

## Evacuation from a Room with two (contiguous) Exits

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### Abstract

Building regulations require that more than one exit should be available for a safe evacuation during a panic situation. The faster is slower effect is expected to occur close to each exit door. However, rooms having contiguous doors not always improve the evacuation performance. Our research examines the statistical behaviour of escaping pedestrians when two contiguous exits are available, but placed at different separation distances. We found that the evacuation time can be improved if the separation exceeds a threshold distance. This threshold distance is related to changes in the clogging dynamics close to the doors.

**Key Words:** Panic, evacuation, clogging, time delays

### 1. Introduction

Current regulations claim that if more than two doors are required for a room, the distance between two of them must be at least one-half or one-third of the room diagonal distance (OSHA, 2015; FBC, 2010). This leaves some space for placing the extra openings (*i.e.* those above two exits) at an arbitrary separation distance. Thus, it is possible to place a couple of doors on the same side of the room. The special case of two contiguous doors has been examined throughout the literature (Kirchner, 2002; Perez, 2002; Daoliang, 2006; Huan-Huan, 2015).

Kirchner and Schadschneider studied the pedestrians evacuation process through two contiguous doors using a cellular automaton model (Kirchner, 2002). The agents were able to leave the room under increasing panic situations. The evacuation time was found to be independent of the separation distance between doors. However, this conclusion was not in complete agreement with the investigation acknowledged by Perez *et al.* (Perez, 2002). These authors assert that the total number of pedestrians leaving the room per unit time slows-down for separation distances (between doors) smaller than the distance necessary to distinguish two independent groups of pedestrians, each one surrounding the nearest door. They identified the slow-down with a disruptive interference effect due to pedestrians crossing in each other's path.

Although the above results were obtained for very narrow doors (*i.e.* single individual width), further investigation showed that they also apply to doors allowing two simultaneous leaving pedestrians (Daoliang, 2006).

From the results reported by Huan-Huan *et al.* (Huan-Huan, 2015), the evacuation time depends on the total width of the openings (if both doors have the same width). But, for a fixed total width of the opening, it appears that the optimal location of the exits depends on

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the doors separation distance.

Our investigation focuses on symmetric configurations with equally sized doors. Alternatively to the above mentioned literature, we examine the evacuation dynamics by means of the Social Force Model (SFM). An overview of this model can be found in Section 2.

In Section 3 we describe the specific settings for the evacuation processes. The measurement conditions for the simulations can also be found there.

In Section 4 we examine the case of two separated doors. We explore the effect of increasing the separation distance  $d_g$  until the clogging areas close to each door become almost independent.

Section 5 resumes the pedestrians behavioural patterns, and its consequences on the evacuation performance, for the different door separation scenarios.

## 2.

### 2.1 The Social Force Model

The ‘‘social force model’’ (SFM) states that our tendency to avoid overcrowded environments acts as a repulsive force, changing our dynamics, although our desire to reach some target point. Both effects (repulsion and desire) operate as *social forces* in pedestrian dynamics. Additionally, friction between people (and walls) is also a very important issue in crowd dynamics. Thus, the three forces (repulsion, desire and friction) are present in the equation of motion for any individual

$$m_i \frac{d\mathbf{v}^{(i)}}{dt}(t) = \mathbf{f}_d^{(i)}(t) + \sum_j \mathbf{f}_s^{(ij)}(t) + \sum_j \mathbf{f}_g^{(ij)}(t) \quad (1)$$

where  $m_i$  is the mass of the pedestrian  $i$ , and  $\mathbf{v}_i$  is its corresponding velocity. The subscript  $j$  represents all other pedestrians (excluding  $i$ ) and the walls.  $\mathbf{f}_d$ ,  $\mathbf{f}_s$  and  $\mathbf{f}_g$  are the desire force, the social (repulsion) force and the friction (or granular) force, respectively. See Refs. (Helbing, 2000; Parisi, 2005 and 2007; Frank 2011 and 2015) for details.

The expression for each kind of forces are as follows

$$\left\{ \begin{array}{l} \mathbf{f}_d^{(i)}(t) = m_i \frac{\mathbf{v}_d^{(i)}(t) - \mathbf{v}_i(t)}{\tau} \\ \mathbf{f}_s^{(ij)} = A_i e^{(r_{ij} - d_{ij})/B_i} \mathbf{n}_{ij} \\ \mathbf{f}_g^{(ij)} = \kappa g(r_{ij} - d_{ij}) \Delta \mathbf{v}_{ij} \cdot \mathbf{t}_{ij} \end{array} \right. \quad (2)$$

where  $\mathbf{v}_d^{(i)}$  is the desired velocity for pedestrian  $i$ ,  $\mathbf{v}^{(i)}$  is the current velocity, and  $\tau$ ,  $A_i$ ,  $B_i$  and  $\kappa$  are fixed parameters. The magnitude  $r_{ij} = r_i + r_j$  is the sum of the pedestrian’s

radius, while  $d_{ij}$  corresponds to the inter-pedestrian distance. Further details on each parameter can be found in Refs. (Parisi, 2005 and 2007; Frank 2011 and 2015).

## 2.2 Clustering structures

The time delays during an evacuation process are related to clustering people as explained in Refs. (Parisi, 2005 and 2007). Groups of pedestrians can be defined as the set of individuals that for any member of the group (say,  $i$ ) there exists at least another member belonging to the same group ( $j$ ) in contact with the former. That is,

$$i \in \mathcal{G} \Leftrightarrow \exists j \in \mathcal{G} / d_{ij} < r_i + r_j \quad (3)$$

where  $\mathcal{G}$  corresponds to any set of individuals. This kind of structure is called a *human cluster*.

From all human clusters appearing during the evacuation process, those that are simultaneously in contact with the walls on both sides of the exit are the ones that possibly *block* the way out. Thus, we are interested in the minimum number of contacting pedestrians belonging to this *blocking cluster* that are able to link both sides of the exit. We call this minimalistic group as a *blocking structure*. Any blocking structure is supposed to work as a barrier for the pedestrians in behind.

## 2.3 The local pressure on the pedestrians

The pressure on a single pedestrian (say,  $i$ ) is defined as (see Helbing, 2000)

$$P_i = \frac{1}{2\pi r_i} \sum_{j=1}^{N-1} \mathbf{f}_s^{(ij)} \cdot \mathbf{n}_{ij} \quad (4)$$

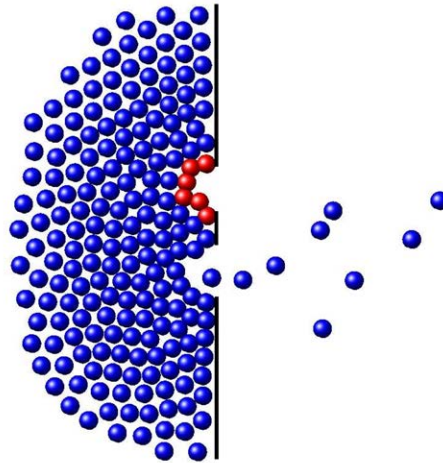
where  $\mathbf{n}_{(ij)}$  corresponds to the unitary vector pointing from the individual  $j$  to the individual  $i$ . Likewise,  $\mathbf{f}_s^{(ij)}$  are the forces acting on the individual  $i$  due to the other individuals. Recall that these forces point from any individual  $j$  to the individual  $i$ , and thus, the products  $\mathbf{f}_s^{(ij)} \cdot \mathbf{n}_{ij}$  are always positive.

## 3. Simulations

We simulated different evacuation processes for room sizes of 20 m  $\times$  20 m, 30 m  $\times$  30 m and 40 m  $\times$  40 m. The rooms had one or two exit doors on the same wall, as shown in Fig. 1. The doors were placed symmetrically from the mid position of the wall, in order to avoid corner effects. Both doors had also the same width.

The evacuation process ran for 3000 s or until 80% of the occupants left the room, whatever occurred first. All positions and velocities were sampled at time intervals of 0.1  $\tau$ . No re-entering mechanism was allowed.

The simulations ran from relaxed situations ( $v_d < 2$  m/s) to very stressing rushes ( $v_d = 6$  m/s). We registered the individuals positions and velocities for each evacuation process. Thus, we were able to compute the ‘‘social pressure’’ through out the process and



**Figure 1:** Snapshot of an evacuation process from a  $20\text{ m} \times 20\text{ m}$  room, with two doors. In red we can see a blocking structure around the upper door. The desired velocity was  $v_d = 4\text{ m/s}$ .

to trace the pedestrians behavioural pattern.

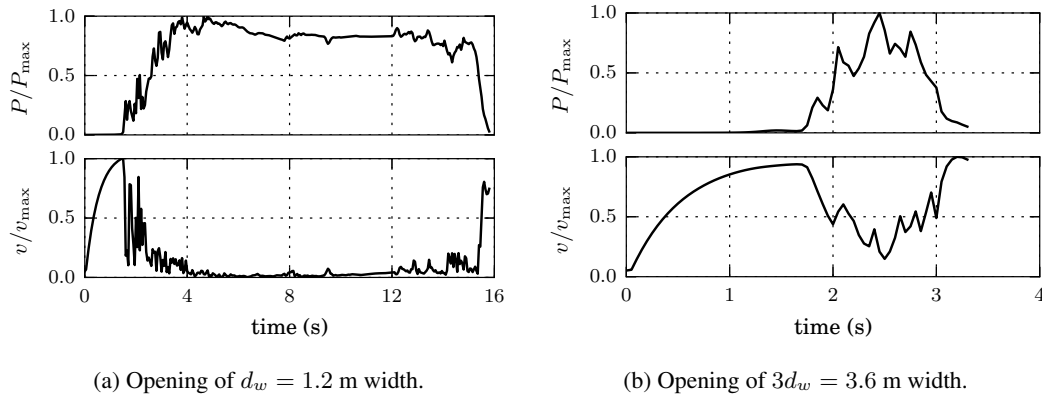
The simulations were supported by LAMMPS molecular dynamics simulator with parallel computing capabilities (Plimpton, 1995). The time integration algorithm followed the velocity Verlet scheme with a time step of  $10^{-4}\text{ s}$ . It was assumed that all the individuals had the same radius ( $r_i = 0.3\text{ m}$ ) and weight ( $m_i = 70\text{ kg}$ ). We ran 30 processes for each panic situation, in order to get enough data for mean values computation.

## 4. Results

### 4.1 The single door vs. wider openings

Fig. 2 illustrates on how the evacuation performance improves as the opening becomes wider. Fig. 2a corresponds to the single door ( $d_w = 1.2\text{ m}$ ), while Fig. 2b corresponds to a wider opening ( $3d_w = 3.6\text{ m}$ ), resembling a multi-leaf opening. Both figures represent the time evolution of a single pedestrian during an evacuation process. We can see the (normalized) pressure acting on the pedestrian and his (her) corresponding velocity. The pressure was computed as defined in Ed. (4).

The pedestrian represented in Fig. 2 increases his (her) velocity towards an asymptotic value at the beginning of the processes. This value corresponds to the desired velocity  $v_d = 4\text{ m/s}$ . But close to  $t = 2\text{ s}$ , the pedestrian suddenly stops because of the clogging around the exit. Clogging is also responsible for the pressure increase, as shown in both Fig. 2a and Fig. 2b. This can be checked over by means of Eq. (2) because when the velocity of the pedestrian vanishes, the desire force  $\mathbf{f}_d$  attains a maximum (in panic situations only).



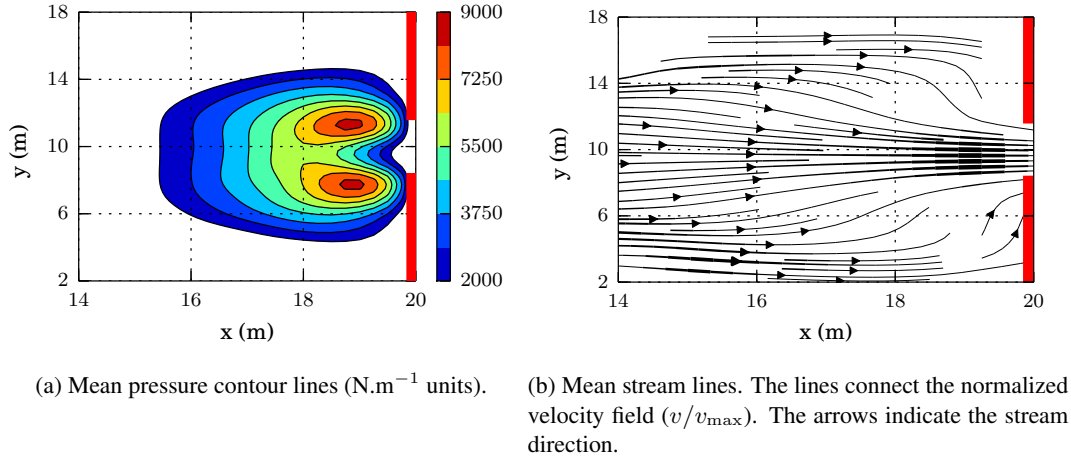
**Figure 2:** Normalized pressure and velocity on a single pedestrian during an evacuation process. Data was recorded from the the initial position at  $x = 12.35$  m and  $y = 8.45$  m, until the individual left the room ( $x > 20$  m). The pedestrians desired velocity was  $v_d = 4$  m/s. Two situations are shown: (a) evacuation through a single door of width  $d_w = 1.2$  m. (b) evacuation through an opening of  $3d_w = 3.6$  m.

The maximum pressure values  $P_{\max}$  in Fig. 2a and Fig. 2b are  $8550 \text{ N.m}^{-1}$  and  $6475 \text{ N.m}^{-1}$ , respectively. The corresponding mean pressure values (after the first 2 s) are 80% and 55% of the respective maximum values. This means that the mean pressure value for the  $3d_w$  situation is lower than the corresponding mean value for the  $d_w$  situation. That is, the wider opening seems to release pressure from time to time. Consequently, the stop-and-go processes are somehow different for the single door with respect to the the wider opening.

For a better understanding on how the pedestrians are (intermittently) released from high pressures in the wide opening situation, we pictured the whole scene into a pressure contour map and a mean stream path map for all the individuals. Fig. 3a shows the pressure levels ( $P_i$ ) for the clogging area. The warm colors are associated to high pressure values. Thus, the warm regions define the places where the pedestrians slow down most of the time. They are expected to get released only for short periods of time. On the contrary, the regions represented in cold colors (low mean pressure) are those where the individuals are able to get released for longer time periods.

Fig. 3b represents the mean stream lines during the evacuation process. It completes the stop-and-go picture since it exhibits the released paths for leaving the room. Notice that the stream lines pass through the low pressure regions. That is, it can be seen in Fig. 3b that the stream lines gather along the middle of the clogging area, where “cold” pressure colors can be found (cf. Fig. 3a). The “warm” pressure colors are placed on the sides of this region.

The above data lets us conclude that the widening of the single door increases the pedestrian’s flux. In the narrow situation (see Fig. 2a), the pedestrians experience a slow down. The corresponding time delays are associated to blocking structures (Parisi, 2005; Parisi, 2007) and causes the pressure acting on the nearby individuals to rise. However, as the opening widens, the pressure pattern changes qualitatively (see Fig. 3a), allowing the pedestrians in the middle of the clogging area to make a pathway to the exit. This pathway



**Figure 3:** Mean pressure and stream lines computed from 30 evacuation processes until 100 pedestrians left the room ( $20\text{ m} \times 20\text{ m}$  size). Data was recorded on a square grid of  $1\text{ m} \times 1\text{ m}$  and then splined to get smooth curves. The red lines at  $x = 20\text{ m}$  represent the walls on the right of the room. There is only one opening of  $3d_w = 3.6\text{ m}$  width (null separation distance between doors of width  $3d_w/2$ ). The pedestrian's desired velocity was  $v_d = 4\text{ m/s}$ .

corresponds to the breaking of the blocking structures.

## 4.2 Separated doors

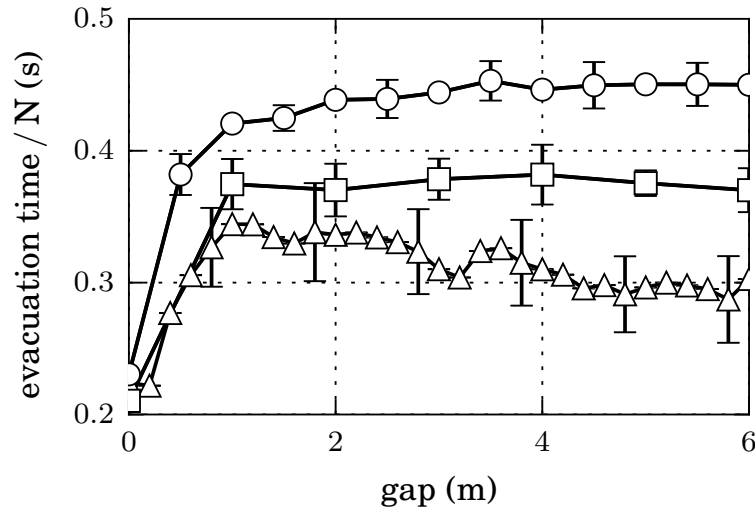
We will now analyze the case in which the evacuation process is through two doors, symmetrically placed on the same side of the room. We will explore the dependence of such a process on the doors separations. We will assume that each door width is  $d_w = 1.2\text{ m}$ .

Fig. 4 exhibits the mean evacuation time per pedestrian as a function of the separation distance (*i.e.* gap or  $d_g$ ). We divided the evacuation time by the total number of pedestrians for visualization reasons.

The results shown in Fig. 4 were not expected. The evacuation time settles to an asymptotic value for separation distances  $d_g > 5\text{ m}$ . The mean evacuation time becomes almost independent of the separation distances  $d_g$  despite that the clogging areas around the doors might still overlap.

Fig. 4 also shows that the slope not always changes sign at  $d_g \simeq 1\text{ m}$ . Furthermore, as the number of pedestrians is increased for  $d_g > 1\text{ m}$ , the evacuation time slope raises to positive values. The greater the number of pedestrians, the worse the evacuation time (per individual). This appears to occur for  $d_g > 1\text{ m}$ , regardless of the crowd size. That is, according to Fig. 4, there exists a separation distance value  $d_g \simeq 1\text{ m}$  where the evacuation slope changes sharply to negative or positive values (for  $d_g > 1\text{ m}$ ). This phenomenon has not been studied in the literature, to our knowledge.

We can resume the results in Fig. 4 in the following way: the evacuation time rises when the doors separation increases from a wide opening (null separation distance) to the distance  $d_g \simeq 1\text{ m}$ . At this gap, the evacuation time slope changes notably, entering a much



**Figure 4:** Mean evacuation time per total number of pedestrians that left the room ( $N$ ), as a function of the doors separation distance. Mean values were computed from 30 evacuation processes. Each door was  $d_w = 1.2$  m width for non-vanishing gaps. The null gap means a single door of  $2d_w$  width. Three situations are shown:  $\triangle$  corresponds to the  $20 \times 20$  m room when 160 pedestrians left the room,  $\square$  corresponds to  $30 \times 30$  m room when 530 pedestrians left the room, and  $\circ$  corresponds to  $40 \times 40$  m room when 865 pedestrians left the room. The desired velocity was  $v_d = 4$  m/s.

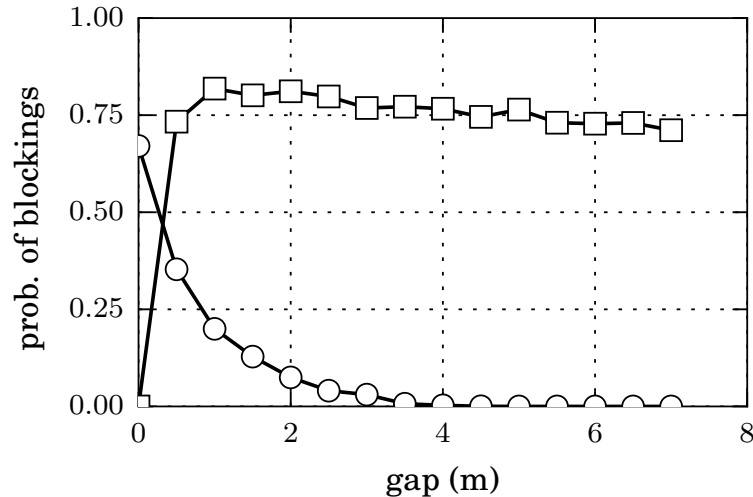
slowly varying regime towards an asymptotic value (for  $d_g \gg 1$  m). The former can be identified as a regime for small values of  $d_g$ , while the latter is valid for moderate to large values of  $d_g$ . The fact that a sharp change occurs at  $d_g \simeq 1$  m, no matter the crowd size, suggests that both regimes are somehow different in nature. This moved us to explore the two regimes separately.

#### 4.2.1 The regime for $d_g < 1$ m

We first examined the *blocking probability* for this regime, that is, the ratio between the time that each door remains blocked (due to a *blocking cluster*) with respect to the total evacuation time. Fig. 5 presents two kinds of blockings: the simultaneous blocking of both doors, and the blocking of a single door (say, the one on the left). The former connects the left most wall with the right most wall, but does not contact the separation wall in the middle of the walls. The latter connects the walls on both sides of the selected door (say, the one on the left).

According to Fig. 5, the single door blockings are not relevant until  $d_g \simeq 1$  m, while the simultaneous blockings weaken as the gap (separation distance  $d_g$ ) increases.

After a close examination of the data and the evacuation animations, we realized that the single door blockings hold if the gap is large enough to accommodate at least two pedestrians. That is, any blocking structure enclosing a single door can hold for some time if the pedestrians at the end of the structure (and in contact with the walls) do hardly leave the structure. Two pedestrians are needed at the gap wall to ensure that both doors remain blocked.



**Figure 5:** Ratio between time steps including blocking structures and the total number of time steps for 30 evacuation processes, as a function of the doors separation distance. The room size was  $20 \times 20$  m with 225 occupants. Each door was  $d_w = 1.2$  m width for non-vanishing gaps. The null gap means a single door of  $2d_w$  width. The desired velocity was  $v_d = 4$  m/s.  $\circ$  corresponds blocking structures connecting both the left side wall of the left door with the right side wall of the right door (see text for details).  $\square$  corresponds to blocking structures connecting both sides of a single door (see text for details).

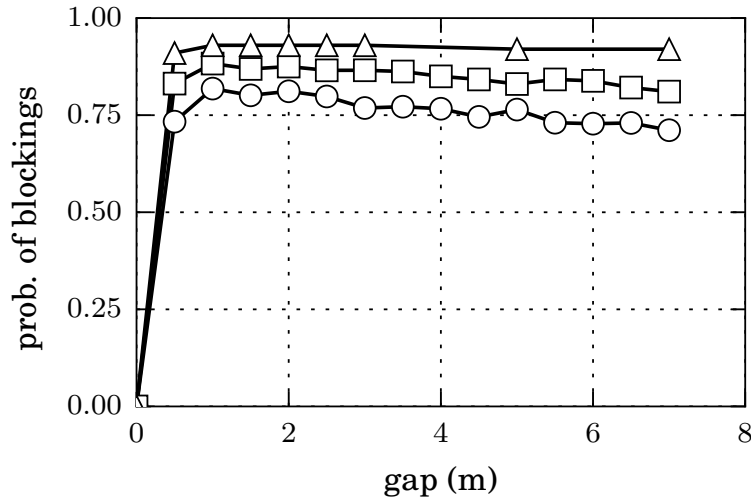
We conclude from the analysis of small gaps ( $d_g < 1$  m) that a door separation distance roughly equal to two pedestrian widths is critical. This distance allows persistent single door blockings. Small distances (close to the null separation) do not actually allow single door blockings to hold for long time. Thus, the role of  $d_g = 2r_{ij}$  (two pedestrian's width) is decisive to move the evacuation process from one regime to another.

#### 4.2.2 The regime for $d_g > 1$ m

Fig. 5 shows that the single door blockings (see Section 4.2.1) remains around 75% of the total evacuation time for  $d_g > 1$  m (225 individuals in the room). We also computed this magnitude for situations with increasing number of individuals (see Fig. 6). The probability of single door blockings approaches unity as the crowd size increases. This means, according to our definition of blocking probability, that the blocking time raises as the number of individuals increases. The gap distance, however, does not play a significant role for  $d_g > 1$  m.

There is a noticeable difference between the evacuation time shown in Fig. 4 and the blocking probability exhibited in Fig. 6. Fig. 4 presents the evacuation time for three different room sizes and increasing number of pedestrians. The slope of the evacuation curve is negative for the  $20 \times 20$  m room, it vanishes for the  $30 \times 30$  m situation and it becomes slightly positive for the  $40 \times 40$  m room (for  $d_g > 1$  m). Thus, as the number of pedestrians increases, the slope of the evacuation time changes sign. However, this does not occur for the blocking probability (see Fig. 6). The slope of the blocking probability remains always negative for an increasing number of pedestrians (and desire velocities). Therefore, the





**Figure 6:** Ratio between time steps including blocking structures and the total number of time steps for 30 evacuation processes, as a function of the doors separation distance. The only blocking structures considered were those connecting both sides of one single door (see text for details). Each door was  $d_w = 1.2$  m width for non-vanishing gaps. The null gap means a single door of  $2d_w$  width. Three scenarios are shown: ○ corresponds to the room of size  $20 \times 20$  m with 225 occupants and a desired velocity of  $v_d = 4$  m/s. □ corresponds to the room of size  $20 \times 20$  m with 225 occupants and a desired velocity of  $v_d = 6$  m/s. △ corresponds to the room of size  $40 \times 40$  m with 961 occupants and a desired velocity of  $v_d = 4$  m/s.

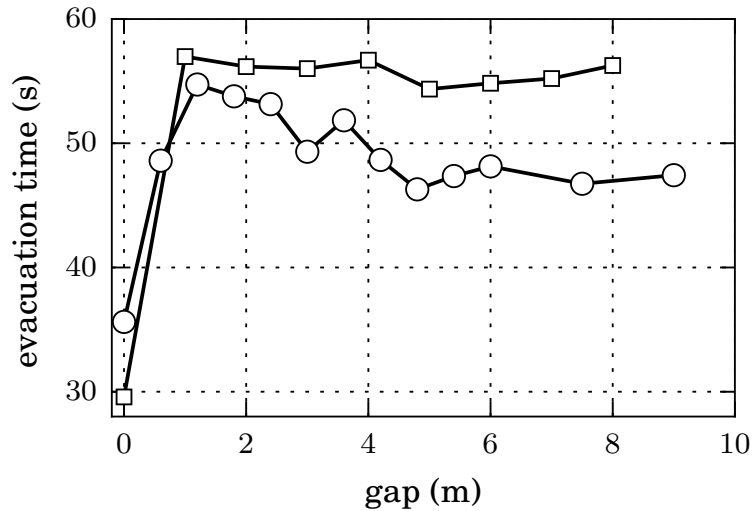
changes in the slope observed in Fig. 4 cannot be explained by changes in the blocking time (*i.e.* blocking probability).

We checked the pressure patterns for the situations represented in Fig. 4. We came to the conclusion that since the evacuation slope in Fig. 4 changes with an increasing number of individuals, the whole bulk should be involved in this phenomenon.

Notice that the pressure of the bulk can vary in two possible ways: if the desire force of the individuals (*i.e.* anxiety levels) changes, or, if the crowd size changes. In the case that the anxiety levels raise, the pedestrians push harder to get out, increasing the individuals social force (social repulsion). If the number of pedestrians  $N$  is increased, the total repulsion is also increased. An inspection of Eq. (4) shows that both situations increase the individuals pressure  $P_i$ .

Fig. 4 exhibits the evacuation time for an increasing number of pedestrians. But, an increase in the pedestrians anxiety level should resemble similar results, if the above reasonings are true. Fig. 7 shows the evacuation time as a function of the separation distance for two different desired velocities. As expected, the sharp change in the slope occurs around  $d_g = 2r_{ij}$ . Also the slope changes as the desired velocity ( $v_d$ ) is increased (*i.e.* higher anxiety level). This confirms that the social pressure is responsible the slope behaviour shown in Fig. 4.

We conclude from the analysis of large gaps ( $d_g > 1$  m) that the evacuation time is controlled by the social pressure in the bulk. The crowd size and the desired velocity  $v_d$



**Figure 7:** Mean evacuation time for 225 pedestrians (room of  $20 \times 20$  m size) as a function of the doors separation distance. Mean values were computed from 30 evacuation processes until 160 pedestrians left the room. Each door was  $d_w = 1.2$  m width for non-vanishing gaps. The null gap means a single door of  $2d_w$  width. ○ corresponds to pedestrians with desired velocity of  $v_d = 4$  m/s. □ corresponds to pedestrians with desired velocity of  $v_d = 8$  m/s.

affects the pressure acting on the pedestrians. For  $d_g > 5$  m in our simulations, the evacuation time is very close to the corresponding asymptotic value, although the bulks around each door are not completely independent. This means that the mixing of both crowds (that is, the fact that the bulks are in contact) do not affect strongly the evacuation performance.

## 5. Conclusions

We examined in detail the evacuation of pedestrians for the situation where two contiguous doors are available for leaving the room. Throughout Section 4 we presented results on the evacuation performance under high anxiety levels and increasing number of pedestrians. Both conditions exhibit the novel result that a worsening in the evacuation time exists as the door separation distance  $d_g$  increases from the null value to roughly the width of two pedestrians. Special situations may enhance the evacuation performance for larger values of  $d_g$ .

The range from  $d_g = 0$  to  $d_g \gg d_w$  was inspected. In the interval  $0 \leq d_g \leq 2r_{ij}$  (two pedestrian's width), the evacuation performance worsened for all the explored situations, as the separation distance between doors  $d_g$  increased. But, from  $d_g > 2r_{ij}$  the evacuation time enhanced for relatively small crowds and moderate anxiety levels. We realized that the sharp change in the evacuation behaviour at  $d_g = 2r_{ij}$  corresponded to qualitative differences in the pedestrian dynamics close to the exits.

After a detailed comparison of the dynamics for the single door situation and for two doors very close to each other (that is,  $d_g < 2r_{ij}$ ), we concluded that the blocking structures (*i.e.* blocking arcs) around the openings were released intermittently, allowing the pedes-

trians to leave the room in a stop-and-go process. As the separation distance approached  $2r_{ij}$ , the blocking arcs around each door, resembled the blocking situation of two single doors. This changes only affected the local dynamics (close to the doors), while the crowd remained gathered into a single clogging area.

For  $d_g > 2r_{ij}$  the single door blocking structures become relevant even for large values of  $d_g$  (see Fig. 5). No further qualitative changes were observed locally around each door. However, increasing the crowd size ( $N$ ) or the pedestrian's anxiety level ( $v_d$ ) slowed down the evacuation. Both magnitudes are linked to the pressure acting on the pedestrians, and therefore, enhanced the "faster is slower" effects.

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