An Evaluation of Different Small Area Estimators and Benchmarking for the Annual Survey of Public Employment & Payroll[1]

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Abstract

The Annual Survey of Public Employment & Payroll (ASPEP), conducted by the Economic Statistical Methods Division (ESMD) and the Economy-Wide Statistics Division (EWD) of the U.S. Census Bureau, provides statistics on the number of federal, state, and local government civilian employees and their gross payrolls. Different small area estimators can be produced using the ASPEP data and auxiliary information from the preceding Census of Governments Employment. We develop a design-based Monte Carlo simulation experiment in which we draw repeated samples from the 2012 Census of Governments Employment data using the ASPEP sampling design and compute a wide range of estimates that use the generated sample and the 2012 Census of Government Employment data. We then compare simulated design-based biases, variances, and mean squared errors of these estimators. The estimators covered under our simulation study includes: Horvitz-Thompson, SPREE, traditional composite and empirical Bayes (EB) and hierarchical Bayes methods. Lastly, we present different benchmarking approaches and compare their performances.

Keywords: Government Units, Monte Carlo Simulation, Composite Estimator, Horvitz-Thompson, Empirical, SPREE.

1. Introduction

Over the last few decades, the U.S. Census Bureau has pioneered in developing innovative small area methodologies in different programs. In one of the most cited papers in small area estimation (SAE) literature, Fay and Herriot (1979) developed a parametric empirical Bayes method to estimate per-capita income of small places with population less than 1000 and demonstrated, using the Census data, that their method was superior to both direct design-based and synthetic methods. More recently, researchers at the U.S. Census Bureau implemented both empirical and hierarchical Bayes methodologies in the context of Small Area Income and Poverty Estimates (SAIPE) and Small Area Health Insurance Estimates (SAHIE) programs; see Bell et al. (2007) and Bauder et al. (2008).

Besides the Census Bureau's well-known SAIPE and SAHIE programs, researchers in the ESMD are actively pursuing state-of-the-art small area estimation techniques to improve the current estimation methodologies for small areas. Some results on the ASPEP estimation were presented at 2013 SAE conference in Thailand, and 2014 SAE in

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Poznan, Poland. There is a large number of small area estimators available in the literature. These estimators typically use either implicit or explicit models to combine survey data with different administrative and Census records. The properties of such estimators are usually studied using the model used to derive the estimator. However, the design-based properties of small area estimators, which are most appealing to the survey practitioners, are largely unknown. In this paper, we show different small area estimators that can be produced using the ASPEP data and auxiliary information from the preceding Census of Governments Employment. We develop a design-based Monte Carlo simulation experiment in which we draw repeated samples from the 2012 Census of Governments data using the ASPEP sampling design and compute a wide range of estimates that use the generated sample and the 2002 Census of Government data. We then compare simulated design-based biases, variances, and mean squared errors of these estimators. The estimators covered under our simulation study include Horvitz-Thompson, SPREE, traditional composite and empirical Bayes.

The Economy-Wide Statistics Division of the U.S. Census Bureau conducts Censuses of about 90,000 federal, state and local government units every five years in order to collect data on the number of full-time federal, and full-time and part-time state and local government employees and payroll. Between two consecutive Censuses (years ending with 2 and 7, e.g., (2007, and 2012), the EWD also conducts the Annual Survey of Public Employment & Payroll, a nationwide sample survey covering all federal, state and local governments in the United States, which include five types of governments: counties, cities, townships, special districts, and school districts. The first three types of government are referred to as general-purpose government, because they generally provide multiple government activities. Activities in ASPEP are designated by function codes (see Appendix). School districts cover only education functions. Special districts usually provide only one function, but can provide two or three functions, like Natural Resources, or Sewerage. ASPEP is the only source of public employment data by program function and full-time and part-time break. Data on employment include the number of full-time and part-time employees and gross pay as well as hours worked for part-time employees. All data are reported for the government's pay period covering March 12. Data collection begins in March and continues for about ten months. For more information on the survey, we refer to http://www.Census.gov/govs/apes.

In 2009, ASPEP was redesigned and the old sample design was replaced by a stratified probability proportional-to-size (PPS) with modified cut-off sample design in order to reduce sample size and respondent burden for small townships and special district governments. At the same time the goal was to improve the precision of the estimates and data quality. The sample design was implemented in multiple steps. First, large governments were made initial certainties. Next, in the first stage of the design, a state-by-governmental type stratified PPS sample was selected, where size was taken as the total payroll (the sum of full-time pay and part-time pay from the 2007 Census of Government: Employment). In the second stage, a cut-off point was constructed to distinguish small and large government units in municipal and special district strata. Lastly, the strata with small-size government units were subsampled using a simple random sampling design. In 2009, we selected 1200 out of 2000 small-size units.

Five years later in 2014, a new sample for ASPEP was selected based on the 2012 Census of Governments: Employment. The sample design was changed slightly. Initial certainty criteria were not used, and more sample was allocated to school district strata of small states. Instead of regular PPS, systematic PPS sampling was performed in all strata after sorting by population (for general-purpose governments), enrollment (for school districts), and function (for special districts). In the second stage of the design, we again used modified cut-off sampling to select a subsample of small-size units.

The ASPEP survey is designed to produce reliable estimates of the number of full-time and part-time employees and payroll at the national level and for large domains (e.g., government functions such as elementary and secondary education, higher education, police protection, fire protection, financial administration, judicial and legal, etc., at the national level, and states aggregates of all function codes). However, it is also required to estimate the parameters for individual function codes within each state. This requirement leads us to explore small area estimation methodology that borrows strength from previous Census data instead of collecting expensive additional data for small cells. We refer to Rao (2003) and Jiang and Lahiri (2006) for a comprehensive account of small area estimation theory and applications. In Section 2, we briefly describe our method. In Section 3, we present our findings from our data analysis.

2. Estimation Methods

2.1 Proposed Methods

In this paper, the variable of interest is the number of full-time employees. Our data is skewed; therefore, we transformed the variable in a log scale (see Figure 2). We proposed two models: area-level model and unit-level on the auxiliary variable (see model (2) and model (5) below).

Area-level Model

Let y_{ij} denote the number of full-time employees for the j$^{\text{th}}$ governmental unit within the i$^{\text{th}}$ small area ($i = 1,\cdots,m;\ j = 1,\cdots,N_i$). The small area in this paper we refer to the cell (state, function). In this paper, we are interested in estimating the total number of fulltime employees for the i$^{\text{th}}$ small area given by $Y_i = \sum_{i=1}^{N_i} y_{ij}\ (i = 1,\cdots,m).$ An estimator of Y_i is given by:

$$\hat{Y}_i^{EB} = N_i \left[f_i \overline{y}_i + (1-f_i)\hat{\overline{Y}}_{ir} \right] \tag{1}$$

where $\overline{y}_i = n_i^{-1}\sum_{j=1}^{n_i} y_{ij}$ is the sample mean; $f_i = n_i/N_i$, N_i and n_i are the sampling fraction, number of government units in the population and sample for area i, respectively; $\hat{\overline{Y}}_{ir}$ is a model-dependent predictor of the mean of the non-sampled part of area i $(i = 1,\cdots,m)$.

In this paper, we obtain $\hat{\bar{Y}}_{ir}$ using the following nested error regression model on the logarithm of the number of full-time employees at the government unit level:

$$\log(y_{ij}) = \beta_0 + \beta_1 \log(\bar{X}_i) + v_i + \varepsilon_{ij}, \tag{2}$$

$$v_i \overset{iid}{\sim} N(0,\tau^2) \text{ and } \varepsilon_{ij} \overset{iid}{\sim} N(0,\sigma^2), \tag{3}$$

where $\bar{X}_i$ is the average number of full-time employees for the i^{th} small area obtained from the previous Census; β_0 and β_1 are unknown intercept and slope, respectively; v_i are small area specific random effects. The distribution of the random effects describes deviations of the area means from values $\beta_0 + \beta_1 \log(\bar{X}_i)$; ε_{ij} are errors in individual observations $(j = 1,..., N_i;\ i = 1,..., m)$. The random variables v_i and ε_{ij} are assumed to be mutually independent. We assume that sampling is non-informative for the distribution of measurements y_{ij} $(j = 1,..., N_i;\ i = 1,..., m)$. A similar model without logarithmic transformation can be found in Battese et al. (1988). The logarithmic transformation is taken to reduce the extent of heteroscedasticity in the employment data. Similar model using unit level auxiliary information was considered by Bellow and Lahiri (2012) in the context of estimating total hectare under corn for U.S. counties. We use the following model-based predictor of $\bar{Y}_{ir}$:

$$\hat{\bar{Y}}_{ir} \approx \exp\left[\hat{\beta}_0 + \hat{\beta}_1 \log(\bar{X}_i) + \hat{v}_i + \frac{1}{2}(\hat{\sigma}^2 + \hat{\delta}_i^2)\right] \tag{4}$$

where $\hat{\beta}_0, \hat{\beta}_1, \hat{v}_i, \hat{\sigma}^2, and\ \hat{\delta}_i^2$ (standard error of $\hat{v}_i$) are obtained by fitting (2) using PROC MIXED of SAS. We obtain our estimate of total number of full-time employees in area i using equations (1) and (4).

Unit-level Model

Besides area-level (model 2), we also performed the unit-level (X_{ij}) model as below.

$$\log(y_{ij}) = \beta_0 + \beta_1 \log(X_{ij}) + v_i + \varepsilon_{ij}, \tag{5}$$

$$v_i \overset{iid}{\sim} N(0,\tau^2) \text{ and } \varepsilon_{ij} \overset{iid}{\sim} N(0,\sigma^2), \tag{6}$$

After estimating the models parameters, the estimate will be obtained by two different ways: simple back transformed, and log-normal back transformed given as follows:

Simple

$$\hat{Y}_i^{EB} = \sum_{j \in S_i} y_{ij} + \sum_{j \notin S_i} \exp(\hat{\beta}_0 + \hat{\beta}_1 \log(X_{ij}) + \hat{v}_i) \text{ (simple)} \tag{7}$$

Log-Normal Back Transformation

$$\hat{Y}_i^{EB} = N_i(f_i \bar{y}_i + (1 - f_i)\hat{\bar{Y}}_{ir} \tag{8}$$

where $\hat{\bar{Y}}_{ir} = \hat{\alpha}_{ir} \exp(\hat{v}_i + \frac{1}{2}(\hat{\sigma}^2 + \hat{\delta}_i^2))$,

and $\hat{\alpha}_{ir} = (N_i - n_i)^{-1} \sum_{j \notin S_i} \exp(\hat{\beta}_0 + \hat{\beta}_1 \log(X_{ij}))$

2.2 Other Models

2.2.1 Direct Estimator (Horvitz-Thompson)

A general estimation formula for estimating the total in small area i[th] is:

$$\hat{t}_i = \sum_{j \in S} w_{ij} y_{ij} \tag{9}$$

where the weight, $w_{ij} = \dfrac{1}{\pi_{ij}}$, and π_{ij} is the inclusion probability for unit j in small area i. In our research, small area i= (state, function).

2.2.2 Decision-based Estimator

The Decision-based (DB) method helps to estimate the synthetic in each cell by providing a stable state total as a reliable estimator in a large area covering all small areas, states by function code level. In other words, it was used for estimating the aggregates. DB was a process of testing the possibility of combining the strata in order to get a better estimate of the total. This method strengthened the statistical models for the area of estimation. The state total was estimated by a single stratum weighted regression (GREG) estimator specified as below.

$$\hat{t}_{y,GREG} = \hat{t}_{y,\pi} + \hat{b}(t_x - \hat{t}_{x,\pi}) \tag{10}$$

where $t_x = \sum_{i \in U} x_i$, $\hat{t}_{x,\pi} = \sum_{i \in S} \dfrac{x_i}{\pi_i}$, $\hat{t}_{y,\pi} = \sum_{i \in S} \dfrac{y_i}{\pi_i}$, $\hat{b} = \dfrac{\sum_{i \in S} (x_i - \overline{x})(y_i - \overline{y}) / \pi_i}{\sum_{i \in S} (x_i - \overline{x})^2 / \pi_i}$,

π_i is the inclusion probability, and x_i is the auxiliary data from the Census of Governments: Employment for government unit *i*. The slope $\hat{b}$ was obtained by the Decision-based (DB) process (Cheng et al., 2009). The DB method improved the precision of estimates and reduced the mean square error of weighted survey total estimates. The idea was to test the equality of linear regression lines to determine whether we can combine data in different substrata. The null hypothesis $H_0 : b_1 = b_2$, that is, the equality of the frame population regression slopes for two substrata. In large samples, $\hat{b}$ is approximately normally distributed, $\hat{b} \sim N(b, \Sigma\ )$. Under the null hypothesis, with two sub-strata U_1, U_2 (large and small) from samples S_1, S_2 of sizes n_1 and n_2, we have $\hat{b}_1 - \hat{b}_2 \sim N(0, \Sigma_{1,2})$ where $\hat{b}_1 \sim N(b, \Sigma_1)$, $\hat{b}_2 \sim N(b, \Sigma_2)$, and $\Sigma_{1,2} = \Sigma_1 + \Sigma_2$. Therefore, the test statistic is

$$(\hat{b}_1 - \hat{b}_2) \Sigma_{1,2}^{-1} (\hat{b}_1 - \hat{b}_2) \sim \chi_1^2 \tag{11}$$

Our research showed that it was unnecessary to test the hypothesis for the intercept equality because our data analysis showed that we never rejected the null hypothesis of equality of intercepts when we could not reject the null hypothesis of equality of slopes.

The critical value for a test based on (9) was obtained from a chi-squared distribution with 1 degree of freedom. The test was performed with a significance level of $\alpha = 0.05$. If we could not reject the null hypothesis, then the slopes estimated in sub-strata S_1 and S_2 were accepted as the same, and the Decision-based estimator was equal to the GREG estimator for the union of two sample sets, that is, for $S = S_1 \cup S_2$. Otherwise, the Decision-based estimator would be the sum of two separate GREG estimators of stratum totals, that is,

$$\hat{t}_{y,DB} = \begin{cases} \hat{t}_{y,greg} & \text{if } H_0 \text{ is accepted} \\ \sum_{h=1}^{2} \hat{t}^{h}_{y,greg} & \text{if } H_0 \text{ is rejected.} \end{cases} \tag{12}$$

where $\hat{t}_{y,greg}$ denotes the GREG estimator from the combined stratum S, while $\hat{t}^{h}_{y,greg}$ denotes the GREG estimator from substratum h from sample S_h. DB produced 51 (50 states and Washington D.C.) totals for each key variable.

2.2.3 Synthetic Estimator

Synthetic estimation assumes that small areas have the same characteristics as large areas, and there is a reliable estimate for large areas. There are many advantages of synthetic estimation. They are accurate, simple and intuitive, aggregated estimates, that can be applied to all sample designs, and borrow strength from similar small areas. Synthetic estimation can even provide estimates for areas with no sample from the sample survey, and it does not need a study model.

The general idea for synthetic estimation is that if we have a reliable estimate for a large area and this large area covers many small areas, then we can use this estimate to produce an estimate for a small area. The key element for calculating the synthetic estimation for a small area (state by function code level) is to estimate the proportion of that small area of interest within the large state area. This estimate for the small area is known as the synthetic estimate.

The synthetic estimator for function code *f* of state *g* is:

$$\hat{t}^{syn}_{if} = \frac{x_{if}}{\sum_{f} x_{if}} \hat{t}^{DB}_{i} \tag{13}$$

where x_{if} is the auxiliary variable which is obtained from the Census of Governments: Employment. In our research x_{if} is the full-time employee in the previous Censuses in state *i* and function *f*. $\hat{t}^{DB}_{i}$ is obtained by the Decision-based (DB) estimate from equation (10).

2.2.4 Composite Estimation

To balance the potential bias of the synthetic estimator, $\hat{t}_{if}^{syn}$, against the instability of the design-based direct estimate, $\hat{t}_{if}^{HT}$, we take a weighted average of two estimators. Thus, the composite estimate was applied on the PPS sample for each cell (state by function). Generally, it has the form below.

$$\hat{t}_{if}^{composite} = \hat{\phi}_i \hat{t}_{if}^{HT} + (1-\hat{\phi}_i)\hat{t}_{if}^{syn} \tag{14}$$

where $\hat{\phi}_i = 1 - \frac{\sum \widehat{var}(\hat{t}_{if}^{HT})}{\sum(\hat{t}_{if}^{syn} - \hat{t}_{if}^{HT})^2}$ (see Rao 4.4.3) (15)

This estimator could give negative weight. In that case we make $\hat{\phi}_i = 0.5$.

2.2.5 Structure PREserving Estimation (SPREE)

SPREE is a synthetic estimation method that uses the method of iterative proportional fitting (IPF) to adjust the cell counts of multi-way table in such a way that the adjusted counts agree with the specified margins. For detail procedures, please see Rao (2003). In our research we construct a two-way table, state by function with a dimension of 49 x 29 of full-time employee counts from a Census data. Between two Censuses we collect sample data. Therefore, the margins are updated by SPREE. The margins could be obtained by reliable direct survey estimates, or by the Decision-based total estimates. In our research we use direct survey estimates.

3. Benchmarking

In this paper we present two different kinds of benchmarking that preserve the design consistency: Ratio, and Additive.

Ratio

$$r_g = \frac{\hat{Y}_g^{\mathrm{HT}}}{\sum_f \hat{Y}_{gf}^{\mathrm{EB}}},\ where\ g = state, f = function$$

$$\hat{y}_{gf}^{\mathrm{Ratio}} = (r_g)(\hat{y}_{gf}^{\mathrm{EB}}) \tag{16}$$

Additive

$$\hat{Y}_i^{Additive} = \hat{Y}_i^{HT} + \lambda_i\left(\sum_{i=1}^m \hat{Y}_i^{HT} - \sum_{i=1}^m \hat{Y}_i^{EB}\right) \tag{17}$$

where $\lambda_i = \frac{(1-f_i)}{\sum_{i=1}^m (1-f_i)}$ $(could\ have\ alternative\ choices\ for\ \lambda_i)$

4. Results & Evaluations

We have three Census of Governments: 2002, 2007 and 2012 from which we create two universes: 2002 data intersects with 2007 data (U_1), and 2007 intersects with 2012 data (U_2). For simplicity, we make U_1 and U_2 containing non-zero values of the variables of interest (full-time employees). We apply production sample designs to draw 1000 replicated samples on each U_2. On each replicate we estimated the full-time employee totals for states and functions (49 states, and 29 functions) using the estimators: HT, SPREE, empirical Bayes (model 2), empirical Bayes (model 5), and the composite. We computed $\widehat{mse}$, $\widehat{var}$, and $\widehat{bias}$ for each estimator from 200 replicates. The analysis covered 49 states, excluding Washington D.C, and Hawaii because we collected all of their data.

Figure 1 and Figure 2 show the data for California before and after log transform respectively.

Figure 1: Skewed Data

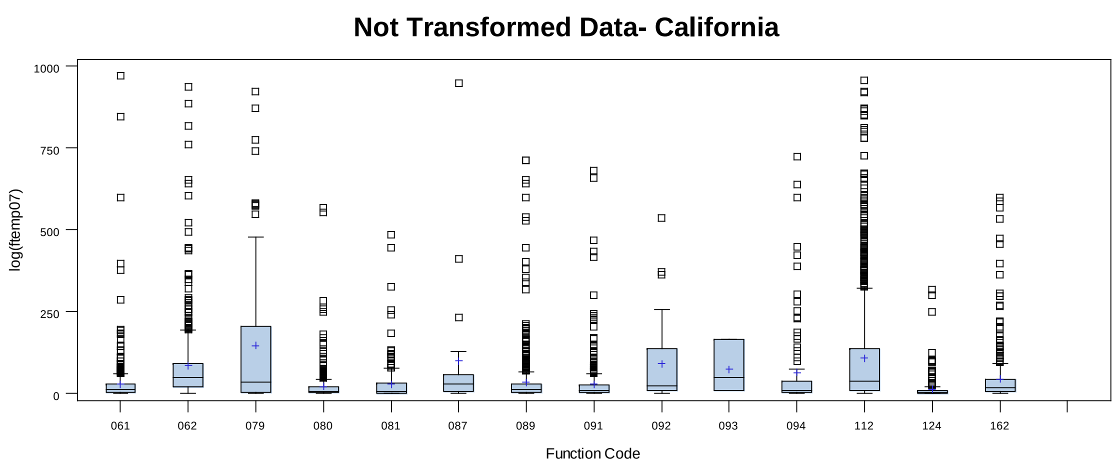

Figure 2: The Data after Log Transformed

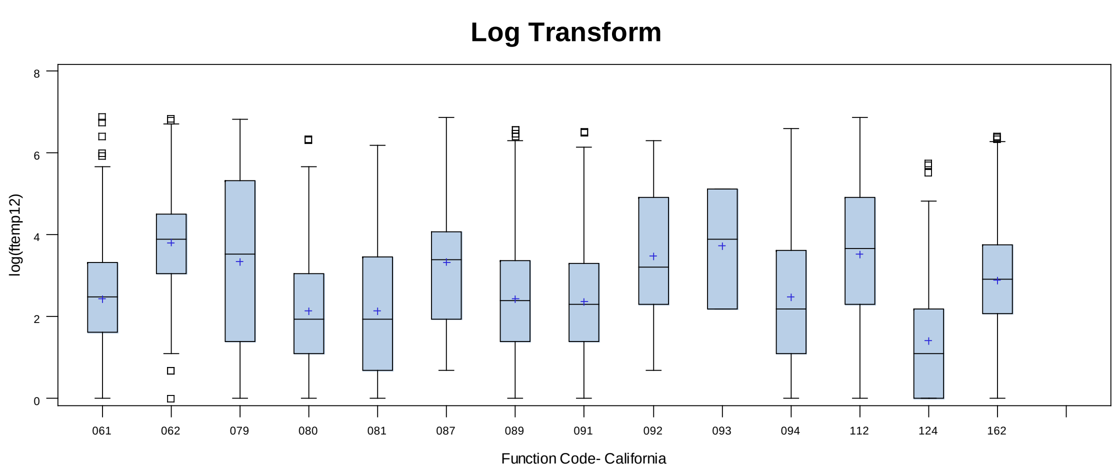

Figure 3 shows the distribution of the residuals after log transformation. As we can see the normality assumption in the model is satisfied very well.

Figure 3: Normality of the Residuals

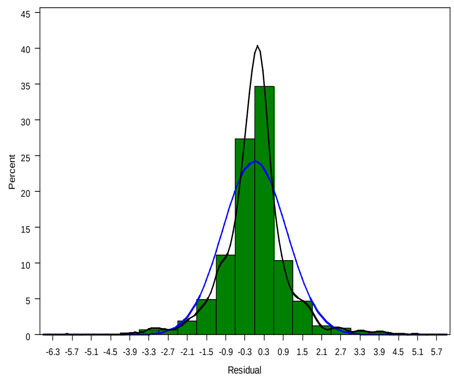

Table 1 shows the number and percentage of times an estimator performs the best among rival estimators in terms of absolute relative errors by domain sample sizes. As we can see the unit-level model outperforms all the other models, even with the correct back transformed model. This could cause by the normality of the error was not met perfectly after the log transformation.

Table 1: Number and Percentage of Times an Estimator Perform the Best among Rival Estimators in terms of Absolute Relative Errors by Domain Sample Sizes

Sample Size	EB Unit Level Simple Transform	EB Unit Level Log Normal Transform	EB Area Level Simple Transform	EB Area Level Log Normal Transform	Composite	Horvitz-Thompson	SPREE	Number of Domains
$n \leq 2$	44 35.5%	26 21.0%	21 16.9%		2 1.6%	19 15.3%	12 9.7%	124
$3 \leq n<5$	40 28.0%	33 23.1%	28 19.6%		1 0.7%	31 21.7%	10 7.0%	143
$5 \leq n < 15$	97 31.4%	72 23.3%	51 16.5%			71 23.0%	18 5.8%	309
$15 \leq n < 50$	196 39.0%	114 22.7%	46 9.2%			121 24.1%	25 5.0%	502 (88.0%)
$50 \leq n$	78 51.3%	28 18.4%	7 4.6%			38 25.0%	1 0.7%	152 (12.0%)
	455 37.0%	273 22.2%	153 12.4%	0 0.0%	3 0.2%	280 22.8%	66 5.4%	

Table 2 shows the number and percentage of domains an estimator performs the best among all rival estimators in terms of simulated relative root mean squared error (RRMSE). As we can see the unit-level model with simple transformation still consistently outperforms the other models.

Table 2

EB Unit Level Simple Transform	EB Unit Level Log Normal Transform	EB Area Level Simple Transform	EB Area Level Log Normal Transform	Composite	Horvitz-Thompson	SPREE
551 44.8%	195 15.8%	88 7.1%	209 17.0%	144 11.7%	6 0.48%	38 3.1%

Table 3 averages RRMSE over 1000 simulated samples. The unit-level with simple transformation produces the smallest RRMSE.

Table 3

EB Unit Level Simple Transform	EB Unit Level Log Normal Transform	EB Area Level Simple Transform	EB Area Level Log Normal Transform	Composite	Horvitz-Thompson	SPREE
1.71%	3.02%	10.7%	3.65%	5.43%	5.43%	32.6%

Table 4 shows the number of positive relative errors (the percentage of the errors above the 0 line) which indicates the randomness of the errors (no pattern). Again, the unit-level with simple transformation shows no pattern in the errors.

Table 4

EB Unit Level Simple Transform	**EB Unit Level Log Normal Transform**	**EB Area Level Simple Transform**	**EB Area Level Log Normal Transform**	**Composite**	**Horvitz-Thompson**	**SPREE**
50.1%	*84.7%*	*71.6%*	*39.1%*	*39.8%*	*46.8%*	*83.6%*

We compare the unit-level with simple back transformation to the two benchmarking methods. Table 5 shows the relative errors of the three estimators. We also investigate the average absolute relative errors on all domains (see Table 6) and percentage of positive relative errors (Table 7).

Table 5

Function	Relative Errors		
	EB Unit-level	Ratio Benchmark	Additive Benchmark
Air Transportation	0.27%	0.28%	-4.90%
Corrections	-0.01%	0.06%	1.05%
Elementary and Secondary - Instruction	-1.19%	-0.74%	-1.16%
Higher Education - Other	0.05%	0.06%	-67.60%
Higher Education - Instructional	0.49%	0.51%	-11.10%
Financial Administration	0.71%	0.76%	0.67%
Fire Protection -Firefighters	-0.80%	-0.77%	-0.59%
Judicial & Legal	0.08%	0.12%	-0.15%
Other Government Administration	0.17%	0.21%	0.25%
Health	-0.21%	-0.15%	-2.09%
Hospitals	0.53%	0.56%	2.12%
Highways	0.49%	0.55%	0.57%
Housing & Community Development	-0.59%	-0.57%	0.23%
Libraries	0.87%	0.88%	1.72%
Natural Resources	3.28%	3.28%	-11.20%
Parks & Recreation	-0.01%	0.02%	-0.08%
Police Protection - Persons With Power Of Arrest	0.25%	0.35%	0.31%
Welfare	0.92%	0.97%	15.90%
Sewerage	-0.15%	-0.12%	-0.19%
Solid Waste Management	-0.09%	-0.07%	-1.42%
Water Transport & Terminals	5.22%	5.23%	127%
All Other & Unallocable	0.10%	0.14%	-0.14%
Water Supply	0.06%	0.09%	0.02%
Electric Power	-0.72%	-0.69%	14.60%
Gas Supply	-3.36%	-3.36%	-11.20%
Transit	-0.91%	-0.83%	-4.16%
Elementary and Secondary - Other	1.13%	1.40%	1.15%
Fire Protection- Other	-0.02%	-0.02%	10.80%
Police Protection -Other	0.51%	0.54%	0.75%

Table 6: Average Absolute Relative Errors on All Domains

EB Unit-level	Ratio Benchmark	Additive Benchmark
0.8%	0.8%	10.1%

Table 7: Percentage of Positive Relative Errors

EB Unit-level	Ratio Benchmark	Additive Benchmark
0.8%	0.8%	10.1%

Lastly, we want to see the performance of the unit-level model with simple back transformed on large domains. We selected the first twenty largest domains, and listed them in Table 8.

Table 8: Relative Errors of the 20 Largest Domains

	Relative Error									
itemcode	EB	Ratio	Additive	HT		EB_	ratio_	additive_	HT_	Domain Sizes
Elementary and Secondary - Instruction	-0.21%	-0.48%	-0.22%	0.00%		.	.	.	1	152
Elementary and Secondary - Other Total	0.85%	0.73%	0.82%	-1.00%		.	1	.	.	152
Streets & Highways	-0.46%	0.04%	4.00%	4.81%		.	1	.	.	133
Police Protection - Officers	0.41%	1.31%	2.70%	1.10%		1	.	.	.	123
Streets & Highways	1.37%	1.31%	0.07%	2.46%		.	.	1	.	119
Police Protection - Officers	1.39%	1.15%	1.10%	-1.11%		.	.	1	.	118
Financial Administration	-3.53%	-3.27%	3.38%	-4.73%		.	1	.	.	118
Other Government Administration	-5.75%	-5.49%	1.09%	-7.98%		.	.	1	.	116
Financial Administration	3.01%	2.96%	1.40%	0.40%		.	.	.	1	113
Streets & Highways	0.65%	1.05%	6.38%	9.19%		1	.	.	.	109
Police-Other	0.06%	0.30%	7.00%	5.34%		1	.	.	.	105
Other Government Administration	-1.32%	-0.99%	5.05%	-5.34%		.	1	.	.	102
Financial Administration	-3.57%	-3.31%	3.92%	-0.24%		.	.	.	1	101
Other Government Administration	0.74%	0.70%	-0.85%	-1.19%		.	1	.	.	100
Elementary and Secondary - Instruction	2.60%	0.33%	2.49%	0.43%		.	1	.	.	99
Elementary and Secondary - Other Total	4.08%	3.06%	3.83%	2.23%		.	.	.	1	99
Police-Other	-0.16%	-0.20%	-1.56%	-2.65%		1	.	.	.	96
Police Protection - Officers	-1.53%	-0.73%	1.03%	-6.62%		.	1	.	.	96
Firefighters	-5.97%	-5.47%	-2.82%	8.17%		.	.	1	.	92
Financial Administration	-0.37%	-0.44%	-3.37%	9.52%		1	.	.	.	92
AVERAGE ABSOLUTE	1.90%	1.60%	2.65%	3.73%	SUM	5	7	4	4	

Conclusion

The HT estimator performed poorly in cells where sample sizes are relatively small. The Battese-Harter-Fuller (BHF) model, not shown in this paper, does not work well because the data are very skewed. Our proposed estimator (model 7) dominantly outperformed the other four estimators on small areas and on some large areas as well. Besides, the area-level model also works well; therefore, when unit-level covariates are not available then the area-level is a good choice. Ratio benchmark outperforms the additive benchmark for a selected choice for λ_i.

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Appendix

Function	**Description**
000	Totals for Government
001	Air Transportation
002	Space Research & Technology (Federal)
005	Corrections
006	National Defense & International Relations (Federal)
012	Elementary and Secondary - Instructional
014	Postal Service (Fed)
016	Higher Education - Other
018	Higher Education - Instructional
021	Education- Other (State)
022	Social Insurance Administration (State)
023	Financial Administration
024	Fire Protection - Firefighters
025	Judicial & Legal
029	Other Government Administration
032	Health
040	Hospitals
044	Highways
050	Housing & Community Development (Local)
052	Libraries
059	Natural Resources
061	Parks & Recreation
062	Police Protection - Persons with Power of Arrest
079	Public Welfare
080	Sewerage
081	Solid Waste Management
087	Water Transport & Terminals
089	All Other & Unallocable
090	State Liquor Stores
091	Water Supply
092	Electric Power
093	Gas Supply
094	Transit
112	Elementary and Secondary - Other
124	Fire Protection - Other
162	Police Protection -Other