Time Series Models of Supply Chain Inventory Data

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Abstract

The main objective of the present paper is to demonstrate the power of statistical time series for analyzing supply chain inventory data. Our specific goal addresses two critical management needs that relate to the development of training tools for new corporate hires to our audit management team and providing quantitative methodologies for capturing or recognizing the underlying nature of supply chain inventory data.

This current effort will involve developing several robust autoregressive integrated moving average (ARIMA) time series models that describe the dynamical behavior of our inventory data. We will highlight the efficacy of the resulting models using generated forecast estimates. An array of classical statistical metrics will also be used to evaluate the sensitivity and stability of generated forecast estimates. In addition, we will provide suggestions about how such models may be used to improve audit verification, minimize total inventory cost and identify potential product shortages or demand change points.

Key Words: Time series, ARIMA, Supply chain inventory, Forecast estimates, Robust, Audit

1. Introduction

Supply chain inventory models address many business concerns, play a major role in the development of effective business strategies and impact profitability. Forecasts about such information are needed across the organization. The ability to model is essential to effective business management. It is the starting point for making decisions in an ever changing business climate that is affected by many external economic factors. The limited application of supply chain models is perhaps understandable because of the large expense associated with building complex, comprehensive forecast models. Forecasting techniques are classified into three basic categories - causal models, smoothing techniques or time series models. Causal models are derived using regression analysis, the smoothing techniques employ exponential fitting while the time series methods utilize Box-Jenkins strategies. The focus of the current inquiry is operational and thus we are interested in establishing correlations among events occurring over a narrowly defined business cycle. In the present study the appropriate Box-Jenkins methodologies will be used to devise robust time series model that are parsimonious, reliable and possess decidability.

Specifically, this paper addresses two major business issues regarding our clients inventory data. First, we are interested in identifying apparent trends within the dataset and addressing management's goal of devising a model that can forecast future inventory demands. The client supporting this study is in the procurement business and there are several advantages to be gained if one is able to predict demand cycle requirements with reasonable certainty and lead time. Typically, inventory has to be stored which means there are costs associated with space rental and any reduction in inventory storage cost directly improves the bottom line. Additionally, given the cyclic nature of the business, management wants to predict the number of employees to hire to conduct the inventory tasks and to use the resulting model as an inventory training tool to be used by these new employees.

1.1 Approach

The technical approach involves using Box-Jenkins statistical time series techniques to gleam possible patterns from the supplied inventory transaction data that covers a year of business purchases. This initial inquiry was to focus on developing an adequate time series model that represented monthly variation in inventory demand. The model was developed using only the daily electronic transactions data for a fiscal year period provided by management. An expressed aim was to discern whether monthly variations in transactions dollars are random or serially correlated in time. Our initial findings would be instrumental in our decision to expand our investigation and construct an appropriate time series for the supplied transaction data.

A time series plot of the running transactions data (in dollars) for only month five are shown in Figure 1A. The dollar values shown are adjusted to protect client identity. The data range is quite large and varies over three orders of magnitude. Positive transaction values represent inventory addition while negative values represent removal of items from inventory. A normal probability plot (Figure 1B) of the transaction dollars reveals that the upper tail of the data occurs at small dollar levels and suggests that filtering the data below a certain dollar threshold level would have negligible impact on the analysis. Clearly these transaction values skew the data.

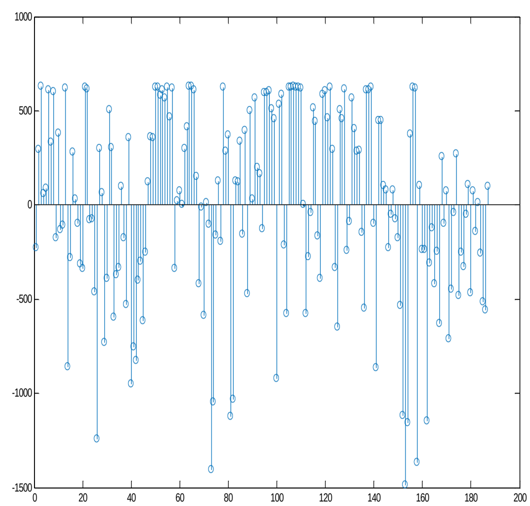

Figure 1A: Time Series Plot of Transaction Data for Month 5

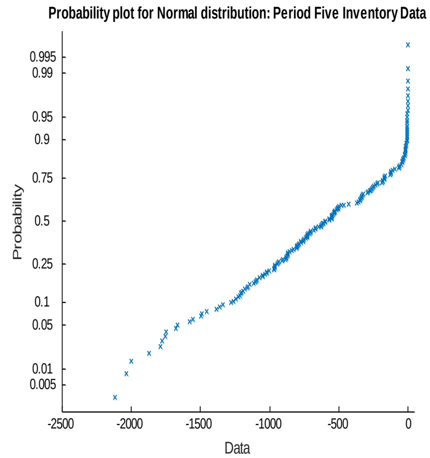

Figure 1B: Normal Probability Plot of the Transaction Dollars for Month 5

Figure 2 reveals that the filtered data appears to be normally distributed. A plot of the mean monthly transaction levels over a fiscal year is highlighted in Figure 3. A sinusoidal model was fitted to show the cyclic nature of the data and disclosed an outlier during period 8 of the fiscal year. Figure 4 exposed the large variation in the number of monthly transactions and showed that most of the transactions occur between periods 4 and 6.

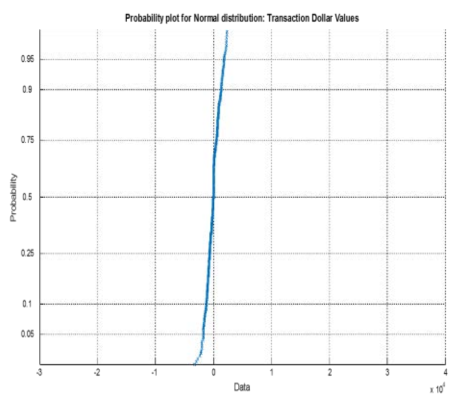

Figure 2: Normal Probability Plot of Filtered Transaction Data

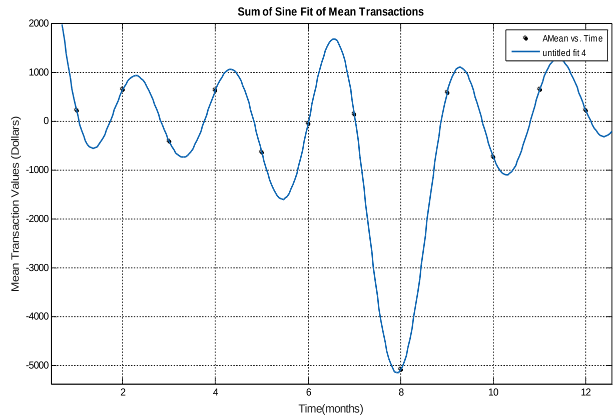

Figure 3: Mean monthly transaction levels over a fiscal year

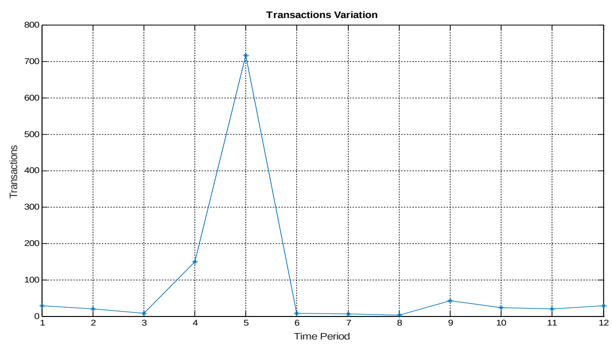

Figure 4: Number of transactions per month over a fiscal year

Our proposed supply chain modeling begins with the initial filtering of the original transaction data through a low pass device that removes transaction values below a critical absolute value. This preprocessing step separate the data into two streams. One stream is sent to an autoregressive integrated moving average (ARIMA) processor for analysis while the other is forwarded to a moving average process unit. After this processing, the outputs from these respective analyses are recombined into a single predictive transaction model for inventory flow/demand.

2. Results and Discussion

In the model development phase, the autocorrelation (ACF) and partial autocorrelation (PACF) plots were used to establish the respective order of the autoregressive and moving average terms. A careful review of the plots suggested that an ARIMA (1-0-1) model was appropriate. No seasonal differencing was required. The non-seasonal

ARIMA model (1-0-1) described in Table 1 consists of one autoregressive parameter and one moving average parameter both of order one. The Minitab (version 16) software package was used to estimate the model parameters. The p-values in Table 1 reveal that the only significant terms in the model are the autoregressive and moving average terms with p-values less that 0.05.

Table 1. ARIMA Model				
Final Estimates of Parameters				
Type	**Coefficient**	**SE Coefficient**	**T**	**P**
AR 1	0.3798	0.1652	2.30	0.022
MA 1	0.0259	0.1783	0.15	0.880
Constant	-469.90	32.93	4.27	0.000
Mean	-757.70	53.10		
Number of Observations = 250				
Residuals: SS=70566949 MS=285696 DF=247 (backforecasts excluded)				
Modified Box-Pierce (Ljung-Box) Chi-Square statistic				
Lag	12	24	36	48
Chi-Square	11.4	25.0	35.4	48.2
DF	9	21	33	45
p-value	0.247	0.247	0.356	0.343

A normal probability plot of the residuals from the fitted ARIMA model is shown in Figure 5. The plot is slightly skewed in both tails and the absolute value of most of the residuals is less that $1000 dollars. Future studies will focus on improving the ARIMA modeling.

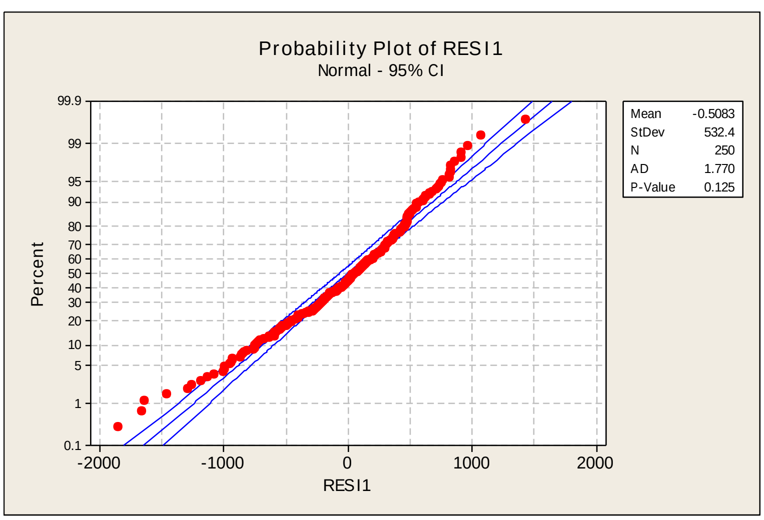

Figure 5: Normal Probability Plot of the ARIMA (1-0-1) Model Residuals

3. Conclusion

For the data supplied by management, the majority of transactions occurred during periods four through six. Removing transaction values below an absolute threshold limit yields a filtered population that is normally distributed. A major outlier in the mean transactional values is observed at month/period eight. An interesting finding in this analysis was the failure to identify the need for a seasonal term in this model. This can be explained in part due to the fact that most of the transactions occur during periods four through six. Additional fiscal year data is needed to test model efficacy.

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