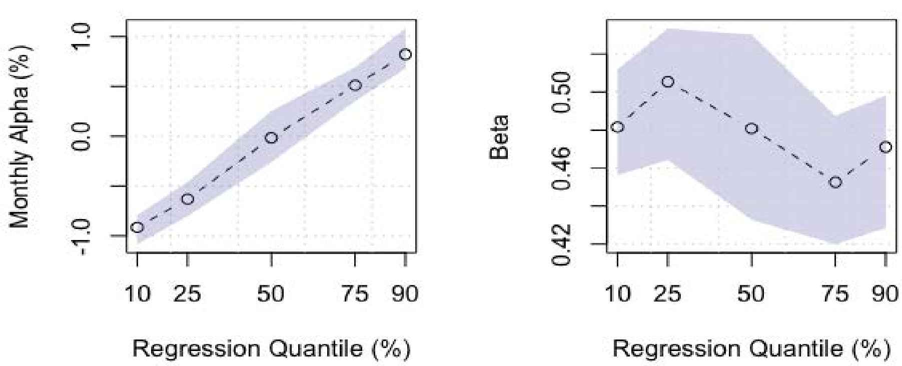

Figure 12: Alpha and beta using Bayesian quantile regressions: CS Long/Short Equity vs. MSCI World Equity from 01/2010 to 12/2011

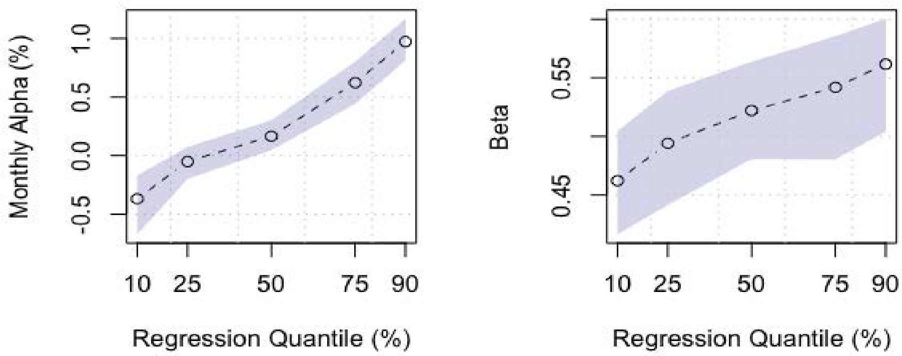

Figure 13: Alpha and beta using Bayesian quantile regressions: CS Long/Short Equity vs. MSCI World Equity from 01/2012 to 12/2013

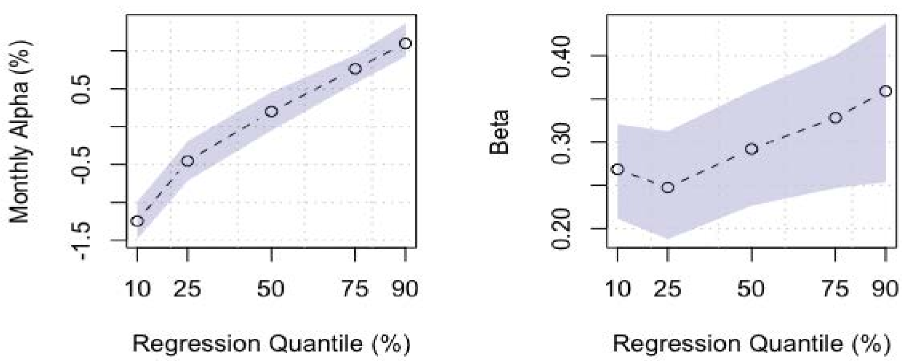

Figure 14: Alpha and beta using Bayesian quantile regressions: CS Long/Short Equity vs. MSCI World Equity from 04/2015 to 03/2016

3. Concluding Remarks

This paper has demonstrated that slowly time-varying regression models are able to capture dynamic factor risks of hedge funds. Most hedge fund strategies show dynamic market factor sensitivities. Industry knowledge is useful in guiding market factor selections. This is also true when the technique is applied to individual hedge fund factor analysis. Both Bayesian dynamic regression models and dynamic linear models are quite capable of modeling dynamic factor risks. The DLM approach is very efficient and provides good model parameter estimates. The Bayesian regression method is more flexible and provides robust results which are also intuitive according to industry experience. The Bayesian quantile regression method helps reveal more risk behaviors outside of average conditions but requires more effort to extract insights from the regression results.

Though the dynamic models studied here can be applied to individual hedge fund analysis, the natural next step is to extend the research to dynamic *hierarchical* Bayesian regression models with partial parameter shrinkage to incorporate multiple fund returns while estimating factor risks in each strategy group. Another area of interest is to study the time-varying properties of variance/covariance coefficients of model parameters.

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