A Revisit To Two-Deck Randomized Response Model

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Abstract

In this paper, we revisit the two deck randomized response model due to Odumade and Singh (2009). We adjust the Odumade and Singh (2009) model following Su, Sedory and Singh (2016) for protection and efficiency. The adjustment makes use of known proportions of unrelated characteristics.

Key words: Two-deck randomized response model, sensitive characteristic, unrelated characteristic, protection and efficiency.

1.Introduction

In this paper, we suggest an improvement over the efficient use of two-decks of cards in randomized response model due to Odumade and Singh (2009) from further efficiency and protection points of views. The model due to Odumade and Singh (2009) is adjustable to be more efficient than the Warner (1965), Mangat and Singh (1990) and Mangat (1994) methods by selecting certain parameters of the proposed randomization device. The collection of data through personal interview surveys on sensitive issues such as induced abortions, drug abuse, and family income is a serious issue; see for example Fox (2016), Chaudhuri, Christofides and Rao (2016), Singh (2014), Chaudhuri and Christofides (2013), Chaudhuri (2011), Fox and Tracy (1986), Lee, Sedory and Singh (2013a, 2013b), and Gjestvang and Singh (2006, 2009).

Odumade and Singh (2009) developed a model where each respondent in a simple random and with replacement (SRSWR) sample of n is provided with two decks of cards marked as Deck-I and Deck-II as shown in Figure 1.1.

$I \in A$ with probability P	$I \in A$ with probability T
$I \in A^c$ with probability $(1-P)$	$I \in A^c$ with probability $(1-T)$
Deck-I	Deck-II

Fig. 1.1. Two decks of cards

Each respondent is requested to draw two cards simultaneously, one card from reach deck of cards, and read the statements in order. The respondent first matches his/her status with the statement written on the first deck of cards, and then he/she matches his/her status with the statement written on the second deck of cards. Let π be the true proportion of respondents in the population that possesses the characteristic A.

Consider a situation that the selected respondent belongs to group A: Now if he/she draws first card with statement $I \in A$ with probability P from the first deck of cards and second card with statement $I \in A$ with probability T from the second deck of cards, then he/she is requested to report: (Yes, Yes).

Consider another situation that the selected respondent belongs to group A^c: Now if he/she draws first card with statement $I \in A^c$ with probability $(1-P)$ from the first deck of cards and second card with statement $I \in A^c$ with probability $(1-T)$ from the second deck of cards, then he/she is also requested to report: (Yes, Yes). Thus the response (Yes, Yes) can come from both types of respondents either belonging to the group A or A^c and hence their privacy will be maintained.

Thus, the probability of getting (Yes, Yes) response is given by:

$$P(\text{Yes},\text{Yes}) = \theta_{11} = PT\pi + (1-P)(1-T)(1-\pi) = (P+T-1)\pi + (1-P)(1-T) \qquad (1.1)$$

Now consider a situation that the selected respondent belongs to group A: Now if he/she draws first card with statement $I \in A$ with probability P from the first deck of cards and second card with statement $I \in A^c$ with probability $(1-T)$ from the second deck of cards, then he/she is requested to report: (Yes, No).

Consider another situation that the selected respondent belongs to group A^c: Now if he/she draws first card with statement $I \in A^c$ with probability $(1-P)$ from the first deck of cards and second card with statement $I \in A$ with probability T from the second deck of cards, then he/she is also requested to report: (Yes, No). Thus the response (Yes, No) can come from both types of respondents either belonging to the group A or A^c and hence their privacy will not be disclosed.

Thus, the probability of getting (Yes, No) response is given by:

$$P(\text{Yes}, \text{No}) = \theta_{10} = P(1-T)\pi + (1-P)T(1-\pi) = (P-T)\pi + T(1-P) \tag{1.2}$$

Now consider a situation that the selected respondent belongs to group A: Now if he/she draws first card with statement $I \in A^c$ with probability $(1-P)$ from the first deck of cards and second card with statement $I \in A$ with probability T from the second deck of cards, then he/she is requested to report: (No, Yes).

Consider another situation that the selected respondent belongs to group A^c: Now if he/she draws first card with statement $I \in A$ with probability P from the first deck of cards and second card with statement $I \in A^c$ with probability $(1-T)$ from the second deck of cards, then he/she is also requested to report: (No, Yes). Thus the response (No, Yes) can come from both types of respondents either belonging to the group A or A^c and hence their privacy will not be disclosed.

Thus, the probability of getting (No, Yes) response is given by:

$$P(\text{No}, \text{Yes}) = \theta_{01} = (1-P)T\pi + P(1-T)(1-\pi) = (T-P)\pi + P(1-T) \tag{1.3}$$

Now consider a situation that the selected respondent belongs to group A: Now if he/she draws first card with statement $I \in A^c$ with probability $(1-P)$ from the first deck of cards and second card with statement $I \in A^c$ with probability $(1-T)$ from the second deck of cards, then he/she is requested to report: (No, No).

Consider another situation that the selected respondent belongs to group A^c: Now if he/she draws first card with statement $I \in A$ with probability P from the first deck of cards and second card with statement $I \in A$ with probability T from the second deck of cards, then he/she is also requested to report: (No, No). Thus the response (No, No) can come from both types of respondents either belonging to the group A or A^c and hence their privacy will not be disclosed.

Thus, the probability of getting (No, No) response is given by:

$$P(\text{No}, \text{No}) = \theta_{00} = (1-P)(1-T)\pi + PT(1-\pi) = (1-P-T)\pi + PT \tag{1.4}$$

The responses from the n respondents can be classified in to 2×2 contingency table as shown in Table 1.1.

Responses	*Yes*	*No*
Yes	n_{11}	n_{10}
No	n_{01}	n_{00}

Table 1.1. The 2×2 contingency table.

Such that:

$$n_{11} + n_{10} + n_{01} + n_{00} = n\,. \tag{1.5}$$

The true probabilities of (Yes, Yes), (Yes, No), (No, Yes) and (No, No) responses in the population can be classified in a 2×2 contingency table as shown in Table 1.2.

Two Decks	True probabilities	Deck – II	
		T	$(1-T)$
Deck-I	P	θ_{11}	θ_{10}
	$(1-P)$	θ_{01}	θ_{00}

Table 1.2. The 2×2 contingency table.

where θ_{11}, θ_{10} , θ_{01} and θ_{00} are given in (1.1), (1.2), (1.3) and (1.4) respectively, such that

$$\theta_{11} + \theta_{10} + \theta_{01} + \theta_{00} = 1 \tag{1.6}$$

Remember that our aim is to estimate the unknown population proportion π of the respondents belonging to the group A.

Let $\hat{\theta}_{11} = n_{11}/n$, $\hat{\theta}_{10} = n_{10}/n$, $\hat{\theta}_{01} = n_{01}/n$ and $\hat{\theta}_{00} = n_{00}/n$ be the observed proportions of (Yes, Yes), (Yes, No), (No, Yes) and (No, No) responses. We define the least square distance between the observed proportions and the true proportions as:

$$\begin{aligned} D &= \frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1}\left(\theta_{ij} - \hat{\theta}_{ij}\right)^2 \\ &= \frac{1}{2}\left[(P+T-1)\pi + (1-P)(1-T) - \hat{\theta}_{11}\right]^2 + \frac{1}{2}\left[(P-T)\pi + T(1-P) - \hat{\theta}_{10}\right]^2 \\ &\quad + \frac{1}{2}\left[(T-P)\pi + P(1-T) - \hat{\theta}_{01}\right]^2 + \frac{1}{2}\left[(1-P-T)\pi + PT - \hat{\theta}_{00}\right]^2 \end{aligned} \tag{1.7}$$

We decided to choose π such that the least square distance D is minimum. Thus, to find such a choice of π we set:

$$\frac{\partial D}{\partial \pi} = 0$$

or

$$\begin{aligned} &\left[(P+T-1)\pi + (1-P)(1-T) - \hat{\theta}_{11}\right](P+T-1) + \left[(P-T)\pi + T(1-P) - \hat{\theta}_{10}\right](P-T) \\ &+ \left[(T-P)\pi + P(1-T) - \hat{\theta}_{01}\right](T-P) + \left[(1-P-T)\pi + PT - \hat{\theta}_{00}\right](1-P-T) = 0 \end{aligned}$$

or

$$2\pi\left[(P+T-1)^2+(P-T)^2\right]-(P+T-1)^2-(P-T)^2=(P+T-1)\left[\hat{\theta}_{11}-\hat{\theta}_{00}\right]+(P-T)\left[\hat{\theta}_{10}-\hat{\theta}_{01}\right]$$

or

$$\pi=\frac{1}{2}+\frac{(P+T-1)\left[\hat{\theta}_{11}-\hat{\theta}_{00}\right]+(P-T)\left[\hat{\theta}_{10}-\hat{\theta}_{01}\right]}{2\left[(P+T-1)^2+(P-T)^2\right]}$$

By the method of moments, we have the following theorem:

Theorem 1.1. An unbiased estimator of the population proportion π is given by:

$$\hat{\pi}_{os}=\frac{1}{2}+\frac{(P+T-1)\left[\hat{\theta}_{11}-\hat{\theta}_{00}\right]+(P-T)\left[\hat{\theta}_{10}-\hat{\theta}_{01}\right]}{2\left[(P+T-1)^2+(P-T)^2\right]} \tag{1.8}$$

Proof. Note that:

$$E\left(\hat{\theta}_{11}\right)=\theta_{11},\quad E\left(\hat{\theta}_{10}\right)=\theta_{10},\quad E\left(\hat{\theta}_{01}\right)=\theta_{01},\ \text{and}\ E\left(\hat{\theta}_{01}\right)=\theta_{00}$$

Now taking the expected values on both sides of (1.8) we have:

$$E(\hat{\pi}_{os})=\frac{1}{2}+\frac{(P+T-1)\left[E\left(\hat{\theta}_{11}\right)-E\left(\hat{\theta}_{00}\right)\right]+(P-T)\left[E\left(\hat{\theta}_{10}\right)-E\left(\hat{\theta}_{01}\right)\right]}{2\left[(P+T-1)^2+(P-T)^2\right]}$$

$$=\frac{(P+T-1)[\theta_{11}-\theta_{00}]+(P-T)[\theta_{10}-\theta_{01}]}{2\left[(P+T-1)^2+(P-T)^2\right]}$$

$$=\frac{1}{2}+\frac{(P+T-1)[2(P+T-1)\pi-(P+T-1)]+(P-T)[2(P-T)\pi-(P-T)]}{2\left[(P+T-1)^2+(P-T)^2\right]}$$

$$=\frac{1}{2}+\frac{2\pi\left[(P+T-1)^2\pi+(P-T)^2\right]-\left\{(P+T-1)^2-(P-T)^2\right\}}{2\left[(P+T-1)^2+(P-T)^2\right]}$$

$$=\frac{1}{2}+\pi-\frac{1}{2}=\pi$$

which proves the theorem.

Now using the concept of multinomial distribution, we have:

$$V\left(\hat{\theta}_{11}\right)=\theta_{11}(1-\theta_{11})/n,\quad Cov\left(\hat{\theta}_{11},\hat{\theta}_{10}\right)=-\theta_{11}\theta_{10}/n,\quad Cov\left(\hat{\theta}_{10},\hat{\theta}_{01}\right)=-\theta_{10}\theta_{01}/n$$

$$V\left(\hat{\theta}_{10}\right)=\theta_{10}(1-\theta_{10})/n,\quad Cov\left(\hat{\theta}_{11},\hat{\theta}_{01}\right)=-\theta_{11}\theta_{01}/n,\quad Cov\left(\hat{\theta}_{10},\hat{\theta}_{00}\right)=-\theta_{10}\theta_{00}/n$$

$$V\left(\hat{\theta}_{01}\right)=\theta_{01}(1-\theta_{01})/n,\quad Cov\left(\hat{\theta}_{11},\hat{\theta}_{00}\right)=-\theta_{11}\theta_{00}/n,\quad Cov\left(\hat{\theta}_{01},\hat{\theta}_{00}\right)=-\theta_{01}\theta_{00}/n$$

and $V\left(\hat{\theta}_{00}\right)=\theta_{00}(1-\theta_{00})/n$.

The use of multinomial in randomized response sampling can also be had from Kim and Warde (2004, 2005).

Now we have the following theorem:

Theorem 1.2. The variance of the estimator $\hat{\pi}_{os}$ is given by:

$$V(\hat{\pi}_{os}) = \frac{(P+T-1)^2\{PT+(1-P)(1-T)\}+(P-T)^2\{T(1-P)+P(1-T)\}}{4n\left[(P+T-1)^2+(P-T)^2\right]^2} - \frac{(2\pi-1)^2}{4n} \qquad (1.9)$$

Proof: We have

$$V(\hat{\pi}_{os}) = \frac{1}{4\left[(P+T-1)^2+(P-T)^2\right]^2}\Big[(P+T-1)^2\cdot V\left(\hat{\theta}_{11}-\hat{\theta}_{00}\right)+(P-T)^2\cdot V\left(\hat{\theta}_{10}-\hat{\theta}_{01}\right)$$
$$+2(P+T-1)(P-T)\cdot Cov\left(\hat{\theta}_{11}-\hat{\theta}_{00},\hat{\theta}_{10}-\hat{\theta}_{01}\right)\Big]$$

$$= \frac{1}{4\left[(P+T-1)^2+(P-T)^2\right]^2}\Big[(P+T-1)^2\left\{V\left(\hat{\theta}_{11}\right)+V\left(\hat{\theta}_{00}\right)-2Cov\left(\hat{\theta}_{11},\hat{\theta}_{00}\right)\right\}$$
$$+(P-T)^2\left\{V\left(\hat{\theta}_{10}\right)+V\left(\hat{\theta}_{01}\right)-2Cov\left(\hat{\theta}_{10},\hat{\theta}_{01}\right)\right\}$$
$$+2(P+T-1)(P-T)\Big\{Cov\left(\hat{\theta}_{11},-\hat{\theta}_{10}\right)-Cov\left(\hat{\theta}_{00},-\hat{\theta}_{10}\right)$$
$$-Cov\left(\hat{\theta}_{11},-\hat{\theta}_{01}\right)+Cov\left(\hat{\theta}_{00},-\hat{\theta}_{01}\right)\Big\}\Big]$$

$$= \frac{1}{4\left[(P+T-1)^2+(P-T)^2\right]^2}\left[(P+T-1)^2\left\{\frac{\theta_{11}(1-\theta_{11})}{n}+\frac{\theta_{00}(1-\theta_{00})}{n}+2\frac{\theta_{11}\theta_{00}}{n}\right\}\right.$$
$$+(P-T)^2\left\{\frac{\theta_{10}(1-\theta_{10})}{n}+\frac{\theta_{01}(1-\theta_{01})}{n}+2\frac{\theta_{10}\theta_{01}}{n}\right\}$$
$$\left.+2(P+T-1)(P-T)\left\{-\frac{\theta_{11}\theta_{10}}{n}+\frac{\theta_{10}\theta_{00}}{n}+\frac{\theta_{11}\theta_{01}}{n}-\frac{\theta_{00}\theta_{01}}{n}\right\}\right]$$

$$= \frac{1}{4n\left[(P+T-1)^2+(P-T)^2\right]^2}\Big[(P+T-1)^2\{PT+(1-P)(1-T)\}+(P-T)^2\{T(1-P)+P(1-T)\}$$
$$-(2\pi-1)^2\left\{(P+T-1)^2+(P-T)^2\right\}^2\Big]$$

$$= \frac{(P+T-1)^2\{PT+(1-P)(1-T)\}+(P-T)^2\{T(1-P)+P(1-T)\}}{4n\left[(P+T-1)^2+(P-T)^2\right]^2} - \frac{(2\pi-1)^2}{4n}$$

which proves the theorem.

Now we have the following theorem:

Theorem 1.3. An estimator to estimate the variance of $\hat{\pi}_{os}$ is given by:

$$\hat{v}(\hat{\pi}_{os}) = \frac{1}{4(n-1)}\left[\frac{(P+T-1)^2\{PT+(1-P)(1-T)\}+(P-T)^2\{T(1-P)+P(1-T)\}}{\{(P+T-1)^2+(P-T)^2\}} - (2\hat{\pi}_{os}-1)^2\right] \tag{1.10}$$

Proof. Taking expected on both sides of (1.10) we have:

$$E[\hat{v}(\hat{\pi}_{os})] = \frac{1}{4(n-1)}\left[\frac{(P+T-1)^2\{PT+(1-P)(1-T)\}+(P-T)^2\{T(1-P)+P(1-T)\}}{\{(P+T-1)^2+(P-T)^2\}} - E(2\hat{\pi}_{os}-1)^2\right]$$

$$= \frac{n}{(n-1)}\left[\frac{(P+T-1)^2\{PT+(1-P)(1-T)\}+(P-T)^2\{T(1-P)+P(1-T)\}}{4n\{(P+T-1)^2+(P-T)^2\}}\right.$$

$$\left. - \frac{4\{V(\hat{\pi}_{os})+\pi^2\}+1-4\pi}{4n}\right] = \frac{n}{(n-1)}\left[V(\hat{\pi}_{os}) - \frac{V(\hat{\pi}_{os})}{n}\right] = V(\hat{\pi}_{os}) .$$

which proves the theorem.

Now we have the following corollary:

Corollary 1.1. If $T = P = P_0$ (say), then the variance of the proposed estimator $\hat{\pi}_{os}$ in (1.9) becomes:

$$V(\hat{\pi}_{os})_{P=T=P_0} = \frac{\pi(1-\pi)}{n} + \frac{P_0(1-P_0)}{2n(2P_0-1)^2} = V(\hat{\pi}_w)_{q=2} \text{ (say)} \tag{1.11}$$

which is same variance if each respondent is requested to use the Warner (1965) device twice.

Proof. On setting $T = P = P_0$ in (1.8), we have:

$$V(\hat{\pi}_{os})_{P=T=P_0} = \frac{(2P_0-1)^2\left[P_0^2+(1-P_0)^2\right]}{4n\{(2P_0-1)^2\}^2} - \frac{(2\pi-1)^2}{4n}$$

$$= \frac{P_0^2+(1-P_0)^2}{4n(2P_0-1)^2} - \frac{(2\pi-1)^2}{4n} = \frac{P_0^2+1+P_0^2-2P_0}{4n(2P_0-1)^2} - \frac{(2\pi-1)^2}{4n}$$

$$= \frac{\pi(1-\pi)}{n} + \frac{P_0(1-P_0)}{2n(2P_0-1)^2}$$

which proves the corollary.

Singh and Sedory (2011, 2012) suggested a new log-likelihood estimator of the population proportion π and developed a lower bound on the variance in this randomized response sampling setup. They consider the problem of maximizing the likelihood function, which is defined as:

$$L = \binom{n}{n_{11}, n_{10}, n_{01}, n_{00}}\theta_{11}^{n_{11}}\theta_{10}^{n_{10}}\theta_{01}^{n_{01}}\theta_{00}^{n_{00}} \tag{1.12}$$

On setting $\frac{\partial \log(L)}{\partial \pi} = 0$, the maximum likelihood estimate $\hat{\pi}_{\text{mle}}$ of π is given by a solution to the following equation:

$$\frac{\hat{\theta}_{11}(P+T-1)}{\theta_{11}} + \frac{\hat{\theta}_{10}(P-T)}{\theta_{10}} + \frac{\hat{\theta}_{01}(T-P)}{\theta_{01}} + \frac{\hat{\theta}_{00}(1-P-T)}{\theta_{00}} = 0 \qquad (1.13)$$

By the well known Cramer-Rao inequality, the lower bound of the variance of the maximum likelihood estimate $\hat{\pi}_{mle}$ of π is given by:

$$V(\hat{\pi}_{mle}) \geq \frac{1}{n\left\{\frac{(P+T-1)^2}{\theta_{11}} + \frac{(P-T)^2}{\theta_{10}} + \frac{(T-P)^2}{\theta_{01}} + \frac{(1-P-T)^2}{\theta_{00}}\right\}} \qquad (1.14)$$

Singh and Sedory (2011, 2012) have shown that at equal protection of respondents that the lower bound of variance in (1.14) is always smaller than the variance of the Odumade and Singh (2009) estimator.

In the next section, we propose a new model which instantly makes use of two decks of cards out of four decks of cards.

2. Proposed Adjusted Randomized Response Model

In the Odumade and Singh (2009) model, the two questions from two decks of cards are related to the same sensitive questions or to the perfectly negatively associated with each other; that is, one group is the complement of the other group in the population of interest. However, it is intuitively evident that to protect the confidentiality of a respondent it is not necessary that the two questions be complementary, for example one might use two unrelated questions (Do you belong to group A/ Do you belong to group Y?) In fact, it is sufficient to make use of some unrelated non-sensitive characteristic in the randomization device, as suggested by Greenberg *et al.* (1969), where they proposed the unrelated questions model. Su, Sedory and Singh (2016) suggest an adjustment to the original Kuk (1990) model by making use of two non-sensitive characteristics, say Y_1 and Y_2. We extend here the same idea of making adjustment with two non-sensitive characteristics in the pioneer Odumade and Singh (2009) model, and investigate if any adjustment can be made to increase efficiency and protection of the respondents.

Let π_{y_1} and π_{y_2} be the known proportions of two non-sensitive characteristics in a population. For example π_{y_1} be the known proportion of those respondents in the population whose mothers were born during day time, and π_{y_2} be the proportion of those respondents in the population whose fathers were born day time. The proposed randomized response model makes instant use of two-decks of cards out of four- decks of cards.

The first-pair of decks consists of two-decks as shown in Figure2.1.

Deck-I	Deck-II
$I \in A$ with probability T	$I \in Y_1$ with probability W_2
$I \in A^c$ with probability $(1-T)$	$I \in Y_1^c$ with probability $(1-W_2)$

Fig. 2.1. First Pair of Decks

In the first pair:

The first-deck consists of two types of cards bearing statements

(i.) I am a member of the sensitive group A
(ii) I am not a member of the sensitive group A

with relative frequencies P and $(1-P)$ respectively.

The second-deck consists of two types of cards bearing statements

(i.) I possess the unrelated characteristic Y_1
(ii) I do not possess the unrelated characteristic Y_1

with relative frequencies W_1 and $(1-W_1)$ respectively.

The second-pair of decks consists of two-decks as shown in Figure2.2.

Deck-I	Deck-II
$I \in A$ with probability P	$I \in Y_1$ with probability W_1
$I \in A^c$ with probability $(1-P)$	$I \in Y_1^c$ with probability $(1-W_1)$

Fig 2.2. Second Pair of Decks

In the second pair:

The first-deck consists of two types of cards bearing statements

(i.) I am a member of the sensitive group A
(ii) I am a member of the non-sensitive group A^c

with relative frequencies T and $(1-T)$ respectively.

The second-deck consists of two types of cards bearing statements

(i.) I possess the unrelated characteristic Y_2
(ii) I do not possess the unrelated characteristic Y_2

with relative frequencies W_2 and $(1-W_2)$ respectively.

Now every respondent selected in a simple random and with replacement sample (SRSWR) of n respondents is instructed to use the above two ordered-pairs of decks as follow: Without asking or disclosing the membership of a respondent, he/she is instructed to use the first ordered pair of decks if he/she belongs to group A; and the second ordered pair of decks if he/she belongs to the non-sensitive group A^c.

Consider a situation that the selected respondent belongs to group A and possess the unrelated characteristic Y_1: Now if he/she draws first card with statement $I \in A$ with probability P from the first deck and second card with statement $I \in Y_1$ with probability W_1 from the second deck, then he/she is requested to report: (Yes, Yes).

Consider a situation that the selected respondent belongs to group A and does not possess the unrelated characteristic Y_1: Now if he/she draws first card with statement $I \in A$ with probability P from the first deck and second card with statement $I \in Y_1^c$ with probability $(1-W_1)$ from the second deck, then he/she is requested to report: (Yes, Yes).

Consider a situation that the selected respondent belongs to non-sensitive group A^c and possess the unrelated characteristic Y_2: Now if he/she draws first card with statement $I \in A^c$ with probability $(1-T)$ from the first deck and second card with statement $I \in Y_2$ with probability W_2 from the second deck, then he/she is requested to report: (Yes, Yes).

Consider a situation that the selected respondent belongs to non-sensitive group A^c and does not possess the unrelated characteristic Y_2: Now if he/she draws first card with statement $I \in A^c$ with probability $(1-T)$ from the first deck and second card with statement $I \in Y_2^c$ with probability $(1-W_2)$ from the second deck, then he/she is requested to report: (Yes, Yes).

Thus, in the proposed model the probability of (Yes, Yes) answer is given by:

$$\theta_{11}^* = \pi P\left[W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\right] + (1-\pi)(1-T)\left[W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\right] \tag{2.1}$$

Likewise:

The probability of (Yes, No) answer is given by:

$$\theta_{10}^* = \pi P\left[W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\right] + (1-\pi)(1-T)\left[W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\right] \tag{2.2}$$

The probability of (No, Yes) answer is given by:

$$\theta_{01}^* = \pi(1-P)\left[W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\right] + (1-\pi)T\left[W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\right] \tag{2.3}$$

and, the probability of (No, No) answer is given by:

$$\theta_{00}^* = \pi(1-P)\left[W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\right] + (1-\pi)T\left[W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\right] \tag{2.4}$$

We consider the likelihood function as:

$$L_1 = \binom{n}{n_{11}, n_{10}, n_{01}, n_{00}} (\theta_{11}^*)^{n_{11}} (\theta_{10}^*)^{n_{10}} (\theta_{01}^*)^{n_{01}} (\theta_{00}^*)^{n_{00}} \tag{2.5}$$

Taking log on both sides, we have:

$$\begin{aligned}
\log(L_1) = \log\binom{n}{n_{11}, n_{10}, n_{01}, n_{00}} \\
&+ n_{11} \log\left[\pi P\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} + (1-\pi)(1-T)\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right] \\
&+ n_{10} \log\left[\pi P\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} + (1-\pi)(1-T)\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right] \\
&+ n_{01} \log\left[\pi(1-P)\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} + (1-\pi)\, T\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right] \\
&+ n_{00} \log\left[\pi(1-P)\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} + (1-\pi)\, T\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]
\end{aligned} \tag{2.6}$$

There are three situations:

Case- I. When π_{y_1} and π_{y_2} are known
Case-II. When π_{y_1} is known and π_{y_2} is unknown, (or π_{y_1} is unknown and π_{y_2} is known)
Case-III. When π_{y_1} and π_{y_2} are also unknown.

In this paper, we consider only the first case. The other two cases are also interesting and can be considered in future studies. With one sample, it is not possible to estimate more than one parameter, so alternative methods could be developed.

3. Situation- I : When π_{y_1} and π_{y_2} are known

In a situation when π_{y_1} and π_{y_2} are known, then there is only one parameter of interest π. The maximum likelihood estimate of π is given by a solution to the non-linear equation given by

$$\begin{aligned}
\frac{\partial \log L_1}{\partial \pi} = {} & \frac{n_{11}\left[P\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - (1-T)\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]}{\theta_{11}^*} \\
&+ \frac{n_{10}\left[P\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - (1-T)\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]}{\theta_{10}^*} \\
&+ \frac{n_{01}\left[(1-P)\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - T\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]}{\theta_{01}^*} \\
&+ \frac{n_{00}\left[(1-P)\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - T\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]}{\theta_{00}^*} = 0
\end{aligned} \tag{3.1}$$

Thus for a given values of π_{y_1} and π_{y_2}, the Cramer-Rao lower-bound of variance by following Singh and Sedory (2011, 2012) is given by

$$V(\hat{\pi}_{new}) \geq \frac{1}{-E\left(\frac{\partial^2 \log(L_1)}{\partial \pi^2}\right)} \tag{3.2}$$

Now

$$\frac{\partial^2 \log L_1}{\partial \pi^2} = -\frac{n_{11}\left[P\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - (1-T)\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]^2}{(\theta_{11}^*)^2}$$
$$-\frac{n_{10}\left[P\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - (1-T)\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]^2}{(\theta_{10}^*)^2}$$
$$-\frac{n_{01}\left[(1-P)\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - T\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]^2}{(\theta_{01}^*)^2}$$
$$-\frac{n_{00}\left[(1-P)\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - T\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]^2}{(\theta_{00}^*)^2}$$

$$= -\frac{n\hat{\theta}_{11}^*\left[P\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - (1-T)\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]^2}{(\theta_{11}^*)^2}$$
$$-\frac{n\hat{\theta}_{10}^*\left[P\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - (1-T)\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]^2}{(\theta_{10}^*)^2}$$
$$-\frac{n\hat{\theta}_{01}^*\left[(1-P)\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - T\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]^2}{(\theta_{01}^*)^2}$$
$$-\frac{n\hat{\theta}_{00}^*\left[(1-P)\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - T\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]^2}{(\theta_{00}^*)^2} \tag{3.3}$$

On using (3.3) in (3.2), the Cramer-Rao lower-bound of variance of the new maximum likelihood estimate $\hat{\pi}_{new}$ is given by

$$V(\hat{\pi}_{new}) \geq \frac{1}{n}\left[\frac{\left[P\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - (1-T)\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]^2}{(\theta_{11}^*)}\right.$$
$$+\frac{\left[P\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - (1-T)\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]^2}{(\theta_{10}^*)}$$
$$+\frac{\left[(1-P)\{W_1\pi_{y_1} + (1-W_1)(1-\pi_{y_1})\} - T\{W_2\pi_{y_2} + (1-W_2)(1-\pi_{y_2})\}\right]^2}{(\theta_{01}^*)}$$
$$\left.+\frac{\left[(1-P)\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\} - T\{W_2(1-\pi_{y_2}) + (1-W_2)\pi_{y_2}\}\right]^2}{(\theta_{00}^*)}\right]^{-1} \tag{3.4}$$

Following Lanke (1975, 1976), and recently Lee, Su, Mondragon, Salinas, Zamora, Sedory and Singh (2016), for the Odumade and Singh (2009) (or equivalently Singh and Sedory (2011, 2012)), method of using of two decks of cards, we compute four conditional probabilities as follows:

$$P[A \mid (\text{Yes}, \text{Yes})] = \frac{PT\pi}{\theta_{11}} \tag{3.5}$$

$$P[A \mid (\text{Yes}, \text{No})] = \frac{P(1-T)\pi}{\theta_{10}} \tag{3.6}$$

$$P[A \mid (\text{No}, \text{Yes})] = \frac{(1-P)T\pi}{\theta_{01}} \tag{3.7}$$

and

$$P[A \mid (\text{No}, \text{No})] = \frac{(1-P)(1-T)\pi}{\theta_{00}} . \tag{3.8}$$

Then the least protection in the OS model due to Odumade and Singh (2009), or equivalently Singh and Sedory (2011, 2012), is given by:

$$\text{Prot(OS Model)} = Max\left[P\{A \mid (\text{Yes}, \text{Yes})\}, P\{A \mid (\text{Yes}, \text{No})\}, P\{A \mid (\text{No}, \text{Yes})\}, P\{A \mid (\text{No}, \text{No})\}\right] \tag{3.9}$$

Again following Lee et al. (2016), we compute the same four conditional probabilities as follows:

$$P^*[A \mid (\text{Yes}, \text{Yes})] = \frac{P\, \pi\{W_1 \pi_{y_1} + (1-W_1)(1-\pi_{y_1})\}}{\theta^*_{11}} \tag{3.10}$$

$$P^*[A \mid (\text{Yes}, \text{No})] = \frac{P\, \pi\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\}}{\theta^*_{10}} \tag{3.11}$$

$$P^*[A \mid (\text{No}, \text{Yes})] = \frac{(1-P)\, \pi\{W_1 \pi_{y_1} + (1-W_1)(1-\pi_{y_1})\}}{\theta^*_{01}} \tag{3.12}$$

and

$$P^*[A \mid (\text{No}, \text{No})] = \frac{(1-P)\, \pi\{W_1(1-\pi_{y_1}) + (1-W_1)\pi_{y_1}\}}{\theta^*_{00}} . \tag{3.13}$$

Then the least protection in the proposed model is given by:

$$\text{Prot(Proposed Model)} = Max\left[P^*\{A \mid (\text{Yes}, \text{Yes})\}, P^*\{A \mid (\text{Yes}, \text{No})\}, P^*\{A \mid (\text{No}, \text{Yes})\}, P^*\{A \mid (\text{No}, \text{No})\}\right] \tag{3.14}$$

In the next section we compare the proposed model with the Odumade and Singh (2009) model through numerical illustrations by taking care into both the protection and efficiency of the proposed model as pointed out by Christofides (2010)and Guerriero and Sandri (2007).

4. Relative Efficiency and Relative Protection

For different choices of W_1, W_2, π_{y_1} and π_{y_2}, we investigate where the proposed estimate $\hat{\pi}_{\text{new}}$ is more efficient and more protective than the maximum likelihood estimate $\hat{\pi}_{\text{mle}}$ of Singh and Sedory (2011, 2012) for at least equal protection of the respondents. Singh and Sedory (2011, 2012) have already shown that the lower bond of variance remains below the variance of the Odumade and Singh (2009) estimator. Based on lower bounds of variance, the percent relative efficiency (RE) of the proposed estimator $\hat{\pi}_{new}$ with respect to the Singh and Sedory (2011, 2012) estimator $\hat{\pi}_{mle}$ is defined as:

$$\text{RE} = \frac{V(\hat{\pi}_{mle})}{\text{V}(\hat{\pi}_{new})} \times 100\% \tag{4.1}$$

We also compute the percent relative protection (RP) of the proposed model with respect to the Singh and Sedory (2011, 2012) (or equivalently Odumade and Singh (2009) model) as:

$$\text{RP} = \frac{\text{Prot(OS Model)}}{\text{Prot(Proposed Model)}} \times 100\% \tag{4.2}$$

If $\text{RP} \geq 100\%$, then the proposed model is more protective than the or Singh and Sedory (2011, 2012) model or Odumade and Singh (2009) model.

We wrote FORTRAN code as given in Appendix-B, and after executing the code the results are given in Appendix-A in Table A. In Table A, we considered $P = T = 0.7$ in the Odumade and Singh (2009) model and also the choice of parameters in the proposed model. Note that both the RE and RP are independent of sample size. Thus we change the values of the sensitive proportion π in the range 0.1 to 0.9 with a step of 0.1. We also change the values of W_1, W_2, π_{y_1} and π_{y_2} in the range of 0.1 to 0.9 each with a step of 0.1. We recorded those parameters of the randomization deice to be used in a survey where RE is greater than 100% and the RP is also greater than 100%. We found that for each choice of π in the range 0.1 to 0.9 with a step of 0.1 there exist a few choices of other parameters W_1, W_2, π_{y_1} and π_{y_2} such that both criterion are met. Stepwise analysis shows that: for $\pi = 0.1$ the value of maximum RE is 107.71% and maximum RP is 105.07% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; for $\pi = 0.2$ the value of maximum RE is 109.11% and maximum RP is 103.44% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; for $\pi = 0.3$ the value of maximum RE is 110.01% and RP is 102.44% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; for $\pi = 0.4$ the value of maximum RE is 110.76% and RP is 101.76% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; for $\pi = 0.5$ the value of maximum RE is 111.61% and RP is 101.26% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; for $\pi = 0.6$ the value of maximum RE is 112.82% and RP is 100.89% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; for $\pi = 0.7$ the value of maximum RE is 114.83% and RP is 100.59% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and

$\pi_{y_2} = 0.7$; for $\pi = 0.8$ the value of maximum RE is 118.48% and RP is 100.36% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$; and for $\pi = 0.8$ the value of maximum RE is 126.03% and RP is 100.16% for $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$. This stepwise analysis shows that if one makes use of the proposed model with known values of the device parameters as $W_1 = 0.1$, $W_1 = 0.2$, $\pi_{y_1} = 0.1$ and $\pi_{y_2} = 0.7$, then proposed new model is always more efficient and more protective than the Odumade and Singh (2009) model, and has lower bond of variance lower than the Singh and Sedory (2011, 2012) lower bounds of variance for the Odumade and Singh (2009) model with the choice $P = T = 0.7$. In the same way other choices of to make a randomization device to be used efficiently and protective than the Odumade and Singh (2009) model can be simulated with the FORTRAN code provided.

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Appendix-A

Table A. Percent relative efficiency (RE) and percent relative protection (RP) for different choice of parameters ($P = T = 0.7$) .

π	W_1	W_2	π_{y_1}	π_{y_2}	RE	RP
0.1	0.1	0.2	0.1	0.7	107.71	105.07
0.1	0.1	0.2	0.2	0.8	104.28	100.56
0.1	0.1	0.2	0.8	0.2	104.28	100.56
0.1	0.1	0.2	0.9	0.3	107.71	105.07
0.1	0.1	0.3	0.1	0.8	107.71	105.07
0.1	0.1	0.3	0.9	0.2	107.71	105.07
0.1	0.1	0.7	0.1	0.2	107.71	105.07
0.1	0.1	0.7	0.9	0.8	107.71	105.07
0.1	0.1	0.8	0.1	0.3	107.71	105.07
0.1	0.1	0.8	0.2	0.2	104.28	100.56
0.1	0.1	0.8	0.8	0.8	104.28	100.56
0.1	0.1	0.8	0.9	0.7	107.71	105.07
0.1	0.2	0.2	0.1	0.8	104.28	100.56
0.1	0.2	0.2	0.9	0.2	104.28	100.56
0.1	0.2	0.8	0.1	0.2	104.28	100.56
0.1	0.2	0.8	0.9	0.8	104.28	100.56
0.1	0.8	0.2	0.1	0.2	104.28	100.56
0.1	0.8	0.2	0.9	0.8	104.28	100.56
0.1	0.8	0.8	0.1	0.8	104.28	100.56
0.1	0.8	0.8	0.9	0.2	104.28	100.56
0.1	0.9	0.2	0.1	0.3	107.71	105.07
0.1	0.9	0.2	0.2	0.2	104.28	100.56
0.1	0.9	0.2	0.8	0.8	104.28	100.56
0.1	0.9	0.2	0.9	0.7	107.71	105.07
0.1	0.9	0.3	0.1	0.2	107.71	105.07
0.1	0.9	0.3	0.9	0.8	107.71	105.07
0.1	0.9	0.7	0.1	0.8	107.71	105.07
0.1	0.9	0.7	0.9	0.2	107.71	105.07
0.1	0.9	0.8	0.1	0.7	107.71	105.07
0.1	0.9	0.8	0.2	0.8	104.28	100.56
0.1	0.9	0.8	0.8	0.2	104.28	100.56
0.1	0.9	0.8	0.9	0.3	107.71	105.07
0.2	0.1	0.2	0.1	0.7	109.11	103.44
0.2	0.1	0.2	0.2	0.8	104.34	100.38
0.2	0.1	0.2	0.8	0.2	104.34	100.38
0.2	0.1	0.2	0.9	0.3	109.11	103.44
0.2	0.1	0.3	0.1	0.8	109.11	103.44

0.2	0.1	0.3	0.9	0.2	109.11	103.44
0.2	0.1	0.7	0.1	0.2	109.11	103.44
0.2	0.1	0.7	0.9	0.8	109.11	103.44
0.2	0.1	0.8	0.1	0.3	109.11	103.44
0.2	0.1	0.8	0.2	0.2	104.34	100.38
0.2	0.1	0.8	0.8	0.8	104.34	100.38
0.2	0.1	0.8	0.9	0.7	109.11	103.44
0.2	0.2	0.2	0.1	0.8	104.34	100.38
0.2	0.2	0.2	0.9	0.2	104.34	100.38
0.2	0.2	0.8	0.1	0.2	104.34	100.38
0.2	0.2	0.8	0.9	0.8	104.34	100.38
0.2	0.8	0.2	0.1	0.2	104.34	100.38
0.2	0.8	0.2	0.9	0.8	104.34	100.38
0.2	0.8	0.8	0.1	0.8	104.34	100.38
0.2	0.8	0.8	0.9	0.2	104.34	100.38
0.2	0.9	0.2	0.1	0.3	109.11	103.44
0.2	0.9	0.2	0.2	0.2	104.34	100.38
0.2	0.9	0.2	0.8	0.8	104.34	100.38
0.2	0.9	0.2	0.9	0.7	109.11	103.44
0.2	0.9	0.3	0.1	0.2	109.11	103.44
0.2	0.9	0.3	0.9	0.8	109.11	103.44
0.2	0.9	0.7	0.1	0.8	109.11	103.44
0.2	0.9	0.7	0.9	0.2	109.11	103.44
0.2	0.9	0.8	0.1	0.7	109.11	103.44
0.2	0.9	0.8	0.2	0.8	104.34	100.38
0.2	0.9	0.8	0.8	0.2	104.34	100.38
0.2	0.9	0.8	0.9	0.3	109.11	103.44
0.3	0.1	0.1	0.1	0.6	100.97	105.85
0.3	0.1	0.1	0.9	0.4	100.97	105.85
0.3	0.1	0.2	0.1	0.7	110.01	102.44
0.3	0.1	0.2	0.2	0.8	104.38	100.27
0.3	0.1	0.2	0.8	0.2	104.38	100.27
0.3	0.1	0.2	0.9	0.3	110.01	102.44
0.3	0.1	0.3	0.1	0.7	100.97	105.85
0.3	0.1	0.3	0.1	0.8	110.01	102.44
0.3	0.1	0.3	0.9	0.2	110.01	102.44
0.3	0.1	0.3	0.9	0.3	100.97	105.85
0.3	0.1	0.4	0.1	0.9	100.97	105.85
0.3	0.1	0.4	0.9	0.1	100.97	105.85
0.3	0.1	0.6	0.1	0.1	100.97	105.85
0.3	0.1	0.6	0.9	0.9	100.97	105.85

0.3	0.1	0.7	0.1	0.2	110.01	102.44
0.3	0.1	0.7	0.1	0.3	100.97	105.85
0.3	0.1	0.7	0.9	0.7	100.97	105.85
0.3	0.1	0.7	0.9	0.8	110.01	102.44
0.3	0.1	0.8	0.1	0.3	110.01	102.44
0.3	0.1	0.8	0.2	0.2	104.38	100.27
0.3	0.1	0.8	0.8	0.8	104.38	100.27
0.3	0.1	0.8	0.9	0.7	110.01	102.44
0.3	0.1	0.9	0.1	0.4	100.97	105.85
0.3	0.1	0.9	0.9	0.6	100.97	105.85
0.3	0.2	0.2	0.1	0.8	104.38	100.27
0.3	0.2	0.2	0.9	0.2	104.38	100.27
0.3	0.2	0.8	0.1	0.2	104.38	100.27
0.3	0.2	0.8	0.9	0.8	104.38	100.27
0.3	0.8	0.2	0.1	0.2	104.38	100.27
0.3	0.8	0.2	0.9	0.8	104.38	100.27
0.3	0.8	0.8	0.1	0.8	104.38	100.27
0.3	0.8	0.8	0.9	0.2	104.38	100.27
0.3	0.9	0.1	0.1	0.4	100.97	105.85
0.3	0.9	0.1	0.9	0.6	100.97	105.85
0.3	0.9	0.2	0.1	0.3	110.01	102.44
0.3	0.9	0.2	0.2	0.2	104.38	100.27
0.3	0.9	0.2	0.8	0.8	104.38	100.27
0.3	0.9	0.2	0.9	0.7	110.01	102.44
0.3	0.9	0.3	0.1	0.2	110.01	102.44
0.3	0.9	0.3	0.1	0.3	100.97	105.85
0.3	0.9	0.3	0.9	0.7	100.97	105.85
0.3	0.9	0.3	0.9	0.8	110.01	102.44
0.3	0.9	0.4	0.1	0.1	100.97	105.85
0.3	0.9	0.4	0.9	0.9	100.97	105.85
0.3	0.9	0.6	0.1	0.9	100.97	105.85
0.3	0.9	0.6	0.9	0.1	100.97	105.85
0.3	0.9	0.7	0.1	0.7	100.97	105.85
0.3	0.9	0.7	0.1	0.8	110.01	102.44
0.3	0.9	0.7	0.9	0.2	110.01	102.44
0.3	0.9	0.7	0.9	0.3	100.97	105.85
0.3	0.9	0.8	0.1	0.7	110.01	102.44
0.3	0.9	0.8	0.2	0.8	104.38	100.27
0.3	0.9	0.8	0.8	0.2	104.38	100.27
0.3	0.9	0.8	0.9	0.3	110.01	102.44
0.3	0.9	0.9	0.1	0.6	100.97	105.85

0.3	0.9	0.9	0.9	0.4	100.97	105.85
0.4	0.1	0.1	0.1	0.6	102.07	104.21
0.4	0.1	0.1	0.2	0.7	100.04	101.56
0.4	0.1	0.1	0.8	0.3	100.04	101.56
0.4	0.1	0.1	0.9	0.4	102.07	104.21
0.4	0.1	0.2	0.1	0.7	110.76	101.76
0.4	0.1	0.2	0.2	0.8	104.46	100.19
0.4	0.1	0.2	0.8	0.2	104.46	100.19
0.4	0.1	0.2	0.9	0.3	110.76	101.76
0.4	0.1	0.3	0.1	0.7	102.07	104.21
0.4	0.1	0.3	0.1	0.8	110.76	101.76
0.4	0.1	0.3	0.2	0.9	100.04	101.56
0.4	0.1	0.3	0.8	0.1	100.04	101.56
0.4	0.1	0.3	0.9	0.2	110.76	101.76
0.4	0.1	0.3	0.9	0.3	102.07	104.21
0.4	0.1	0.4	0.1	0.9	102.07	104.21
0.4	0.1	0.4	0.9	0.1	102.07	104.21
0.4	0.1	0.6	0.1	0.1	102.07	104.21
0.4	0.1	0.6	0.9	0.9	102.07	104.21
0.4	0.1	0.7	0.1	0.2	110.76	101.76
0.4	0.1	0.7	0.1	0.3	102.07	104.21
0.4	0.1	0.7	0.2	0.1	100.04	101.56
0.4	0.1	0.7	0.8	0.9	100.04	101.56
0.4	0.1	0.7	0.9	0.7	102.07	104.21
0.4	0.1	0.7	0.9	0.8	110.76	101.76
0.4	0.1	0.8	0.1	0.3	110.76	101.76
0.4	0.1	0.8	0.2	0.2	104.46	100.19
0.4	0.1	0.8	0.8	0.8	104.46	100.19
0.4	0.1	0.8	0.9	0.7	110.76	101.76
0.4	0.1	0.9	0.1	0.4	102.07	104.21
0.4	0.1	0.9	0.2	0.3	100.04	101.56
0.4	0.1	0.9	0.8	0.7	100.04	101.56
0.4	0.1	0.9	0.9	0.6	102.07	104.21
0.4	0.2	0.1	0.1	0.7	100.04	101.56
0.4	0.2	0.1	0.9	0.3	100.04	101.56
0.4	0.2	0.2	0.1	0.8	104.46	100.19
0.4	0.2	0.2	0.9	0.2	104.46	100.19
0.4	0.2	0.3	0.1	0.9	100.04	101.56
0.4	0.2	0.3	0.9	0.1	100.04	101.56
0.4	0.2	0.7	0.1	0.1	100.04	101.56
0.4	0.2	0.7	0.9	0.9	100.04	101.56

0.4	0.2	0.8	0.1	0.2	104.46	100.19
0.4	0.2	0.8	0.9	0.8	104.46	100.19
0.4	0.2	0.9	0.1	0.3	100.04	101.56
0.4	0.2	0.9	0.9	0.7	100.04	101.56
0.4	0.8	0.1	0.1	0.3	100.04	101.56
0.4	0.8	0.1	0.9	0.7	100.04	101.56
0.4	0.8	0.2	0.1	0.2	104.46	100.19
0.4	0.8	0.2	0.9	0.8	104.46	100.19
0.4	0.8	0.3	0.1	0.1	100.04	101.56
0.4	0.8	0.3	0.9	0.9	100.04	101.56
0.4	0.8	0.7	0.1	0.9	100.04	101.56
0.4	0.8	0.7	0.9	0.1	100.04	101.56
0.4	0.8	0.8	0.1	0.8	104.46	100.19
0.4	0.8	0.8	0.9	0.2	104.46	100.19
0.4	0.8	0.9	0.1	0.7	100.04	101.56
0.4	0.8	0.9	0.9	0.3	100.04	101.56
0.4	0.9	0.1	0.1	0.4	102.07	104.21
0.4	0.9	0.1	0.2	0.3	100.04	101.56
0.4	0.9	0.1	0.8	0.7	100.04	101.56
0.4	0.9	0.1	0.9	0.6	102.07	104.21
0.4	0.9	0.2	0.1	0.3	110.76	101.76
0.4	0.9	0.2	0.2	0.2	104.46	100.19
0.4	0.9	0.2	0.8	0.8	104.46	100.19
0.4	0.9	0.2	0.9	0.7	110.76	101.76
0.4	0.9	0.3	0.1	0.2	110.76	101.76
0.4	0.9	0.3	0.1	0.3	102.07	104.21
0.4	0.9	0.3	0.2	0.1	100.04	101.56
0.4	0.9	0.3	0.8	0.9	100.04	101.56
0.4	0.9	0.3	0.9	0.7	102.07	104.21
0.4	0.9	0.3	0.9	0.8	110.76	101.76
0.4	0.9	0.4	0.1	0.1	102.07	104.21
0.4	0.9	0.4	0.9	0.9	102.07	104.21
0.4	0.9	0.6	0.1	0.9	102.07	104.21
0.4	0.9	0.6	0.9	0.1	102.07	104.21
0.4	0.9	0.7	0.1	0.7	102.07	104.21
0.4	0.9	0.7	0.1	0.8	110.76	101.76
0.4	0.9	0.7	0.2	0.9	100.04	101.56
0.4	0.9	0.7	0.8	0.1	100.04	101.56
0.4	0.9	0.7	0.9	0.2	110.76	101.76
0.4	0.9	0.7	0.9	0.3	102.07	104.21
0.4	0.9	0.8	0.1	0.7	110.76	101.76

0.4	0.9	0.8	0.2	0.8	104.46	100.19
0.4	0.9	0.8	0.8	0.2	104.46	100.19
0.4	0.9	0.8	0.9	0.3	110.76	101.76
0.4	0.9	0.9	0.1	0.6	102.07	104.21
0.4	0.9	0.9	0.2	0.7	100.04	101.56
0.4	0.9	0.9	0.8	0.3	100.04	101.56
0.4	0.9	0.9	0.9	0.4	102.07	104.21
0.5	0.1	0.1	0.1	0.6	102.95	103.03
0.5	0.1	0.1	0.2	0.7	100.31	101.12
0.5	0.1	0.1	0.8	0.3	100.31	101.12
0.5	0.1	0.1	0.9	0.4	102.95	103.03
0.5	0.1	0.2	0.1	0.7	111.61	101.26
0.5	0.1	0.2	0.2	0.8	104.62	100.14
0.5	0.1	0.2	0.8	0.2	104.62	100.14
0.5	0.1	0.2	0.9	0.3	111.61	101.26
0.5	0.1	0.3	0.1	0.7	102.95	103.03
0.5	0.1	0.3	0.1	0.8	111.61	101.26
0.5	0.1	0.3	0.2	0.9	100.31	101.12
0.5	0.1	0.3	0.8	0.1	100.31	101.12
0.5	0.1	0.3	0.9	0.2	111.61	101.26
0.5	0.1	0.3	0.9	0.3	102.95	103.03
0.5	0.1	0.4	0.1	0.9	102.95	103.03
0.5	0.1	0.4	0.9	0.1	102.95	103.03
0.5	0.1	0.6	0.1	0.1	102.95	103.03
0.5	0.1	0.6	0.9	0.9	102.95	103.03
0.5	0.1	0.7	0.1	0.2	111.61	101.26
0.5	0.1	0.7	0.1	0.3	102.95	103.03
0.5	0.1	0.7	0.2	0.1	100.31	101.12
0.5	0.1	0.7	0.8	0.9	100.31	101.12
0.5	0.1	0.7	0.9	0.7	102.95	103.03
0.5	0.1	0.7	0.9	0.8	111.61	101.26
0.5	0.1	0.8	0.1	0.3	111.61	101.26
0.5	0.1	0.8	0.2	0.2	104.62	100.14
0.5	0.1	0.8	0.8	0.8	104.62	100.14
0.5	0.1	0.8	0.9	0.7	111.61	101.26
0.5	0.1	0.9	0.1	0.4	102.95	103.03
0.5	0.1	0.9	0.2	0.3	100.31	101.12
0.5	0.1	0.9	0.8	0.7	100.31	101.12
0.5	0.1	0.9	0.9	0.6	102.95	103.03
0.5	0.2	0.1	0.1	0.7	100.31	101.12
0.5	0.2	0.1	0.9	0.3	100.31	101.12

0.5	0.2	0.2	0.1	0.8	104.62	100.14
0.5	0.2	0.2	0.9	0.2	104.62	100.14
0.5	0.2	0.3	0.1	0.9	100.31	101.12
0.5	0.2	0.3	0.9	0.1	100.31	101.12
0.5	0.2	0.7	0.1	0.1	100.31	101.12
0.5	0.2	0.7	0.9	0.9	100.31	101.12
0.5	0.2	0.8	0.1	0.2	104.62	100.14
0.5	0.2	0.8	0.9	0.8	104.62	100.14
0.5	0.2	0.9	0.1	0.3	100.31	101.12
0.5	0.2	0.9	0.9	0.7	100.31	101.12
0.5	0.8	0.1	0.1	0.3	100.31	101.12
0.5	0.8	0.1	0.9	0.7	100.31	101.12
0.5	0.8	0.2	0.1	0.2	104.62	100.14
0.5	0.8	0.2	0.9	0.8	104.62	100.14
0.5	0.8	0.3	0.1	0.1	100.31	101.12
0.5	0.8	0.3	0.9	0.9	100.31	101.12
0.5	0.8	0.7	0.1	0.9	100.31	101.12
0.5	0.8	0.7	0.9	0.1	100.31	101.12
0.5	0.8	0.8	0.1	0.8	104.62	100.14
0.5	0.8	0.8	0.9	0.2	104.62	100.14
0.5	0.8	0.9	0.1	0.7	100.31	101.12
0.5	0.8	0.9	0.9	0.3	100.31	101.12
0.5	0.9	0.1	0.1	0.4	102.95	103.03
0.5	0.9	0.1	0.2	0.3	100.31	101.12
0.5	0.9	0.1	0.8	0.7	100.31	101.12
0.5	0.9	0.1	0.9	0.6	102.95	103.03
0.5	0.9	0.2	0.1	0.3	111.61	101.26
0.5	0.9	0.2	0.2	0.2	104.62	100.14
0.5	0.9	0.2	0.8	0.8	104.62	100.14
0.5	0.9	0.2	0.9	0.7	111.61	101.26
0.5	0.9	0.3	0.1	0.2	111.61	101.26
0.5	0.9	0.3	0.1	0.3	102.95	103.03
0.5	0.9	0.3	0.2	0.1	100.31	101.12
0.5	0.9	0.3	0.8	0.9	100.31	101.12
0.5	0.9	0.3	0.9	0.7	102.95	103.03
0.5	0.9	0.3	0.9	0.8	111.61	101.26
0.5	0.9	0.4	0.1	0.1	102.95	103.03
0.5	0.9	0.4	0.9	0.9	102.95	103.03
0.5	0.9	0.6	0.1	0.9	102.95	103.03
0.5	0.9	0.6	0.9	0.1	102.95	103.03
0.5	0.9	0.7	0.1	0.7	102.95	103.03

0.5	0.9	0.7	0.1	0.8	111.61	101.26
0.5	0.9	0.7	0.2	0.9	100.31	101.12
0.5	0.9	0.7	0.8	0.1	100.31	101.12
0.5	0.9	0.7	0.9	0.2	111.61	101.26
0.5	0.9	0.7	0.9	0.3	102.95	103.03
0.5	0.9	0.8	0.1	0.7	111.61	101.26
0.5	0.9	0.8	0.2	0.8	104.62	100.14
0.5	0.9	0.8	0.8	0.2	104.62	100.14
0.5	0.9	0.8	0.9	0.3	111.61	101.26
0.5	0.9	0.9	0.1	0.6	102.95	103.03
0.5	0.9	0.9	0.2	0.7	100.31	101.12
0.5	0.9	0.9	0.8	0.3	100.31	101.12
0.5	0.9	0.9	0.9	0.4	102.95	103.03
0.6	0.1	0.1	0.1	0.6	103.92	102.13
0.6	0.1	0.1	0.2	0.7	100.58	100.79
0.6	0.1	0.1	0.8	0.3	100.58	100.79
0.6	0.1	0.1	0.9	0.4	103.92	102.13
0.6	0.1	0.2	0.1	0.7	112.82	100.89
0.6	0.1	0.2	0.2	0.8	104.93	100.10
0.6	0.1	0.2	0.8	0.2	104.93	100.10
0.6	0.1	0.2	0.9	0.3	112.82	100.89
0.6	0.1	0.3	0.1	0.7	103.92	102.13
0.6	0.1	0.3	0.1	0.8	112.82	100.89
0.6	0.1	0.3	0.2	0.9	100.58	100.79
0.6	0.1	0.3	0.8	0.1	100.58	100.79
0.6	0.1	0.3	0.9	0.2	112.82	100.89
0.6	0.1	0.3	0.9	0.3	103.92	102.13
0.6	0.1	0.4	0.1	0.9	103.92	102.13
0.6	0.1	0.4	0.9	0.1	103.92	102.13
0.6	0.1	0.6	0.1	0.1	103.92	102.13
0.6	0.1	0.6	0.9	0.9	103.92	102.13
0.6	0.1	0.7	0.1	0.2	112.82	100.89
0.6	0.1	0.7	0.1	0.3	103.92	102.13
0.6	0.1	0.7	0.2	0.1	100.58	100.79
0.6	0.1	0.7	0.8	0.9	100.58	100.79
0.6	0.1	0.7	0.9	0.7	103.92	102.13
0.6	0.1	0.7	0.9	0.8	112.82	100.89
0.6	0.1	0.8	0.1	0.3	112.82	100.89
0.6	0.1	0.8	0.2	0.2	104.93	100.10
0.6	0.1	0.8	0.8	0.8	104.93	100.10
0.6	0.1	0.8	0.9	0.7	112.82	100.89

0.6	0.1	0.9	0.1	0.4	103.92	102.13
0.6	0.1	0.9	0.2	0.3	100.58	100.79
0.6	0.1	0.9	0.8	0.7	100.58	100.79
0.6	0.1	0.9	0.9	0.6	103.92	102.13
0.6	0.2	0.1	0.1	0.7	100.58	100.79
0.6	0.2	0.1	0.9	0.3	100.58	100.79
0.6	0.2	0.2	0.1	0.8	104.93	100.10
0.6	0.2	0.2	0.9	0.2	104.93	100.10
0.6	0.2	0.3	0.1	0.9	100.58	100.79
0.6	0.2	0.3	0.9	0.1	100.58	100.79
0.6	0.2	0.7	0.1	0.1	100.58	100.79
0.6	0.2	0.7	0.9	0.9	100.58	100.79
0.6	0.2	0.8	0.1	0.2	104.93	100.10
0.6	0.2	0.8	0.9	0.8	104.93	100.10
0.6	0.2	0.9	0.1	0.3	100.58	100.79
0.6	0.2	0.9	0.9	0.7	100.58	100.79
0.6	0.8	0.1	0.1	0.3	100.58	100.79
0.6	0.8	0.1	0.9	0.7	100.58	100.79
0.6	0.8	0.2	0.1	0.2	104.93	100.10
0.6	0.8	0.2	0.9	0.8	104.93	100.10
0.6	0.8	0.3	0.1	0.1	100.58	100.79
0.6	0.8	0.3	0.9	0.9	100.58	100.79
0.6	0.8	0.7	0.1	0.9	100.58	100.79
0.6	0.8	0.7	0.9	0.1	100.58	100.79
0.6	0.8	0.8	0.1	0.8	104.93	100.10
0.6	0.8	0.8	0.9	0.2	104.93	100.10
0.6	0.8	0.9	0.1	0.7	100.58	100.79
0.6	0.8	0.9	0.9	0.3	100.58	100.79
0.6	0.9	0.1	0.1	0.4	103.92	102.13
0.6	0.9	0.1	0.2	0.3	100.58	100.79
0.6	0.9	0.1	0.8	0.7	100.58	100.79
0.6	0.9	0.1	0.9	0.6	103.92	102.13
0.6	0.9	0.2	0.1	0.3	112.82	100.89
0.6	0.9	0.2	0.2	0.2	104.93	100.10
0.6	0.9	0.2	0.8	0.8	104.93	100.10
0.6	0.9	0.2	0.9	0.7	112.83	100.89
0.6	0.9	0.3	0.1	0.2	112.82	100.89
0.6	0.9	0.3	0.1	0.3	103.92	102.13
0.6	0.9	0.3	0.2	0.1	100.58	100.79
0.6	0.9	0.3	0.8	0.9	100.58	100.79
0.6	0.9	0.3	0.9	0.7	103.92	102.13

0.6	0.9	0.3	0.9	0.8	112.83	100.89
0.6	0.9	0.4	0.1	0.1	103.92	102.13
0.6	0.9	0.4	0.9	0.9	103.92	102.13
0.6	0.9	0.6	0.1	0.9	103.92	102.13
0.6	0.9	0.6	0.9	0.1	103.92	102.13
0.6	0.9	0.7	0.1	0.7	103.92	102.13
0.6	0.9	0.7	0.1	0.8	112.82	100.89
0.6	0.9	0.7	0.2	0.9	100.58	100.79
0.6	0.9	0.7	0.8	0.1	100.58	100.79
0.6	0.9	0.7	0.9	0.2	112.83	100.89
0.6	0.9	0.7	0.9	0.3	103.92	102.13
0.6	0.9	0.8	0.1	0.7	112.82	100.89
0.6	0.9	0.8	0.2	0.8	104.93	100.10
0.6	0.9	0.8	0.8	0.2	104.93	100.10
0.6	0.9	0.8	0.9	0.3	112.83	100.89
0.6	0.9	0.9	0.1	0.6	103.92	102.13
0.6	0.9	0.9	0.2	0.7	100.58	100.79
0.6	0.9	0.9	0.8	0.3	100.58	100.79
0.6	0.9	0.9	0.9	0.4	103.92	102.13
0.7	0.1	0.1	0.1	0.6	105.34	101.42
0.7	0.1	0.1	0.2	0.7	100.96	100.53
0.7	0.1	0.1	0.8	0.3	100.96	100.53
0.7	0.1	0.1	0.9	0.4	105.34	101.42
0.7	0.1	0.2	0.1	0.6	100.89	101.84
0.7	0.1	0.2	0.1	0.7	114.83	100.59
0.7	0.1	0.2	0.2	0.8	105.50	100.07
0.7	0.1	0.2	0.8	0.2	105.50	100.07
0.7	0.1	0.2	0.9	0.3	114.83	100.59
0.7	0.1	0.2	0.9	0.4	100.89	101.84
0.7	0.1	0.3	0.1	0.7	105.34	101.42
0.7	0.1	0.3	0.1	0.8	114.83	100.59
0.7	0.1	0.3	0.2	0.9	100.96	100.53
0.7	0.1	0.3	0.8	0.1	100.96	100.53
0.7	0.1	0.3	0.9	0.2	114.83	100.59
0.7	0.1	0.3	0.9	0.3	105.34	101.42
0.7	0.1	0.4	0.1	0.8	100.89	101.84
0.7	0.1	0.4	0.1	0.9	105.34	101.42
0.7	0.1	0.4	0.9	0.1	105.34	101.42
0.7	0.1	0.4	0.9	0.2	100.89	101.84
0.7	0.1	0.6	0.1	0.1	105.34	101.42
0.7	0.1	0.6	0.1	0.2	100.89	101.84

0.7	0.1	0.6	0.9	0.8	100.89	101.84
0.7	0.1	0.6	0.9	0.9	105.34	101.42
0.7	0.1	0.7	0.1	0.2	114.83	100.59
0.7	0.1	0.7	0.1	0.3	105.34	101.42
0.7	0.1	0.7	0.2	0.1	100.96	100.53
0.7	0.1	0.7	0.8	0.9	100.96	100.53
0.7	0.1	0.7	0.9	0.7	105.34	101.42
0.7	0.1	0.7	0.9	0.8	114.83	100.59
0.7	0.1	0.8	0.1	0.3	114.83	100.59
0.7	0.1	0.8	0.1	0.4	100.89	101.84
0.7	0.1	0.8	0.2	0.2	105.50	100.07
0.7	0.1	0.8	0.8	0.8	105.50	100.07
0.7	0.1	0.8	0.9	0.6	100.89	101.84
0.7	0.1	0.8	0.9	0.7	114.83	100.59
0.7	0.1	0.9	0.1	0.4	105.34	101.42
0.7	0.1	0.9	0.2	0.3	100.96	100.53
0.7	0.1	0.9	0.8	0.7	100.96	100.53
0.7	0.1	0.9	0.9	0.6	105.34	101.42
0.7	0.2	0.1	0.1	0.7	100.96	100.53
0.7	0.2	0.1	0.9	0.3	100.96	100.53
0.7	0.2	0.2	0.1	0.8	105.50	100.07
0.7	0.2	0.2	0.9	0.2	105.50	100.07
0.7	0.2	0.3	0.1	0.9	100.96	100.53
0.7	0.2	0.3	0.9	0.1	100.96	100.53
0.7	0.2	0.7	0.1	0.1	100.96	100.53
0.7	0.2	0.7	0.9	0.9	100.96	100.53
0.7	0.2	0.8	0.1	0.2	105.50	100.07
0.7	0.2	0.8	0.9	0.8	105.50	100.07
0.7	0.2	0.9	0.1	0.3	100.96	100.53
0.7	0.2	0.9	0.9	0.7	100.96	100.53
0.7	0.8	0.1	0.1	0.3	100.96	100.53
0.7	0.8	0.1	0.9	0.7	100.96	100.53
0.7	0.8	0.2	0.1	0.2	105.50	100.07
0.7	0.8	0.2	0.9	0.8	105.50	100.07
0.7	0.8	0.3	0.1	0.1	100.96	100.53
0.7	0.8	0.3	0.9	0.9	100.96	100.53
0.7	0.8	0.7	0.1	0.9	100.96	100.53
0.7	0.8	0.7	0.9	0.1	100.96	100.53
0.7	0.8	0.8	0.1	0.8	105.50	100.07
0.7	0.8	0.8	0.9	0.2	105.50	100.07
0.7	0.8	0.9	0.1	0.7	100.96	100.53

0.7	0.8	0.9	0.9	0.3	100.96	100.53
0.7	0.9	0.1	0.1	0.4	105.34	101.42
0.7	0.9	0.1	0.2	0.3	100.96	100.53
0.7	0.9	0.1	0.8	0.7	100.96	100.53
0.7	0.9	0.1	0.9	0.6	105.34	101.42
0.7	0.9	0.2	0.1	0.3	114.83	100.59
0.7	0.9	0.2	0.1	0.4	100.89	101.84
0.7	0.9	0.2	0.2	0.2	105.50	100.07
0.7	0.9	0.2	0.8	0.8	105.50	100.07
0.7	0.9	0.2	0.9	0.6	100.89	101.84
0.7	0.9	0.2	0.9	0.7	114.83	100.59
0.7	0.9	0.3	0.1	0.2	114.83	100.59
0.7	0.9	0.3	0.1	0.3	105.34	101.42
0.7	0.9	0.3	0.2	0.1	100.96	100.53
0.7	0.9	0.3	0.8	0.9	100.96	100.53
0.7	0.9	0.3	0.9	0.7	105.34	101.42
0.7	0.9	0.3	0.9	0.8	114.83	100.59
0.7	0.9	0.4	0.1	0.1	105.34	101.42
0.7	0.9	0.4	0.1	0.2	100.89	101.84
0.7	0.9	0.4	0.9	0.8	100.89	101.84
0.7	0.9	0.4	0.9	0.9	105.34	101.42
0.7	0.9	0.6	0.1	0.8	100.89	101.84
0.7	0.9	0.6	0.1	0.9	105.34	101.42
0.7	0.9	0.6	0.9	0.1	105.34	101.42
0.7	0.9	0.6	0.9	0.2	100.89	101.84
0.7	0.9	0.7	0.1	0.7	105.34	101.42
0.7	0.9	0.7	0.1	0.8	114.83	100.59
0.7	0.9	0.7	0.2	0.9	100.96	100.53
0.7	0.9	0.7	0.8	0.1	100.96	100.53
0.7	0.9	0.7	0.9	0.2	114.83	100.59
0.7	0.9	0.7	0.9	0.3	105.34	101.42
0.7	0.9	0.8	0.1	0.6	100.89	101.84
0.7	0.9	0.8	0.1	0.7	114.83	100.59
0.7	0.9	0.8	0.2	0.8	105.50	100.07
0.7	0.9	0.8	0.8	0.2	105.50	100.07
0.7	0.9	0.8	0.9	0.3	114.83	100.59
0.7	0.9	0.8	0.9	0.4	100.89	101.84
0.7	0.9	0.9	0.1	0.6	105.34	101.42
0.7	0.9	0.9	0.2	0.7	100.96	100.53
0.7	0.9	0.9	0.8	0.3	100.96	100.53
0.7	0.9	0.9	0.9	0.4	105.34	101.42

0.8	0.1	0.1	0.1	0.6	107.88	100.86
0.8	0.1	0.1	0.2	0.7	101.63	100.32
0.8	0.1	0.1	0.8	0.3	101.63	100.32
0.8	0.1	0.1	0.9	0.4	107.88	100.86
0.8	0.1	0.2	0.1	0.6	102.88	101.11
0.8	0.1	0.2	0.1	0.7	118.48	100.36
0.8	0.1	0.2	0.2	0.8	106.53	100.04
0.8	0.1	0.2	0.8	0.2	106.53	100.04
0.8	0.1	0.2	0.9	0.3	118.48	100.36
0.8	0.1	0.2	0.9	0.4	102.88	101.11
0.8	0.1	0.3	0.1	0.7	107.88	100.86
0.8	0.1	0.3	0.1	0.8	118.48	100.36
0.8	0.1	0.3	0.2	0.9	101.63	100.32
0.8	0.1	0.3	0.8	0.1	101.63	100.32
0.8	0.1	0.3	0.9	0.2	118.48	100.36
0.8	0.1	0.3	0.9	0.3	107.88	100.86
0.8	0.1	0.4	0.1	0.8	102.88	101.11
0.8	0.1	0.4	0.1	0.9	107.88	100.86
0.8	0.1	0.4	0.9	0.1	107.88	100.86
0.8	0.1	0.4	0.9	0.2	102.88	101.11
0.8	0.1	0.6	0.1	0.1	107.88	100.86
0.8	0.1	0.6	0.1	0.2	102.88	101.11
0.8	0.1	0.6	0.9	0.8	102.88	101.11
0.8	0.1	0.6	0.9	0.9	107.88	100.86
0.8	0.1	0.7	0.1	0.2	118.48	100.36
0.8	0.1	0.7	0.1	0.3	107.88	100.86
0.8	0.1	0.7	0.2	0.1	101.63	100.32
0.8	0.1	0.7	0.8	0.9	101.63	100.32
0.8	0.1	0.7	0.9	0.7	107.88	100.86
0.8	0.1	0.7	0.9	0.8	118.48	100.36
0.8	0.1	0.8	0.1	0.3	118.48	100.36
0.8	0.1	0.8	0.1	0.4	102.88	101.11
0.8	0.1	0.8	0.2	0.2	106.53	100.04
0.8	0.1	0.8	0.8	0.8	106.53	100.04
0.8	0.1	0.8	0.9	0.6	102.88	101.11
0.8	0.1	0.8	0.9	0.7	118.48	100.36
0.8	0.1	0.9	0.1	0.4	107.88	100.86
0.8	0.1	0.9	0.2	0.3	101.63	100.32
0.8	0.1	0.9	0.8	0.7	101.63	100.32
0.8	0.1	0.9	0.9	0.6	107.88	100.86
0.8	0.2	0.1	0.1	0.7	101.63	100.32

0.8	0.2	0.1	0.9	0.3	101.63	100.32
0.8	0.2	0.2	0.1	0.8	106.53	100.04
0.8	0.2	0.2	0.9	0.2	106.53	100.04
0.8	0.2	0.3	0.1	0.9	101.63	100.32
0.8	0.2	0.3	0.9	0.1	101.63	100.32
0.8	0.2	0.7	0.1	0.1	101.63	100.32
0.8	0.2	0.7	0.9	0.9	101.63	100.32
0.8	0.2	0.8	0.1	0.2	106.53	100.04
0.8	0.2	0.8	0.9	0.8	106.53	100.04
0.8	0.2	0.9	0.1	0.3	101.63	100.32
0.8	0.2	0.9	0.9	0.7	101.63	100.32
0.8	0.8	0.1	0.1	0.3	101.63	100.32
0.8	0.8	0.1	0.9	0.7	101.63	100.32
0.8	0.8	0.2	0.1	0.2	106.53	100.04
0.8	0.8	0.2	0.9	0.8	106.53	100.04
0.8	0.8	0.3	0.1	0.1	101.63	100.32
0.8	0.8	0.3	0.9	0.9	101.63	100.32
0.8	0.8	0.7	0.1	0.9	101.63	100.32
0.8	0.8	0.7	0.9	0.1	101.63	100.32
0.8	0.8	0.8	0.1	0.8	106.53	100.04
0.8	0.8	0.8	0.9	0.2	106.53	100.04
0.8	0.8	0.9	0.1	0.7	101.63	100.32
0.8	0.8	0.9	0.9	0.3	101.63	100.32
0.8	0.9	0.1	0.1	0.4	107.88	100.86
0.8	0.9	0.1	0.2	0.3	101.63	100.32
0.8	0.9	0.1	0.8	0.7	101.63	100.32
0.8	0.9	0.1	0.9	0.6	107.88	100.86
0.8	0.9	0.2	0.1	0.3	118.48	100.36
0.8	0.9	0.2	0.1	0.4	102.88	101.11
0.8	0.9	0.2	0.2	0.2	106.53	100.04
0.8	0.9	0.2	0.8	0.8	106.53	100.04
0.8	0.9	0.2	0.9	0.6	102.88	101.11
0.8	0.9	0.2	0.9	0.7	118.48	100.36
0.8	0.9	0.3	0.1	0.2	118.48	100.36
0.8	0.9	0.3	0.1	0.3	107.88	100.86
0.8	0.9	0.3	0.2	0.1	101.63	100.32
0.8	0.9	0.3	0.8	0.9	101.63	100.32
0.8	0.9	0.3	0.9	0.7	107.88	100.86
0.8	0.9	0.3	0.9	0.8	118.48	100.36
0.8	0.9	0.4	0.1	0.1	107.88	100.86
0.8	0.9	0.4	0.1	0.2	102.88	101.11

0.8	0.9	0.4	0.9	0.8	102.88	101.11
0.8	0.9	0.4	0.9	0.9	107.88	100.86
0.8	0.9	0.6	0.1	0.8	102.88	101.11
0.8	0.9	0.6	0.1	0.9	107.88	100.86
0.8	0.9	0.6	0.9	0.1	107.88	100.86
0.8	0.9	0.6	0.9	0.2	102.88	101.11
0.8	0.9	0.7	0.1	0.7	107.88	100.86
0.8	0.9	0.7	0.1	0.8	118.48	100.36
0.8	0.9	0.7	0.2	0.9	101.63	100.32
0.8	0.9	0.7	0.8	0.1	101.63	100.32
0.8	0.9	0.7	0.9	0.2	118.48	100.36
0.8	0.9	0.7	0.9	0.3	107.88	100.86
0.8	0.9	0.8	0.1	0.6	102.88	101.11
0.8	0.9	0.8	0.1	0.7	118.48	100.36
0.8	0.9	0.8	0.2	0.8	106.53	100.04
0.8	0.9	0.8	0.8	0.2	106.53	100.04
0.8	0.9	0.8	0.9	0.3	118.48	100.36
0.8	0.9	0.8	0.9	0.4	102.88	101.11
0.8	0.9	0.9	0.1	0.6	107.88	100.86
0.8	0.9	0.9	0.2	0.7	101.63	100.32
0.8	0.9	0.9	0.8	0.3	101.63	100.32
0.8	0.9	0.9	0.9	0.4	107.88	100.86
0.9	0.1	0.1	0.1	0.6	113.16	100.39
0.9	0.1	0.1	0.2	0.7	102.92	100.14
0.9	0.1	0.1	0.8	0.3	102.92	100.14
0.9	0.1	0.1	0.9	0.4	113.16	100.39
0.9	0.1	0.2	0.1	0.6	107.09	100.50
0.9	0.1	0.2	0.1	0.7	126.03	100.16
0.9	0.1	0.2	0.2	0.8	108.53	100.02
0.9	0.1	0.2	0.8	0.2	108.53	100.02
0.9	0.1	0.2	0.9	0.3	126.03	100.16
0.9	0.1	0.2	0.9	0.4	107.09	100.50
0.9	0.1	0.3	0.1	0.6	101.25	100.62
0.9	0.1	0.3	0.1	0.7	113.16	100.39
0.9	0.1	0.3	0.1	0.8	126.03	100.16
0.9	0.1	0.3	0.2	0.9	102.92	100.14
0.9	0.1	0.3	0.8	0.1	102.92	100.14
0.9	0.1	0.3	0.9	0.2	126.03	100.16
0.9	0.1	0.3	0.9	0.3	113.16	100.39
0.9	0.1	0.3	0.9	0.4	101.25	100.62
0.9	0.1	0.4	0.1	0.7	101.25	100.62

0.9	0.1	0.4	0.1	0.8	107.09	100.50
0.9	0.1	0.4	0.1	0.9	113.16	100.39
0.9	0.1	0.4	0.9	0.1	113.16	100.39
0.9	0.1	0.4	0.9	0.2	107.09	100.50
0.9	0.1	0.4	0.9	0.3	101.25	100.62
0.9	0.1	0.6	0.1	0.1	113.16	100.39
0.9	0.1	0.6	0.1	0.2	107.09	100.50
0.9	0.1	0.6	0.1	0.3	101.25	100.62
0.9	0.1	0.6	0.9	0.7	101.25	100.62
0.9	0.1	0.6	0.9	0.8	107.09	100.50
0.9	0.1	0.6	0.9	0.9	113.16	100.39
0.9	0.1	0.7	0.1	0.2	126.03	100.16
0.9	0.1	0.7	0.1	0.3	113.16	100.39
0.9	0.1	0.7	0.1	0.4	101.25	100.62
0.9	0.1	0.7	0.2	0.1	102.92	100.14
0.9	0.1	0.7	0.8	0.9	102.92	100.14
0.9	0.1	0.7	0.9	0.6	101.25	100.62
0.9	0.1	0.7	0.9	0.7	113.16	100.39
0.9	0.1	0.7	0.9	0.8	126.03	100.16
0.9	0.1	0.8	0.1	0.3	126.03	100.16
0.9	0.1	0.8	0.1	0.4	107.09	100.50
0.9	0.1	0.8	0.2	0.2	108.53	100.02
0.9	0.1	0.8	0.8	0.8	108.53	100.02
0.9	0.1	0.8	0.9	0.6	107.09	100.50
0.9	0.1	0.8	0.9	0.7	126.03	100.16
0.9	0.1	0.9	0.1	0.4	113.16	100.39
0.9	0.1	0.9	0.2	0.3	102.92	100.14
0.9	0.1	0.9	0.8	0.7	102.92	100.14
0.9	0.1	0.9	0.9	0.6	113.16	100.39
0.9	0.2	0.1	0.1	0.7	102.92	100.14
0.9	0.2	0.1	0.9	0.3	102.92	100.14
0.9	0.2	0.2	0.1	0.8	108.53	100.02
0.9	0.2	0.2	0.9	0.2	108.53	100.02
0.9	0.2	0.3	0.1	0.9	102.92	100.14
0.9	0.2	0.3	0.9	0.1	102.92	100.14
0.9	0.2	0.7	0.1	0.1	102.92	100.14
0.9	0.2	0.7	0.9	0.9	102.92	100.14
0.9	0.2	0.8	0.1	0.2	108.53	100.02
0.9	0.2	0.8	0.9	0.8	108.53	100.02
0.9	0.2	0.9	0.1	0.3	102.92	100.14
0.9	0.2	0.9	0.9	0.7	102.92	100.14

0.9	0.8	0.1	0.1	0.3	102.92	100.14
0.9	0.8	0.1	0.9	0.7	102.92	100.14
0.9	0.8	0.2	0.1	0.2	108.53	100.02
0.9	0.8	0.2	0.9	0.8	108.53	100.02
0.9	0.8	0.3	0.1	0.1	102.92	100.14
0.9	0.8	0.3	0.9	0.9	102.92	100.14
0.9	0.8	0.7	0.1	0.9	102.92	100.14
0.9	0.8	0.7	0.9	0.1	102.92	100.14
0.9	0.8	0.8	0.1	0.8	108.53	100.02
0.9	0.8	0.8	0.9	0.2	108.53	100.02
0.9	0.8	0.9	0.1	0.7	102.92	100.14
0.9	0.8	0.9	0.9	0.3	102.92	100.14
0.9	0.9	0.1	0.1	0.4	113.16	100.39
0.9	0.9	0.1	0.2	0.3	102.92	100.14
0.9	0.9	0.1	0.8	0.7	102.92	100.14
0.9	0.9	0.1	0.9	0.6	113.16	100.39
0.9	0.9	0.2	0.1	0.3	126.03	100.16
0.9	0.9	0.2	0.1	0.4	107.09	100.50
0.9	0.9	0.2	0.2	0.2	108.53	100.02
0.9	0.9	0.2	0.8	0.8	108.53	100.02
0.9	0.9	0.2	0.9	0.6	107.09	100.50
0.9	0.9	0.2	0.9	0.7	126.03	100.16
0.9	0.9	0.3	0.1	0.2	126.03	100.16
0.9	0.9	0.3	0.1	0.3	113.16	100.39
0.9	0.9	0.3	0.1	0.4	101.25	100.62
0.9	0.9	0.3	0.2	0.1	102.92	100.14
0.9	0.9	0.3	0.8	0.9	102.92	100.14
0.9	0.9	0.3	0.9	0.6	101.25	100.62
0.9	0.9	0.3	0.9	0.7	113.16	100.39
0.9	0.9	0.3	0.9	0.8	126.03	100.16
0.9	0.9	0.4	0.1	0.1	113.16	100.39
0.9	0.9	0.4	0.1	0.2	107.09	100.50
0.9	0.9	0.4	0.1	0.3	101.25	100.62
0.9	0.9	0.4	0.9	0.7	101.25	100.62
0.9	0.9	0.4	0.9	0.8	107.09	100.50
0.9	0.9	0.4	0.9	0.9	113.16	100.39
0.9	0.9	0.6	0.1	0.7	101.25	100.62
0.9	0.9	0.6	0.1	0.8	107.09	100.50
0.9	0.9	0.6	0.1	0.9	113.16	100.39
0.9	0.9	0.6	0.9	0.1	113.16	100.39
0.9	0.9	0.6	0.9	0.2	107.09	100.50

0.9	0.9	0.6	0.9	0.3	101.25	100.62
0.9	0.9	0.7	0.1	0.6	101.25	100.62
0.9	0.9	0.7	0.1	0.7	113.16	100.39
0.9	0.9	0.7	0.1	0.8	126.03	100.16
0.9	0.9	0.7	0.2	0.9	102.92	100.14
0.9	0.9	0.7	0.8	0.1	102.92	100.14
0.9	0.9	0.7	0.9	0.2	126.03	100.16
0.9	0.9	0.7	0.9	0.3	113.16	100.39
0.9	0.9	0.7	0.9	0.4	101.25	100.62
0.9	0.9	0.8	0.1	0.6	107.09	100.50
0.9	0.9	0.8	0.1	0.7	126.03	100.16
0.9	0.9	0.8	0.2	0.8	108.53	100.02
0.9	0.9	0.8	0.8	0.2	108.53	100.02
0.9	0.9	0.8	0.9	0.3	126.03	100.16
0.9	0.9	0.8	0.9	0.4	107.09	100.50
0.9	0.9	0.9	0.1	0.6	113.16	100.39
0.9	0.9	0.9	0.2	0.7	102.92	100.14
0.9	0.9	0.9	0.8	0.3	102.92	100.14
0.9	0.9	0.9	0.9	0.4	113.16	100.39

Appendix-B

Fortran Code

```
! FORTRAN CODE USED IN SIMULATION STUDY (FILE: OS2016.F95)
      USE NUMERICAL_LIBRARIES
      IMPLICIT NONE
      INTEGER II, NN
      REAL P, T, PI, TH11,TH10,TH01,TH00,DENO1,VAROS
      REAL PAYY, PAYN, PANY, PANN
      REAL W1,W2, PY1, PY2, TH11S,TH10S,TH01S,TH00S
      REAL PAYYS, PAYNS, PANYS, PANNS
      REAL TERM1, TERM2, TERM3, TERM4, DENO2, VARL
      REAL RE,RP,PRMAX1,PRMAX2,PROP
      REAL A1, A2, B1, B2, C1, C2, D1, D2
      CHARACTER*20 OUT_FILE
      CHARACTER*20 IN_FILE
      WRITE(*,'(A)') 'NAME OF THE OUTPUT FILE'
      READ(*,'(A20)') OUT_FILE
      OPEN(42, FILE=OUT_FILE, STATUS='UNKNOWN')
      II = 0
      NN = 0
      P = 0.7
      T = 0.7
      DO 2222 PI = 0.1, 0.9, 0.1
      TH11 = P*T*PI + (1-PI)*(1-P)*(1-T)
```

```
      TH10 = P*(1-T)*PI + (1-P)*T*(1-PI)
      TH01 = (1-P)*T*PI + P*(1-T)*(1-PI)
      TH00 = (1-P)*(1-T)*PI + P*T*(1-PI)
      PAYY = P*T*PI/TH11
      PAYN = P*(1-T)*PI/TH10
      PANY = (1-P)*T*PI/TH01
      PANN = (1-P)*(1-T)*PI/TH00
      PRMAX1 = MAX(PAYY,PAYN,PANY,PANN)
      DENO1=(P+T-1)**2/TH11+(P-T)**2/TH10+(T-P)**2/TH01
     1 +(1-P-T)**2/TH00
      VAROS = 1.0/DENO1
      DO 3333 W1 = 0.1, 0.95, 0.1
      DO 3333 W2 = 0.1, 0.95, 0.1
      DO 3333 PY1 = 0.1, 0.95, 0.1
      DO 3333 PY2 = 0.1, 0.95, 0.1
      TH11S = PI*P*(W1*PY1+(1-W1)*(1-PY1))
     1 + (1-PI)*(1-T)*(W2*PY2+(1-W2)*(1-PY2))
      TH10S = PI*P*(W1*(1-PY1)+(1-W1)*PY1)
     1 + (1-PI)*(1-T)*(W2*(1-PY2)+(1-W2)*PY2)
      TH01S = PI*(1-P)*(W1*PY1+(1-W1)*(1-PY1))
     1 + (1-PI)*T*(W2*PY2 + (1-W2)*(1-PY2))
      TH00S = PI*(1-P)*(W1*(1-PY1)+(1-W1)*PY1)
     1 + (1-PI)*T*(W2*(1-PY2)+(1-W2)*PY2)
      PAYYS = PI*P*(W1*PY1+(1-W1)*(1-PY1))/TH11S
      PAYNS = PI*P*(W1*(1-PY1)+(1-W1)*PY1)/TH10S
      PANYS = PI*(1-P)*(W1*PY1+(1-W1)*(1-PY1))/TH01S
       PANNS = PI*(1-P)*(W1*(1-PY1)+(1-W1)*PY1)/TH00S
       PRMAX2 = MAX(PAYYS,PAYNS,PANYS,PANNS)
       A1 = P*(W1*PY1 + (1-W1)*(1-PY1))
       A2 = (1-T)*(W2*PY2 +(1-W2)*(1-PY2))
       B1 = P*(W1*(1-PY1)+(1-W1)*PY1)
       B2 = (1-T)*(W2*(1-PY2)+(1-W2)*PY2)
       C1 = (1-P)*(W1*PY1 +(1-W1)*(1-PY1))
       C2 = T*(W2*PY2+(1-W2)*(1-PY2))
       D1 = (1-P)*(W1*(1-PY1)+(1-W1)*PY1)
       D2 = T*(W2*(1-PY2)+(1-W2)*PY2)
       TERM1 = (A1-A2)**2/TH11S
       TERM2 = (B1-B2)**2/TH10S
       TERM3 = (C1-C2)**2/TH01S
       TERM4 = (D1-D2)**2/TH00S
        DENO2 = TERM1+TERM2+TERM3+TERM4
        VARL = 1.0/DENO2
        RE = VAROS*100/VARL
        RP = PRMAX1*100/PRMAX2
        IF((RE.GT.100).AND.(RP.GE.100))THEN
       WRITE(42,101)P,T,PI,W1,W2,PY1,PY2,RE,RP,PRMAX1,PRMAX2
101    FORMAT(2X,7(F7.3,1X),1X,2(F9.2,2X),1X,2(F7.4,2X))
       II = II+1
       ENDIF
        NN = NN+1
3333   CONTINUE
```

```
2222  CONTINUE
7777  CONTINUE
      PROP = DBLE(II)/DBLE(NN)
      WRITE(42,102)NN,II,PROP
102   FORMAT(2X,I9,2X,I9,2X,F9.5)
      STOP
      END
```