A New Set of Asymmetric Filters For Real Time Trend-Cycle Estimation

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Abstract
For assessing in real time the short-term trend of major economic indicators, official statistical agencies generally rely on asymmetric filters that were developed by Musgrave in 1964. However, the use of the latter introduces revisions as new observations are added to the series and, from a policymaking viewpoint, they are too slow in detecting true turning points. In this paper, we use a reproducing kernel methodology to derive asymmetric filters that converge quickly and monotonically to the corresponding symmetric one. We show theoretically that proposed criteria for time-varying bandwidth selection produce real-time trend-cycle filters to be preferred to the Musgrave filters from the viewpoint of revisions and time delay to detect true turning points. We use a set of leading, coincident and lagging indicators of the US economy to illustrate the potential gains statistical agencies could have by also using our methods in their practice.

Key Words: Recession and recovery analysis, reproducing kernels, seasonally adjusted data, Musgrave filters, time-varying bandwidth selection, US economy.

1. Introduction

The basic approach to the analysis of current economic conditions, known as recession and recovery analysis, is that of assessing the real time trend-cycle of major socio-economic indicators (*leading, coincident and lagging*) using percentage changes, based on seasonally adjusted units, calculated for months and quarters in chronological sequence. The main goal is to evaluate the behaviour of the economic indicators during incomplete phases by comparing current contractions or expansions with corresponding phases in the past. This is done by measuring changes of single time series (mostly seasonally adjusted) from their standing at cyclical turning points with past changes over a series of increasing spans. This differs from business-cycle studies where cyclical fluctuations are measured around a long term trend to estimate complete business-cycles. The real time trend corresponds to an incomplete business-cycle and is strongly related to what is currently happening on the business-cycle stage.

In recent years, statistical agencies have shown an interest in providing trend-cycle or smoothed seasonally adjusted graphs to facilitate recession and recovery analysis. Among other reasons, this interest originated from the recent crisis and major economic and financial changes of global nature which have introduced more variability in the data. The United States entered in recession in December 2007 till June 2009, and this has produced a chain reaction all over the world, particularly, in Europe. There are no evidence of a fast recovery as in previous recessionS: the economic growth is sluggish and with high levels of unemployment. It has become difficult to determine the direction of the short term trend (or trend-cycle) as traditionally done by looking at month to month (quarter to quarter) changes of seasonally adjusted values, particularly to assess the upcoming of a true turning point. Failure in providing reliable real time trend-cycle estimates could give rise to dangerous drift of the adopted policies. Therefore, a consistent prediction is of fundamental importance. It can be done by means of either univariate parametric models or nonparametric techniques. The majority of the statistical agencies use nonparametric seasonally

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adjusted software, such as the Census X11 method and its variants X11/X12ARIMA and X13, and hence this PAPER deals with the real time trend-cycle estimation produced by the Musgrave filters (Musgrave, 1964) available in these software.

As widely discussed the linear filter developed by Henderson (1916) is the most frequently applied and has the property that fitted to exact cubic functions will reproduce their values, and fitted to stochastic cubic polynomials it will give smoother results than those estimated by ordinary least squares. Its Reproducing Kernel Hilbert Space (RKHS) representation consists OF a kernel function obtained as the product of the biweight density function and the sum of its orthonormal polynomials that is particularly suitable when the length of the filter is rather short, say between 5 to 23 terms, which are those often applied by statistical agencies (see also for details Dagum and Bianconcini, 2008).

At the beginning and end of the sample period, the Henderson filter of length, say $2m + 1$ cannot be applied to the m data points, hence only asymmetric filters can be used. The estimates of the real time trend are then subject to revisions produced by the innovations brought by the new data entering in the estimation and the time-varying nature of the asymmetric filters, in the sense of being different for each of the m data points. The asymmetric filters applied to the first and last m observations associated with the Henderson filter were developed by Musgrave (1964) on the basis of minimising the mean squared revision between the final estimates, obtained with the symmetric Henderson weights, and preliminary estimates from the asymmetric weights, subject to the constraint that the sum of these weights is equal to one. The assumption made is that at the end of the series, the seasonally adjusted values do not follow a cubic polynomial, but a linear trend-cycle plus a purely random irregular. Dagum and Bianconcini (2008 and 2013) were the first to introduce a RKHS representation of them.

The RKHS approach presented here is strictly nonparametric and based on a fundamental result due to Berlinet (1993), according to which a kernel estimator of order p can be always decomposed into the product of a reproducing kernel R_{p-1}, belonging to the space of polynomials of degree at most $p - 1$, and a density function f_0 with finite moments up to order $2p$. Given the density function, once the length of the symmetric filter is chosen, let us say, $2m + 1$, the statistical properties of the asymmetric filters are strongly affected by the bandwidth parameter of the kernel function from which the weights are derived. We present time-varying bandwidth parameters because the asymmetric filters are time-varying. Three specific criteria of bandwidth selection are chosen based on the minimisation of (1) the distance between the transfer functions of asymmetric and symmetric filters, (2) the distance between the gain functions of asymmetric and symmetric filters, and (3) the phase shift function over the domain of the signal.

We deal only with the reduction of revisions due to filter changes that depends on how close the asymmetric filters are respect to the symmetric one (Dagum and Laniel, 1987; Dagum, 1996) and do not consider those introduced by the innovations in the new data. Another important aspect dealt with is the capability of the asymmetric filters to signal the upcoming of a true turning point which depends on the time delay for its identification. This is obtained by calculating the number of months (quarters) it takes for the last trend-cycle estimate to signal a true turning point in the same position of the final trend-cycle data. An optimal asymmetric filter should have a time path that converges fast and monotonically to the final estimate as new observations are added to the series.

2. Asymmetric filters and RKHS

Let $\{y_t, t = 1, 2, ..., N\}$ denote the input series, supposed to be seasonally adjusted where trading day variations. Moving holidays and extreme values, if present, have been removed.

It is assumed that it can be decomposed into the sum of a systematic component (signal) g_t, representing the trend-cycle (usually estimated jointly) plus an erratic component u_t (noise), such that

$$y_t = g_t + u_t. \tag{1}$$

The noise u_t is assumed to be white noise, $WN(0, \sigma_u^2)$, or, more generally, a stationary and invertible AutoRegressive Moving Average (ARMA) process. The signal $g_t, t = 1, \cdots, T$, is assumed to be a smooth function of time, such that it can be represented *locally* by a polynomial of degree p in a variable j, which measures the distance between y_t and its neighboring observations $y_{t+j}, j = -m, ..., m$. This is equivalent to estimate the trend-cycle $\hat{g}_t$ as a weighted moving average as follows

$$\hat{g}_t = \sum_{j=-m}^{m} w_j y_{t+j} = \mathbf{w}'\mathbf{y} \qquad t = m+1, \cdots, N-m, \tag{2}$$

where $\mathbf{w}' = \begin{bmatrix} w_{-m} & \cdots & w_0 & \cdots & w_m \end{bmatrix}$ contains the weights to be applied to the input data $\mathbf{y}' = \begin{bmatrix} y_{t-m} & \cdots & y_t & \cdots & y_{t+m} \end{bmatrix}$ to get the estimate $\hat{g}_t$ for each point in time.

The derivation of the symmetric Henderson filter with the RKHS methodology assumes the availability of $2m + 1$ input values centered at t. However, at the end of the sample period, that is $t = N - (m+1), \cdots, N$, only $2m, \cdots, m+1$ observations are available, and asymmetric filters of the same length have to be considered. Hence, at the boundary, the effective domain of the kernel function , say K_4 is $[-1, q^*]$, with $q^* << 1$, instead of $[-1, 1]$ as for any interior point. This implies that the symmetry of the kernel is lost, and it does not integrate to unity on the asymmetric support ($\int_{-1}^{q^*} K_4(t)dt \neq 1$). Furthermore, the moment conditions are not longer satisfied, that is $\int_{-1}^{q*} t^i K_4(t)dt \neq 0$ for $i = 1, 2, 3$. To overcome these limitations, several boundary kernels have been proposed in the literature. In the context of real time trend-cycle estimation, the condition that the kernel function integrates to unity is essential, whereas the unbiasedness property can only be satisfied with a great increase in the variance of the estimates. This is a consequence of the well-known trade-off between bias and variance. This latter becomes very large because most of the contribution to the real time trend-cycle estimates comes from the current observation which gets the largest weight. Based on these considerations, Dagum and Bianconcini (2008, 2013 and 2015) followed the so called "cut and normalize" method (Gasser and Muller, 1979; Kyung-Joon and Schucany, 1998), according to which the boundary kernels $K_4^{q^*}$ are obtained by cutting the symmetric kernel K_4 to omit that part of the function lying between q^* and 1, and by normalizing it on [-1,q^*]. That is,

$$K_4^{q^*}(t) = \frac{K_4(t)}{\int_{-1}^{q^*} K_4(t)dt} = \frac{\det(\mathbf{H}_4^0[1, \mathbf{t}])f_{0B}(t)}{\det(\mathbf{H}_4^0[1, \boldsymbol{\mu}^{q*}])} \qquad t \in [-1, q^*], \tag{3}$$

where $\boldsymbol{\mu}^{q*} = \begin{bmatrix} \mu_0^{q*} & \mu_1^{q*} & \mu_2^{q*} & \mu_3^{q*} \end{bmatrix}$ with $\mu_r^{q*} = \int_{-1}^{q*} t^r f_{0B}(t)dt$ being proportional to the moments of the truncated biweight density f_{0B} on the support $[-1, q^*]$, which from now on we simply refer to as truncated moments.

Applied to real data, the "cut and normalize" method gives the following formula for the asymmetric weights

$$w_{q,j} = \frac{K_4^{q^*}(j/b_q)}{\sum_{j=-m}^{q} K_4^{q^*}(j/b_q)} = \frac{\det(\mathbf{H}_4^0[1, \mathbf{j}/\mathbf{b_q}])(1/b_q)f_{0B}(j/b_q)}{\det(\mathbf{H}_a)} \tag{4}$$

$$j = -m, \cdots, q; q = 0, \cdots, m-1$$

where $\mathbf{H}_4^0[1, \mathbf{j}/\mathbf{b_q}]$ is the Hankel matrix whose elements are the moments of f_{0B}, and where the first column has been substituted by the vector $\mathbf{j}/\mathbf{b_q}' = \begin{bmatrix} 1 & (j/b_q) & (j/b_q)^2 & (j/b_q)^3 \end{bmatrix}$. On the other hand, $\mathbf{H}_a = \mathbf{H}_4^0[1, \mathbf{S}^q]$ with $\mathbf{S}^q = \begin{bmatrix} S_0^q & S_1^q & S_2^q & S_3^q \end{bmatrix}'$, being $S_r^q = \sum_{j=-m}^{q} \frac{1}{b_q}\left(\frac{j}{b_q}\right)^r f_{0B}\left(\frac{j}{b_q}\right)$ the discrete approximation of μ_r^{q*}. Finally, $b_q, q = 0, \cdots, m-1$, is the local bandwidth, specific for each asymmetric filter. It allows to relate the discrete domain of the filter, that is $\{-m, \cdots, q\}$, for each $q = 0, \cdots, m-1$, to the continuous domain of the kernel function, that is $[-1, q^*]$. It can be shown (see Dagum and Bianconcini, 2015 and 2016) that the generic element of $\mathbf{w}_q$ is

$$w_{q,j} = \left[\frac{\mu_4 - \mu_2\left(\frac{j}{b_q}\right)^2}{S_0^q\mu_4 - S_2^q\mu_2}\right]\frac{1}{b_q} f_{0B}\left(\frac{j}{b_q}\right) \tag{5}$$

$$j = -m, \cdots, q; q = 0, \cdots, m-1.$$

2.1 Properties of the asymmetric filters

Since the trend-cycle estimates for the last m data points do not use $2m+1$ observations as for any interior point, but $2m, 2m-1, \cdots, m+1$ data, they are subject to revisions due to new observations entering in the estimation and filters change. In the specific case of the RKHS filters the asymmetric filter weights are related to the symmetric ones and their convergence depends on the relationship between the two discretized biweight density functions, truncated and non-truncated, jointly with the relationship between their respective truncated S_r^q and untruncated S_r discrete moments. The latter provide an approximation of the continuous moments μ_r, which improves as the asymmetric filter length increases. Similarly, the convergence of $S_r^q, q = 0, \cdots, m$, to the corresponding non-truncated moment S_r depends on the length of the asymmetric filter given by q, and on the local bandwidth b_q. It should be noticed that b_q plays a very important role in the convergence property. For the last trend-cycle point weight, $q = 0$, eq. (5) reduces to

$$w_{0,0} = \frac{\mu_4}{S_0^0\mu_4 - S_2^0\mu_2}\frac{15}{16b_0}.$$

It is apparent that the largest b_0 the smaller is the weight given to the last trend-cycle point. Since the sum of all the weights of the last point asymmetric filter, $w_{0,-m}, \cdots, w_{0,0}$, must be equal to one, this implies that the weights for the remaining points must be small and close to one another.

3. Optimal bandwidth selection

The main effects introduced by a linear filter on a given input can be fully described in the frequency domain by its transfer function

$$\Gamma(\omega) = \sum_{j=-m}^{m} w_j \exp(-i2\pi\omega j) \qquad \omega \in [-1/2, 1/2],$$

where, for better interpretation, the frequencies ω are given in cycles for unit of time instead of radians. $\Gamma(\omega)$ represents the Fourier transform of the filter weights, $w_j, j = -m, \cdots, m$, and it relates the spectral density $h_y(\omega)$ and $h_g(\omega)$ of the input and of the output, respectively, by

$$h_g(\omega) = \Gamma(\omega)h_y(\omega).$$

Thus, the transfer function $\Gamma(\omega)$ measures the effect of the filter on the total variance of the input at different frequencies. It is generally expressed in polar coordinates

$$\Gamma(\omega) = G(\omega)\exp(-i2\pi\phi(\omega)) \tag{6}$$

such that the impact of the filter on a particular (complex-valued) series $y_t = \exp(i2\pi\omega t), \omega \in [-1/2, 1/2]$, is given by

$$\hat{g}_t \quad = \quad G(\omega)\exp[i2\pi(\omega t - \phi(\omega))].$$

$G(\omega) = |\Gamma(\omega)|$ is called the gain of the filter and measures the amplitude of the output for a sinusoidal input of unit amplitude, whereas $\phi(\omega)$ is called the phase function and shows the shift in phase of the output compared with the input. Hence, the transfer function plays a fundamental role to measure that part of the total revisions due to filters change.

The quantity $|\Gamma_q(\omega) - \Gamma(\omega)|^2$ is a measure of the revisions due to filters change (Dagum, 1982) and it can be decomposed using the law of cosines as follows:

$$|\Gamma_q(\omega) - \Gamma(\omega)|^2 \quad = \quad |G_q(\omega) - G(\omega)|^2 + 4G_q(\omega)G(\omega)\sin\left(\phi_q\left(\frac{\omega}{2}\right)\right)^2 \tag{7}$$

where the phase shift for the symmetric filter is equal to 0 or $\pm\pi$, and where $1-\cos(\phi_q(\omega)) = 2\sin\left(\phi_q\left(\frac{\omega}{2}\right)\right)^2$. Based on eq. (7), the mean square filter revision error can be expressed as follows

$$2\int_0^{1/2} |\Gamma_q(\omega) - \Gamma(\omega)|^2 d\omega = 2\int_0^{1/2} |G_q(\omega) - G(\omega)|^2 d\omega + 8\int_0^{1/2} G_q(\omega)G(\omega)\sin\left(\phi\left(\frac{\omega}{2}\right)\right)^2 d\omega \tag{8}$$

The first component reflects the part of the total mean square filter error which is attributed to the amplitude function of the asymmetric filter. On the other hand, the second term measures the distinctive contribution of the phase shift. The term $G_q(\omega)G(\omega)$ is a scaling factor which accounts for the fact that the phase function is dimensionless, *i.e.* it does not convey level information (Wildi, 2008).

Once the length of the filter is fixed, the properties of the asymmetric filters derived in RKHS are strongly affected by the choice of the time-varying local bandwidths $b_q, q = 0, \cdots, m-1$. A filter is said to be optimal if it minimises both revisions and time delay to detect a true turning point. The LHS of eq. (8) is a measure of total filter revision that provides the best compromise between the amplitude function of the asymmetric filter (gain) and its phase function (time displacement) (Dagum, 1982; Dagum and Laniel, 1987) . Optimal asymmetric filters in this sense can be derived using local bandwidth parameters selected according to the following criterion

$$b_{q,\Gamma} = \min_{b_q}\sqrt{2\int_0^{1/2} |\Gamma_q(\omega) - \Gamma(\omega)|^2 d\omega}. \tag{9}$$

Based on the decomposition of the total filter revision error provided in eq. (8), further bandwidth selection criteria can be defined by emphasising more the gain or phase shift effects, and/or by attaching varying importance to the different frequency components, depending on whether they appear in the spectrum of the input time series or not. In the context of smoothing a monthly input, the frequency domain $\Omega = \{0 \le \omega \le 0.50\}$ can be partitioned in two main intervals: (1) $\Omega_S = \{0 \le \omega \le 0.06\}$ associated with cycles of 16 months or longer attributed to the signal (trend-cycle) of the series, and (2) $\bar{\Omega}_S = \{0.06 < \omega \le 0.50\}$ corresponding to short cyclical fluctuations attributed to the noise.

A class of optimal asymmetric filters based on bandwidth parameters $b_q, q = 0, \cdots, m-1$ is selected as follows

$$b_{q,G} = \min_{b_q} \sqrt{2 \int_0^{1/2} |G_q(\omega) - G(\omega)|^2 d\omega}. \tag{10}$$

and

$$b_{q,\phi} = \min_{b_q} \sqrt{2 \int_{\Omega_S} G_q(\omega) G(\omega) \left[1 - \cos(\phi_q(\omega))\right]}. \tag{11}$$

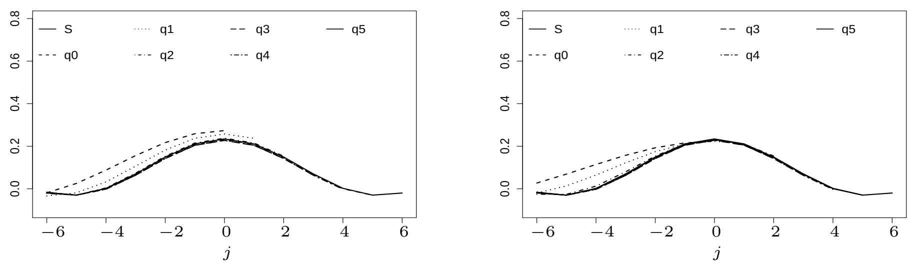
It has to be noticed that the minimisation of the phase error in eq. (11) is very close to minimising the average phase shift in month for the signal, that is

$$b_{q,\phi} = \min_{b_q} \left[\frac{1}{0.06} \int_{\Omega_S} \frac{\phi(\omega)}{2\pi\omega} d\omega \right]. \tag{12}$$

Dagum and Bianconcini (2015) showed that, as q approaches m, the bandwidth parameters selected to optimise the criteria (9) and (10) get closer to $m+1$, which is the global bandwidth considered for the symmetric Henderson filter. Hence, based on the relationships between truncated and untruncated discrete biweight density functions and respective discrete moments previously discussed, the asymmetric filters based on $b_{q,\Gamma}$ and $b_{q,G}$, $q = 0, \cdots, m-1$, should be characterised by a fast convergence to the symmetric filter. This is confirmed by Figure 1 that illustrates, as an example, the time path of these filters corresponding to the 13-term symmetric one. Similar conclusions can be drawn for different filter lengths.

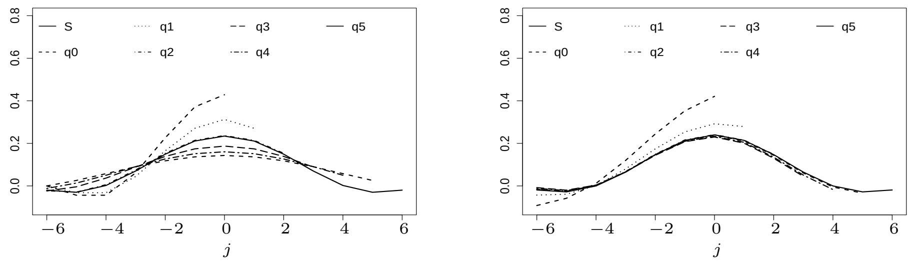
The asymmetric filters based on $b_{q,\Gamma}$ and $b_{q,G}$, $q = 0, \cdots, m-1$, converge very fast to the symmetric filter, particularly after the previous to the last point, with the main differences observed for the last point filters. For these latter, the different behavior is analyzed in the frequency domain in Figure 3, that shows the corresponding gain and phase shift functions. It can be noticed that, as expected, the filter whose bandwidth $b_{0,G}$ is derived as minimizer of eq. (10) shows a gain function closer to that of the symmetric Henderson filter than the one based on $b_{0,\Gamma}$, suppressing more noise at the highest frequencies, and it reproduces very well the signal in the lower frequency band.

Figure 1: Time path of the asymmetric filters based on $b_{q,\Gamma}$ (left), $b_{q,G}$ (right) corresponding to the 13-term symmetric filter.



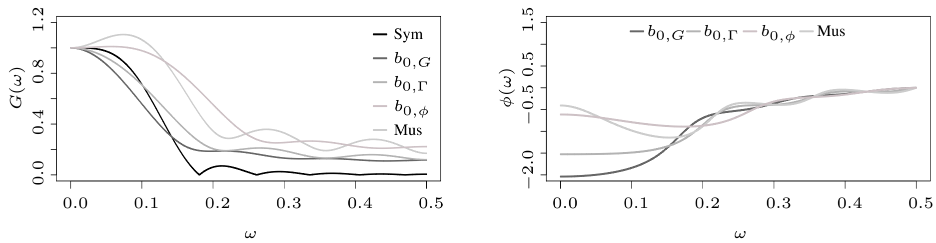
In terms of phase shift or time delay, the filters that behave better are the ones based on the bandwidth parameters selected to minimise the average phase shift in months over the signal domain. However, as shown in Figure 2, their time path is only very close to that of the filters derived by Musgrave (1964) up to $q = 2$ but there is no monotonic convergence of these asymmetric filters to their final oneS. As already said, the Musgrave filters are based on the minimisation of the mean squared revision between the final estimates, obtained

Figure 2: Time path of the asymmetric filters based on $b_{q,\phi}$ (left) and of the Musgrave asymmetric filters (right) corresponding to the 13-term symmetric filter.



by the application of the symmetric filter, and the preliminary estimates, obtained by the application of an asymmetric filter, subject to the constraint that the sum of the weights is equal to one (Laniel, 1985; Doherty, 2001). These filters have the good property of fast detection of turning points. This property is reflected in their phase shift function that, for

Figure 3: Gain (left) and phase shift (right) functions for the last point asymmetric filters based on $b_{0,\Gamma}$, $b_{0,G}$, and $b_{0,\phi}$ compared with the last point Musgrave filter.



the last point filter, is illustrated in Figure 3. As can be seen, both the last point Musgrave filter and the one based on $b_{0,\phi}$ produce almost one half of the phase shift introduced by the filter based on $b_{0,\Gamma}$ and a quarter of the one introduced by the filter based on $b_{0,G}$ at the signal frequency band. However, the reduced phase shift produced by these two filters is compensated by larger revisions introduced in the final estimates.

4. Empirical application

The asymmetric filters previously derived can be applied in many fields, such as, macroeconomic, finance, health, hydrology, meteorology, criminology, physics, labor markets, utilities, and so on. In fact, in any time series where the impact of the trend-cycle is of relevance. A set of leading, coincident, and lagging indicators of the U.S. economy is chosen to corroborate the theoretical conclusions discussed before. The leading indicators are time series that have a turning point before the economy as a whole changes, whereas the coincident indicators change direction approximately at the same time as the whole economy, thereby providing information about the current state of the economy. On the other hand, the lagging indicators are those that usually change direction after the economy as a whole does. The composite indexes are typically reported in the financial and trade press, and the data analysed in this study are from the St. Louis Federal Reserve Bank database, the Bureau of Labor Statistics and the National Bureau of Economic Research (NBER). The asymmetric filters derived following the RKHS methodology versus the Musgrave filters, applied in conjunction with the symmetric Henderson filter, are evaluated as follows.

4.1 Reduction of revision size in real time trend-cycle estimates

The reduction of revisions in real time trend-cycle estimates is very important because the estimates are preliminary and used to assess the current stage of the economy. Statistical agencies and major users of these indicators are reluctant to large revisions because these can lead to erroneous statement concerning the current economic situation. The series considered are all seasonally adjusted, where also trading day variations, moving holidays and extreme values have been removed, if present. The socio-economic indicators are series of different length but the periods selected sufficiently cover the various lengths published for these series.

Here, we study how the filters derived in RKHS and the classical Musgrave estimators respond to the variability of the data. For each series, the length of the filters is selected according to the I/C (noise to signal) ratio, as classically done in the X11/X12ARIMA procedure (Ladiray and Quenneville, 2001). In the sample, the ratio ranges from 0.20 to 1.98, hence filters of length 9 and 13 terms are applied.

The comparisons are based on the relative filter revisions between the final symmetric filter S and the last point asymmetric filter A, that is,

$$R_t = \frac{S_t - A_t}{S_t}, \quad t = 1, 2, ..., N \tag{13}$$

For each series and for each estimator, we calculate the ratio between the Mean Square Percentage Error (MSPE) of the revisions corresponding to the filters derived following the RKHS methodology and those corresponding to the last point Musgrave filter. For all the estimators, the results illustrated in Table 1 indicate that the ratio is always smaller than one, indicating that the kernel last point predictors, based on time-varying bandwidth parameters, introduce smaller revisions than the Musgrave filter. This implies that the estimates obtained by the former will be more accurate than those derived by the application of the latter. In particular, as expected, the best performance is shown by the filter based on the optimal bandwidth $b_{0,G}$ obtained by minimizing the criterion (10). In almost all the series its ratio with the last point Musgrave filter is less than one half and, on average, around 0.480. This implies that when applied to real data, the filter based on $b_{0,G}$ produces a reduction of fifty percent of the revisions introduced in the real time trend-cycle estimates given by the Musgrave filter.

4.2 Turning point detection

It is important that the reduction of revisions in real time trend-cycle estimates is not achieved at the expense of increasing the time lag to detect the upcoming of a true turning point. A turning point is generally defined to occur at time t if (*downturn*):

$$y_{t-k} \leq \cdots \leq y_{t-1} > y_t \geq y_{t+1} \geq \cdots \geq y_{t+m}$$

or (*upturn*)

$$y_{t-k} \geq \cdots \geq y_{t-1} < y_t \leq y_{t+1} \leq \cdots \leq y_{t+m}.$$

Following Zellner et al. (1991), it is selected $k = 3$ and $m = 1$ given the smoothness of the trend cycle data. For each estimator, the time lag to detect the true turning point is affected by the convergence path of its asymmetric filters $\mathbf{w}_q, q = 0, \cdots, m-1$, to the symmetric one $\mathbf{w}$.

Table 1: Ratio of the mean square percentage revision errors of the last point asymmetric filters based on $b_{0,G}$, $b_{0,\Gamma}$ and $b_{0,\phi}$, and the last point Musgrave filter.

Macro-area	Series	$\frac{b_{0,G}}{Mus}$	$\frac{b_{0,\Gamma}}{Mus}$	$\frac{b_{0,\phi}}{Mus}$
Leading	Average weekly overtime hours: manufacturing	0.492	0.630	0.922
	New orders for durable goods	0.493	0.633	0.931
	New orders for nondefense capital goods	0.493	0.633	0.931
	New private housing units authorized by building permits	0.475	0.651	0.927
	S&P 500 stock price index	0.454	0.591	0.856
	M2 money stock	0.508	0.655	0.932
	10-year treasury constant maturity rate	0.446	0.582	0.849
	University of Michigan: consumer sentiment	0.480	0.621	0.912
Coincident	All employees: total nonfarm	0.517	0.666	0.951
	Real personal income excluding current transfer receipts	0.484	0.627	0.903
	Industrial production index	0.477	0.616	0.884
	Manufacturing and trade sales	0.471	0.606	0.869
Lagging	Average (mean) duration of unemployment	0.509	0.649	0.937
	Inventory to sales ratio	0.483	0.618	0.894
	Index of total labor cost per unit of output	0.515	0.663	0.983
	Commercial and industrial loans at all commercial banks	0.473	0.610	0.871

To determine the time lag needed by an indicator to detect a true turning point it is calculated the number of months it takes for the real time trend-cycle estimate to signal a turning point in the same position as in the final trend-cycle series. For the series analysed in this chapter, the time delays for each estimator are shown in Table 2. It can be noticed that the filters based on the bandwidth $b_{q,\phi}$ take almost two months (on average) similar to the Musgrave filters to detect the turning point. This is due to the fact that, even if $b_{q,\phi}$ filters are designed to be optimal in timeliness, their convergence path to the symmetric filter is slower and moreover not monotonic.

On the other hand, the filters based on $b_{q,\Gamma}, q = 0, \cdots, m-1$, and $b_{q,G}, q = 0, \cdots, m-1$, perform strongly better. In particular, whereas the former detect the turning point with an average time delay of 1.67 months, the latter takes 1.27 months.

The fastest the upcoming of a turning point is detected the fastest new policies can be applied to counteract the impact of the business-cycle stage. Failure to recognize the downturn in the cycle or taking a long time delay to detect it may lead to the adoption of policies to curb expansion when in fact, a recession is already underway.

The filters based on local bandwidth parameters selected to minimize criterion (10) are the best, since they drastically reduce the total revisions by one half with respect to the Musgrave filters and, similarly, almost by one half the number of months needed to detect a true turning point.

5. Concluding remarks

This paper deals with the problem of real time trend-cycle estimation where the linear asymmetric filters are developed using the RKHS methodology. Given the length of the RKHS asymmetric filter, its properties strongly depend on the bandwidth parameter of the asymmetric kernel function from which the filter weights are derived. Since the m

Table 2: Time lag in detecting true turning points for the asymmetric filters based on $b_{q,G}$, $b_{q,\Gamma}$, and $b_{q,\phi}$, and the Musgrave filters.

Macro-area	Series	$b_{q,G}$	$b_{q,\Gamma}$	$b_{q,\phi}$	$Musgrave$
Leading	Average weekly overtime hours: manufacturing	1	1	1	1
	New orders for durable goods	1	2	3	2
	New orders for nondefense capital goods	1	2	2	3
	New private housing units authorized by building permits	2	2	3	3
	S&P 500 stock price index	1	2	2	2
	10-year treasury constant maturity rate	1	1	1	2
	University of Michigan: consumer sentiment	1	1	1	1
Coincident	All employees: total nonfarm	1	1	1	2
	Real personal income excluding current transfer receipts	1	1	1	1
	Industrial production index	1	1	1	1
	Manufacturing and trade sales	1	2	3	3
Lagging	Average (mean) duration of unemployment	3	3	4	3
	Inventory to sales ratio	1	1	1	2
	Index of total labor cost per unit of output	2	2	3	2
	Commercial and industrial loans at all commercial banks	1	1	1	1
	Average time lag in months	1.27	1.67	1.93	2.00

asymmetric filters corresponding to a $2m+1$ symmetric filter are time-varying, one for each specific point, local time-varying bandwidth parameters are introduced. The three main criteria of bandwidth parameter selection are minimisation of: (1) the distance between the gain functions of asymmetric and symmetric filters, (2) the distance between the transfer functions of asymmetric and symmetric filters, and (3) the phase shift function over the domain of the signal.

From a theoretical viewpoint, any of the three criteria produces real time trend-cycle filters to be preferred with respect to the currently being used developed by Musgrave (1964) concerning both size of revisions and time delay to detect true turning points. The RKHS asymmetric filters have been applied to a set of leading, coincident, and lagging indicators of the U.S. economy to corroborate the theoretical conclusions. The real time trend-cycle filter calculated with the bandwidth parameter that minimises the distance between the asymmetric and symmetric filters gain functions is to be preferred. This last point trend-cycle filter reduces around one half the size of the total revisions as well as the time delay to detect a true turning point with respect to the Musgrave filter. For illustrative purposes, Table 3 and Table 4 give the weight systems of these filters for 9- and 13-term symmetric filters.

Table 3: Weights corresponding to the 9-term symmetric filter.

0.31218	0.28804	0.22278	0.13330	0.044036	0.00000	0.00000	0.00000	0.00000
0.27101	0.31845	0.27101	0.15289	0.02630	-0.03965	0.00000	0.00000	0.00000
0.10931	0.25652	0.32009	0.25652	0.10931	-0.01705	-0.03470	0.00000	0.00000
-0.01544	0.11250	0.25957	0.32281	0.25957	0.11250	-0.01544	-0.03605	0.00000
-0.03907	-0.01074	0.12023	0.26574	0.32767	0.26574	0.12023	-0.01074	-0.03907
0.00000	-0.03605	-0.01544	0.11250	0.25957	0.32281	0.25957	0.11250	-0.01544
0.00000	0.00000	-0.03470	-0.01705	0.10931	0.25652	0.32009	0.25652	0.10931
0.00000	0.00000	0.00000	-0.03965	0.02630	0.15289	0.27101	0.31845	0.27101
0.00000	0.00000	0.00000	0.00000	0.04404	0.13330	0.22278	0.28804	0.31184

Table 4: Weights corresponding to the 13-term symmetric filter.

0.22362	0.21564	0.19266	0.157478	0.11444	0.06902	0.02714	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.21065	0.22352	0.21065	0.174523	0.12230	0.06460	0.01357	-0.01982	0.00000	0.00000	0.00000	0.00000	0.00000
0.15391	0.21013	0.23100	0.21013	0.15391	0.08000	0.01250	-0.02600	-0.025570	0.00000	0.00000	0.00000	0.00000
0.06338	0.14212	0.20452	0.22808	0.20452	0.14212	0.06338	-0.00217	-0.02978	-0.01617	0.00000	0.00000	0.00000
-0.00258	0.06319	0.14245	0.20533	0.22909	0.20533	0.14245	0.06319	-0.00258	-0.02996	-0.01593	0.00000	0.00000
-0.02983	0.00060	0.06762	0.14651	0.20848	0.23179	0.20848	0.14651	0.06762	0.00060	-0.02983	-0.01855	0.00000
-0.01986	-0.02982	0.00217	0.07010	0.14921	0.21106	0.23429	0.21106	0.149208	0.07010	0.00217	-0.02982	-0.01986
0.00000	-0.01855	-0.02983	0.00066	0.06762	0.14651	0.20848	0.23179	0.20848	0.14651	0.06762	0.00060	-0.02983
0.00000	0.00000	-0.01593	-0.02996	-0.00258	0.06319	0.14245	0.20533	0.22909	0.20533	0.14245	0.06319	-0.00258
0.00000	0.00000	0.00000	-0.01617	-0.02978	-0.00217	0.06338	0.14212	0.20452	0.22808	0.20452	0.14212	0.06338
0.00000	0.00000	0.00000	0.00000	-0.02557	-0.02600	0.01250	0.08000	0.15391	0.21013	0.23100	0.21013	0.15391
0.00000	0.00000	0.00000	0.00000	0.00000	-0.01982	0.01357	0.06460	0.12230	0.17452	0.21065	0.22352	0.21065
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.02714	0.06902	0.11444	0.15748	0.19267	0.21564	0.22362

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