An Application of TOPSIS Method to Rank the Instructors Based on Students' Performances

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Abstract
Multiple Criteria Decision Making (MCDM) has recently been recognized as an efficient statistical technique to describe situations where there is a need for integration of the results of different studies to make an overall judgement. Many case studies and applications are available covering different domains of MCDM methods. In this study, we apply TOPSIS method—one of the most classical MCDM methods that was first developed by Hwang and Yoon—to rank the instructors based on several criteria, such as, students' attendance, quiz, project, and exam. Data were collected from an introductory statistics course taught at IUPUI. Twelve sections of the same course were offered and taught by the three instructors for the academic year 2014-15. The course was run under the supervision of a course coordinator, and instructors used the same standard PowerPoint lecture notes and followed the same evaluation criteria. Due to the variations among the scores under each evaluation criteria, we obtained entropy weights of these criteria and then incorporated into the TOPSIS technique to calculate an overall 'composite index' for the instructors to arrive at their individual rankings.

Key Words: Multiple Criteria Decision Making, Entropy Method, Evaluation Criteria, Composite Index

1. Introduction

STAT 30100 is an introduction to statistical methods course offered in the Department of Mathematical Sciences at Indiana University-Purdue University Indianapolis (IUPUI). This course mainly introduces statistical methods with applications to diverse fields and an emphasis on understanding and interpreting standard techniques. Topics include: data analysis for one and several variables, design of samples and experiments, basic probability, sampling distributions, confidence intervals and hypothesis tests for means and proportions, and correlation and regression. In principal, the goal of this course is to help students develop the ability for statistical thinking that is necessary to formulate research questions, to collect or identify appropriate data, to select correct statistical methods, to process the data to gain insight, and to use that insight to make informed decisions.

Students from different backgrounds and disciplines are enrolled in this course every semester. Teaching an introductory statistics course to such a diverse population is always a challenging task. Some students find this course very difficult and thus, worry about passing the course with a good grade. At times, students discuss this course with students who already taken this course before they enroll in a particular section. On occasion, students try to choose an instructor for their class that is most capable and professional teacher. Sometimes a teacher with outstanding academic record may not teach well in an introductory level course. The purpose of this paper is to apply *The Technique for Order of Preference by Similarity to Ideal Solution* (TOPSIS) method, one of the Multiple Criteria Decision Making (MCDM) techniques advocated by Hwang and Yoon (1981), Zeleny (1982), and Yoon and Hwang (1995), to rank the instructors based on students' performances, such as students' attendance, quiz, project, and exam grades.

The paper is organized as follows. In Section 2, we start with a brief description of the computational algorithm underlying the TOPSIS method in a theoretical framework. We introduce the dataset and apply the TOPSIS method to rank the instructors in Section 3. We close the paper with some remarks in Section 4.

2. TOPSIS Method

MCDM has recently been recognized as an efficient statistical technique to describe situations where there is a need for integration of the results of different studies to make an overall judgement. Many case studies and applications are available covering different domains of MCDM methods. One of the most classical MCDM methods was first developed by Hwang and Yoon (1981), called the TOPSIS method, using the intuitive principle that the best alternatives should have the shortest distance from the ideal alternative and the farthest distance from the negative-ideal alternative. This method along with another less popular ELECTRE method thoroughly is discussed in Hwang and Yoon (1981), Zeleny (1982), and Yoon and Hwang (1995). The main reason of choosing TOPSIS method among all the other simple MCDM methods is that the decisions made by TOPSIS maximize the profit and minimize the harm, just like a wise businessman (see, for example, Hwang and Yoon (1981), Pakpour *et al*. (2013)).

Assume there are '*m*' instructors (or alternatives) and there are '*n*' evaluation criteria (or attributes) to evaluate the instructors. Each instructor is judged by 8 evaluation criteria that include students' attendance, quiz, MLP HW, Project, Exam 1, Exam 2, Exam 3, and Final exam.

We denote by $X = ((x_{ij}))$ the positive-valued score matrix of order $m \times n -$ representing the instructors as rows of the matrix *X* and the evaluation criteria as the columns of the matrix *X*. In order that an instructor is adjudged the best with respect to a specific evaluation criterion, it is tacitly assumed that the score for this particular instructor has to *exceed* those of all others in the list. Note that the objective of the study is to arrive at an 'over-all' ranking of the instructors, by taking into account their performance across all the evaluation criteria. In fine, one has to ensure that all the scores for each evaluation criterion have the same interpretation in terms of 'max-to-min' going hand-in-hand with 'best-to-worst'. At times, the *X*-matrix is also termed as 'Decision Matrix', see Table 1.

It is clear that for one single evaluation criterion, the ranking of the instructors is trivial. Also as and when all the criteria values exhibit same relative positions of different instructors, the solution is easy to arrive at. Non-trivial situations arise when there are 'wave-like' patterns in the data and this is most expected scenario in practice with real data.

One natural and simple-minded approach has been to work out the average score for each instructor—by averaging the scores across all the evaluation criteria. That means, we simply compute the row averages in the *X*-matrix of scores and use them for ranking of instructors. There are obvious limitations to this approach since it does not take into account the variations among the scores [of different instructors] under each evaluation criterion. It deals with one method at a time. Apart from this, the point to be noted is that while we are working out the average score, we are assuming that all the evaluation criteria are equally important and hence they possess the same weight. This has been a point of concern to scientists and data analysts who have worked out a solution to this problem. Naturally, we should call upon these 'subject experts' and utilize their knowledge in ascertaining relative weights of the different evaluation criteria. Data-driven techniques have been suggested in the literature if we do not access to these experts' inputs. One such common technique is based on the Shannon Entropy Measure (Shannon, 1948), nicely explained in (Pakpour *et al*., 2013). We will discuss and apply the Shannon Entropy Measure for the evaluation of the weights of different evaluation criteria. Once the weights of this criteria is determined using the Shannon Entropy Measure, then we incorporate it into the TOPSIS technique to calculate an overall 'composite index' for the instructors to arrive at their individual rankings.

In general, the TOPSIS method evaluates the following decision matrix which contains *m* alternatives $A_1, A_2, \dots, A_m$ associated with *n* attributes or criteria $C_1, C_2, \dots, C_n$, x_{ij} is the numerical outcome of the i^{th} alternative with respect to the j^{th} criterion, and w_j is the weight of criterion C_j.

Table 1: Decision matrix in MCDM

Alternatives	Criteria (Weights	C_1 w_1	C_2 w_2	C_3 w_3	$\cdots$ $\cdots$	C_n w_n)
A_1		x_{11}	x_{12}	x_{13}	$\cdots$	x_{1n}
A_2		x_{21}	x_{22}	x_{23}	$\cdots$	x_{2n}
A_3		x_{31}	x_{32}	x_{33}	$\cdots$	x_{3n}
$\vdots$		$\vdots$				
A_m		x_{m1}	x_{m2}	x_{m3}	$\cdots$	x_{mn}

We consider the following steps to calculate the entropy weight:

Step 1. Transferring the decision matrix to the normalized mode:

In order to compute the entropy measure for the j^{th} criterion, the related values in the decision matrix are first normalized as p_{ij} :

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad \text{for } \; i = 1,2,3,\dots,m \; \text{ and } \; j = 1,2,3,\dots,n \tag{1}$$

Step 2. Calculating the entropy of dataset for each criterion:

In this step, the entropy of the j^{th} criterion, e_j, is calculated as follows:

$$e_j = -\alpha \sum_{i=1}^{m} p_{ij} \ln p_{ij} \qquad \text{for} \quad j = 1,2,3,\ldots,n \tag{2}$$

where, α represents a constant: $\alpha = 1/\ln(m)$, which guarantees that $0 \le e_j \le 1$.

Next, the operation of subtraction is used to measure the degree of diversity relative to the corresponding anchor value (unity), d_j, using the following formula:

$$d_j = 1 - e_j \qquad \text{for} \quad j = 1,2,3,\ldots,n \tag{3}$$

<u>Step 3. Defining criteria weights:</u>

The entropy weight $W = (w_1, w_2, \ldots, w_n)$ is calculated using

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \qquad \text{for} \quad j = 1,2,3,\ldots,n \tag{4}$$

Once the weights are chosen using the entropy method, these weights are then incorporated into the so-called TOPSIS method to calculate an overall score. The algorithm of this technique is summarized as follows:

i) Construct the normalized decision matrix *R*:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad \text{for} \quad i = 1,2,3,\ldots,m \ \text{ and } \ j = 1,2,3,\ldots,n \tag{5}$$

ii) Construct the weighted the normalized decision matrix *V*:

$$v_{ij} = r_{ij} \times w_j \qquad \text{for} \quad i = 1,2,3,\ldots,m \ \text{ and } \ j = 1,2,3,\ldots,n \tag{6}$$

iii) Determine the "Positive-ideal Row" (IDR) that one with the largest observed value for each column:

$$IDR = (\max_i v_{i1}, \max_i v_{i2}, \ldots, \max_i v_{in}) = (v_1^+, v_2^+, \ldots, v_n^+) \ \text{ for } \ i = 1,2,3,\ldots,m \tag{7a}$$

Similarly, the "Negative-ideal Row" $(NIDR)$ that one with the smallest observed value for each column:

$$NIDR = (\min_i v_{i1}, \min_i v_{i2}, \ldots, \min_i v_{in}) = (v_1^-, v_2^-, \ldots, v_n^-) \ \text{ for } \ i = 1,2,3,\ldots,m \tag{7b}$$

iv) Measure the distance, d_i^+ for $i = 1,2,3,\ldots,m$, of each alternative from the positive ideal one:

$$d_i^+ = \left[\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2 \right]^{\frac{1}{2}} \quad \text{for } i = 1,2,3\ldots,m \qquad 8(a)$$

Similarly, measure the distance, d_i^- for $i = 1,2,3,\ldots,m$, of each alternative from the negative ideal one:

$$d_i^- = \left[\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2 \right]^{\frac{1}{2}} \text{for } i = 1,2,3\ldots,m \qquad (8b)$$

The distance measures used in equations 8a and 8b are referred to as 'Euclidian distance' or 'Euclidian Norm', denoted by L_2.

v) Calculate the relative closeness of alternatives to ideal solution by computing what is known as 'Composite Index [CI]':

$$CI_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad \text{for } i = 1,2,3,\ldots,m \qquad (9)$$

Where $0 \le CI_i \le 1$. These composite indices are used for final ranking of the methods, the rule being: max–to–min for ranks 1–to–*m*.

3. Data and Results

Twelve sections of the same course were offered and taught by three instructors for the academic year 2014-15. The course was run under the supervision of a course coordinator, and the instructors used the same standard PowerPoint lecture notes and followed the same evaluation criteria. We collected the average score for each evaluation criteria for each instructor for the academic year 2014-15. If an instructor taught more than one section of the course during the academic year, we consider the average score for all these sections. The final dataset is shown in the following Table 2. Due to the privacy, we kept the instructor names anonymous and labeled them as Instructor I, II, and III.

Table 1: Students' evaluation criteria by Instructors

Instructor	*Attendance [45]*	*Quiz [80]*	*HW [100]*	*Project [75]*	*Exam1 [100]*	*Exam2 [100]*	*Exam3 [100]*	*Final [200]*
I	42.3	61.0	85.8	60.9	77.5	78.2	75.5	142.5
II	41.2	65.1	82.6	70.6	80.9	79.8	80.1	145.6
III	40.6	65.4	80.2	69.0	81.2	78.7	79.3	144.0

We consider the data displayed in Table 2 is as a given decision matrix to the MCDM method where we can apply the TOPSIS method. Before applying the TOPSIS-MCDM technique, we calculate the entropy weights for all eight criteria. After normalizing the decision matrix using equation (1), we calculate the indices e_j and d_j using formulas provided in equations (2) and (3), respectively. Finally, we use these indices to calculate the entropy weights for all criteria using equation (4). Table 3 summarizes the results of all necessary indices including the weights for all eight criteria.

Table 3: Calculating the entropy (e_j), degree of diversity (d_j) and criteria weight (w_j) of data for each decision criterion

Indices	*Attendance [45]*	*Quiz [80]*	*HW [100]*	*Project [75]*	*Exam1 [100]*	*Exam2 [100]*	*Exam3 [100]*	*Final [200]*
e_j	0.99986	0.99954	0.99966	0.99815	0.99980	0.99997	0.99969	0.99996
d_j	0.00014	0.00046	0.00034	0.00185	0.00020	0.00003	0.00031	0.00004
w_j	0.04274	0.13559	0.10119	0.54926	0.05965	0.01014	0.09102	0.01042

The weights in Table 3 obtained by entropy method are then incorporated into the TOPSIS technique (equations 5-9) to calculate an overall 'composite score' for each instructor. The step-by-step calculations of the TOPSIS method are shown in the Tables 4-6. The final rankings of all instructors are the descending order of the TOPSIS score CI_i , which are shown in the last column of Table 6. According to Table 4, the assessment result of TOPSIS based on entropy method is Instructor II>Instructor III> Instructor I.

Table 4: Summary of normalized decision matrix data for TOPSIS method using eight decision criteria for each instructor.

Instructor	*Attendance [45]*	*Quiz [80]*	*HW [100]*	*Project [75]*	*Exam1 [100]*	*Exam2 [100]*	*Exam3 [100]*	*Final [200]*
I	0.59092	0.55134	0.59734	0.52525	0.56014	0.57209	0.55632	0.57112
II	0.57475	0.58860	0.57533	0.60883	0.58479	0.58408	0.59064	0.58355
III	0.56611	0.59125	0.55873	0.59451	0.58674	0.57582	0.58452	0.57732

Table 5: Summary of the weighted normalized decision matrix data for TOPSIS method using eight decision criteria for each instructor.

Instructor	*Attendance [45]*	*Quiz [80]*	*HW [100]*	*Project [75]*	*Exam1 [100]*	*Exam2 [100]*	*Exam3 [100]*	*Final [200]*
I	0.02525	0.07475	0.06044	0.28850	0.03341	0.00580	0.05064	0.00595
II	0.02456	0.07981	0.05822	0.33441	0.03488	0.00592	0.05376	0.00608
III	0.02419	0.08017	0.05654	0.32654	0.03500	0.00584	0.05320	0.00601
*IDR	0.02525	0.08017	0.06044	0.33441	0.03500	0.00592	0.05376	0.00608
+NIDR	0.02419	0.07475	0.05654	0.28850	0.03341	0.00580	0.05064	0.00595

*Positive ideal row/solution; +Negative ideal row/solution

Table 6: Summary of the positive d_i^+ and negative distance d_i^- and the rank based on final TOPSIS scores for each instructor

Instructor	d_i^+	d_i^-	CI_i	Rank
I	0.04636111	0.004048009	0.080303115	3
II	0.002361881	0.046348707	0.951511955	1
III	0.008868242	0.038541653	0.812945341	2

4. Conclusion

Choice of an instructor for the STAT 30100 course is an important decision for many students and choosing the best instructor might help them pass this course successfully. In this paper, the TOPSIS multi criteria decision making technique was discussed for ascertaining the over-all rankings of instructors when judged against several alternative decision criteria. Eight main evaluation criteria indicating students' performances were used to rank the instructors. Note that in this paper we have considered weight using the entropy method only; however, there are several other methods proposed in the literature that one can use to calculate the weight (see, for example, Zou *et al*., 2006; Lertprapai and Tiensuwan, 2009; Cascales and Lamata, 2012). It would be interesting to see whether the ranks differ or not using not only TOPSIS method but also some other MCDM methods.

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