Success Indicators in the USMA Advanced Core Mathematics Program

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Abstract
In this paper we study the relationship between Advanced Placement scores on the Calculus AB/BC exams and the success of cadets in the Advanced Core Mathematics Program at The United States Military Academy. The purpose of this study is to analyze whether cadets with satisfactory AP scores on the Calculus examinations should be offered admittance in the Advanced Core Mathematics Program (ACMP), without having to take the Summer Validation Exam. We also consider other indicators, such as ACT, SAT and CEER scores, and determine, once again, if a cadet should be instantly admitted into the ACMP. This would not only greatly reduce the workload of instructors administering the Summer Validation Exam, but more importantly it would reduce superfluous testing during Cadet Basic Training, an important transitional period for incoming cadets.

Key Words: AP Exams, ACT, CEER, SAT, Validation Exams, Advanced Core Mathematics Program

1. Introduction

The Advanced Core Mathematics Program (ACMP) at the United States Military Academy is a voluntary two-semester advanced mathematics sequence for selected cadets who have validated single variable calculus and demonstrated strength in the mathematical sciences. It is designed to provide a foundation for the continued study of mathematics, technology, sciences, and engineering. Students in the program study Advanced Multivariable Calculus (MA153) during their first semester and Mathematical Modeling and Introduction to Differential Equations (MA255) in their second semester. Course descriptions are located in the appendix. After the successful completion of these two courses, cadets return to the core mathematics program to take Probability and Statistics (MA206) in the fall of sophomore year - one semester before cadets in the regular core Math Program.

A benefit of enrolling in the Advanced Core Mathematics Program is the ability for cadets to satisfy their core math requirement in three semesters, rather than the standard four semester requirement. Thus allowing cadets to take an additional course while at West Point. The second, and most important, benefit is that cadets are exposed to topics not covered in the Core Mathematics Program, such as advanced vector calculus and a full semester devoted to differential equations. This is particularly important for those cadets considering a math, science, or engineering major as they will be better prepared for the advanced classes in their second, third and fourth years.

The current process by which cadets are selected for the Advanced Core Mathematics Program is primarily based on a cadets' Summer Validation Exam score. This 3 hour exam tests cadets'

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comprehension of single-variable calculus concepts (typically topics covered in college Calculus I & II courses) without the use of technology. Cadets must take this exam, which is offered during the first week of Cadet Basic Training (CBT), in order to be considered for the ACMP. Cadets are then placed into a series of tiers according to their validation exam score. Cadets scoring a minimum set value on the validation exam are automatically offered seats in the ACMP. Those not scoring above the threshold, are furthered considered based on other indicators, such as AP scores, SAT scores, ACT scores, class rank, and CEER scores (a function of the previously mentioned variables). Those cadets who score below a lower threshold on the validation exam, but have strong success indicators are often provided the opportunity to be a part of the ACMP. It is the program's intent to select cadets who show promise of passing the two ACMP course sequence with at least a B- in each course.

Our goal in this study is to analyze when cadets with satisfactory AP scores on the Calculus examinations should be offered admittance in the ACMP, without having to take the Summer Validation Exam during Cadet Basic Training. This would not only greatly reduce the workload of instructors administering the tests, but more importantly it would relieve pressure on cadets who may wish to validate other courses. In addition, we consider what other indicators, such as ACT, SAT and CEER scores, can help determine, once again, if a cadet should be offered a seat in the ACMP.

2. Background

As part of our study we looked into the use of AP exam scores in the department of Mathematical Sciences at the United States Naval Academy and the United States Air Force Academy. The following is a compilation of this information as provided on the respective academy's website, [3] and [4].

2.1 USNA

Validating Calculus I: Midshipmen who have finished the online pre-calculus exam have the opportunity to take an online multiple choice Calculus I exam. This exam is updated each year to reflect any changes in our first calculus course. Calculators are not allowed on the Calculus exams. A score of roughly 70% is needed to validate the course. Calculus I can also be validated by means of AP exams. Those students who took AP Calculus in high school and scored a 4 or 5 on the AB exam (or on the AB portion of the BC exam) validate Calculus I. High scores on an IB exam also count as validation.

Validating Calculus II: Those who validate Calculus I and who have taken a course in Calculus II are invited back to take a Calculus II validation exam. The Calculus II exam is also multiple choice. Calculus I and II can also be validated by means of BC exams. Those who scored a 4 or 5 on the BC exam validate Calculus I and Calculus II.

2.2 USAFA

When a cadet enters the Academy, they take several validation tests offered by the various academic departments. Successful completion of a test will enable cadets to be placed in an accelerated or advanced course, or perhaps to receive validation credit and substitute another course. Representatives from each department review transcripts of new cadets who have prior college credit. Credit may be awarded for any satisfactorily completed college course that is equivalent to a course in the Academy curriculum. A cadet who passes a validation examination or who makes an acceptable score on a College Board Advanced Placement examination may also earn validation credit. Departments certify this credit to the Office of Registrar, Customer Service.

2.3 USMA

USMA policy states, [2]:

"Cadets may be excused from (validate) certain core courses if they have sufficient knowledge of a subject to meet appropriate department's standards. Credit earned in other colleges, Advanced Placement Examination scores, and tests administered at the Military Academy are considered in validation decisions. Advanced Placement Examination's scores may be used in mathematics, physics, chemistry, history, social sciences, and foreign languages. Validation of core courses allows a cadet to substitute an additional elective in place of a core course. If a cadet shows unusual ability or has prior knowledge of the subject, but does not validate the course, he/she may be enrolled in an advanced or accelerated version."

However, a satisfactory score on the AP calculus exams is not a guarantee of placement in the Advanced Core Mathematics Program. As of now, acceptance into the program is based on a combination of factors, which include a cadet's Summer Validation Exam score, SAT/ACT Math scores, college level calculus grades, and AP Calculus scores. Moreover, the Summer Validation Exam is offered during the first week of Cadet Basic Training (CBT) and signifies a cadet's intent to enroll in the Advanced Core Mathematics Program. Therefore, cadets must take this exam in order to be considered for the program.

In light of this, there is interest in analyzing the performance of USMA cadets who enter the academy having received satisfactory scores on AP exams. Take for example the study conducted by Steven Cho, an instructor in the USMA Physics Department, who was interested in the study of predictors for success in Physics, see [5]. These underlying issues are further motivation for our study.

3. About the AP Exams

The following information was found on the College Board Course Description for Calculus AB and Calculus BC, [1]:

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

In general, the AP composite score points are set so that the lowest raw score needed to earn an AP Exam score of 5 is equivalent to the average score among college students earning grades of A in the college course. Similarly, AP Exam scores of 4 are equivalent to college grades of A-, B+, and B. AP Exam scores of 3 are equivalent to college grades of B-, C+, and C.

AP Score	Meaning
5	Extremely Well Qualified
4	Well Qualified
3	Qualified
2	Possibly Qualified
1	No recommendation

Table 1: AP Scores and meaning

Table 1 provides a meaning for each respective score on an AP examination. Given the above information related to the AB and BC exams, most of our analysis is based on comparing AB exam scores to performance on MA153 and performance on the BC exam to that in MA255.

4. Methods and Results

The data set used in the following sections consists of seven years of historical data, covering the ACMP cadets from the class of 2009 through the class of 2015. The data set includes the cadet's Advanced Placement score (both AB and BC, if provided) and their final scores in MA153 and MA255. We additionally have some data for their standardized test scores on ACT, SAT and CEER.

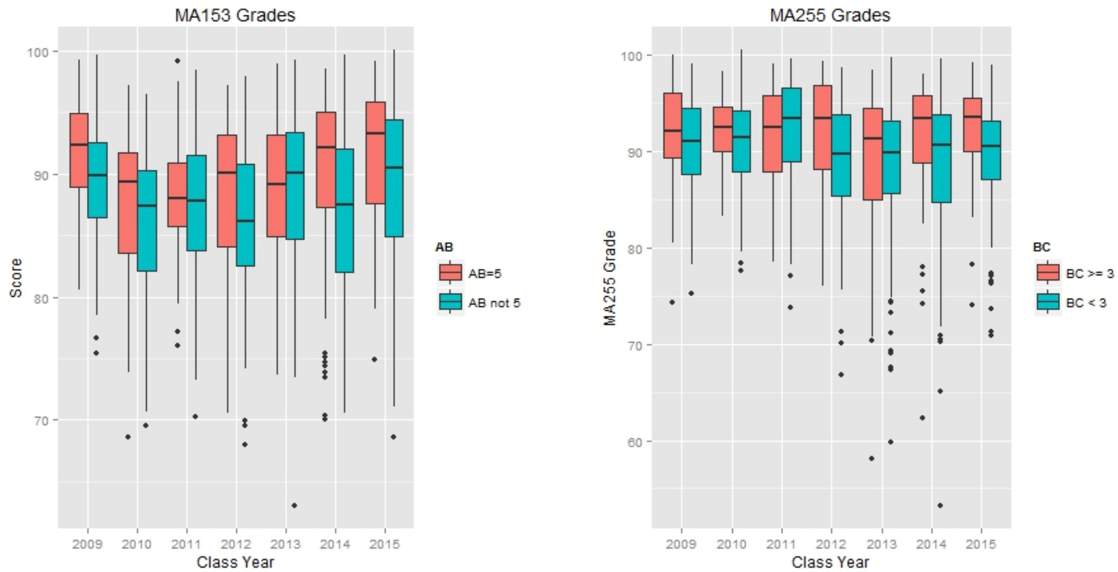
There are 1392 cadets in our data set with 936 of them having at least one AP score on file. So, roughly two-thirds of our ACMP cadets had an AP score on file. Table 2 shows the breakdown by year for each class. As you will notice, the population of the course has grown from year to year, with the exception of 2011 to 2012. For academic year 2013, which is not included in this study, our MA153 course started with 310 cadets but finished with 301 cadets. Of the 936 cadets with AP scores on file, 218 of them had scores for both tests.

Class	# of Cadets	# with AP Scores	% with AP Scores
2009	153	94	61.4%
2010	184	124	67.4%
2011	172	100	58.1%
2012	172	140	81.4%
2013	191	149	78.0%
2014	250	158	63.2%
2015	270	171	63.3%

Table 2: Class Breakdown

Figure 1 gives box plots depicting the distribution of MA153 grades and MA255 grades with the given parameters related to the AB and BC exam scores, respectively. Inspection of the boxplots shows visual indication of greater average scores in MA153 and MA255 for cadets that have an AB score of 5 and BC scores above 3, respectively.

Using the above mentioned data we find some results through the computation of conditional probabilities, testing hypothesis and by running tree regressions. The results and analysis for these processes are found in sections 4.1, 4.2, and 4.3, respectively.



(a) MA153 final grades and AB score of 5 (b) MA255 final grades and BC scores over 3

Figure 1: Box plots associated with MA153/255 final grades and AB/BC exam scores

4.1 Probability Analysis

First notice that our study relates the AP score on the AB exam with a final grade in MA153, while we relate a cadets' AP score on the BC exam to a final grade in MA255. In addition, we define success as achieving a final grade of at least 80% in the ACMP courses. This is the equivalent of earning a letter grade of at least a B-. We will look at the number of students who did not achieve this mark.

Table 3 shows the number of cadets that did not achieve at least an 80% as final course grades in the ACMP separated according to their respective AB scores. Table 4 shows the same information, except using BC scores. Note that the second column in Tables 3 and 4 titled "# of cadets" may include two values. This accounts for cadets that completed MA153 but did not complete MA255. These cadets may have decided to leave the academy, dropped out of the ACMP to take a different math course, or some may have validated MA255. Also shown in these tables is the percentage of the population that did not achieve an 80% in the course.

Tables 3 and 4 shows us that, generally speaking, the better a cadet performed on the AP test, the less chance there is that they scored less than 80% in the ACMP. We should expect this outcome, as the AP examinations are an indication of the cadet's ability to perform at the college level. We can also see that the number of unsuccessful students decreased as they transitioned from MA153 to MA255.

4.2 Testing Hypothesis

In this section we provide the results related to the hypothesis testing conducted to determine if there is a significant difference in the final course average between cadets who earned a specific

AB Exam					
Score	Total # of cadets	# with MA153 final grade < B-	% with MA153 final grade < B-	# with MA255 final grade < B-	% with MA255 final grade < B-
5	436/431	40	9.2%	27	6.3%
4	265/256	41	15.5%	21	8.2%
3	75/73	15	20.0%	7	9.6%
2	11	2	18.2%	2	18.2%
1	3/2	1	33.3%	0	0.0%

Table 3: Cadets with AB Scores achieving less than 80% in MA153 and MA255

BC Exam					
Score	Total # of cadets	# with MA153 final grade < B-	% with MA153 final grade < B-	# with MA255 final grade < B-	% with MA255 final grade < B-
5	208/203	5	2.4%	4	2.0%
4	79	7	8.9%	8	10.0%
3	66/64	9	13.6%	4	6.3%
2	5	1	20.0%	0	0.0%
1	6	1	16.7%	0	0.0%

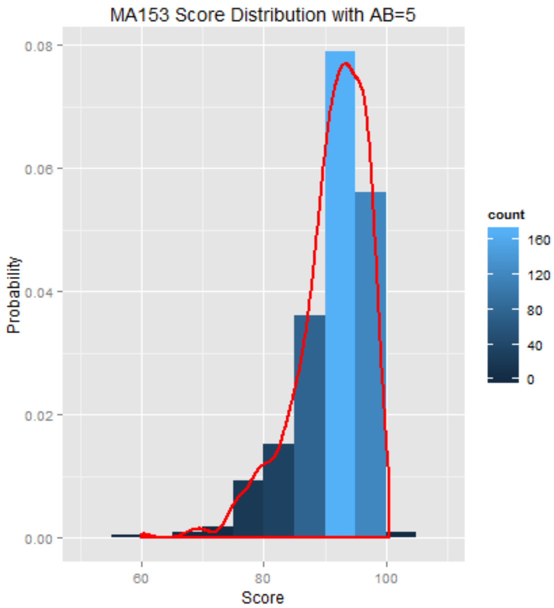
Table 4: Cadets with BC Scores achieving less than 80% in MA153 and MA255.

exam score on the AP calculus exams in comparison to the final class averages for other cadets in the course. First observe that Figure 2 provides histograms associated to the frequency of the final grades in MA153 when categorized by specific AB scores. Similarly, Figure 3 provides histograms associated to the frequency of the final grades in MA255 when categorized by specific BC scores.

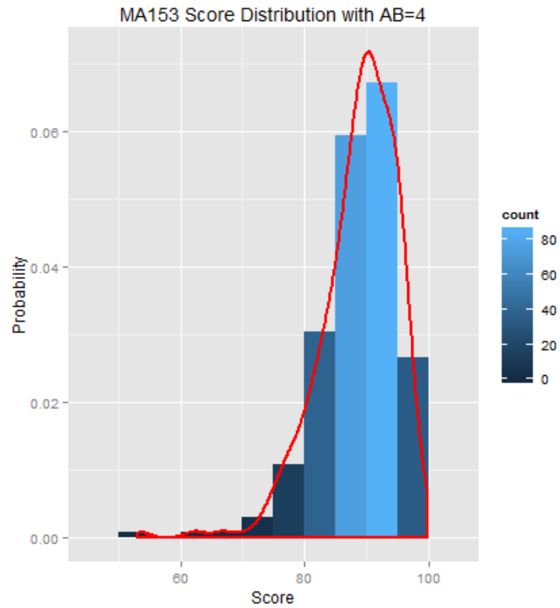
Our goal with hypothesis testing is to not only determine whether cadets with certain scores on the AP Calculus exams do better than average on the final course grades, but in fact to determine when they do no worse than the final class average grades. This is important because the main goal is to determine when cadets are taking superfluous exams. So in fact we are not necessarily trying to determine if they perform better than average, but instead we would like to know when they are doing well enough to pass the course with a minimum of an 80%.

With the above in mind we set out to test the initial hypothesis that cadets with an AB score of 3 or higher would have a final MA153 course mean grade less than or equal to other cadet's course final grade mean. Running a 1-tail, 2-sample $t-$test, there is marginal significant evidence ($p-$value of .075) and we fail to reject the null hypothesis. Hence we cannot say that cadets with an AB score of 3 or higher do better than other cadets in MA153 in terms of final course grade. Another 1-tail, 2-sample t-test was run to determine whether in fact cadets with an AB score of 5 would have a final MA153 course mean grade less than or equal to other cadet's course final grade mean. We found statistically significant evidence ($p-$value of 0.000233709) to reject this initial hypothesis, and hence support that cadets who score a 5 on the AB exam in fact do better than average on their MA153 final grade.

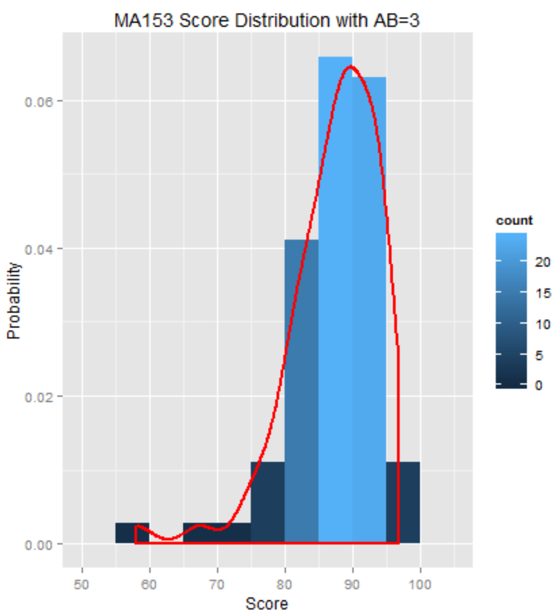
Given that the BC exam covers more material than the AB we also considered how a BC exam score could affect a cadets MA153 final grade average. So we set out to test the initial hypothesis



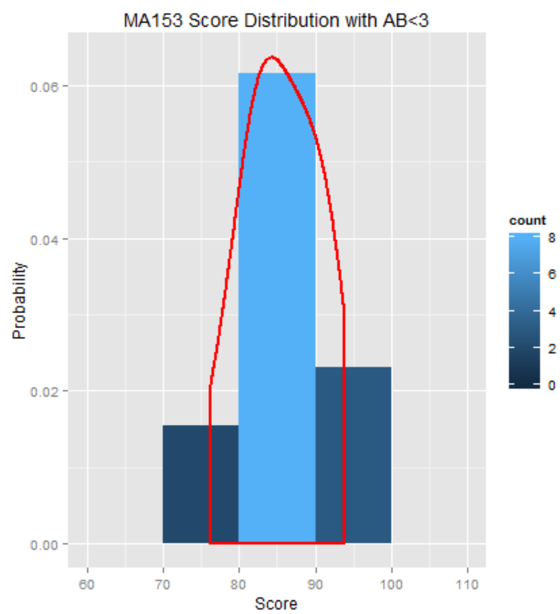
(a) AB Score of 5



(b) AB Score of 4

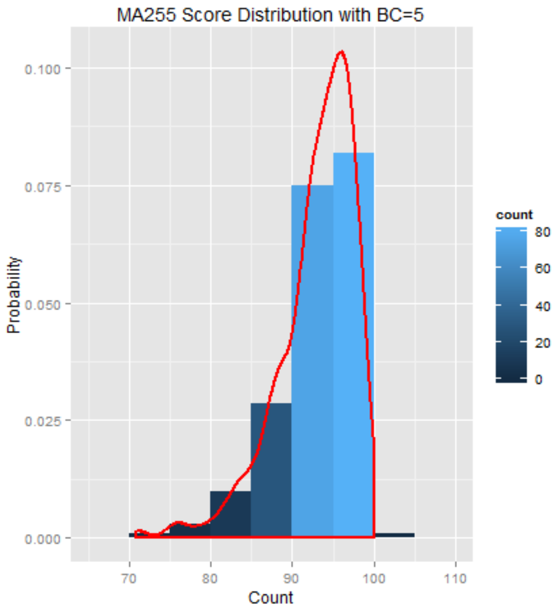


(c) AB Score of 3

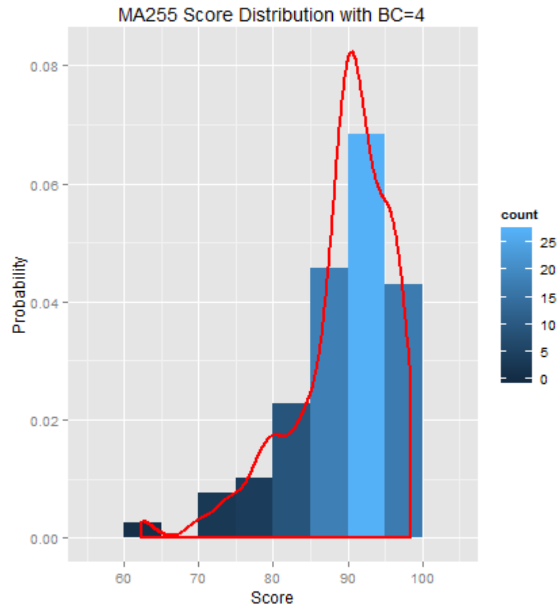


(d) AB Score of less than 3

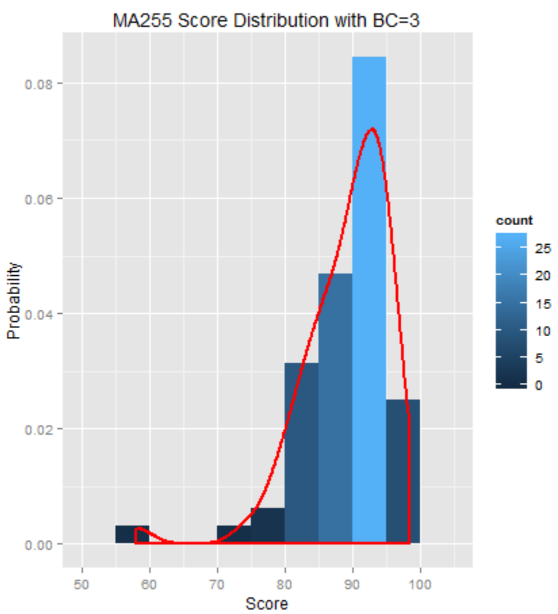
Figure 2: Frequency histograms of MA153 final grades given a specific AB score



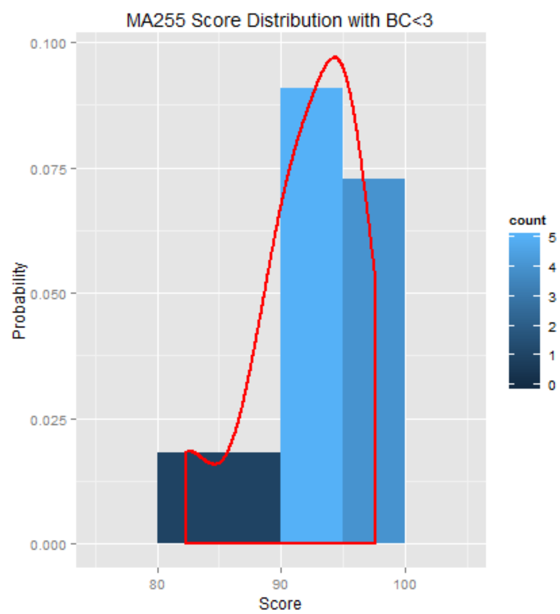
(a) BC Score of 5



(b) BC Score of 4



(c) BC Score of 3



(d) BC Score of less than 3

Figure 3: Frequency histograms of MA255 final grades given a specific BC score

that cadets with an BC exam score of 3 or higher would have a final MA153 course mean grade less than or equal to other cadet's course final grade mean. Running a 1-tail, 2-sample $t-$test, there is very strong evidence ($p-$value of 1.39×10^{-13}) to reject the null hypothesis and support the claim that cadets with a BC score of 3 or higher in fact do better than other cadets in MA153 in terms of final course grade.

In a similar fashion we set out to test the initial hypothesis that cadets with an BC score of 3 or higher would have a final MA255 course mean grade less than or equal to other cadet's course final grade mean. Running a 1-tail, 2-sample $t-$test, there is statistically significant evidence ($p-$value of 0.00022895) to reject the null hypothesis and hence support the statement that cadets who score a 3 or higher on the BC exam have a higher final course average than other cadets in the course.

4.3 Tree Regression

The primary objective of this study is to predict the success of a student in MA153 and MA255. We have shown that the AB and BC test scores are good predictors for success, but we were interested in establishing a better prediction using other variables at our disposal. To begin with, we established the correlation between the student's MA153 and MA255 grades and all of the other pre-admission variables available to us. Table 6 shows the results of the correlation.

	MA153	MA255
SATM	0.14193	0.12002
ACTM	0.32775	0.27673
HSP	-0.24296	-0.22035
CEER	0.37064	0.31337
AB Score	0.29626	0.23499
BC Score	0.38419	0.22494
MA153	1.00000	0.65691
MA255	0.65691	1.00000

Table 6: Correlation

With the available pre-acceptance variables, we ran a recursive partitioning algorithm through the statistical programming language R to produce a tree regression model. Both the BC and the AB scores were removed from this first model to gage the ability of the other available variables to predict success in either MA153 or MA255. Figure 4 shows the first tree regression grow from the variables, with the first split occurring at a CEER score of 654.5.

Given that the CEER score is calculated using most of the variables we have in our tree regression, we ran another tree regression to determine if there is a significant difference if CEER score was omitted from the regression. Figure 5 shows the new tree with the first split now at SAT Math (SATM) score of 715.

Finally, a tree regression was modeled where we again considered the the AB and BC scores. Figure 6 shows the final tree regression with the first split at CEER score of 654.5 with a model accuracy of 0.891523.

Table 7 provides the results of comparing the three models as well as how accurate the models were with the original data. Recall that we defined success to be a final grade of at least 80% in MA153 and MA255. All three models have an 89% accuracy, but also have a large false positive

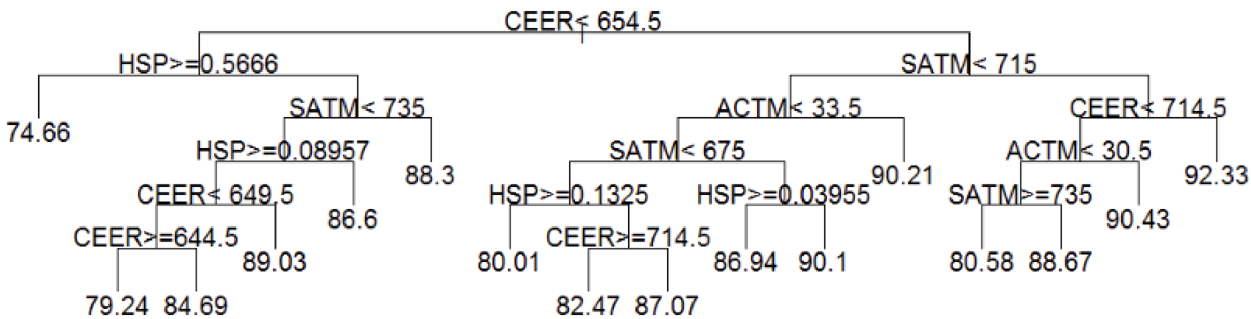


Figure 4: Tree Regression with CEER score

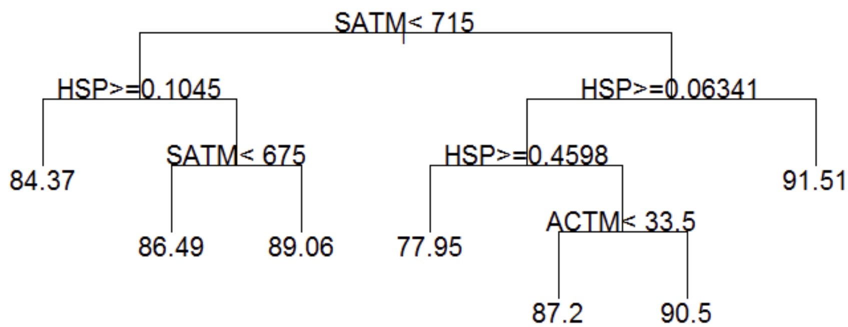


Figure 5: Tree Regression without CEER score

rate.

Predicted \ Actual	Grade less than 80	Grade 80 or higher	Model accuracy
Model with CEER Grade < 80	7	7	0.89009
Model with CEER Grade ≥ 80	147	1231	
Model w/o CEER Grade < 80	5	4	0.88937
Model w/o CEER Grade ≥ 80	149	1234	
Final Model Grade < 80	10	7	0.891523
Final Model Grade ≥ 80	144	1231	

Table 7: Model Accuracy

4.4 Testing Model Adequacy

It was of great interest to this study to determine how accurate the model is at predicting the success rate of future USMA cadets in the Advanced Core Mathematics Program. To do so we used a data set from Fall 2013 with MA153 final grades to test the validity of the tree regressions. It is important to note that this data was not used to produce the models and was entirely independent from the data set used in the previous sections.

By defining success as achieving an 80% or highest final MA 153 course grade, the regression correctly predicted 267 out of the 268 successes in MA153. Meaning that of 268 cadets who achieved

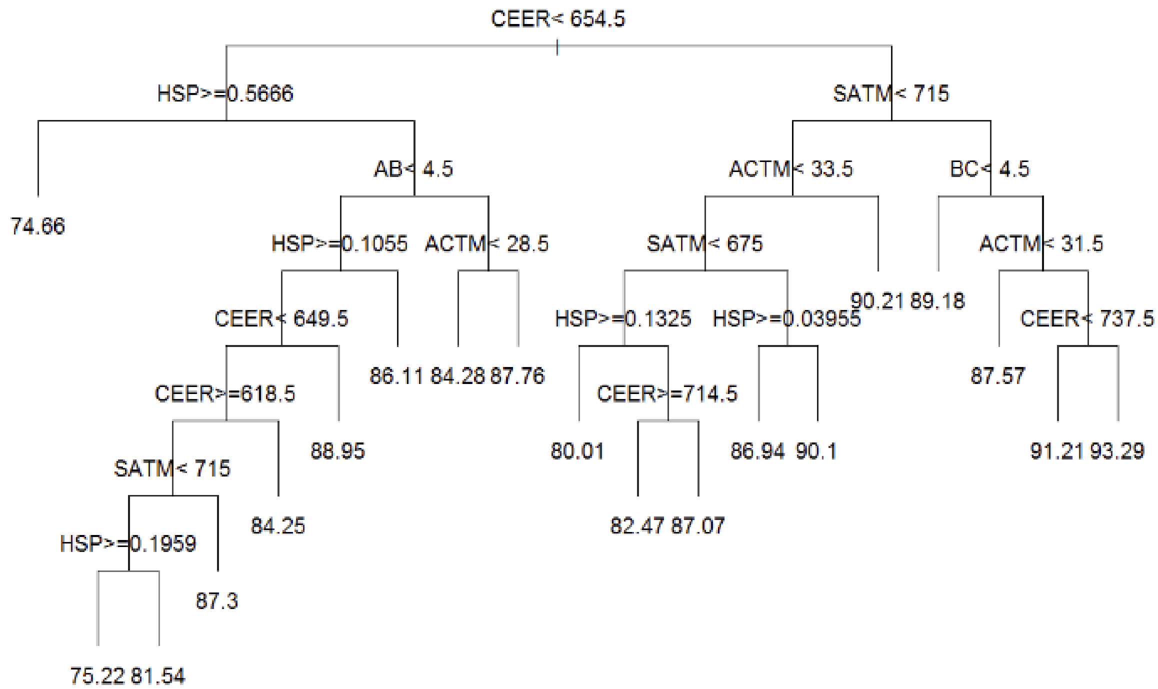


Figure 6: Final Tree Regression

the success threshold of 80% or higher in MA153, our model predicted one cadet would fail, yet this cadet in fact achieved the success mark. Moreover, the model was extremely inaccurate in predicting failures in MA153, meaning those cadets earning less than an 80% final grade. In fact our model predicted zero failures, when there were 37 cadets with a final grade lower than 80%. Based on these results, we do not recommended the use of our models to predict whether or not a particular student will be successful in MA153. However, what our models do show is that the most valuable variables for predicting success in MA153 are the CEER score, SAT-Math, and AP scores. These values (set to a certain threshold) can be used as indicators to determine success in MA153.

We would like to note that at the time of submission we did not have access to the Spring 2014 MA255 final grades since that course was in progress. However, it would be of interest, in the near future, to determine how well our model predicted success for that particular course.

5. Recommendations

Based on the results found in Sections 4.1, 4.2, 4.3, and 4.4 we make the following recommendation.

The following cadets should, without need for retesting, be immediately offered a seat in the Advanced Core Mathematics Program:

1. Those cadets with an AP score of 4 or 5 on the AB Calculus exam.

2. Those cadets with an AP score of 3, 4 or 5 on the BC Calculus exam.

3. Those cadets with a CEER score greater than 654.5, assuming they have taken at least a single variable calculus course.
4. Those cadets with a SAT-Math score greater than 715, assuming they have taken at least a single variable calculus course.

Those cadets who do not satisfy any of the above requirements should continue to be required to take the Summer Validation Exam in order to be accepted into the Advanced Core Mathematics Program.

6. Acknowledgements

The authors would like to thank LTC Donald Outing, Program Director for the Advanced Core Mathematics Program, for his support and encouragement throughout this project. In addition, the authors would like to express deep gratitude to all of the previous MA153 and MA255 course directors who kept excellent records of the cadets in the Advanced Core Mathematics Program.

A. Appendix

MA153 – Advanced Multivariable Calculus
This course consists of an advanced coverage of topics in multivariable calculus. Topics include a detailed study of vectors and geometry of space, vector functions, partial derivatives, multiple integrals and vector calculus. An understanding of course material is enhanced through the use of Mathematica (computer algebra system).

MA255 – Math Modeling and Intro to Differential Equations
This course emphasizes the interaction between mathematics and the physical sciences through modeling with differential equations. Topics include a study of first order differential equations, first order difference equations, Laplace transforms, second order differential equations, series solution techniques, systems of first order linear equations, numerical methods, and nonlinear equations and stability. An understanding of course material is enhanced through the use of Mathematica (computer algebra system) and Excel (spreadsheet).

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