# Making Better Decisions on Placement in Mathematics Courses

W. Robert Stephenson[1], Chen Hua[2], Elgin Johnston[3], Wolfgang Kliemann[4]

**Abstract**

ALEKS (Assessment and LEarning in Knowledge Spaces) is educational software based on knowledge space theory (Doignon and Falmagne, 1999, 2011; Falmagne, Koppen, Villano, Doignon, and Johanessen, 1990). ALEKS software claims to be able to assess an individual's mastery of certain types of problems and to be able to indicate what topics the individual is prepared to learn. One of the ALEKS assessments looks at various areas of mathematics with the intent of assessing mastery of those areas and indicating readiness for further study. The ALEKS mathematics assessment is increasingly being used to assist in the placement of students into appropriate courses that require prerequisite knowledge of certain areas of mathematics. This paper looks at the relationship between students' ALEKS mathematics assessment scores and their success in mathematics and statistics courses. Logistic regression is used to model the relationship between the ALEKS mathematics assessment score, or sub-score, and the probability of "success" in a specific mathematics or statistics course. Different levels of "success", from simply passing the course to attaining a specific grade are investigated.

**Keywords:** ALEKS, Logistic Regression, Mathematics Placement, Statistics Placement

## 1. Introduction

Assessment and LEarning in Knowledge Spaces (ALEKS) is an assessment and learning system. It is Web-based and uses artificially intelligent adaptive questioning to assess what a student knows and doesn't know. A student can then work through course material, on line, and ALEKS will reassess the student's knowledge to make sure that what the student is exposed to in the course material is learned and retained. Although designed to be both an assessment and a learning system, many colleges and universities are using ALEKS as a means of evaluating students' prerequisite knowledge, especially in mathematics, so as to place those students in the appropriate mathematics or mathematics related courses. See for example Carpenter and Hanna (2007), Feldman, Bullock and Callahan (2012), Reddy and Harper (2013a, 2013b). The use of ALEKS as a placement tool is the focus of this paper.

The ALEKS mathematics assessment tool is personalized for each student. The student is presented 20 to 30 questions on each of several aspects of mathematics. As the student works through each question, ALEKS begins to understand what the student knows and doesn't know. ALEKS chooses questions adaptively, based on how the student does on previous questions. Upon completion of the assessment, ALEKS constructs a multicolor

[1] Department of Statistics, Iowa State University, Ames, IA 50011
[2] Department of Statistics, Iowa State University, Ames, IA 50011
[3] Department of Mathematics, Iowa State University, Ames, IA 50011
[4] Department of Mathematics, Iowa State University, Ames, IA 50011

pie chart. Each "slice" of the "pie" corresponds to a particular area in the mathematics assessment. The version of the ALEKS mathematics assessment used for this paper has eight areas:

- Equations and Inequalities (EQUAT)
- Exponentials and Logarithms (LOG)
- Exponents and Polynomials (PNMAL)
- Geometry and Trigonometry (TRIG)
- Linear and Quadratic Functions (FNCTN)
- Radical Expressions (RADCL)
- Rational Expressions (RATL)
- Real Numbers (NUM)

Each "slice" is filled by a darker color which represents how much the student has mastered that area; ALEKS sub-score (0 to 100). An example of the pie chart is presented in Figure 1.

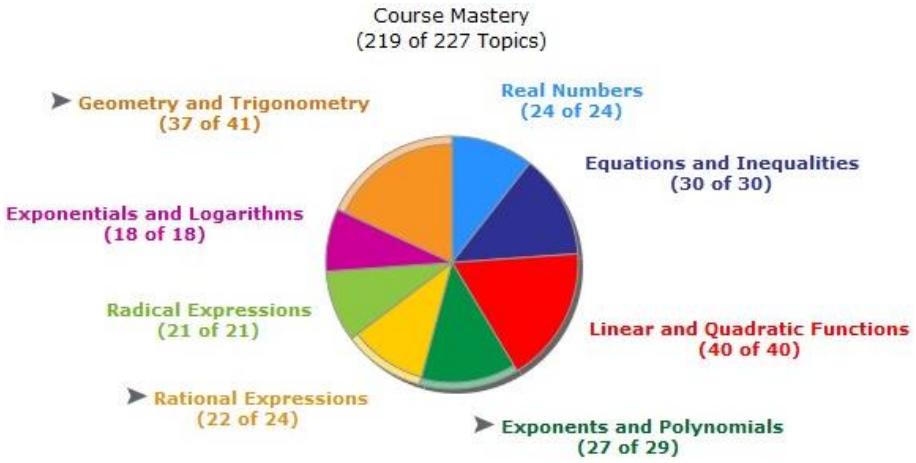


**Figure 1**: Example of a pie chart created for the ALEKS mathematics assessment

The pie chart reflects the student's current knowledge in each of the areas. ALEKS then makes recommendations for further study to enhance the student's knowledge (the learning part of ALEKS). In this paper we examine the use of an overall ALEKS score (a weighted combination of sub-score on a scale from 0 to 100), or sub-score, to make placement recommendations so that students enroll in mathematics, and mathematics related, courses for which they are prepared.

All engineering students entering Iowa State University in fall 2010 were required to take the ALEKS assessment in mathematics. If a student was not satisfied with their first overall ALEKS score, they could take the assessment a second time. In subsequent years, more and more students entering Iowa State University were required to take the ALEKS assessment in mathematics.

A suggested placement in mathematics or mathematics related courses was developed using the overall ALEKS score. Placement recommendations for courses that will be the focus of this paper are summarized in Table 1.

**Table 1:** Placement recommendations based on the overall ALEKS score.

| ALEKS | 35 or higher | 65 or higher |
|---|---|---|
| Course | Introductory Statistics | Calculus I |

Although the overall ALEKS score was used to make placement recommendations, these were not binding and students could enroll in the mathematics or mathematics related courses they chose even though their overall ALEKS score indicated their lack of preparation for that level course.

In order to evaluate the effectiveness of the overall ALEKS score, or sub-score, for placement in mathematics or mathematics related courses, we propose using logistic regression to model the relationship between the ALEKS score or sub-score and "success" in the first mathematics or mathematics related course taken by students.

"Success" can be defined in several ways. "Success" can be defined as simply completing the course with a passing (D– or higher) grade. Alternatively, "success" can be defined as achieving a particular letter grade e.g. A, B, C or D.

Logistic regression is used to predict the probability of "success" based on one or more numerical explanatory (predictor) variables. In simple logistic regression the logarithm of the odds for success is modeled as a linear function of the explanatory (predictor) variable. For example, if "success" is defined as completing the course with a passing (D–) grade, then let

$$Prob(Pass) = \quad p$$
$$Prob(Fail) = 1 - p$$

The simple logistic regression model is

$$log\left(\frac{p}{1-p}\right) = \alpha + \beta x$$

where *x* is a numerical explanatory (predictor) variable, like the overall ALEKS score.

This model can be rewritten as

$$p = \frac{e^{\alpha+\beta x}}{1 + e^{\alpha+\beta x}}$$

Maximum likelihood estimation is used to obtain estimates of α and β, which in turn are used to estimate the probability of success, $p$.

## 2. Results

In this section we investigate how effective the ALEKS score is at predicting "success" in several courses; a first semester calculus course and introductory statistics courses taught at Iowa State University. Although each of these courses has a recommended minimum ALEKS score for placement, students often take a course even though they do not meet the minimum recommended ALEKS score. For each course, the distribution of ALEKS scores for those students taking the course will be described. The overall distribution of grades and distribution of grades conditional on ranges of ALEKS scores will be presented. Simple logistic regression equations will be fit using the ALEKS score as the explanatory (predictor) variable and various definitions of "success" for the response. How well the logistic regression fits the data will be evaluated.

### 2.1 Calculus I

This course has a prerequisite of two semesters of algebra, one year of geometry and one semester of trigonometry in high school or a college level course: Preparation for Calculus. The course covers differential calculus, applications of the derivative and an introduction to integral calculus. The recommended minimum ALEKS score for placement in Calculus I is 65.

The data set consists of 1099 students who took Calculus I during the fall semester 2011. Most (72%) of these students were classified as freshman and 26% were classified as sophomores. Most (78%) of these students were from the College of Engineering and another 15% were from the College of Liberal Arts and Sciences.

Figure 2 shows the distribution of ALEKS scores for the 1099 students. The mean ALEKS score was 72.95, the median ALEKS score was 75.0 and the standard deviation of the ALEKS scores was 14.92. The distribution is skewed to the left. Approximately 23% of the students who took Calculus I had an ALEKS score below the minimum recommended score of 65.

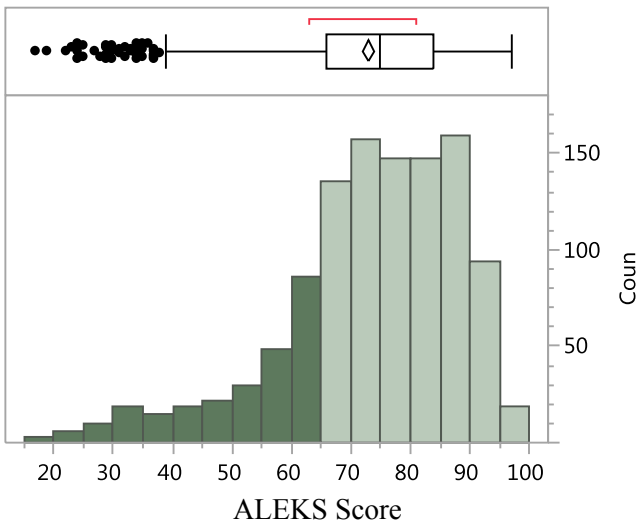


**Figure 2***:* Distribution of ALEKS scores for students enrolled in Calculus I

Figure 3 shows the distribution of final grades in Calculus I Although a +/– system of grading was used we have chosen to concentrate on the whole letter grade, therefore a B could be a B– or B or B+. The grade of X indicates that a student dropped the course after the first week of classes.

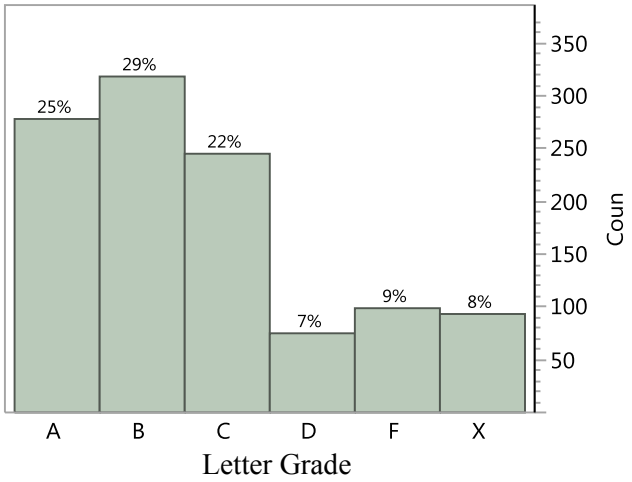


**Figure 3:** Distribution of letter grades for students enrolled in Calculus I

In order to investigate the relationship between the ALEKS score and the letter grade earned in Calculus I we first look at the distribution of letter grades conditional on ALEKS score categories. The ALEX score categories are less than 50, 50 to 60, 60 to 70, 70 to 80, 80 to 90 and 90 to 100. The numbers of students, along with the percentages in parentheses, receiving each letter grade within each ALEKS category are given in Table 2.

**Table 2:** Numbers and percentages of students receiving each letter grade within each ALEKS category.

| Grade | < 50 | 50 to 60 | 60 to 70 | 70 to 80 | 80 to 90 | 90 to 100 | |
|---|---|---|---|---|---|---|---|
| X | 19 (21.8) | 12 (15.8) | 20 (9.1) | 21 (7.0) | 16 (5.3) | 4 (3.6) | 92 |
| F | 21 (24.1) | 9 (11.8) | 22 (10.1) | 21 (7.0) | 21 (6.9) | 3 (2.7) | 97 |
| D | 7 (8.1) | 3 (4.0) | 20 (9.1) | 23 (7.6) | 19 (6.3) | 1 (0.9) | 73 |
| C | 21 (24.1) | 24 (31.6) | 61 (27.9) | 71 (23.5) | 47 (15.5) | 20 (18.0) | 244 |
| B | 10 (11.5) | 23 (30.3) | 52 (23.8) | 108 (35.8) | 92 (30.3) | 31 (27.9) | 316 |
| A | 9 (10.3) | 5 (6.6) | 44 (20.1) | 58 (19.2) | 109 (35.9) | 52 (46.9) | 277 |
| ALEKS | 87 | 76 | 219 | 302 | 304 | 111 | 1099 |

Inspection of Table 2 reveals two trends: as the ALEKS score increases the proportion of students receiving A's tends to increase and the proportion of students receiving F's of dropping the course, X's, tends to decrease. Although students are interested in what

letter grade they receive, they are often interested in getting a minimum grade, e.g, a D– or better so that they pass the course or a C– or better so that they can take the next course in the calculus sequence, Calculus II. Table 3 presents the numbers of students, along with the percentages in parentheses, receiving at least a specific grade within each ALEKS category

**Table 3:** Cumulative numbers and percentages of students receiving at least each grade within each ALEKS category.

| | ALEKS Category | | | | | | |
|---|---|---|---|---|---|---|---|
| Grade | < 50 | 50 to 60 | 60 to 70 | 70 to 80 | 80 to 90 | 90 to 100 | |
| F or X | 40 (46.0) | 21 (27.6) | 42 (19.2) | 42 (13.9) | 37 (12.2) | 7 (6.3) | 97 |
| D– or better | 47 (54.0) | 55 (72.4) | 177 (80.8) | 260 (86.1) | 267 (87.8) | 104 (93.7) | 73 |
| C– or better | 40 (46.0) | 52 (68.4) | 157 (71.7) | 237 (78.5) | 248 (81.6) | 103 (92.8) | 244 |
| B– or better | 19 (21.8) | 28 (36.8) | 96 (43.9) | 166 (55.0) | 201 (66.1) | 83 (74.8) | 316 |
| A– or better | 9 (10.3) | 5 (6.6) | 44 (20.1) | 58 (19.2) | 109 (35.9) | 52 (46.9) | 277 |
| | 87 | 76 | 219 | 302 | 304 | 111 | 1099 |

From the percentages presented in Table 3, the relationship between ALEKS scores and receiving a particular grade, or better, is apparent. As the ALEKS score increases the percentage of students receiving that grade, or better, increases. The opposite is true for grades of F or X, as the ALEKS score increases, the percentage of students receiving an F or dropping the course, X, tends to decrease. These trends are presented visually in the mosaic plot in Figure 4.

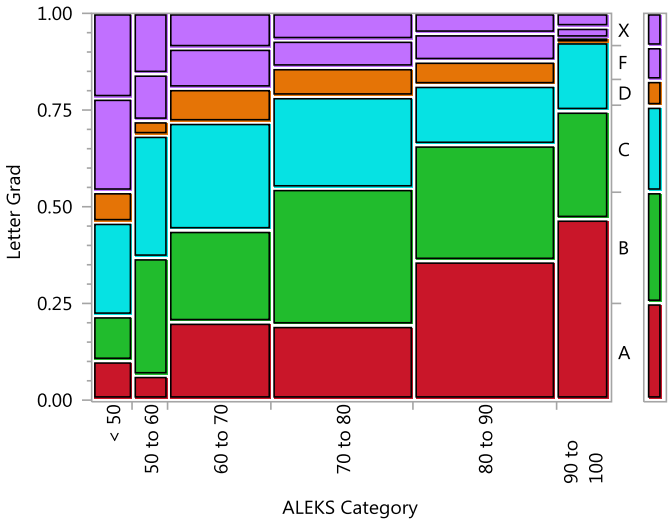


**Figure 4:** Mosaic plot of letter grades within each ALEKS category

Given the general trend seen between the ALEKS score and the cumulative percentage of students receiving a specific grade, or better, a logistic regression is fit using the ALEKS

score as the explanatory (predictor) variable and the categories A– or better, B– or better, C– or better and D– or better a passing grade.

The fitted equations for the logarithm of the odds, $log\left(\frac{p}{1-p}\right)$, where $p$ is the probability of "success" together with the test statistic and P-value for the significance of the ALEKS variable are given in Table 4. The predicted probabilities are plotted in Figure 5.

**Table 4:** Fitted equation for the logarithm of the odds of success.

| "success" | Fitted equation for $log\left(\frac{p}{1-p}\right)$, | $\chi^2$ | P-value |
|---|---|---|---|
| Pass | –1.4304 + 0.04291*ALEKS | 70.32 | < 0.0001 |
| C– or better | –1.6536 + 0.03969*ALEKS | 69.88 | < 0.0001 |
| B– or better | –3.1620 + 0.04546*ALEKS | 89.64 | < 0.0001 |
| A– or better | –4.7025 + 0.04791*ALEKS | 60.23 | < 0.0001 |

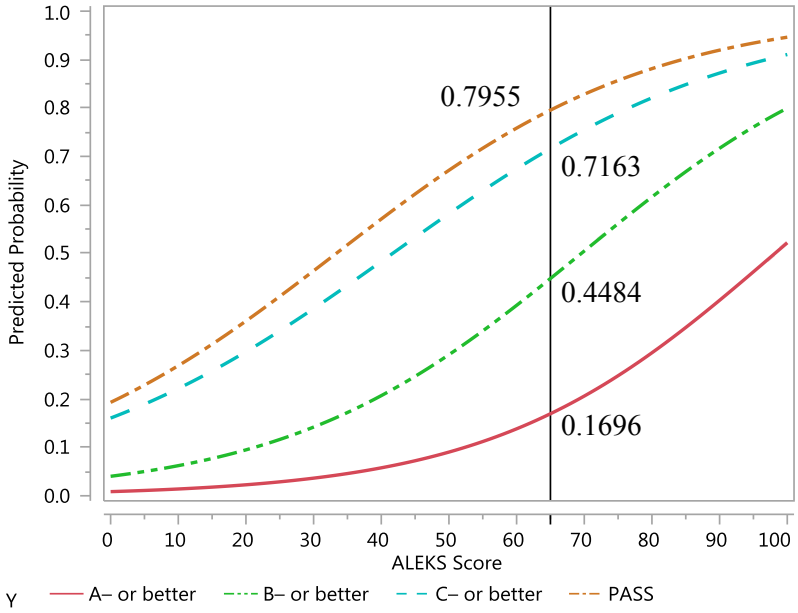


**Figure 5:** Graphs of the predicted probabilities for various grades, or better in Calculus I based on ALEKS score.

Table 5 compares the observed cumulative probabilities for each of the grades, or better, with the predicted probabilities, calculated at the midpoint, for each ALEKS category. In general, there is good agreement between the observed and predicted cumulative probabilities. The one exception is that the logistic equation does not fit well for the students with ALEKS scores less than 50 or between 50 to 60 who received an A– or better. For the other grades the fitted logistic equations provide reasonable predictions for the probability of success, as defined as achieving that specified grade or better.

**Table 5:** Observed and predicted, in parentheses, cumulative probabilities for receiving at least each grade within each ALEKS category.

| Grade | < 50 | 50 to 60 | 60 to 70 | 70 to 80 | 80 to 90 | 90 to 100 |
|---|---|---|---|---|---|---|
| F or X | 0.4598 (0.3380) | 0.2763 (0.2830) | 0.1918 (0.2045) | 0.1391 (0.1434) | 0.1217 (0.0983) | 0.0631 (0.0662) |
| D– or better | 0.5402 (0.5178) | 0.7237 (0.7170) | 0.8082 (0.7955) | 0.8609 (0.8566) | 0.8783 (0.9017) | 0.9369 (0.9338) |
| C– or better | 0.4598 (0.4342) | 0.6842 (0.6293) | 0.7169 (0.7163) | 0.7848 (0.7897) | 0.8158 (0.8481) | 0.9279 (0.8925) |
| B– or better | 0.2184 (0.1721) | 0.3684 (0.3403) | 0.4384 (0.4484) | 0.5497 (0.5615) | 0.6612 (0.6686) | 0.7477 (0.7607) |
| A– or better | 0.1034 (0.0463) | 0.0658 (0.1123) | 0.2009 (0.1696) | 0.1921 (0.2480) | 0.3586 (0.3474) | 0.4685 (0.4622) |
| | 87 | 76 | 219 | 302 | 304 | 111 |

Chi square goodness of fit tests are performed comparing the observed number of students to the predicted number of students in each category for each specified grade. Table 6 summarizes the results of those tests.

**Table 6:** Goodness of fit test statistics and P-values.

| Grade | $\chi^2$ test statistic | P-value |
|---|---|---|
| D– or better | 0.33 | 0.9993 |
| C– or better | 1.35 | 0.9688 |
| B– or better | 1.41 | 0.9652 |
| A– or better | 12.91 | 0.0445* |

All of the fits, with the exception of A– or better, are remarkably good. The lack of fit for the students who received an A– or better comes from those with ALEKS scores less than 50 or between 50 and 60. This lack of fit could be due, in part, to the low number of students in those categories. If the two categories are combined, the fit is much better ($\chi^2$ = 5.41, P-value = 0.3679).

The ALEKS score can accurately predict the probability of success in Calculus I for grades of B– or better, C– or better and D– or better. This can be very useful in advising students, especially those who do not meet the recommended minimum ALEKS score of 65.

### 2.2 Introductory Statistics

Introductory statistics courses have a prerequisite of 1½ years of high school algebra. The courses cover descriptive statistics and graphical displays of data, relationships between two variables, data collection, elementary probability, estimation and testing. The recommended minimum ALEKS score for placement in introductory statistics is 35.

The data set consists of 156 students who took an introductory statistics course from fall 2011 to spring 2013. Many more students took these courses during that time frame but

most of the students were in majors that did not require them to take the ALEKS test. The two colleges with the most students enrolled were the College of Liberal Arts and Sciences, 59% and the College of Agriculture and Life Sciences, 25%.

Figure 6 shows the distribution of ALEKS scores for the 156 students. The mean ALEKS score was 50.1, the median ALEKS score was 48 and the standard deviation of ALEKS scores was 23.28. The distribution is skewed to the right. Approximately 32% of the students who took introductory statistics had an ALEKS score below the minimum recommended score of 35. Based on the ALEKS scores, the mathematical preparation of students taking an introductory statistics course is much lower than those taking Calculus I.

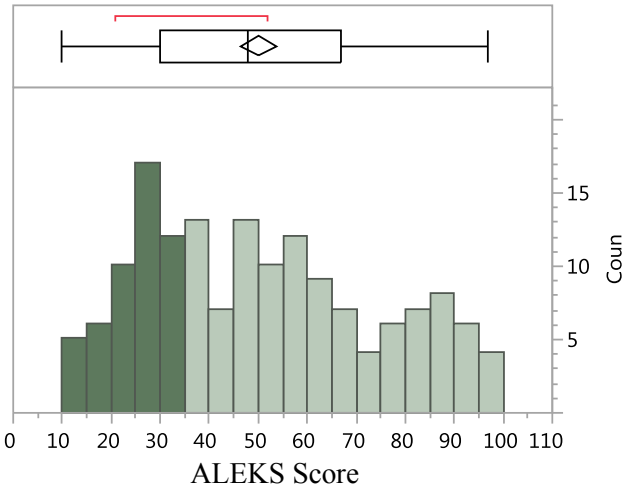


**Figure 6:** Distribution of ALEKS scores for students enrolled in introductory statistics

Figure 7 shows the distribution of final grades in introductory statistics. Although a +/– system of grading was used we have chosen to concentrate on the whole letter grade, therefore a B could be a B– or B or B+. The grade X indicates that a student dropped the course after the first week of classes. The distribution of final grades in the introductory statistics courses is much different than that for calculus I. A greater proportion of students are "successful" in an introductory statistics course compared to Calculus I.

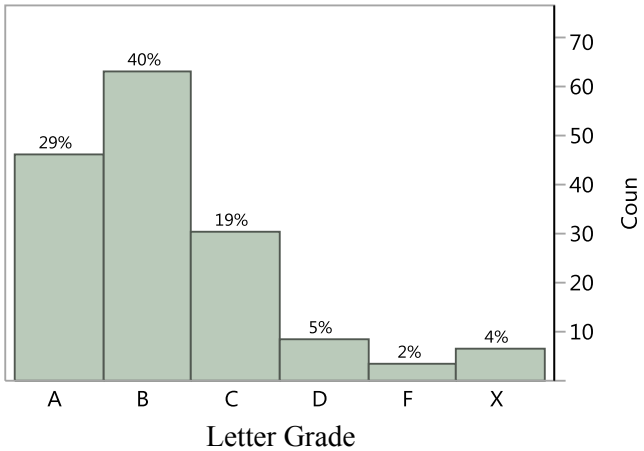


**Figure 7:** Distribution of letter grades for students enrolled in introductory statistics

Table 7 presents the numbers of student, along with the percentages in parentheses, receiving at least a specific grade in introductory statistics within each ALEKS category.

**Table 7:** Cumulative numbers and percentages of students receiving at least each grade within each ALEKS category.

| | ALEKS Category | | | | | | |
|---|---|---|---|---|---|---|---|
| Grade | < 30 | 30 to 40 | 40 to 50 | 50 to 60 | 60 to 80 | 80 to 100 | |
| F or X | 4<br>(10.5) | 1<br>(4.0) | 1<br>(5.0) | 1<br>(4.5) | 2<br>(7.7) | 0<br>(0.0) | 9 |
| D– or better | 34<br>(89.5) | 24<br>(96.0) | 19<br>(95.0) | 21<br>(95.5) | 24<br>(92.3) | 25<br>(100.0) | 147 |
| C– or better | 30<br>(78.9) | 22<br>(88.0) | 18<br>(90.0) | 21<br>(95.5) | 23<br>(88.5) | 25<br>(100.0) | 139 |
| B– or better | 22<br>(57.9) | 13<br>(52.0) | 16<br>(80.0) | 17<br>(77.3) | 19<br>(73.1) | 22<br>(88.0) | 109 |
| A– or better | 5<br>(13.2) | 5<br>(20.0) | 6<br>(30.0) | 5<br>(22.7) | 9<br>(34.6) | 16<br>(64.0) | 46 |
| | 38 | 25 | 20 | 22 | 26 | 25 | 156 |

Unlike for Calculus I, the trends between ALEKS scores and the percentage of students getting a specific grade, or better, in introductory statistics are not as clear. This is apparent from the mosaic plot in Figure 8 where cumulative probabilities tend to level off for ALEKS scores between 40 and 80. There are not a lot of data and the 2 students who had reasonably good ALEKS scores (60 to 80) but earned grades of F have a substantial effect when using the ALEKS score to model the probability a student will pass the course. There seems to be a fairly good relationship between the probability that a student does well in the course, A– or better, and the ALEKS score. For grades of B– or better and C– or better, there appears to be a threshold value for the ALEKS score where a value above that threshold indicates a greater probability of attaining those grades.

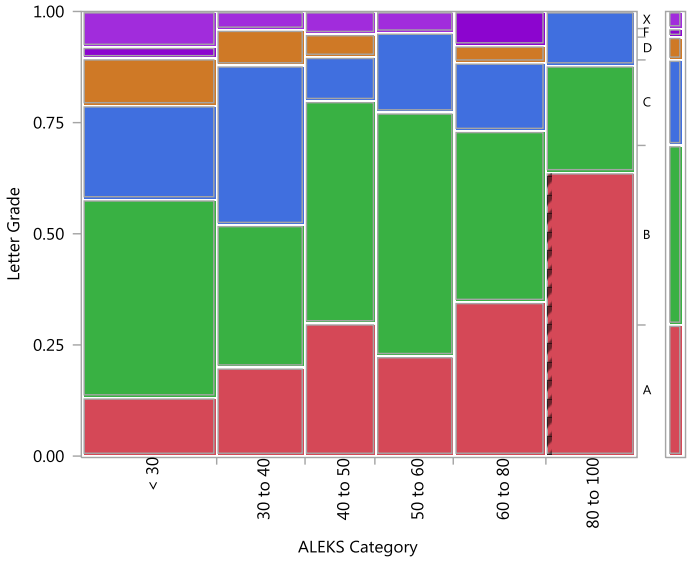


**Figure 8:** Mosaic plot of letter grades in introductory statistics within each ALEKS category

Logistic regression models are fit using the ALEKS score as the explanatory (predictor) variable and the categories A– or better, B– or better, C– or better and D– or better a passing grade.

The fitted equations for the logarithm of the odds, $log\left(\frac{p}{1-p}\right)$, where $p$ is the probability of "success" together with the test statistic and P-value for the significance of the ALEKS variable are given in Table 8. The predicted probabilities are plotted in Figure 9.

**Table 8:** Fitted equation for the logarithm of the odds of success in introductory statistics.

| "success" | Fitted equation for $log\left(\frac{p}{1-p}\right)$, | $\chi^2$ | P-value |
|---|---|---|---|
| Pass | 1.5007 + 0.02962*ALEKS | 3.17 | 0.0750 |
| C– or better | 0.3504 + 0.04132*ALEKS | 9.85 | 0.0017 |
| B– or better | –0.4128 + 0.02656*ALEKS | 10.82 | 0.0010 |
| A– or better | –2.7652 + 0.03533*ALEKS | 19.91 | < 0.0001 |

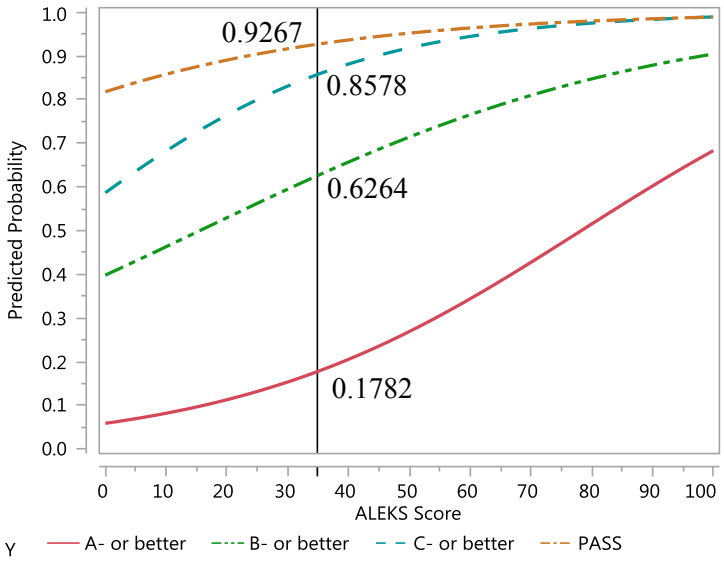


**Figure 9:** Predicted probabilities for various grades, or better, in introductory statistic based on the ALEKS score.

Table 9 compares the observed cumulative probabilities for each of the grades, or better, with the predicted probabilities, calculated at the midpoint of each ALEKS category. There is not as good agreement between the observed and predicted probabilities as was seen with the Calculus I data. This is seen in the fairly flat nature of the observed probabilities for the observed cumulative probabilities for ALEKS scores between 30 and 80. However, the small sample size and the two students who got reasonable ALEKS scores (60 to 80) but failed the course may explain some of the discrepancies.

**Table 9:** Observed and (predicted) cumulative probabilities for various grades, or better.

| | ALEKS Score Categories | | | | | |
|---|---|---|---|---|---|---|
| Grade | < 30 | 30 to 40 | 40 to 50 | 50 to 60 | 60 to 80 | 80 to 100 |
| F or X | 0.1053 (0.1098) | 0.0400 (0.0753) | 0.0500 (0.0571) | 0.0455 (0.0431) | 0.0769 (0.0273) | 0.0000 (0.0000) |
| D– or better | 0.8947 (0.8902) | 0.9600 (0.9247) | 0.9500 (0.9429) | 0.9545 (0.9569) | 0.9231 (0.9727) | 1.0000 (0.9847) |
| C– or better | 0.7895 (0.7644) | 0.8800 (0.8526) | 0.9000 (0.8974) | 0.9545 (0.9297) | 0.8846 (0.9624) | 1.0000 (0.9832) |
| B– or better | 0.5789 (0.5296) | 0.5200 (0.6202) | 0.8000 (0.6805) | 0.7727 (0.7353) | 0.7308 (0.8095) | 0.8800 (0.8785) |
| A– or better | 0.1316 (0.1132) | 0.2000 (0.1731) | 0.3000 (0.2296) | 0.2273 (0.2979) | 0.3462 (0.4275) | 0.6400 (0.6021) |
| | 38 | 25 | 20 | 22 | 26 | 25 |

## 3. Sub-scores

The ALEKS score is made up of sub-scores for various areas of mathematics. The eight areas are:

- Real Numbers (Num)
- Equations and Inequalities (Equat)
- Linear and Quadratic Functions (Fnctn)
- Exponents and Polynomials (Pnmal)
- Rational Expressions (Ratl)
- Radical Expressions (Radcl)
- Exponentials and Logarithms (Log)
- Geometry and Trigonometry (Trig)

While mastery of all of these areas may be important for success in Calculus I, not all of these areas are important for doing well in an introductory statistics course. Because the ALEKS score is the compilation of the sub-scores and it provides reasonable predictions of various levels of success in calculus, using the ALEKS score for placement in Calculus I makes sense.

The predictions based on the ALEKS score for various levels of success in introductory statistics are not as good. This leads us to investigate if sub-scores might do a better job than the full ALEKS score.

### 3.1 Introductory Statistics

Using the data on the 156 students who took the ALEKS test and an introductory statistics course, simple logistic regression using Rational Expressions (Ratl) provided almost as good a fit as using the full ALEKS score for A– or better and B– or better and a

better fit to predicting the probability of C– or better or for passing an introductory statistics course than the overall ALEKs score.

Figure 10 displays the distribution of the Rational Expressions sub-scores for the 156 students who took an introductory statistics course.

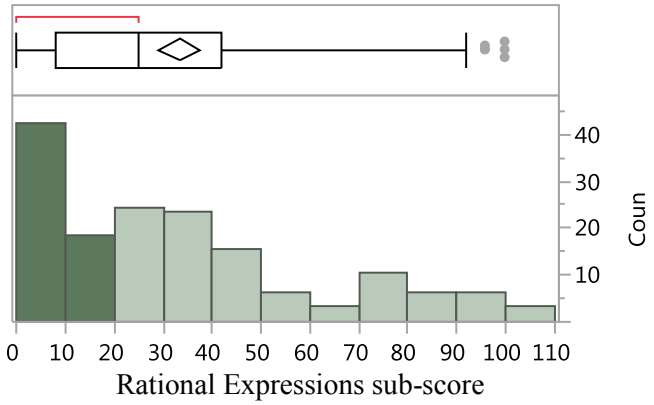


**Figure 10:** Distribution of Rational Expressions sub-score for students taking introductory statistics.

Table 11 presents the numbers of student, along with the percentages in parentheses, receiving at least a specific grade in introductory statistics within each Rational Expressions sub-score category.

**Table 11:** Cumulative numbers and percentages of students receiving at least each grade within each Rational Expressions sub-score category.

| | Rational Expressions sub-score Category | | | | | |
|---|---|---|---|---|---|---|
| Grade | < 20 | 20 to 40 | 40 to 60 | 60 to 80 | 80 to 100 | |
| F or X | 5<br>(8.3) | 3<br>(6.4) | 1<br>(4.8) | 0<br>(0.0) | 0<br>(0.0) | 9 |
| D– or better | 55<br>(91.7) | 44<br>(93.6) | 20<br>(95.2) | 13<br>(100.0) | 15<br>(100.0) | 147 |
| C– or better | 49<br>(81.7) | 43<br>(91.5) | 20<br>(95.2) | 12<br>(92.3) | 15<br>(100.0) | 139 |
| B– or better | 32<br>(53.3) | 36<br>(76.6) | 17<br>(81.0) | 12<br>(92.3) | 12<br>(80.0) | 109 |
| A– or better | 8<br>(13.3) | 14<br>(29.8) | 8<br>(38.1) | 7<br>(53.8) | 9<br>(60.0) | 46 |
| | 60 | 47 | 21 | 13 | 15 | 156 |

The association between the cumulative percentage of students receiving at least each grade and increasing values for the Rational Expressions sub-score is apparent in Figure 11. Because of the smaller number of students a change of only a couple of students achievement would rectify the dip in cumulative percentages seen for higher Rational Expression sub-score values and the B– or better and C– or better levels of success.

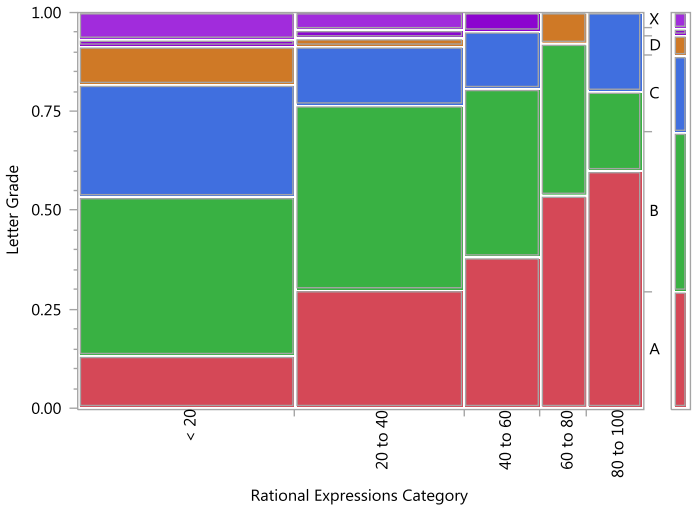


**Figure 11:** Mosaic plot of letter grades in introductory statistics within each Rational Expression sub-score category

The predicted probabilities for the logarithm of the odds, $log\left(\frac{p}{1-p}\right)$, where $p$ is the probability of "success" are plotted in Figure 12.

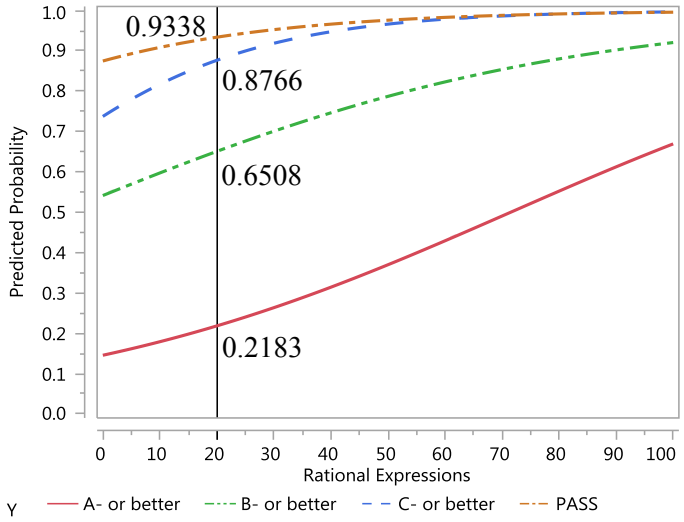


**Figure 12:** Graphs of the predicted probabilities for various grades, or better, in introductory statistics using the Rational Expressions sub-score.

There is better agreement between the observed and predicted probabilities using the Rational Expressions sub-score compared to the ALEKS score for the different levels of success. Because of the smaller number of students, greater fluctuations between observed and expected probabilities are to be expected. It should be noted that the two students with fairly high ALEKS scores who failed the class did not do as well on the Rational Expressions area of ALEKS as they did on other areas in ALEKS that may not be as important for an introductory statistics class.

## 4. Conclusions

There are statistically significant relationships between the probability of success in Calculus I and the ALEKS score. The agreement between observed and predicted probabilities of success is quite good, except for when success is defined as A– or better and ALEKS scores below 60. Even though there is a statistically significant relationship, the ALEKS score does not account for a great deal of the variation in success of students. The generalized $R^2$ for the fits are from 0.10 to 0.12. This means that, although the relationships are statistically significant, they are not overly strong. This is not surprising as mathematical preparation is only one contributing factor for success in an academic subject. However, it is clear from the data that lower ALEKS score tend to be associated with lower probabilities of success in Calculus I. It seems reasonable to use the ALEKS score as a guide for the mathematical preparation for Calculus I. Students scoring below a 65 should be advised that they may be less likely to be successful compared to students scoring 65 or above.

There are also statistically significant relationships between the probability of success in an introductory statistics course and the ALEKS score, except for when success is defined as passing the course. The relationships are not as strong with generalized $R^2$ values ranging from 0.06 to 0.12 except when success is defined as A– or better when the generalize $R^2 = 0.17$. One can get almost as good, and sometimes better, predictions of the probability of success in an introductory statistics course by using the Rational Expressions sub-score. However, given the small number of students in introductory statistics courses who took the ALEKS assessment it may be premature to conclude whether ALEKS, or a sub-score like Rational Expressions, should be used for placement in statistics.

## References

Carpenter, Jenna and Hanna, Ruth Ellen (2007). "Predicting student success in calculus," *Proceedings of the 2007 American Society for Engineering Education Conference*.

Doignon, J.-P. & Falmagne, J.-C. (1999). *Knowledge Spaces.* Springer-Verlag.

Doignon, J.-P. & Falmagne, J.-C. (2011). *Learning Spaces*. Springer-Verlag.

Falmagne, J.-C., Koppen, M., Villano, M., Doignon, J.-P. and Johanessen, L., (1990), "Introduction to Knowledge Spaces: How to Build, Test, and Search Them," *Psychological Review*, 97, pp. 201 – 224.

Feldman, Alex, Bullock, Doug, and Callahan, Janet (2012). "Using online assessment and practice to achieve better retention and placement in pre-calculus and calculus," *Proceedings of the 2012 American Society for Engineering Education Conference*.

Reddy, Alison Ahlgren and Harper, Mark (2013a). "Mathematics Placement at the University of Illinois," *PRIMUS*, 23(8): 683-702.

Reddy, Alison Ahlgren and Harper, Mark (2013b). "ALEKS-based placement at the University of Illinois." Contributed Chapter, *Knowledge Spaces: Applications in Education*. Springer-Verlag, Berlin Heidelberg.