A Cross-Discipline Modeling Capstone Experience

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Abstract
The Mathematical Association of America and the American Statistical Association have both updated and revised their curriculum guidelines. The guidelines of both associations recommend that students engage in a "capstone" experience, be exposed to applications, and have opportunities to communicate mathematical and statistical analysis, among other recommendations [1, 15]. In this article, we describe a mathematics and statistics capstone course, implemented at a small liberal arts college. We provide evidence that this course accomplishes the goals set forth by the MAA and ASA. We include guidance for other institutions on implementing a similar course.

Key Words: statistics education, mathematics education, mathematical modeling

1. Introduction

Since 2000, the National Survey of Student Engagement (NSSE) has tracked various student engagement activities. They define a "culminating senior experience" as capstone course, senior project or thesis, comprehensive exam, or portfolio [11]. They classify such a culminating experience as an "enriching experience", along with activities like working with a faculty member on a research project, studying abroad, and internships and field experiences. The NSSE and the Association of American Colleges and Universities both use the term "high-impact practices" for these experiences [5]. According to the NSSE 2013 report,

> "High-impact practices share several traits: they demand considerable time and effort, provide learning opportunities outside of the classroom, require meaningful interactions with faculty and students, encourage interaction with diverse others, and provide frequent and meaningful feedback. Participation in these practices can be life-changing." [11, p. 21]

According to the NSSE 2013 report, participation in culminating senior experiences and research projects is associated with both size of institution and major field of study. Approximately 70% of physical sciences seniors report that they have participated or will participate in such an experience.

The NSSE's definition of a culminating senior experience is written so it can apply to various fields of study. Certainly mathematics and statistics departments at both large and small institutions have implemented culminating experiences for undergraduates using all of these approaches: research projects, theses, capstone courses, and comprehensive exams, or a combination thereof (see [9] for an excellent summary of these approaches with statistics undergraduates). But we may ask: What are the qualities of a successful culminating experience in mathematics and/or statistics? Are there essential elements that should be included, regardless of the type of experience?

Brown and Kass [2] present four recommendations in the training of (graduate-level) statisticians. However, we (and others; see [9]) believe their recommendations are relevant to the elements of a successful undergraduate capstone experience as well. The recommendations are: 1) minimize prerequisites to research; 2) identify ways to foster statistical thinking; 3) require real-world problem solving; 4) encourage deep cross-disciplinary

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knowledge. They stress the need to move beyond just methodology courses and encourage true "statistical thinking", which is more than just a collection of methods. In their response to Brown and Kass, Nolan *et al.* [12] add to this list, stressing the importance of computing and experiential learning. Levine [8] makes his own recommendations as part of the "senior year experience". Students should have opportunities to: 1) integrate their existing knowledge; 2) add breadth to their existing knowledge base 3) use their expert knowledge to produce a substantial product; 4) prepare them for the transition from college to the world beyond.

In this paper, we describe a capstone experience for mathematics and statistics majors at a small liberal arts college. We hope that our capstone course addresses many, if not all, of the recommendations made by [2], [12], and [8], applied to both disciplines. In Sections 2.2 and 3, we address many of these recommendations, along with those made by the American Statistical Association [1] and the Mathematical Association of America [15]. First, however, we will present some history of our capstone course (Section 2.1) and some details about coverage and structure of the course (Section 2.2). Finally, in Section 4, we present some student feedback about the course, along with suggestions for other institutions who wish to implement a similar course.

2. History and Structure of this Course

2.1 Background

Gustavus Adolphus College is a selective liberal arts college of approximately 2500 students located in southern Minnesota. The Mathematics, Computer Science, and Statistics (MCS) Department has twelve full-time faculty, currently consisting of two Ph.D. statisticians, three Ph.D. computer scientists, and seven Ph.D. mathematicians. In addition, the department employs an adjunct with an M.S. in Statistics who teaches a 2/3 load of mostly statistics courses. The department graduates 17 math majors per year, on average, along with an average of 14 computer science majors. In Spring 2016, the department graduated the first class of 5 statistics majors; in Spring 2017 we expect 10 statistics graduates.

All mathematics, computer science, and statistics majors must graduate with a "capstone" course. By far the most popular capstone course is MCS 358: Mathematical Model Building. The description of this course is as follows:

> "This course provides an introductory study of the formulation of mathematical models to represent, predict, and control real-world situations, especially in the social and biological sciences. The course will use ideas from calculus, linear algebra, differential equations, probability, and statistics to describe processes that change in time in some regular manner, which may be deterministic or stochastic.
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> A mathematical model is a mathematical representation of some physical process or system. Since real-world phenomena are often too complex to model exactly, there are always simplifications and assumptions that one must make in building a mathematical model. In this course we will look at the model-building process and how to critique and refine models."

Our desire in this course is to have students see two perspectives of mathematical modeling: the deterministic perspective, and the stochastic perspective. We believe strongly that this course successfully provides the elements of a good capstone experience for both math and statistics majors.

Gustavus Adolphus College requires each student to complete two January terms (formally known a the *January Interim Experience* or IEX) during their residency at the college. IEX emphasizes experiential learning and these experiences range from internships to study-abroad courses to on-campus project based courses. The Mathematical Model Building course takes this last perspective. The project-oriented, application-focused pedagogy of this course exemplifies the institutional goals for IEX. Students may only complete one IEX at a time (e.g., one course, one internship). Thus, because students are only taking this course, they have the opportunity to focus on these projects in a way that would be difficult during a regular semester. On-campus IEX classes usually meet twenty days and classes must meet for a minimum of two hours per day.

2.2 Course Coverage and Structure

Mathematical Modeling is offered every other January and has almost always been team-taught by a a statistician and an applied mathematician. Since 2000, we have used the text *A Course in Mathematical Modeling* by Douglas Mooney and Randall Swift [10] which we have found has an organization that is conducive to this team-teaching approach. The text, and hence our course, alternates between deterministic modeling techniques and statistical analysis methods. Moreover, each chapter concludes with a set of projects that focus on the techniques discussed in that chapter.

Because MCS-358 is a capstone, it has a relatively long list of prerequisites including

- single and multivariable calculus,
- linear algebra,
- introduction to Computer Science (using *Python*),
- introduction to statistics, and
- the departmental “introduction to proofs” course.

The typical student is nearing graduation and thus usually has additional quantitative experiences such as the completion of our probability and statistics sequence or another mathematical sequence. In addition, we get a large number of double-majors, with the second major typically physics, computer science, economics, or biology.[1]

Because of this significant quantitative background, students have seen many of the mathematical and statistical ideas used in this course previously in their education. The focus of the classroom time is on the application of these ideas and not the underlying mathematical and statistical fundamentals. Additionally, the amount of homework designed to provide experience with any given technique is minimal (or nonexistent, in many cases).

The combination of text organization and student familiarity with statistical and mathematical tools allows for a project oriented structure that basically follows the outline below:

- 2 days of classroom activity led by professor M on chapter n,
- 1 day of student presentations from chapter $n-1$ projects,
- 2 days of classroom activity led by professor S on chapter $n+1$,
- 1 day of student presentations from chapter n projects,

[1] Occasionally, we admit students will math minors and majors in other other departments, such as biology and economics. Sometimes these students don’t have every mathematical prerequisite; we will discuss this further in Section 5.

- repeat.

The course is scheduled with a 10:30-12:30 meeting time; although there are no formal afternoon meeting times, students are strongly encouraged to use afternoons for project work. Both professors hold "office hours" during the afternoon; we spend quite a lot of that time in the computer lab, working with individual groups who have specific questions or issues. All of these factors combine to produce a busy and productive schedule for the student. Mornings are spent in the classroom discussing a deterministic method (for example) while afternoons are spent in the computer lab working on a statistics based project. As the week progresses, this repeats with the roles switched. The fourth week of term is dedicated almost exclusively to student work on their term-long project.

Each chapter project is done in teams of 3 - 4 students with the teams changing from chapter to chapter. Generally we create teams randomly, but on occasion make adjustments based on a variety of concerns including project difficulty, student background, and our desire to have each student work with all other students by the end of the term. Each group needs to prepare both a written report and a short (about 15 minute) presentation on their assigned topic.

In addition to the chapter projects, students self-select teams for a term-long project. We have tried a variety of methods in assigning these term projects but have found that the greatest consistency in interest and difficulty is gained by having students choose from a list of old Mathematical Contest in Modeling (MCM) projects [3]. These are projects of significant length and difficulty and given that the students are busy with the chapter projects, we find it necessary to establish milestones for the term projects.

End of week 1 Choose team and list top 3 project choices.

End of week 2 Submit timeline, outline of approach, and list of possible sources.

Midweek 3 Meet with professors to give a progress report.

End of week 4 Papers due, presentations in class.

3. Skills needed for statisticians and mathematicians, and how they are addressed by this course

The 2014 American Statistical Association Guidelines for Undergraduate Programs in Statistical Science [1] states that statistical programs should provide statistics majors "sufficient background" in five areas: statistical methods and theory, data manipulation and computation, mathematical foundations, statistical practice, and discipline-specific knowledge. The authors of the guidelines maintain that development in these skill areas should follow a progression throughout the student's academic career. Moreover, "[t]hese skills need to be introduced, supported, and reinforced throughout a student's academic program, beginning with introductory courses and augmented in later classes. Ideally such a program would culminate with capstone and/or internship experiences." [1, p. 9].

The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA) published an updated curriculum guide to majors in the mathematical sciences in 2015 [15]. The recommendations of this committee are similar to those outlined above. In particular, they present a list of the four "cognitive goals" for all curricula in the mathematical sciences. These are

Cognitive Recommendation 1 Students should develop effective thinking and communication skills.

Cognitive Recommendation 2 Students should learn to link application and theory.

Cognitive Recommendation 3 Students should learn to use technological tools.

Cognitive Recommendation 4 Students should develop mathematical independence and experience open-ended inquiry.

Additional "content recommendations" are also represented in the report. Because these recommendations focus on the whole of the mathematics curriculum, several of them are not relevant to this capstone course. However, the following recommendations are particularly relevant to our course.

Content Recommendation 3 Mathematical sciences major programs should include concepts and methods from data analysis, computing, and mathematical modeling.

Content Recommendation 8 Students majoring in the mathematical sciences should work, independently or in a small group, on a substantial mathematical project that involves techniques and concepts beyond the typical content of a single course.

In this section, we hope to convince the reader that our capstone course addresses all of the skill areas recommended by the ASA, in addition to the cognitive and content recommendations made by the MAA.

3.1 Statistical Methods and Theory

As described in Section 2.2, students come to this course from a variety of backgrounds: math, statistics, computer science, biology, physics, economics. Although some have quite extensive backgrounds in applied and/or theoretical statistics, they are required only to have taken an introductory statistics course. For some students, their last statistics course was AP Statistics, taken (three or four years ago) in high school. And yet the level of statistical topics covered in the course is quite advanced: random variables, demographic and environmental stochasticity in population dynamics, χ^2 tests for model validation, multiple regression and non-linear modeling, and Poisson processes and queueing theory. Even our statistics majors will enter this course without any previous exposure to some of these topics.

Recall that the focus of this course is mathematical and statistical modeling. Although we discuss χ^2 tests for model validation, and assessing a regression model (F tests, variable selection), we do not spend time on other formal inference procedures. (Of course, our statistics majors see such procedures extensively in their other courses.) That being said, we believe this course provides valuable experience in statistical reasoning. Students must think about the variables involved in the process they are trying to model, which of those variables may vary randomly, what distribution those variables may follow, how to assess their choices, and how to summarize, graph, and report on that variability and on the resulting model. In this way, they practice the modeling process from "start" to "finish". We believe this type of reasoning is extraordinarily valuable to all those who build models to describe processes, regardless of academic major or future career choice.

3.2 Data manipulation and Computation

Depending on the instructors, different programming, mathematical, and statistical packages have been used over the years. In the most recent incarnations of the course, *Mathematica* was used for mathematical work and `R` was used for statistical work. Many students had no (or very limited) previous background in one or both of these packages. All students are expected to have a basic working knowledge of *Python*, since Computer Science I is a pre-requisite and that is the language used in that course.

R and *Mathematica* are parts of the course and students are provided with the introduction and information needed to work in those programs. However, students are encouraged to work with whatever program they feel comfortable. In the past, we have had students use *Maple*, *MatLab*, and even *Excel*. Some students enjoy the challenge of taking a calculation or simulation presented in class, and "translating" it into the language with which they are more comfortable.

Depending on a particular student's assigned chapter projects and his or her choice of final project (see Section 2.2), he or she may encounter more or fewer explicit programming tasks, and more or less data management. Many projects require students to find real data, which they must then clean and format appropriately. Simulation studies are a major part of the course for every student. For example, when discussing environmental and demographic stochasticity, several of the chapter projects require students to change the values of various parameters, to investigate the effect on a population. Again, they see simulations performed in both *Mathematica* and R; some use those packages and some choose to use other programs.

3.3 Mathematical Foundations

As described in Section 2.2, half the course is taught from a mathematical, deterministic point of view. The mathematical modeling techniques covered in this course focus on discrete and continuous time dependent models. On the discrete side, iterative models/discrete dynamical systems are introduced first, and students are encouraged to explore these models both numerically and analytically. Higher-dimensional models such as Markov chain models and age structured models are also investigated using methods of linear algebra. Finally, we discuss continuous modeling using systems of ordinary differential equations. Students are introduced to phase-plane analysis methods including linear-stability analysis of equilibria using eigenvalues and eigenvectors.

3.4 Communication and Collaboration

In [1], "statistical practice" is described thusly: "Graduates should be expected to write clearly, speak fluently, and construct effective visual displays and compelling written summaries. They should demonstrate ability to collaborate in teams and to organize and manage projects."[1, p. 10]. In this course, a huge amount of emphasis is put on written and oral communication, teamwork, and collaboration. As explained in Section 2.2, traditional "homework" is not assigned; rather, assessment of student learning is entirely through team projects. After two days of lecture on a particular chapter, students are randomly assigned to 3- to 5-person teams and assigned a chapter project. They have two to three days before a written and oral report on that project is due. Each team is assigned a separate project, often with very different goals, and requiring different mathematical/statistical/computing skills.

All students in this course have completed at least two writing intensive courses at Gustavus, including at least one in the MCS department. Nonetheless, they come to us with varying abilities and experience in writing mathematics. We have found that explicit instruction in writing is worth a small amount of class time. At the beginning of the term, students are provided a model paper written by one of the instructors, along with a companion document that describes and analyses the organization, structure, and voice. Additionally, we assign F. E. Su's "Guidelines for Good Mathematical Writing" [13], which provides advice about mathematical writing. We see students' writing improve substantially over the term because they are getting constant feedback.

The written and oral reports from the chapter projects are graded independently by each instructor. Projects are assessed on several dimensions, according to rubrics (see Appendix). Written reports are expected to be both thorough and crisp. Oral reports are presented to the entire class, and all group members are expected to contribute. The final grade for the chapter project is determined by combining the two instructors' total grades on the written project rubric, combining the two instructors' total grades on the oral presentation rubric, then adding those two grades. (The written project is worth more in the final grade than the oral presentation.)

For each new chapter, groups are re-arranged. After each project is completed, students are asked to self-assess their group dynamics in two ways. One, they fill out a short questionnaire about the strengths and weaknesses of their group. Secondly, they choose how to divide a (hypothetical) honorarium among their group members, based on contributions to the group. These group assessments are collected and reviewed by the instructors, to keep abreast of any major conflicts that may be brewing or anyone who is not contributing consistently. However, students are strongly encouraged to work out interpersonal conflicts among themselves. We also continually remind students that this setting closely mirrors what they may face in a work environment, with the final product relying on the contributions of all group members.

For the final project, described in Section 2.2, students choose their own 3- to 5-person groups. Oral reports are longer (30 minutes) and presented to not only the class, but also most of the MCS Department faculty. Other interested parties are also invited, such as the students' advisors, the Dean of the Natural Science division, and the head of the college's Marketing department. Professionalism is stressed. In addition, the written report is graded with a more critical eye than the chapter projects. We see the final project as the culmination of the course, so the mathematical content, the quality of the written report, and the quality of the oral presentation are all expected to be more detailed and of higher quality.

3.5 Interdisciplinary Practice

The ASA Guidelines state that the "approach to teaching should ... emphasize authentic real-world data and substantive applications" [1, p. 13]. Similarly, Content Recommendation 3 of the CUPM report recommends that majors "... should include concepts from ... mathematical modeling." This course is, first and foremost, an applied course. There are no rote mathematical or statistical "homework problems," in the traditional sense. Every project is a real-world, open-ended problem to be solved. In addition, the flexible and undetermined nature of scientific investigation is stressed. Although we often expect students working on a certain chapter project to reach certain insights, there is never one "right" answer.

Consistent with the realities of scientific inquiry, students are encouraged to go beyond the textbook project. A paper that clearly explains the goals and background of the project, clearly states any underlying assumptions of the model(s), discusses the advantages and limitations of the model(s), relates the mathematics/statistics to the phenomenon being modeled, and provides a conclusion that clearly summarizes the work . . . this is a 'B' paper. In order to earn an 'A', they must do all this in a thorough and concise way, *and* they must explore possible extensions to the model(s) and/or the project in detail. We provide little or no guidance as to possible extensions, nor do we have expectations about what the "right" extensions are. We want the students to think as scientists: What additional questions have arisen from their investigations? Is there another perspective they could take on this problem? What happens if you relax or change the assumptions?

We feel that a major strength of this course is its focus on application of mathematical/statistical modeling to a variety of areas. Chapter projects cover topics as diverse as chickadee populations, tree ecology, the Dow Jones index, capture-recapture models, human birth and death rates, CPU speeds, traffic models, disease tracking, and predation by vampires. Final projects come from an even larger list of possibilities, including applications from biology, ecology, sociology, anthropology, physics, engineering, and computer science. Since groups choose their own final project (from past MCM projects), they can focus on something that interests them, or about which they have pre-existing knowledge or expertise.

Any student with any major is welcome in this course, as long as they have completed the pre-requisites. Over the years, this has included students majoring in mathematics, computer science, statistics, biology, physics, and economics. More importantly, the variety of student majors, experiences, expertise, and areas of interest brings valued diversity to the course. For example, a biology major whose interest is ecology will have different insight and bring different skills to a project on disease modeling than a computer science major. Because every student will work with every other student at least once during the course, they learn a lot from each other. This includes not just mathematics and computer programming, but also different perspectives on a problem. We believe this starts at the top: the two instructors have two different perspectives (one mathematician and one statistician), and have themselves experienced varied applications. Thus, we are from the beginning modeling the legitimacy and value of approaching projects from different directions.

4. Looking Back and Looking Forward

4.1 Student Responses

In this section we present some student responses to an online anonymous evaluation. The responses are from January 2014, when 16 out of the 22 students responded to the evaluation form.

When asked about their gains in understanding of various course topics, between 12 and 14 students (out of 16) reported "good" or "great" gains in each of the main course topics: discrete dynamical systems, discrete stochasticity, Markov chains, and empirical modeling. For the two topics that come at the end of the term and are given less weight (see Sections 3.1 and 3.3), differential equations and queuing theory, between 6 and 9 students reported "good" or "great" gains in their understanding of the material.

Table 1 below summarizes reported student gains in skills that we believe are important in this course, and Table 2 summarizes reported student gains in integrating skills and ideas. We are especially proud of the gains students report in communication skills, as we feel that is a major benefit of our course structure. While students receive explicit guidance at the beginning of the term on "How to write a modeling paper", they do not receive explicit instructions about oral and non-oral presentation skills. However, they are graded on these skills throughout the semester, they receive frequent feedback, and every few days they see their peers present so they can learn best practices from each other. We also feel that Table 2 shows that the students are clearly integrating and synthesizing the knowledge and skills they learn in this course, as suggested by [2] and [8].

It is worth noting that "critically reading scientific research articles" (Table 1) is not a structured part of the course. However, students often turn to outside sources when working on their chapter projects or researching their final projects. We were surprised and excited to see that so many students reported gains in this area, even though it is not something we focus on. In addition to the skills below, 12 out of the 16 students said they had made "good" or "great" gains in their comfort level in working with complex ideas. 12 students said they had made "good" or "great" gains in their comfort with computer programming.

Skills	no gains	little gain	moderate gain	good gain	great gain
Formulating questions, interpreting problems, and thinking about existing problems in a more mathematical/algorithmic way	0	0	2	4	10
Writing documents in discipline-appropriate style and format	1	0	0	2	13
Critically reading scientific research articles	0	3	3	5	5
Using `R`	0	1	4	2	9
Using *Mathematica*	0	4	1	7	4
Oral communication	0	2	3	3	8
Non-oral presentation skills (such as making appropriate slides, or timing presentations)	0	3	2	4	7
Effectively working in groups	0	0	4	2	10

Table 1: Reported student gains in skills.

Gains made in integrating the following:	no gains	little gain	moderate gain	good gain	great gain
Connecting key class ideas with other knowledge	0	1	2	5	8
Applying what I learned in this class in other situations	0	1	3	5	7
Using systematic reasoning in my approach to problems	0	1	0	9	6
Using a critical approach to information and arguments I encounter in daily life	0	1	1	9	5
Communicating my work (either in writing or orally) in a way others can understand	0	1	0	5	10

Table 2: Reported student gains in integration of learning.

4.2 Suggestions for Other Departments

As mentioned in Section 2.1, our department is a combined mathematics, statistics, and computer science department. Mathematics and statistics majors must take a capstone course, of which this course is one option. However, it is by far the most popular option, with approximately half of our math majors in any given year choosing this course as their capstone. Course enrollments in previous years have ranged up to 32 students, with an average of 22.

The 2014 ASA Guidelines state that "[w]henever possible, the undergraduate experience should include opportunities for internships, senior-level capstone courses, consulting experiences, research experiences, or a combination." [1, p. 13] This is similar to CUPM Content Recommendation 8. For small institutions and/or small departments that want to revise or build new statistics majors (as we have), it may be daunting to develop a new capstone course. Especially in departments with only one or two statistics faculty, staffing pressures may be such that faculty feel they have neither the mental space nor the course availability to create such a course. We appreciate these concerns, and we feel that a course such as ours could be a solution for many other small undergraduate institutions.

First of all, offering the course only once every other year decreases the pressure on the rest of the teaching load. Requiring a relatively short list of pre-requisite courses means that any on-track junior or senior student can take the course, which reduces the logistical problems inherent in every-other-year courses. The large number of topics, both mathematical and statistical, could be daunting to a single statistician or a single mathematician. We have found great success in team-teaching the course. Not only does this break up the course topics, it also spreads out the course management load. Instead of a work load that is overwhelming, we find the course to be very manageable for two people and allows both instructors to really enjoy and immerse themselves in the course, as the students do. Obviously, the commitment of two faculty rather than one increases the pressure on the rest of the teaching load. For departments that can not feasibly dedicate two faculty to one course, we suggest choosing one professor as the instructor of record, and bringing in colleagues for guest lectures on the topics with which the instructor is not as comfortable. Finally, we encourage departments to consider a combined math/stat capstone such as this for the interdisciplinary and cross-disciplinary value it offers. As we argued in Section 3.5, we feel that students benefit from seeing the perspectives of their fellow students in other disciplines, in addition to the different perspectives offered by the two instructors.

We also recognize that the existence of January term (IEX) is relatively unusual, and colleagues at many other institutions will not have the opportunity to run such a course over a "mini-mester". Obviously, some changes would need to be made to organize a course like this one over a full semester. We believe the most important *logistical* aspect of the course is the two-hour class period. This block of class time allows plenty of time for activities, discussions, lecture, and computer work. Much of the background material is review for the students, but it may have been a few years since they last saw the material. A two-hour class period gives us time to review (usually in the context of some real-world example), discuss extensions or interesting cases within that example, introduce new concepts that build on their existing knowledge, and practice those concepts through lab work or activities. Another important benefit of the two-hour class time is the ability to have several student presentations on the same day, all of substantial length, with time for questions between presentations.

Hence, for departments that want to implement a course like this one within a standard semester, we suggest a two days per week, two hours per day schedule. In a 12- to 15-week semester, this would result in 24 to 30 two-hour class days (compared to our 20 two-hour class days). As mentioned in Section 2.2, although there are no formal afternoon meeting times, students usually spend afternoons in the computer lab, working on their group chapter projects, or their final projects. In fact, there is a lot of (informal, but important) teaching that happens during this time! We can expect this dedication because the students have very few other demands on their time, as they are not taking any other courses. During a regular semester, of course, it is unreasonable to expect students to spend all of their free time in the lab, and it is more difficult for students to jockey their schedules to find time for their entire group to meet. Thus, during a full semester, some class days could be dedicated to work days: one day (or more) per chapter, depending on the length of the semester. Or considerable time could be spent on the final two topics, differential equations and queueing theory, which as we have mentioned we tend to cut short due to time constraints. The topics within this course are so varied and deep, we have no doubt that our colleagues who implement such a course during a standard semester will have no problems filling the "extra" time.

4.3 Open Questions

A course is never a "finished product", however, and we do still struggle with open questions about how to improve the course. One such question regards students from outside majors: that is, those without a mathematics, statistics, or computer science (MCS) major.

In the past ten years, we have allowed a handful of such students into the course, even if they did not have the full list of prerequisites. In all cases, we felt that they contributed to the course in an important and different way than the MCS majors. Since their major fields of study are not math, statistics, or computer science, they have insight on real-world applications in their disciplines, and often approach problem-solving differently. Of course, the students who want to take such a course outside of their major are an exceptionally bright and driven bunch, so there is a self-selection bias at work. However, an open question for us as instructors is how to be more intentional about encouraging these types of students to take on the challenge, and how to evaluate *their* experiences. Our belief is that they may see a value in the course that is different than the value our MCS students recognize.

5. Conclusion

As stated in Section 1, we believe that the capstone course we have implemented here at Gustavus Adolphus addresses the suggestions made not just by [8] and others in the last fifteen years, but also those skills stressed by the ASA and MAA in their guidelines [1, 15]. We also have evidence that the students recognize the contribution this course makes to their future, in both academic and non-academic contexts. When asked on the anonymous evaluation at the end of January 2014, "What will you carry with you into other classes or other aspects of your life?", these are responses of two students:

> "The mindset developed here will carry with me during problems I am trying to solve. The final project questions were extremely open ended, and that led to needing to approach the problem in steps. The use of breaking a problem down is going to be needed further in my academic career."
>
> "I think this class opened my mind to using math in a modeling context. My whole life math has been about solving equations but this class challenged individuals to solve real life problems with math."

Recall that the National Survey for Student Engagement defines "high-impact" practices as those that "demand considerable time and effort, provide learning opportunities outside of the classroom, require meaningful interactions with faculty and students, encourage interaction with diverse others, and provide frequent and meaningful feedback" [11, p. 21]. Our Mathematical Modeling course does all of things and more, thus providing our students with a truly transformative experience.

References

[1] American Statistical Association Undergraduate Guidelines Workgroup (ASA Guidelines Workgroup), 2014, *2014 curriculum guidelines for undergraduate programs in statistical science*, Alexandria, VA: American Statistical Association. http://www.amstat.org/education/curriculumguidelines.cfm.

[2] Brown, E. N. and Kass, R. E., 2009. What is Statistics?, *The American Statistician*.

[3] Consortium for Mathematics and Its Applications, (COMAP), 2014. http://www.comap.com/undergraduate/contests/

[4] Hoerl, Roger, 2014. *Roadmap for smaller statistical programs to incorporate 2014 guidelines for undergraduate statistics education*, White Paper.

[5] Kuh, George D., 2008. High-Impact Educational Practices: What They Are, Who Has Access to Them, and Why They Matter. *Association of American Colleges and Universities.*

[6] Lazar, N. A. and Reeves, J. and Franklin, C., 2011. A capstone course for undergraduate statistics majors, *The American Statistician*.

[7] Legler, J. and Roback, P. and Ziegler-Graham, K. and Scott, J. and Lane-Getaz, S. and Richey, M., 2010. A model for an interdisciplinary undergraduate research program, *The American Statistician.*

[8] Levine, A., 1998. *A president's personal and historical perspective* in *The Senior Year Experience: Facilitating Integration, Reflection, Closure, and Transition*, Jossey-Bass Publishers.

[9] Malone, Christopher, 2014. *Capstones in the undergraduate statistics curricula*, White Paper.

[10] Mooney, D. and Swift, R., 1999. *A Course in Mathematical Modeling*, The Mathematical Association of America.

[11] National Survey of Student Engagement (NSSE), 2013. *A Fresh Look at Student Engagement – Annual Report 2013*, Indiana University Center for Postsecondary Research.

[12] Nolan, D. and Temple Lang, D., 2009. Comment to 'What is Statistics?', *The American Statistician*.

[13] Su, F. E, 2015. Some guidelines for good mathematical writing, *MAA Focus*, 35, 4.

[14] Spurrier, J., 2001. A capstone course for undergraduate statistics majors, *Journal of Statistics Education*

[15] Zorn, P, ed 2015. *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences.* The Mathematical Association of America. http://www.maa.org/sites/default/files/pdf/CUPM/pdf/CUPMguide_print.pdf.

A. Resources: Grading Rubrics

NAMES:__________________________________ PROJECT:_________________

MCS358: PRESENTATION RUBRIC

CRITERIA	Beginning	Intermediate	Exemplary	Score
Content Knowledge & Preparation	Little grasp of subject; seems confused or uncertain	At ease with subject; can provide basic information	Mastery of subject; provides additional information with ease	(10)
Content Coverage	Includes little essential or new information	Includes essential information; audience mostly understands the topic	Main ideas very clear, developed, and integrated	(10)
Mechanics	Voice not clear, too fast or too slow; no audience interaction	Usually clear and well-paced, with one or two exceptions; audience engages	Clear and well-paced; exceptional audience interaction	(5)
Time	Less than 7 minutes	7-12 minutes or 16-20 minutes	12 -15 minutes	(3)
Organization	Ill-prepared, just reads notes with distracting mannerisms.	Generally prepared, with occasional reliance on notes, minimal distracting mannerisms	Well-prepared, little reliance on notes, no distracting mannerisms	(3)
Slide Appearance	Text size varies; slides hard to read; few or inappropriate graphics	Text is uniform and readable; slide appearance consistent; graphics ok	Text is uniform and readable; slide appearance consistent; concise text; helpful graphics	(9)

Comments:

NAMES:_________________________________ **PROJECT:**________________

MCS358: WRITING RUBRIC

	Beginning	**Intermediate**	**Exemplary**	Score
Abstract	Missing	Mostly describes model, method, and results or too long	Thorough and terse	(5)
Introduction & Background	Does not give any information about what to expect in the report	Explains project goals with insufficient background	Presents a concise description of project goals with sufficient background	(5)
Model Description	Does not describe variables and model	Descriptions are understandable; some lack detail or are confusing	Descriptions are understandable and adequately detailed	(10)
Assumptions	Does not describe assumptions	Adequate assumptions are made but are lacking grounds or detail	Assumptions are well-grounded, detailed, and comprehensive	(10)
Model Analysis	Missing	Fairly complete but some details lacking	Complete and detailed	(10)
Tables and Figures	No tables and figures	Tables and figures are correct, but are missing information or confusing	Tables and figures are correct and complete	(5)
Conclusion/Discussion	Presents an illogical explanation for findings and is incomplete	Presents a partial, but logical explanation for findings	Presents a full and logical explanation for findings	(5)
Grammar & Spelling	Very frequent grammar and/or spelling errors.	Only one or two errors.	All grammar and spelling are correct.	(2)
Citations	None	Incomplete references in body; inappropriate choice of sources	Complete references in body; appropriate sources	(3)
Difficulty/Extensions	Does the project as written (without advanced techniques)	Brief exploration of extensions or analysis	Sophisticated technique or detailed extension	(5)

Comments: