An Investigation of the Day-of-the-Week Effect on the Volatility and Returns of Individual S&P 500 Sectors

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Abstract

Previous studies have shown that returns associated with the stock market or foreign exchange's futures show variations across the day of the week. On such study, that employs a modified GARCH model for estimation, shows that returns associated with the S&P 500 stock index exhibit highest volatility on Fridays and lowest on Wednesdays. In this study we investigate whether this day-of-the-week effect on returns and volatility is present in the different sectors that constitute the S&P 500 Index. The data set used provide daily returns from February 2005 to February 2015 and is more recent than the data employed in the original study on the S&P Index. Results show that in general, Tuesdays show high volatility for a majority of the sectors, Wednesdays show high returns for most sectors, and that this effect tapers down over the week with Mondays not exhibiting any increase in volatility or returns. Results also show that that the nature of the day-of-the-week effect is not consistent across sectors.

Key Words: Conditional Heteroskedasticity, Stock Returns, GARCH Models, Financial Time Series, Time Varying Volatility

1. Introduction

Some authors, such as Cross (1973), contested the assumption that the mean returns would remain constant across the five days of the trading week. Others, such as Osborn and Smith (1989) as well as Harvey and Huang (1991), argued that the assumption of constant unconditional variance is violated by some empirical series. Of particular interest is a paper by Berument and Kiymaz (2001) who analyzed 6,409 daily observations from Standard and Poor's 500 (S&P 500) Index taken from January 3, 1973 through October 20, 1997. While the above authors studied the composite S&P 500 index, in our study, daily returns from the ten different sectors included in the S&P 500 Index are studied to determine if similar "day-of-the-week" effect exists in both mean returns and their volatility in individual sectors and whether such patterns are consistent across sectors.

The first study of the day-of-the-week effect on returns was carried out by Cross (1973), who analyzed returns on the S&P 500 Index covering the years 1953 through 1970. The findings indicate that the mean return on Fridays is higher than that on Mondays. French (1980) found a similar pattern on the S&P 500 Index over the period 1953-1977. Gibbons and Hess (1981) analyzed 30 selected stocks from the Dow Jones Industrial Index and found negative returns for Mondays. Additional analysis was carried out by Keim and Stambaugh (1984), finding patterns similar to those found by the previous studies.

Of particular interest to researchers was the Monday returns, which some suggested should be higher than returns for other days because of the gap that exists between Friday trading and Monday activities. For example French (1980) suggested that Monday returns should be higher than returns for other days. Other publications that investigated related issues are Gibbons and Hess (1981), Lakonishok and Levi (1982), and Rogalski (1984). In addition, Jaffe and Westerfield (1985) studied the day-of-the-week effect in stock markets in Australia, Canada, Japan, and U.K. while Solnik and Bousquet (1990) studied such effects for stocks traded in the Paris Bourse (a historic Paris stock exchange renamed Euronext Paris in 2000). The former study found the lowest returns for the Japanese and Australian stock markets to occur on Tuesdays. The latter study found negative returns on Tuesdays for the Paris market.

As mentioned before, Berument and Kiymaz (2001) found a day-of-the-week effect that increased volatility on Fridays and lowered it on Wednesdays. Other investigatiors also found such effects. For example, Harvey and Huang (1991), who studied interest rate and foreign exchange futures market, found higher volatilities on Fridays while Ederington and Lee (1993) found such effects in the bond and stock markets. Choudhry (2000) studied data from seven Asian stock markets and found evidence of day-of-the-week effects on volatility, but these effects were not alike across the countries under study. Rodriguez (2012) who studied volatilities in the Latin American stock markets found Monday to have lower than normal volatility with Friday showing a higher than normal effects.

Investigations on the relationship between returns and volatility were carried out by French, Schwert, and Stambaugh (1987) who found that unusual stock market returns are negatively corrected with unexpected volatility changes. Campbell and Hentschel (1992) suggested that increase in volatility lowers stock prices. Others who studied the relationship between stock returns and volatility are: Baillie and DeGennaro (1990), Chan, Karolyi, and Stulz (1992), Glosten, Jagannathan and Runkle (1993), Corhay and Rad (1994), and Theodossiou and Lee (1995). These studies do not directly investigate the presence of a day-of-the-week effect on stock market volatility but looked at the relationship between stock price and volatility.

2. An Autoregressive – GARCH Model

One way to introduce a non-constant unconditional variance is to use the formulation adopted by Choudhry (2000) as well as by Berument and Kiymaz (2001). In this formulation, the constant term α_0 found in the GARCH model is replaced by terms specific to each day. This modified GARCH model is as follows:

$$\varepsilon_t = \sigma_t e_t \text{, for } t = 0, \ \pm 1, \ \pm 2, \ \ldots\text{, where } e_t \sim i.i.d.\ N(0,1)$$

and
$$\sigma_t^2 = \sum_{k=1}^{5} \delta_k d_k + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \text{ for } t = 0, \ \pm 1, \ \pm 2, \ \ldots \quad (2.1)$$

where $\sigma_t = \sqrt{\sigma_t^2}$, $\alpha_j \geq 0$ for $j = 1, 2, ..., q$, and $\beta_i \geq 0$ for $i = 1, 2, ..., p$, with the d_k representing a dummy variable for the k^{th} trading day of the week, k=1, 2, 3, 4, 5.

The authors did not restrict the δ_k to be positive. The additional condition $\sum_{j=1}^{q}\alpha_j + \sum_{i=1}^{p}\beta_i < 1$ is required for the time series $\{\varepsilon_t\}$ to be covariance stationary.

Given the closing value X_t of a stock on day *t*, it is common to compute the return, R_t, for day *t* by $R_t = \ln(X_t / X_{t-1})$. The above GARCH processes are zero mean processes because it can be shown easily that $E[\varepsilon_t] = 0$ for all values of *t* and this may be too restrictive to model the returns of a given stock. Researchers such as Berument and Kiymaz (2001) as well as Rodriguez (2012) extended this model to an Autoregressive Model (AR) with a mean that varies with the day-of-the-week, with errors that are a GARCH process given by (2.1). Their formulation for R_t, the return observed on day *t*, is given by

$$R_t = \sum_{k=1}^{5}\gamma_k d_k + \sum_{l=1}^{m}\phi_l R_{t-l} + \varepsilon_t \tag{2.2}$$

with $\varepsilon_t = \sigma_t e_t$, for $t = 0, \ \pm 1, \ \pm 2, \ldots.$, where $e_t \sim i.i.d.\ N(0,1)$

and $\sigma_t^2 = \alpha_0 + \sum_{k=1}^{4}\delta_k d_k + \sum_{j=1}^{q}\alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p}\beta_i \sigma_{t-i}^2$ for $t = 0, \ \pm 1, \ \pm 2, \ldots..$

Observe that $\sigma_t = \sqrt{\sigma_t^2}$, $\alpha_0 > 0$, $\alpha_j \ge 0$ for $j = 1, 2, \ldots, q$, and $\beta_i \ge 0$ for $i = 1, 2, \ldots, p$, with the d_k representing a dummy variable for the k^{th} trading day of the week, *k*=1, 2, 3, 4, 5. The additional condition $\sum_{j=1}^{q}\alpha_j + \sum_{i=1}^{p}\beta_i < 1$ is required for the time series $\{\varepsilon_t\}$ to be covariance stationary. Only four of the five d_k terms are included in the intercept term of the GARCH portion of Equation (2.2) because including all five dummy variables together with the constant term α_0 will result in collinearity. All five dummy variables were, however, fitted in the regression portion of Equation (2.2) which has no intercept.

The above formulation is used in this study to model the returns computed from the S&P 500 sector indices. One advantage of the above formulation is that it allows the modeling of returns as an autoregressive process and also account for the conditional heteroskedasticity of the error process. It also accounts for any day-of-the-week effect on the returns as well as on volatility. Another advantage is that this model can be fitted to data using existing software such as the Statistical Analysis System (SAS©).

3. Standard and Poor's 500 Stock Index and the Data

The Standard and Poor's 500 (S&P 500) Index is based on the weighted stock prices of 500 large companies. The index consists of companies that can be broadly categorized into ten sectors: (1) Consumer Discretionary, (2) Consumer Staples, (3) Energy, (4) Financials, (5) Health Care, (6) Industrials, (7) Materials, (8) Technology, (9) Telecommunications Services, and (10) Utilities. Based on this standard, the above sectors consist of the industries given in Table 1.

Table 1. List of Industries Belonging to S&P 500 Sectors

Sector	Industry
Consumer Discretionary	Auto Components, Automobiles, Household Durables, Leisure Equipment & Products, Textiles Apparel & Luxury Goods, Hotels, Restaurants & Leisure, Diversified Consumer Services, Media, Distributors, Internet and catalog Retail, Multiline Retail, Specialty Retail
Consumer Staples	Food staples and Retailing, Beverages, Food Products, Tobacco, Household Products, Personal Products
Energy	Energy Equipment & Services, Oil, Gas, & Consumable Fuels
Financials	Commercial Banks, Thrift & Mortgage Finance, Diversified Financial Services, Consumer Finance, Capital Markets, Insurance, Real Estate (discontinued effective 04/30/2006), Real Estate Investment Trusts, Real Estate Management & Development
Healthcare	Healthcare Providers & Services, Healthcare Equipment & Supplies, Healthcare Technology, Biotechnology, Pharmaceuticals, Life Sciences Tools & Services
Industrials	Aerospace & Defense, Building Products, Construction & Engineering, Electrical Equipment, Industrial Conglomerates, Machinery, Trading Companies & Distributors, Commercial Services & Supplies, Professional Services, Air Freight & Logistics, Airlines, Marine, Road & Rail, Transportation Infrastructure
Information Technology	Internet Software & Services, IT Services, Software, Communications Equipment, Computers & Peripherals, Electronic Equipment & Components, Office Electronics, Semiconductor Equipment and Products (discontinued effective 04/30/2003), Semiconductors & Semiconductor Equipment
Materials	Chemicals, Construction Materials, Containers & Packaging, Metals & Mining, Paper & Forest Products
Telecommunications Services	Diversified Telecommunication Services, Wireless Telecommunication Services
Utilities	Electric Utilities, Gas Utilities, Multi-Utilities, Water Utilities, Independent Power Producers & Energy Traders

It is important to note that sometimes financial analysts consider Consumer Staples and Discretionary Sectors as one. Also some combine Materials and Industrial sectors. The ten-sector classification given above is defined based on the Global Industry Classification Standard (GICS®) which was jointly developed by Standard and Poor's and MSCI Barra in 1999 (S&P Indices (2008)).

3.1 Description of the Data and the Data Source

The price data for each sector was obtained from the website http://us.spindices.com/indices/equity/sp-500. Sector breakdowns can be obtained from the same site (see Table 2). The index data for the composite S&P 500 index as well as the individual sectors provide daily prices computed using total returns, which include dividends, and prices based on total net returns, which do not count dividends. The analysis conducted in this research used total net returns series.

The ten-year observation period of the data is the longest span available from the website. The website also provides data from other indices such as S&P 100, S&P Small Caps 600, S&P 900, S&P 1000 and S&P Composite 1500.

Table 2. Breakdown of S&P 500 Sectors as a Percentage of the Aggregate Index

Sector	Percentage
Consumer Discretionary	12.9%
Consumer Staples	9.7%
Energy	7.3%
Financials	16.6%
Healthcare	15.2%
Industrials	9.9%
Information Technology	20%
Materials	2.9%
Telecommunications Services	2.4%
Utilities	3%

3.2 Pre-Processing of the Data

The data set for each sector was first pre-processed to include the day of the week using an algorithm that employed the calendar date to determine the day. The returns, R_t, for day *t* was computed using the formula $R_t = \ln(P_t) - \ln(P_{t-1})$, where P_t is the price for day *t*. Note that the return for the first day in the price series, February 14, 2005, could not be computed because the price of the index for the previous day was not available in the data set.

The graphs of the returns for two of the sectors, namely Energy and Financial, over a ten-year period from February 15, 2005 through February 12, 2015 are given in Figures 1 and 2. Note that the horizontal axis is labeled starting at one through 2,517 to reflect the 2,517 returns computed from 2,518 prices. Since the 2008/2009 financial crisis affected all stocks in some way or another, the behavior of the returns during that time may be of interest. September 2, 2008 corresponds to data point 894 (*t*=894). October 1, 2008 corresponds to *t*=915 and the corresponding *t* value for December 31, 2008 is 978.

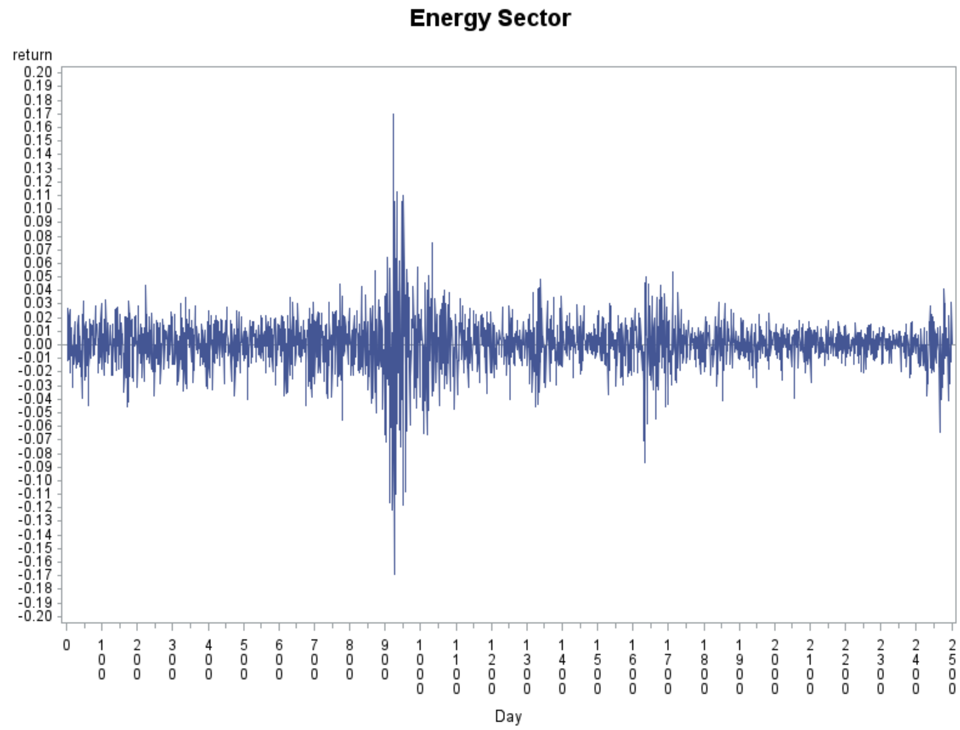

Figure 1. Returns for the Energy Sector by Day

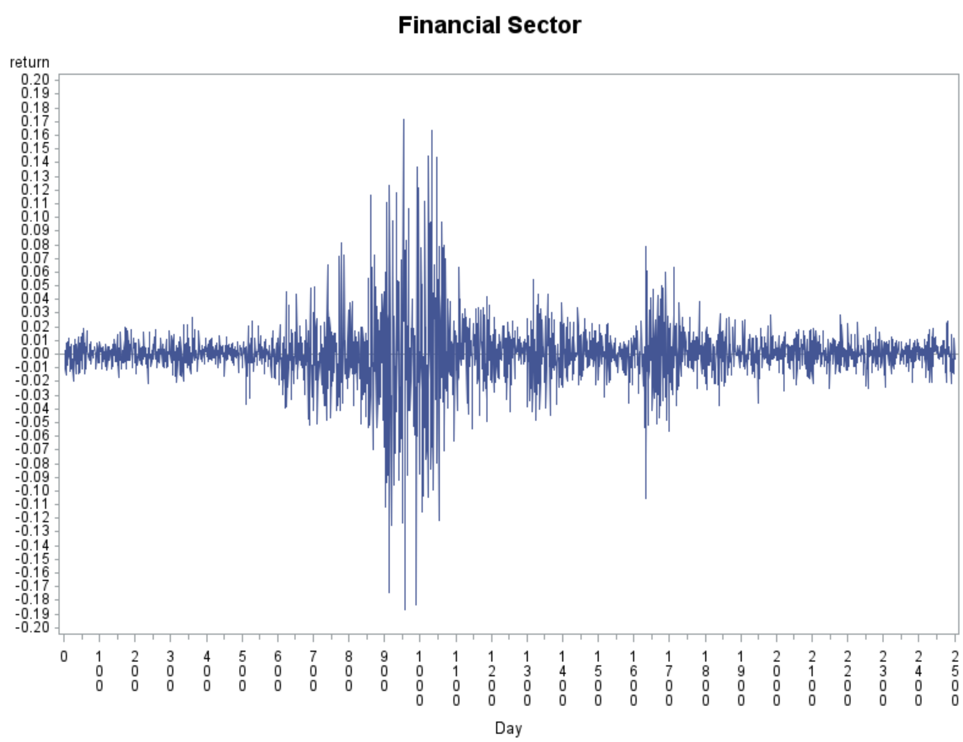

Figure 2. Returns for the Financial Sector by Day

4. The Modeling Procedure and Results

The volatility was modeled using the Autoregressive-GARCH formulation given in Equation Set (2.2). The AUTOREG Procedure available in SAS (Version 9.4) was employed to carry out the model fitting. The conditions $\sum_{j=1}^{q}\alpha_j + \sum_{i=1}^{p}\beta_i < 1$, which is sufficient to ensure the covariance stationary assumption, was imposed and the assumption $e_t \sim i.i.d.\ N(0,1)$ was initially made for the underlying innovations e_t that drive the GARCH process. In addition, the orders of the GARCH process was assumed to be p=1 and q=1 as is commonly done. Inspection of the Akaike information criterion (AIC) and the corrected Akaike information criterion (AICC) showed that assuming the e_t to be independently distributed as t random variables gave a better fit except for one sector. Note that the AUTOREG procedure in SAS automatically determines the degrees of freedom associated with the t-distribution. Fitting the full model created estimability problems because the full model was over-parameterized. Therefore, a two-by-step approach was employed to do the modeling. Details of this procedure are given below.

4.1 The Two-Step Modeling Procedure

First Model (2.2) was fitted without the GARCH component. That is, the error terms were assumed to be conditionally homoscedastic. Then the insignificant terms in the model $R_t = \sum_{k=1}^{5}\gamma_k d_k + \sum_{l=1}^{m}\phi_i R_{t-l} + \varepsilon_t$ were eliminated using significance level 0.05 as the cut-off criteria. This elimination was done one term at a time, with the most insignificant term (that with the highest p-value) considered first for elimination. When two terms had p-values close to one another, each of the terms were eliminated in two separate runs and the AIC values for each model were compared. The elimination that reduced the AIC by the most amount was then selected.

Once the model was reduced in this manner, the GARCH portion $\sigma_t^2 = \alpha_0 + \sum_{k=1}^{4}\delta_k d_k + \sum_{j=1}^{q}\alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{p}\beta_i \sigma_{t-i}^2$ of the model (with p=1 and q=1) was added to the remaining Autoregressive (AR) part. The terms $\sum_{k=1}^{4}\delta_k d_k$ were introduced into the model using the HETERO command available in SAS. Then the dummy variables d_k that were not significant at 0.05 level were eliminated. Fitting of these dummy variables sometimes caused identification problems. Therefore, these terms were fitted one at a time. First the significant term that reduced the AIC by the most amount was fitted. Then another term was considered for inclusion using the significance level and AIC value as criteria.

4.2 Results

The complete results from the above analysis for each individual sector are not reported here for brevity. To aide comparison of significant influences of the days of the week on both returns and volatility, the statistically significant effects are summarized in Table 3 given below. Percent change in returns was computed as the ratio of the mean change in return for a given day-of-the-week (as indicated by the coefficient of the corresponding dummy in the AR portion of the model) to the mean of the absolute daily return for that sector, multiplied by 100. Percent change in volatility is computed as 100 times the coefficient of the respective dummy variable in the GARCH portion divided by the unconditional volatility computed for that sector.

Table 3. Days of the Week with Significant Differences in Returns and Volatility

Sector		Day of the Week				
		Monday	Tuesday	Wednesday	Thursday	Friday
Consumer Discretionary	Percentage Change in Return			12.0482%	10.2609%	
	Percentage Change in Volatility		20.2908%			
Consumer Staples	Percentage Change in Return		10.6%	11.2%	13.0%	
	Percentage Change in Volatility					
Energy	Percentage Change in Return		9.7971%			11.2659%
	Percentage Change in Volatility					
Financial	Percentage Change in Return		6.5369%	9.2996%		
	Percentage Change in Volatility		5.4219%			
Health Care	Percentage Change in Return		14.1262%	11.7858%	13.3321%	
	Percentage Change in Volatility		16.1218%			
Industrials	Percentage Change in Return			10.0121%	9.8537%	9.6108%
	Percentage Change in Volatility		14.4586%			
Information Technology	Percentage Change in Return		11.1687%	16.9764%	9.2860%	
	Percentage Change in Volatility					
Materials	Percentage Change in Return			12.6571%		13.3500%
	Percentage Change in Volatility		6.6704%			
Telecommunication Services	Percentage Change in Return				10.0708%	
	Percentage Change in Volatility		29.3133%			
Utilities	Percentage Change in Return		12.9303%			12.0616%
	Percentage Change in Volatility					

Note: Blank entries indicate statistically insignificant effects.

The change in return was computed as a percentage of the mean absolute return rather than the mean return because, averaged over many days, the mean return is quite small and using that as the denominator can give the impression that the shifts are quite large. Computation of the day-of-the-week effects as a percentage of the mean absolute return gives one some idea of the effect sizes with respect to the average daily absolute “shifts” in log price. Also note that the day-of-the-week effects on returns are computed not as relative shifts from an overall mean but as actual changes in returns attributed to a given day.

The statistically significant dummy variables d_k included in the AR portion of the model all had positive coefficients, suggesting that the corresponding days had higher returns than the other days of the week, which acted as the base-line return in the estimated regression model. This is similar to the results Berument and Kiymaz (2001) obtained, where all the significant dummy variables have positive coefficients. Thus, Monday, for example, was not associated with returns higher than the baseline-level. So is Tuesday and Friday for Consumer Discretionary Sector. This sector showed higher than base-line return for Wednesdays and Thursdays. Tuesday had a positive effect on returns on six out of the ten sectors, with the highest effect at 14% for the Healthcare Sector. Wednesdays affected seven out of the ten sectors producing higher than base-line returns, the highest being an almost 17% increase for the Information Technology Sector. Thursdays positively affected the returns of six of the ten sectors, while Friday affected only four of the sectors.

The reasons why certain days had more impact on some sectors and not on others is a question that needs insight into the trading strategies and how various markets react to events and is best left to researchers with more familiarity with such issues. One major observation that can be made based on this research results on returns is that Monday had no positive effect on the returns of any sector and Wednesday seems to affect the returns positively for most sectors. This is somewhat similar to the results obtained by Berument and Kiymaz (2001) who studied the S&P 500 returns (aggregated over all sectors) from January 1973 through October 1997 and found lowest returns on Monday and highest on Wednesday. They, however, found a different pattern when data from October 1987 to October 1997 were studied.

As for volatility, six of the ten sectors had higher volatility on Tuesdays, with Telecommunications sector showing a 29% increase in volatility on Tuesdays. Mondays Wednesdays, Thursdays and Fridays did not increase the volatility level over the base-line. This is contrary to the results of Berument and Kiymaz (2001) who found higher volatility on Fridays. However, when the above authors studied the data for the period January 1973 through October 1987, they found highest volatility on Tuesdays. The difference in the results may be due to the time period under study. The period over which the present research was conducted includes the recession of 2008/2009 which may have changed the way the market reacts to economic shocks.

The above results must be interpreted with caution because the complex model formulation used in this study can be sensitive to small changes in the prescribed model as well as changes in the period over which the data is gathered. In addition, the inclusion of data from the recession of 2008/2009 can bias the results from what one would have obtained otherwise. Moreover, it was assumed that any day-of-the-week effect that was present before the recession remained the same after 2009. What can be said with some certainty is that there is evidence that a day-of-the-week effects exists for volatilities on Tuesdays and that the Tuesday effect seems to cut across many sectors of the S&P 500 index. In addition, returns seem to be high on Wednesdays across a majority of the sectors. What should not be deducted from these results is that the exact effects presented in Table 3 are figures reliable enough to base ones investment strategies on. These should be taken as results of an initial step with more confirmatory analysis based on additional data required before firm conclusions can be made.

5. Conclusions and Future Work

Data from the ten sectors of S&P 500 indices were investigated for the presence of the day-of-the-week effect on returns and volatility. Period of the study spans from the February 2005 to the February 2015. None of the sectors showed a significance change in return or volatility on Monday but a clear day-of-the-week effect on Tuesdays and Wednesdays on volatility and returns, respectively. The effect of each day of the week differed across the type of sector studied. The inclusion of data from the 2008/2009 recession may have affected the results and further analysis on this issue is needed. The first author is currently engaged in studying the data from 2010 and beyond using a different formulation of the AR-GARCH model. Further analysis on select individual stock prices would shed additional light on this day-of-the-week phenomenon. Investigation of other stock indices would also be a fruitful exercise.

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References

Baillie, R.T. and DeGennaro, R.P. (1990). "Stock Returns and Volatility," *Journal of Quantitative and Financial Analysis*, 25, 203-214.

Berument, H. and Kiymaz, H. (2001). "The Day of the Week Effect on Stock Market Volatility," *Journal of Economics and Finance*, 25(2), 181-193.

Campbell, J. Y., and Hentschel, L. (1992). "No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns," *Journal of Financial Economics*, 31, 281-318.

Chan, K.C., Karolyi, A., and Stulz, R.M. (1992). "Global financial markets and the risk premium on U.S. equity," *Journal of Financial Economics*, 32, 137-167.

Choudhry, T. (2000). "Day of the week effect in emerging Asian stock markets: evidence from the GARCH model," *Applied Financial Economics*, 10, 235-242.

Corhay, A., and Rad, A.T. (1994). "Statistical Properties of Daily Returns: Evidence from European Stock Markets," *Journal of Business Finance and Accounting*, 21, 271-282.

Cross, F. (1973). "The Behavior of Stock Prices on Fridays and Mondays," *Financial Analysts Journal*, 29: 67-69.

Ederington, L. H., and Lee, J.H. (1993). "How Markets Process Information: News Releases and Volatility," Journal *of Finance*, 48: 1161-1191.

French, K. R. (1980). “Stock Returns and The Weekend Effect,” *Journal of Financial Economics*, 8, 55-69.

French, K. R., Schwert, G. W., and Stambaugh, R. F. (1987). “Expected Stock Returns and Volatility,” *Journal of Financial Economics*, 19, 3-29.

Gibbons, M. and Hess, P. (1981). “Day of the Week Effects and Asset Returns,” *Journal of Business*, 54, 579-596.

Harvey, C., and R. Huang. (1991). “Volatility in Foreign Exchange Futures Markets,” *Review of Financial Studies*, 4, 543-570.

Jaffe, J. and Westerfield, R. (1985). “The Weekend Effect in Common Stock Returns: The International Evidence,” *Journal of Finance*, 40, 433-454.

Glosten, L.R., Jagannathan, R., and Runkle, D.E. (1993). “On the Relation between the Expected value and the Volatility of the nominal Excess Return on Stocks,” *The Journal of Finance,* 48(5), 1779-1801.

Keim, D. B., and Stambaugh, F. (1984). “A Further Investigation of Weekend Effects in Stock Returns,” *Journal of Finance*, 39, 819-840.

Lakonishok, J., and Levi, M. (1982). “Weekend Effects on Stock Returns: A Note,” *Journal of Finance*, 37, 883-889.

Osborn, D.R. and Smith, J.P. (1989). “The Performance of Periodic Autoregressive Models in Forecasting Seasonal U.K. Consumption,” *Journal of Business and Economic Statistics*, 7, 117-127.

Rodriguez, W.K. (2012). “Day of the week effect in Latin American stock markets,” *Revista de Análisis Económico*, 27(1), 71-89.

Rogalski, R. J. (1984). “New Findings Regarding Day-of-the-Week Returns over Trading and Non-Trading Periods: A Note,” *Journal of Finance*, 35, 1603-1614.

S&P Dow Jones Indices, (2008). *S&P Indices Methodology*, < http://www.spindices.com/documents/methodologies/methodology-sp-equity-indices-policies-practices.pdf>.

Solnik, B. and Bousquet, L. (1990). “Day-of-the-Week Effect on the Paris Bourse,” *Journal of Banking and Finance*, 14, 461-468.

Theodossiou, P., and Lee, U. (1995). “Relationship between Volatility and Expected Returns Across International Stock Markets,” *Journal of Business Finance and Accounting*, 22, 289- 300.