If Everyone Is an Indexer, Who Sets Prices? Combining Valuation-Driven and Capital-Driven Asset Demand

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Abstract
A classical objection to the efficient markets hypothesis is "if it's optimal for everyone to index, who will set prices?" We introduce a macro-demand model with endogenous valuation driven speculative demand and exogenous valuation insensitive capital flows. We address the computation of price, capital, and valuation elasticities of individual assets. Key statistical and computational issues are identifiability and essential nonlinearity with phase transitions. We illustrate the phase transition phenomena within the macro-demand model. Model and estimator behavior is different in the capital constrained (frozen/solid) phase, the valuation driven (liquid) phase, and the abundant capital, outlier valuation driven (gas/bubble) phase; moreover, the under- and the over-capitalized phases differ from the liquid phase asymmetrically.

Key Words: efficient markets hypothesis, indexing, phase transition, bubble, macro demand, capital assets

1. Introduction

1.1. Equilibrium Model

Our capital asset valuation equilibrium model combines two deviations from the efficient markets framework:

1. **Heterogenous Beliefs:** investors hold private valuations for the asset in question. The valuation may be informed by the market price, but need not agree with the market price. In one limiting case, investors agree on a single valuation, which then becomes the market price. If the market price deviates from the consensus, all stand ready to trade in the opposing direction.

   The general case is a proper distribution of private valuations, which might have a mean/median/mode below or above the market value. We also consider the opposite extreme case, complete disagreement about the valuation, as a limit as the scale of the distribution becomes large.

2. **Exogenous Capital Shocks:** the amount of capital available to be invested in the particular asset is finite. Even if market participants believe an asset to be mispriced, there is a limited amount of capital they can allocate to that bet. The amount of capital available is subject to short-term change for reasons unrelated to the value of the underlying assets.

Neither of these deviations, by itself, readily forms an equilibrium valuation model. We will demonstrate that the two deviations fit conveniently together into a model that manifests or illustrates a number of phenomena, such as bubbles and frozen markets.

1.2. Methodology

A preview of the methods we use:

1. **Microeconomics:** our key tool is the analysis of compensated elasticities of demand functions.

2. **Statistics and Distributions:** we borrow from auction theory the trick of expressing demand functions in terms of distributions of private valuations. We observe qualitatively different behavior within the model based on whether these distributions are heavy- or light-tailed.

3. **Phases of Matter:** in physical chemistry and statistical thermodynamics, an equation of state, such as the van der Waals equation, describes the relationship between a set of state variables. In the limit as one or more state variables tends to an extreme, the state equation becomes dominated by some effects, and other effects become negligible.

   In different limits, as different effects become dominant, the behavior of the system becomes different in a qualitative sense. Thus the three phases of matter can be seen as limiting cases of a single general pressure/volume/temperature relationship. We adapt a similar metaphor of phases and phase transitions.

1.3. Three Phases

We blend existing economic terminology (liquid/frozen/bubble) with an analogy to physical chemistry, where the phases of matter can be seen as limiting cases of a single general pressure/volume/temperature relationship. The three phases of the equilibrium are:

1. **Liquid:** this is the classical case following the Efficient Markets Hypothesis. Traditionally, liquid markets are those where large trades can be executed without market prices. In our liquid phase, the market is a valuation machine, and derives a consensus valuation. At equilibrium, the market price is that valuation; and any deviations are opposed by large capital flows in the opposing direction.

   The liquid phase occurs when there is little heterogeneity of valuation beliefs, and the amount of accessible capital is neither insufficient nor excessive to support an equilibrium market price at that valuation.

2. **Frozen or Solid:** here, capital availability matters more than valuations. The solid phase occurs in two distinct limiting cases.

   When capital is scarce and insufficient to support the median valuation, there is effective consensus that the asset is underpriced. Any incremental capital exposed to this market is immediately invested in the asset, and bids up the price. On the other hand, marginal changes in the consensus valuation are irrelevant, since they don’t change the general agreement that the asset is underpriced.

The other limiting case is extreme disagreement over the valuation, represented by a distribution with a scale parameter tending to infinity. In whatever way equilibrium prices are formed in this situation, we find the equilibrium price sensitive to capital but not valuation shifts, as in the above case.

3. **Bubble or Gas:** traditionally a bubble is a situation when asset prices are above typical or consensus valuations. We make no claim that this is due to irrational behavior, or limited information. We do not claim that a bubble is unsustainable or self reverting. There is disagreement on private valuations, and those with the highest valuations have sufficient capital to make up the entirety of demand. Awareness of others' valuations, and their relationship to the market price, may (exogenously) alter valuations. For our purposes the valuations are a given, and the bubble is declared only if it remains after such feedback considerations.

   This behavior can also occur in at least two circumstances. If price sensitive investors are highly capitalized, the small fraction of extreme valuation optimists can form enough demand to sustain an equilibrium at a price in their range. Alternatively, demand can be dominated by price insensitive investors, such as those who believe in the efficient hypothesis. Then only a small supply of shares remaining to divide among price sensitive investors. The moderate capital of the valuation optimists is then sufficient to sustain an equilibrium at a high price.

2. The Model

2.1. Heterogeneous Beliefs

"It were not best that we should all think alike; it is difference of opinion that makes horse-races."

- Mark Twain, Pudd'nhead Wilson's Calendar, 1894

Borrowing from the auction literature, the demand for the asset is described in terms of a distribution of private valuations. Our decomposition of demand into price sensitive and insensitive components is analogous to the way short-term and long-term investors are treated separately in the market segmentation theory of term structure. Our assumptions are:

1. Demand consists of price sensitive and and price insensitive components. Unlike supply of the asset, which is fixed and denominated in shares, the demand is naturally denominated in dollars.

2. Price sensitive, or speculative, demand, is the aggregate of individual investor demands. Each price sensitive investor, or speculator, has a private valuation of the asset. The speculator invests all of his capital if price is below the private valuation, none if price is too high. This assumption can be relaxed by giving individual investors a less abrupt allocation function, based e.g. on portfolio optimization.

3. The log valuations are distributed with CDF $F(\cdot)$, up to location $\ln\mu$ and scale σ. There are infinitely many infinitely small speculators, with total capital K_S dollars.

Thus the total dollar demand from speculators is $K_S\left(1 - F\left(\frac{\ln p - \ln \mu}{\sigma}\right)\right)$.

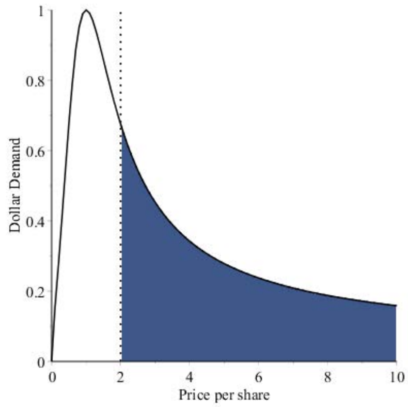

Figure 1: The price-sensitive component of demand function as a function of price is the area under the curve, containing the capital of investors who consider the asset undervalued.

4. Price insensitive demand is a given dollar amount, K_i. The exogenous shocks described below affect both K_i and K_s.

5. No short selling. This is implied in the above decomposition of demand. We may extend the model by incorporating the negative demand to borrow and sell shares into the price sensitive component of demand. Alternatively, the exogenous action of short sellers may be interpreted as changing the supply of shares.

2.2. Exogenous Capital Shocks

Above, we treat the amount of capital available to either price sensitive (speculative) and price insensitive demand as an exogenous given. Given, but not fixed. The amount of capital available to invest in an asset class is set and changed by forces having nothing to do with valuation. We are interested in the sensitivity of the equilibrium to such changes.

Some examples of situations which can be described in these terms:

1. The S&P adds or removes a stock from the S&P 500. Index funds buy or sell accordingly. This has a well-documented impact on the price, without an obvious informative effect; indeed, anticipating and bidding on such index moves is a recognized active trading strategy.

2. An IPO participant allocates \$1 mln to a new issue, buying shares at market or offering price, relying on price discovery by the rest of market participants. The supply of an IPO is fixed in shares, but demand in dollars is aggregated across multiple investors; there may be more or less interest or awareness, and therefore more or less dollar demand, by investors who do not choose to perform their own valuation analysis. Over/undersubscription and “first day pops” are common phenomena. The phenomenon is especially overt in a Dutch Auction IPO, but even when the offering price is discretionary, supply and demand must balance once trading is permitted.

3. A pension fund decides to lower emerging markets allocation from 5% to 2% because of change in tax policy or institutional risk or liquidity appetite. Allocation changes by large asset managers can have large impact on small companies or markets, for reasons unrelated to the particular company or market's value.

4. An individual receives a $500 tax refund, invests it in index fund with no view on market. The individual investor follows the EMH in not attempting to time the market or form a valuation opinion. The classical paradox applies: if the EMH holds, who is on the other side of the trade? Does the trade occur without moving the market?

5. A German bank takes losses on US CDO's, first sells Australian bonds for liquidity, then decreases exposure to Indonesian bonds to lower VaR. At each stage of the sequence, the allocation of a portfolio to an asset class changes for reasons having nothing to do with the valuation of that asset class.

In the above examples, both price-sensitive (Speculator) and insensitive (Indexer? Allocator?) capital are subject to exogenous shocks. In particular, we are interested in the phenomenon of allocators making trades in a particular asset class for reasons that are not connected to valuation opinions for that specific asset class. We would expect EMH-believing indexers to act in this way when dealing with wealth or risk tolerance changing, but EMH belief is not required for the behavior to occur. The traditional EMH paradox may be formulated thus: if everybody was an indexer, nobody would set prices. A more subtle version is: if everybody was an indexer with capital inflows/outflows, who sells to accommodate the inflow? The assumption of heterogeneous beliefs allows for the resolution of this paradox, by allowing the other side of the trade to be taken by investors with differing opinions.
We classify the EMH believers as price insensitive investors, and speculators with private valuation opinions as price sensitive investors, the two components of the demand side of the model. The speculators are necessary to set the prices; but in different model states, the influence of these beliefs on the price may be higher or lower.

2.3. Model Construction

Given the supply of a capital asset fixed at $s = s_0$ shares, and demand as described above, we write, in equilibrium:

$$\text{shares} \times \text{price} = \text{dollar demand for the asset}$$

If there are no capital insensitive investors,

$$ps = K_S\left(1 - F\left(\frac{\ln p - \ln \mu}{\sigma}\right)\right) \tag{1}$$

If there are only capital insensitive investment, then we find the absurd situation,

$$ps = K_i \tag{2}$$

With both investor types,

$$ps = K_i + K_S\left(1 - F\left(\frac{\ln p - \ln \mu}{\sigma}\right)\right) \tag{3}$$

Here

- p is the price per share
- s is the number of shares in the hands of the speculators only
- K_s is the maximum capital investable by the speculators
- $F(\cdot)$ is the cumulative distribution function of the speculators' trigger valuations, up to location and scale
- $\ln \mu$ and σ are the log location and scale parameters

3. Elasticity Analysis

3.1. Speculative Demand

In the speculative demand only model, we find, by differentiation and algebra,

$$ps = K_S\left(1 - F\left(\frac{\ln p - \ln \mu}{\sigma}\right)\right)$$

$$\begin{aligned} d\ln p = {} & \frac{1}{1 + \frac{1}{\sigma}\frac{f}{1-F}} d\ln K_S - \frac{1}{1 + \frac{1}{\sigma}\frac{f}{1-F}} d\ln s + \frac{\frac{1}{\sigma}\frac{f}{1-F}}{1 + \frac{1}{\sigma}\frac{f}{1-F}} d\ln \mu \\ & + \frac{\frac{1}{\sigma}\frac{f}{1-F}}{1 + \frac{1}{\sigma}\frac{f}{1-F}} (\ln p - \ln \mu)\, d\ln \sigma \end{aligned}$$

Expressed in terms of hazard ratio-like term $h = \frac{f}{1-F} = \frac{f\left(\frac{\ln p - \ln \mu}{\sigma}\right)}{1-F\left(\frac{\ln p - \ln \mu}{\sigma}\right)}$:

$$d\ln p = \frac{d\ln K_S - d\ln s + \frac{h}{\sigma} d\ln \mu + \frac{h}{\sigma}(\ln p - \ln \mu)\, d\ln \sigma}{1 + \frac{h}{\sigma}} \tag{4}$$

3.2. Non-Speculative Demand

To add in non-speculative (price insensitive) demand, we cannot simply differentiate $ps_0 = K_i + K_S\left(1 - F\left(\frac{\ln p - \ln \mu}{\sigma}\right)\right)$ and obtain a tractable set of elasticities. Rather, we let $ps = K_S\left(1 - F\left(\frac{\ln p - \ln \mu}{\sigma}\right)\right)$ as before, so that $ps_0 = K_i + ps$. We use the previous derivation, and then eliminate s and s_0 in favor or K_i and K_s. We thus obtain a total differential treating s_0 as a constant, p as the dependent variable, and K_S, K_i, μ, and σ as the independent variables. This corresponds to the situation where the share supply is fixed in the short run, and capital and valuation changes affect the price.

Starting with $ps_0 = K_i + ps$,

$$s_0 = \frac{K_i + ps}{p} = \frac{K_i}{p} + s$$

$$0 = \frac{p\, dK_i - K_i dp}{p^2} + ds$$

$$d\ln s = \frac{K_i}{ps}\ d\ln p - \frac{1}{ps} dK_i$$

Since $ps = (1-F)K_S$,

$$d\ln s = \frac{K_i}{(1-F)K_S}\ d\ln p - \frac{1}{(1-F)K_S} dK_i = \frac{K_i}{K_S}\frac{1}{1-F}(d\ln p - d\ln K_i)$$

Substituting in the $d\ln s$:

$$d\ln p = \frac{1}{1+\frac{h}{\sigma}} d\ln K_S - \frac{1}{1+\frac{h}{\sigma}} d\ln s + \frac{\frac{h}{\sigma}}{1+\frac{h}{\sigma}} d\ln\mu + \frac{\frac{1}{\sigma}\frac{f}{1-F}}{1+\frac{h}{\sigma}}(\ln p - \ln\mu)\ d\ln\sigma$$

$$d\ln p = \frac{1}{1+\frac{h}{\sigma}} d\ln K_S - \frac{1}{1+\frac{h}{\sigma}}\frac{K_i}{K_S}\frac{1}{1-F}(d\ln p - \ln K_i)$$
$$+ \frac{\frac{h}{\sigma}}{1+\frac{h}{\sigma}} d\ln\mu + \frac{\frac{h}{\sigma}}{1+\frac{h}{\sigma}}(ln\, p - ln\,\mu)\ d\, ln\,\sigma$$

$$\left(1 + \frac{1}{1+\frac{h}{\sigma}}\frac{K_i}{K_S}\frac{1}{1-F}\right) d\ln p = \frac{1}{1+\frac{h}{\sigma}} d\ln K_S + \frac{1}{1+\frac{h}{\sigma}}\frac{K_i}{K_S}\frac{1}{1-F} d\ln K_i$$
$$+ \frac{\frac{h}{\sigma}}{1+\frac{h}{\sigma}} d\ln\mu + \frac{\frac{h}{\sigma}}{1+\frac{h}{\sigma}}(\ln p - \ln\mu)\ d\, ln\,\sigma$$

The resulting elasticities are

$$\begin{aligned}
&d\, ln\, p = \\
&= \frac{1}{1+\frac{h}{\sigma}+\frac{K_i}{K_S}\frac{1}{1-F}} d\ln K_S \\
&+ \frac{1}{1+\frac{h}{\sigma}+\frac{K_i}{K_S}\frac{1}{1-F}}\frac{K_i}{K_S}\frac{1}{1-F} d\ln K_i \\
&+ \frac{1}{1+\frac{h}{\sigma}+\frac{K_i}{K_S}\frac{1}{1-F}}\frac{h}{\sigma} d\ln\mu \\
&+ \frac{1}{1+\frac{h}{\sigma}+\frac{K_i}{K_S}\frac{1}{1-F}}\frac{h}{\sigma}(\ln p - \ln\mu)\ d\ln\sigma
\end{aligned}$$

3.3. Closed Form Solution for Logistic Distribution

Let $F(\cdot)$ be the logistic distribution with mean 0 and stdev 1, letting $q = \frac{\sqrt{3}}{\pi} \approx 0.55$ for convenience. Then

$$F(x) = \frac{1}{1 + \exp\left(-\frac{x}{q}\right)};\ f(x) = \frac{\exp\left(-\frac{x}{q}\right)}{q\left(1 + \exp\left(-\frac{x}{q}\right)\right)^2}$$

Algebraic manipulation gives

$$\frac{f(x)}{1 - F(x)} = \frac{1 + \exp\left(-\frac{x}{q}\right)}{\exp\left(-\frac{x}{q}\right)} \frac{\exp\left(-\frac{x}{q}\right)}{q\left(1 + \exp\left(-\frac{x}{q}\right)\right)^2} = \frac{1}{q\left(1 + \exp\left(-\frac{x}{q}\right)\right)} = \frac{1}{q}F(x)$$

Then, for the model with speculative demand only,

$$d\ln p = \frac{d\ln K_S - d\ln s}{1 + \frac{F}{q\sigma}} + \frac{d\ln\mu + (\ln p - \ln\mu)\, d\ln\sigma}{1 + \frac{q\sigma}{F}}$$

This gives convenient forms for the capital elasticity

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{1}{1 + \frac{F}{q\sigma}} = \frac{q\sigma}{q\sigma + F}$$

and valuation elasticity

$$\frac{\partial \ln p}{\partial \ln \mu} = \frac{1}{1 + \frac{q\sigma}{F}} = \frac{F}{F + q\sigma}$$

The general speculative-only form is then

$$d\ln p = \frac{d\ln K_S - d\ln s}{1 + \frac{F}{q\sigma}} + \frac{d\ln\mu + (\ln p - \ln\mu)\, d\ln\sigma}{1 + \frac{q\sigma}{F}}$$

Conveniently,

$$\frac{\partial \ln p}{\partial \ln K_S} + \frac{\partial \ln p}{\partial \ln \mu} = 1$$

When $F \approx 0$, price is only sensitive to capital availability, and not valuation:

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{q\sigma}{q\sigma + F} \approx \frac{q\sigma}{q\sigma + 0} = 1$$

$$\frac{\partial \ln p}{\partial \ln \mu} = 1 - \frac{\partial \ln p}{\partial \ln K_S} = 0$$

For any F, as $\sigma \to \infty$,

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{q\sigma}{q\sigma + F} \approx \frac{q\sigma}{q\sigma} = 1$$

If $F \gg 0$ and $\sigma \approx 0$:

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{q\sigma}{q\sigma + F} \approx \frac{0}{F} = 0$$

At a representative value of $\sigma = 0.2$, $q\sigma \approx 0.1$.

4. Phase Limits

4.1. Classical/Liquid Phase

Under the logistic distribution,

$$d \ln p = \frac{d \ln K_S - d \ln s}{1 + \frac{F}{q\sigma}} + \frac{d \ln \mu + (\ln p - \ln \mu)\, d \ln \sigma}{1 + \frac{q\sigma}{F}}$$

Suppose there is little disagreement on the price, so $\sigma \approx 0$. Then with $F \neq 0$, $d \ln p \approx d \ln \mu + (\ln p - \ln \mu)\, d \ln \sigma$. This is the (classical) liquid phase. Valuation matters, and incremental valuation shifts are translated directly into price. On the other hand, small capital shocks do not matter. The valuation elasticity is 1, capital elasticity is 0.

4.2. Solid/Frozen Phase

A qualitatively different situation occurs when $F \approx 0$:

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{q\sigma}{q\sigma + F} \approx \frac{q\sigma}{q\sigma + 0} = 1$$

$$\frac{\partial \ln p}{\partial \ln \mu} = 1 - \frac{\partial \ln p}{\partial \ln K_S} = 0$$

Here valuation elasticity is 0, and capital elasticity is 1. When capital is scarce, every speculator is fully invested; fund managers agree the asset is undervalued, but can only allocate so much. This may have been the situation in the post-2008 mortgage market.

This also happens with $F \neq 0$ and $\sigma \to \infty$:

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{q\sigma}{q\sigma + F} \approx \frac{q\sigma}{q\sigma} = 1$$

Here we have extreme disagreement about valuation; incremental changes in consensus/mean valuation don't matter. Informally, the market can be pushed around by capital shocks, because it doesn't know what the price should be.

4.3. Gas/Bubble Phase

At the opposite extreme, when $F \approx 1$, too many dollars want to be allocated to the hot sector, or the IPO is oversubscribed. For more capital to fit in, it has to elbow out other capital.

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{q\sigma}{q\sigma + F} \approx \frac{q\sigma}{q\sigma + 1} \approx q\sigma$$

Using our representative value of $\sigma = 0.2$, $q\sigma \approx 0.1$, giving a capital elasticity of ≈ 0.1, valuation elasticity of ≈ 0.9. A high $\sigma = 4$ gives a capital elasticity of $\approx 2/3$, valuation elasticity of $\approx 1/3$. Thus the elasticities can range from 0 to 1, and are highly sensitive to the scale of the underlying distribution. Further, this case is especially sensitive to the Logistic Distribution assumption, as represented by the hazard ratio h term. For a general distribution

$$\frac{\partial \ln p}{\partial \ln K_S} = \frac{1}{1 + \frac{h}{\sigma}};\ \frac{\partial \ln p}{\partial \ln \mu} = \frac{\frac{h}{\sigma}}{1 + \frac{h}{\sigma}}$$

At high $F(x) \approx 1$, and therefore high $x = \frac{\ln p - \ln \mu}{\sigma}$, the right asymptotic behavior of the hazard ratio of a given distribution dominates this behavior.

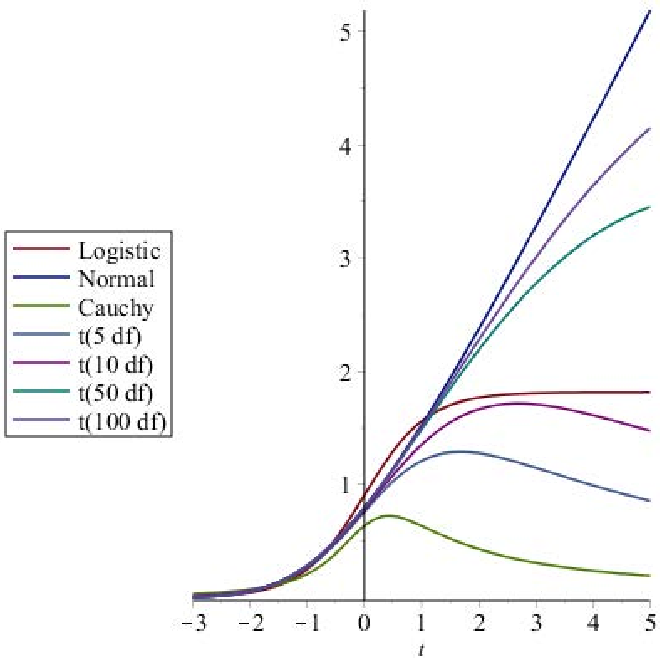

Figure 2: The hazard ratio term for various continuous real line support distributions under standardized parametrizations.

The behavior of the hazard ratio $h = \frac{f}{1-F}$ at the tail of the distribution $F \to 1$ varies dramatically based on the distribution. In Figure 2, we plot the hazard ratio for various distributions. The logistic distribution is flat, approaching a constant as $x \to \infty$. The heavier tailed Student's t and Cauchy distributions decay to zero, whereas the lighter tailed normal distribution grows linearly with x.

Thus, in the bubble case, either a high or low valuation or capital sensitivity is a possibility, depending on the tail properties of the underlying valuation distribution.

4.4. Valuation-Capital Sensitivity Tradeoff

We can generalize the tradeoff between valuation and capital sensitivity to the case including price insensitive investors.

The valuation sensitivity answers the question: if everyone's private valuations increase by 1%, by how many % does the price increase? We shift the log-mean μ of the log-location-scale family by $\Delta\mu$; the valuation sensitivity is the same as the previously computer valuation elasticity, which in the case without price insensitive investors was previously found to be

$$\frac{\partial \ln p}{\partial \ln \mu} = \frac{\frac{h}{\sigma}}{1 + \frac{h}{\sigma}}$$

We introduce the capital injection sensitivity ϵ_C, a modified form of the capital sensitivity. The question this answers is: if an external agent buys 1% of the overall shares, by how many % does the price increase? We will find, in the general case with sensitive and insensitive capital,

$$\epsilon_V + \epsilon_C = 1$$

As above, the position of the elasticities on this 0-to-1 tradeoff scale depend on the phase of the equilibrium:

1) For a liquid market, $\epsilon_V = 1$ and $\epsilon_C = 0$

2) For a solid/frozen market, $\epsilon_V = 0$ and $\epsilon_C = 1$

3) For a gas/bubble market, *it depends*. In particular,

   - ϵ_V tends to 1 if
     1) Tails are light (the valuation distribution is normal) AND
     2) The fraction of capital held by speculators is nonzero.

   - ϵ_C tends to 1 if
     1) Tails are heavy (e.g. Cauchy, Student's t) OR
     2) The fraction of capital held by speculators $\rightarrow 0$.

   - Logistic tails are an intermediate case

The valuation sensitivity is thus a measure of market liquidity. When the amount of capital is commensurate with an agreed upon consensus valuation, markets are *most liquid* and $\epsilon_V \rightarrow 1$. When there is a scarcity of capital or extreme disagreement about valuations, markets are *least liquid*, and $\epsilon_V \rightarrow 0$. When there is an abundance of capital, different behaviors are possible.

4.5. Derivation of the Valuation-Capital Sensitivity Tradeoff

We will derive ϵ_C, answering the question: if an external agent buys 1% of the overall shares, by how many % does the price increase?

Within the general model, including price insensitive capital, we use the previously derived elasticities,

$$d \ln p = \frac{d \ln K_S + \frac{K_i}{K_S}\frac{1}{1-F} d \ln K_i + h\, d \ln \mu + h(\ln p - \ln \mu)\, d \ln \sigma}{1 + h + \frac{K_i}{K_S}\frac{1}{1-F}}$$

Let r be the fraction of shares owned by speculators:

$$r = \frac{K_S(1-F)}{K_i + K_S(1-F)}$$

Then

$$\frac{1}{1 + h + \frac{K_i}{K_S}\frac{1}{1-F}} = \frac{K_S(1-F)}{K_i + K_S(1-F)} \frac{1}{1 + h\frac{K_S(1-F)}{K_i + K_S(1-F)}} = \frac{r}{1+hr}$$

$$\frac{K_i}{K_S}\frac{1}{1-F} = \frac{1-r}{r}$$

$$d \ln p = \frac{r}{1+hr} d \ln K_S + \frac{1-r}{1+hr} d \ln K_i + \frac{hr}{1+hr} d \ln \mu + \frac{hr}{1+hr}(\ln p - \ln \mu)\, d \ln \sigma$$

The partial derivative with respect to a shift in insensitive capital, scaled to market cap, is

$$\begin{aligned}
\epsilon_C &= \frac{\partial \ln p}{\partial K_i / [K_i + K_S(1-F)]} = \frac{\partial \ln p}{\frac{K_i}{K_i + K_S(1-F)} \partial \ln K_i} = \frac{\partial \ln p}{\partial \ln K_i} \frac{K_i + K_S(1-F)}{K_i} \\
&= \frac{1}{1 + h + \frac{K_i}{K_S}\frac{1}{1-F}} \frac{K_i}{K_S}\frac{1}{1-F} \frac{K_i + K_S(1-F)}{K_i} = \frac{K_i + K_S(1-F)}{K_S(1-F) + K_S(1-F)h + K_i} \\
&= \frac{1}{1 + h\frac{K_S(1-F)}{K_i + K_S(1-F)}} = \frac{1}{1+hr}
\end{aligned}$$

The valuation sensitivity is derived directly as an elasticity:

$$\begin{aligned}
\epsilon_V &= \frac{\partial \ln p}{\partial \ln \mu} = \frac{h}{1 + h + \frac{K_i}{K_S}\frac{1}{1-F}} = \frac{hK_S(1-F)}{hK_S(1-F) + K_i + K_S(1-F)} \\
&= h\frac{K_S(1-F)}{K_i + K_S(1-F)} \frac{1}{1 + h\frac{K_S(1-F)}{K_i + K_S(1-F)}} = \frac{hr}{1+hr}
\end{aligned}$$

Thus

$$\epsilon_V = \frac{hr}{1+hr} \tag{5}$$

$$\epsilon_C = \frac{1}{1+hr} \tag{6}$$

$$\epsilon_C + \epsilon_V = 1 \tag{7}$$

Thus hr is also a measure of liquidity; if $hr \gg 1$, $\epsilon_V \to 1$. Both the h and the r terms depend nontrivially on the shape of the distribution and the other parameters of the equilibrium state.

5. Discussion

5.1. Phenomena Treated as Exogenous

We treat changes in private valuations, and capital shocks, as exogenous. This means we explicitly avoid modeling the opposing direction of causality. A change in the market price of the asset is informative and would be expected to affect private valuations. A price change also affects the wealth of asset owners, affecting more strongly those who have higher allocation to the asset. This distorts both the shape and the scale of the demand function. Combining the information and capital effects, a price change may also affect capital availability by attracting capital not currently involved in the particular market.

Expressed in terms of the model abstractions, such as private valuations, the reverse causal effects outside are more subjective and less mechanistic than the phenomena we do model. We may ascribe them to the free will of market participants, or causes too complex to analyze, but too irregular in variability to be treated as random noise.

The choice to externalize these reverse-causal phenomena does not make our model inconsistent with our view of economic reality, but it does make it non-predictive. To predict empirical changes, we would need to separately model the reverse-causal components, rather than treat them as negligible.

5.2. Efficient Market Hypothesis

So do we believe in market efficiency? Instead of Efficient Market Hypothesis, the perspective demonstrated in this paper argues for a Pretty Reasonable Market Hypothesis. The market valuation represents somebody's marginal beliefs. Blindly investing in an asset, that is, joining the price insensitive investor category, is equivalent to investing as a speculator with the same opinion as the marginal investor.

If capital is abundant, the optimists' beliefs prevail. If capital is scarce, pessimists' beliefs prevail. In the long run, returns depend on the shifts in the equilibrium. Changes in capital availability may move the price from the pessimist camp to the optimist camp, without changes in the fundamentals of the underlying asset.

In the long run, returns depend on which camp is right. We should not assume that the optimists tend to overvalue assets, pessimists undervalue them, and moderates are right. This is at best an untested additional empirical assumption. The assumption may not only be untested, but untestable. Three competing hypotheses would lead to radically different investment strategies:

1) **Strong EMH:** the marginal agent is always right. We should not try to determine whether the valuation is high or low relative to the mean/mode/median consensus.

2) **Mean reversion hypothesis:** moderate consensus valuation is better than pessimists or optimists; we should try to determine which group's opinions are driving the price, and buy only if valuation is low relative to consensus.

3) **Momentum hypothesis:** abundant capital reinforces growth, scarce capital causes complete collapse. We should try to determine which group's opinions are driving the price, and buy only if valuation is *high* relative to consensus.

5.3. Implications for Practitioners

The framework we present can help a practitioner identify a bubble or a frozen market, but does not substitute for judgement. We do not asset that bubbles will pop or that frozen markets will melt.

We do asset that markets act differently in the three phases. Liquid markets are classical asset valuation machines. Frozen markets are capital constrained; prices can be pushed around by big trades. Bubble markets are sensitive and potentially unstable. The phases we describe are limiting cases; realistic markets may exist in-between the extremes, combining features of multiple phases. Too much capital and too little are not mirror images of each other; trying to exploit underpriced and overpriced assets requires different analytical skillsets.

The practitioner can look not only for phases, but also for phase transition events. Historical volatilities and correlations change during a phase transition; risk models and trading strategies based on history collected in one phase will become invalid in another.