

Analysis of Seasonal Daily Pattern of CO2 Concentration in Antarctica Using Doubly Cyclic Smoothing Splines

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Abstract

In order to flexibly estimate continuously varying daily pattern of CO2 concentration throughout a year at Syowa station in Antarctica, we propose a doubly cyclic smoothing splines method. This method is defined using the tensor product method with cyclic splines. A doubly cyclic smoothing spline function is a univariate function of time, but the penalty is defined on a torus. For the balanced data, the basis functions can be transformed to trigonometric functions by an orthogonal basis transformation, and it is shown that this method penalizes more on higher frequency basis functions. In other words, it can be considered as a soft thresholding low pass filter in two dimension on torus.

We fit a linear additive model for CO2 concentration at Showa station in Antarctica with temporal trend, annual variation, wind velocity and doubly cyclic smooth term of time with daily and annual cycles. Because of a strong temporal trend and seasonal variation, it has been said that CO2 does not have a daily variation, but our result showed that it has a significant daily variation in summer. We also found that the effect of wind velocity on CO2 depends on season.

Key Words: Cyclic cubic spline smoothing; Smoothing, Tensor product smooth, doubly cyclic smoothing splines, soft thresholding low pass filter

1. Doubly cyclic smoothing splines

1.1 cyclic cubic spline

A cyclic cubic spline function $g(t)$ is a function that satisfies the following properties

- It is periodic. For an positive integer T with $T \geq 2$, $g(t + mT) = g(t)$ for $m = 0, \pm 1, \pm 2, \dots$
- It is a piece-wise polynomial. Let knots be $t^{(0)} < t^{(1)} < \dots < t^{(K-1)} < t^{(K)}$, ($t^{(K)} - t^{(0)} = T$). Then, $g(t) = p_j(t)$ for $t \in [t^{(j-1)}, t^{(j)}]$, $j = 1, 2, \dots, K$ where $p_j(t)$ are a cubic polynomial.
- It is continuous up to second derivative. For $j = 1, 2, \dots, K-1$, $p_j(t^{(j)}) = p_{j+1}(t^{(j)})$, $p'_j(t^{(j)}) = p'_{j+1}(t^{(j)})$, $p''_j(t^{(j)}) = p''_{j+1}(t^{(j)})$.
- The values up to second derivatives are the same for the both ends, $p_1(t^{(0)}) = p_K(t^{(K)})$, $p'_1(t^{(0)}) = p'_K(t^{(K)})$, $p''_1(t^{(0)}) = p''_K(t^{(K)})$.

A cyclic cubic spline function can be parameterized as follows (Wood, 2004) Now let the function values at knots be parameters:

$$\beta_j = g(t^{(j)}) \quad j = 1, 2, \dots, K$$

Then,

$$g(t) = \sum_{j=1}^K \beta_j b_j(t) = \boldsymbol{\beta}^T \mathbf{b}(t)$$

where basis function $b_j(t)$ is a cyclic cubic spline function satisfying

$$b_i(t^{(j)}) = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

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and $\boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_K)^T$, $\mathbf{b}(t) = (b_1(t), b_2(t), \dots, b_K(t))^T$.

Now, we define a simple cyclic cubic spline smoothing model as

$$y_i = g(t_i) + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2), \quad i = 1, \dots, n$$

where $g(t)$ is a cyclic cubic spline function. For given data, we find the cyclic cubic spline function that minimizes the following penalized sum of squared errors.

$$\Omega_\lambda(g) = \sum_{i=1}^n \{y_i - g(t_i)\}^2 + \lambda \int_{t^{(0)}}^{t^{(K)}} g''(t)^2 dt, \quad \lambda > 0 \quad (1)$$

1.2 Tensor product smooths

Tensor product method (Wood, 2004) is a smoothing method that uses tensor product functions defined as follows. Let $a_1(s), a_2(s), \dots, a_{K_s}(s)$ be basis functions on functional space Ω_s , and $b_1(t), b_2(t), \dots, b_{K_t}(t)$ be basis functions on functional space Ω_t . The basis functions for tensor product method are products of two basis functions

$$a_i(s)b_j(t), \quad i = 1, 2, \dots, K_s, \quad j = 1, 2, \dots, K_t$$

and a tensor product function is explained as a linear combination of these functions as

$$f_{st}(s, t) = \sum_{i=1}^{K_s} \sum_{j=1}^{K_t} \beta_{ij} a_i(s) b_j(t). \quad (2)$$

1.2.1 Roughness penalty for tensor product method

Roughness penalty for tensor product smoothing function is defined as

$$J(f_{st}) = \int_{\Omega_s \times \Omega_t} \lambda_s \left(\frac{\partial^2 f_{st}}{\partial s^2} \right)^2 + \lambda_t \left(\frac{\partial^2 f_{st}}{\partial t^2} \right)^2 ds dt.$$

Penalty term for the tensor product smoothing method, when knots are evenly spaced, can be approximated as

$$J(f_{st}) \approx \lambda_s \boldsymbol{\beta}^T (S_s \otimes I_{K_t}) \boldsymbol{\beta} + \lambda_t \boldsymbol{\beta}^T (I_{K_s} \otimes S_t) \boldsymbol{\beta}$$

where $\boldsymbol{\beta}$ is a vector of appropriately rearranged function values at grids (Wood, 2006).

1.3 Doubly cyclic smoothing splines

In order to analyze cyclic variation with seasonal change, we propose doubly cyclic smoothing splines method. Let $f_1^a, f_2^a, \dots, f_{K_a}^a$ be basis functions for the functional space of cyclic functions with one year cycle and let $f_1^d, f_2^d, \dots, f_{K_d}^d$ be basis functions for the functional space of cyclic functions with one day cycle.

We start with a univariate function of time t , defined on a coil, that winds around a torus:

$$f(t) = \sum_{i=1}^{K_a} \sum_{j=1}^{K_d} \beta_{ij} f_i^a(t) f_j^d(t)$$

Then, to have a function that is smooth in two directions, we re-express this as:

$$\tilde{f}(s, t) = \sum_{i=1}^{K_a} \sum_{j=1}^{K_d} \beta_{ij} f_i^a(s) f_j^d(t)$$

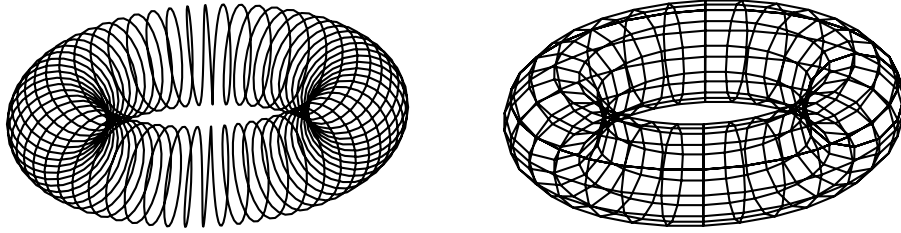


Figure 1: Coil on torus (left) and grid on torus (right).

and consider penalty to this function.

1.4 Mechanism of doubly cyclic cubic spline smoothing method

What does a doubly cyclic cubic smoothing spline do? When knots are evenly spaced, and the numbers of observations are equal for all knots, basis functions can be linearly transformed into the following orthogonal basis functions:

$$\begin{aligned}
 f_{ij}^{cos} &: \text{cyclic cubic spline function whose values at knots are equal to} \\
 &\quad \cos(2\pi t(ih_a + jh_d)) \text{ for } i = 0, \dots, q_a^*, j = 0, \pm 1, \dots, \pm q_d^* \\
 f_{ij}^{sin} &: \text{cyclic cubic spline function whose values at knots are equal to} \\
 &\quad \sin(2\pi t(ih_a + jh_d)) \text{ for } i = 0, \dots, q_a^*, j = 0, \pm 1, \dots, \pm q_d^*
 \end{aligned}$$

where $q_a^* \leq K_d/2 - 1$, $q_d^* \ll K_a/2 - 1$ and for $h_a = 1/K_a$, $h_d = 1/K_d$. Then, the estimated function can be expressed as

$$\begin{aligned}
 \hat{f}(t) = & \sum_{i=0}^{q_a^*} \sum_{j=-q_d^*}^{q_d^*} \left(1 + \lambda_a \frac{(1 - \cos 2\pi i h_a)^2}{2 + \cos 2\pi i h_a} + \lambda_d \frac{(1 - \cos 2\pi j h_d)^2}{2 + \cos 2\pi j h_d} \right)^{-1} \\
 & \times \left(\frac{\langle \mathbf{u}_{ij}^{cos}, \bar{\mathbf{y}} \rangle}{|\mathbf{u}_{ij}^{cos}|^2} f_{ij}^{cos} + \frac{\langle \mathbf{u}_{ij}^{sin}, \bar{\mathbf{y}} \rangle}{|\mathbf{u}_{ij}^{sin}|^2} f_{ij}^{sin} \right)
 \end{aligned}$$

where

- \mathbf{u}_{ij}^{cos} : vectors of values of $\cos(2\pi t(ih_a + jh_d))$ at knots
- \mathbf{u}_{ij}^{sin} : vectors of values of $\sin(2\pi t(ih_a + jh_d))$ at knots
- f_{ij}^{cos} : cyclic cubic spline function with $\cos(2\pi t(ih_a + jh_d))$ as values at knots
- f_{ij}^{sin} : cyclic cubic spline function with $\sin(2\pi t(ih_a + jh_d))$ as values at knots
- $\bar{\mathbf{y}}$: vector of the averages at knots.

The doubly cyclic cubic smoothing spline shrinks the components of

- the basis function with values at knots $\cos(2\pi t(ih_a \pm jh_d))$, and
- the basis function with values at knots $\sin(2\pi t(ih_a \pm jh_d))$

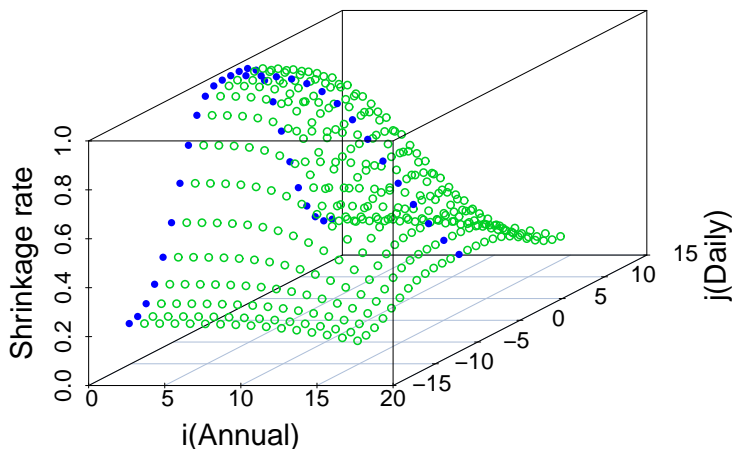


Figure 2: Shrinkage rates.

by multiplying them by the shrinkage rate:

$$\left(1 + \lambda_a \frac{(1 - \cos 2\pi i h_a)^2}{2 + \cos 2\pi i h_a} + \lambda_d \frac{(1 - \cos 2\pi j h_d)^2}{2 + \cos 2\pi j h_d}\right)^{-1}$$

The larger i or j is, the more wiggly the basis function is in either direction.

The more wiggly the basis function is in either direction, the more its coefficient is shrunk.

2. Analysis of CO2 concentration data at Syowa station

In this section, we show the results of analysis for CO2 concentration observed at Syowa station in Antarctica. We have two purposes for this analysis. One is to figure out the daily pattern of CO2 concentration through a year and the other is to obtain some information regarding atmosphere circulation system around Syowa station. CO2 concentration data and weather data were provided from National Institute of Polar Research.

2.1 Syowa station

Syowa station is the observatory established at 69°00'22"S, 39°35'24"E, and 29.18m above sea level. This station is located on an island near the coastal region of the Antarctic continent. The characteristic of island established Syowa station is that the island is connected to the Antarctic continent by sea ice except in summer. Also the weather factors of Syowa station have two characteristics. The first is wind direction. The primary peak is around the northeasterly wind direction and can be attributed to katabatic winds because the Antarctic continent is situated about 4 km to the east of Syowa station. The katabatic winds observed at Syowa station tend to be easterly and slightly northerly due to the Coriolis effect. The katabatic winds is the winds when air stays in the Antarctic Continent, air is gradually cooled by radiational cooling and is risen density, so that the cool air blows when cool air glides down it by gravity to low altitude.

2.2 CO2 concentration data

CO2 concentration data we have were observed every hour from February 3, 1984 to December 31, 2007 at Syowa station. Variables observed include CO2 concentration in ppm (C02con for abbreviation), wind direction with 16 directions (Winddir) and wind speed in $\times 10^{-1}$ m/s(Windspe).

2.3 Model

2.3.1 Model of daily pattern with seasonality

We first consider a model with daily pattern with seasonality and temporal trend.

$$\text{Model 1} \quad y = f_{d,s}(t) + f_{tr}(t) + \epsilon \quad (3)$$

where y is CO2 concentration at Syowa station, $f_{d,s}(t)$ is a doubly cyclic smooth with day and year cycle, $f_s(t)$ is a smooth function for a temporal trend and ϵ is a Gaussian error with variance σ^2 .

2.3.2 Model of daily pattern and Wind direction, Wind speed

In addition to daily pattern and temporal trend, we consider effects of interaction with wind direction and wind velocity.

$$\text{Model 2} \quad y = f_{d,s}(t) + f_{tr}(t) + f_{wd,ws}(D, S) + \epsilon \quad (4)$$

where y , $f_{d,s}(t)$ and $f_{tr}(t)$ are the same as the above and $f_{WD,WS}(D, S)$ is the tensor product of cyclic smooth for wind direction and smoothing spline for wind direction.

2.3.3 Model of daily pattern and Wind speed with seasonality

In addition to daily pattern and temporal trend, we consider wind velocity with seasonality.

$$\text{Model 3} \quad y = f_{d,s}(t) + f_{tr}(t) + f_{ws,s}(S, t) + \epsilon \quad (5)$$

2.4 Results of analysis with three models

2.4.1 Models 1: results and findings regarding daily patterns

Fig.3 illustrates the daily pattern estimated with Model 1. This figure only shows daily pattern at the 4th of each month because of obtaining continuously daily pattern estimated to use doubly cyclic smoothing splines. Furthermore we correct so that the means of each daily pattern are zero in order to remove influence of annual pattern. Fig.4 illustrates daily pattern of CO2 concentration and the 99 % confidence intervals for January and July 4: January 4 is summer and red, July 4 is winter and blue. Each dashed line express estimated value, and each two solid lines are the supremum and infimum of confidence intervals. Seen Fig.4, daily pattern of January 4 has significance, but July 4 doesn't have. Daily pattern has this significance about from early December to early February. Thus summer has significance of daily pattern.

2.4.2 Model 2: results and findings

Model2 is constructed to understand atmosphere circulation system. Fig.5 figures out CO2 concentration variation with wind speed on each directions by model2. The 99 % confidence intervals are all so narrow that we could estimate reliable functions. The predominant wind direction is northeasterly(southwestward) in the frequency distribution. This figure show a weak increase at about wind speed 20m/s of north-northeasterly, northeasterly, and east-northeasterly wind directions, but the estimated functions are almost horizontally: Increasing CO2 concentration at wind speed 0m/s is the

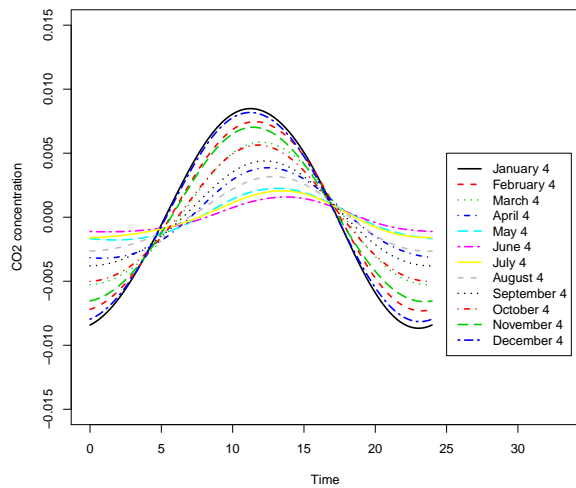


Figure 3: Daily pattern of CO2 concentration at the 4th of each month with model1

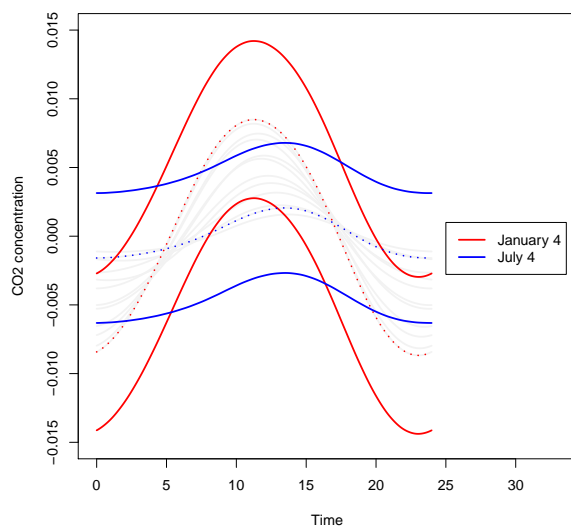


Figure 4: Daily pattern of CO2 concentration and the 99% confidence intervals for January 4 (red) and July 4 (blue).

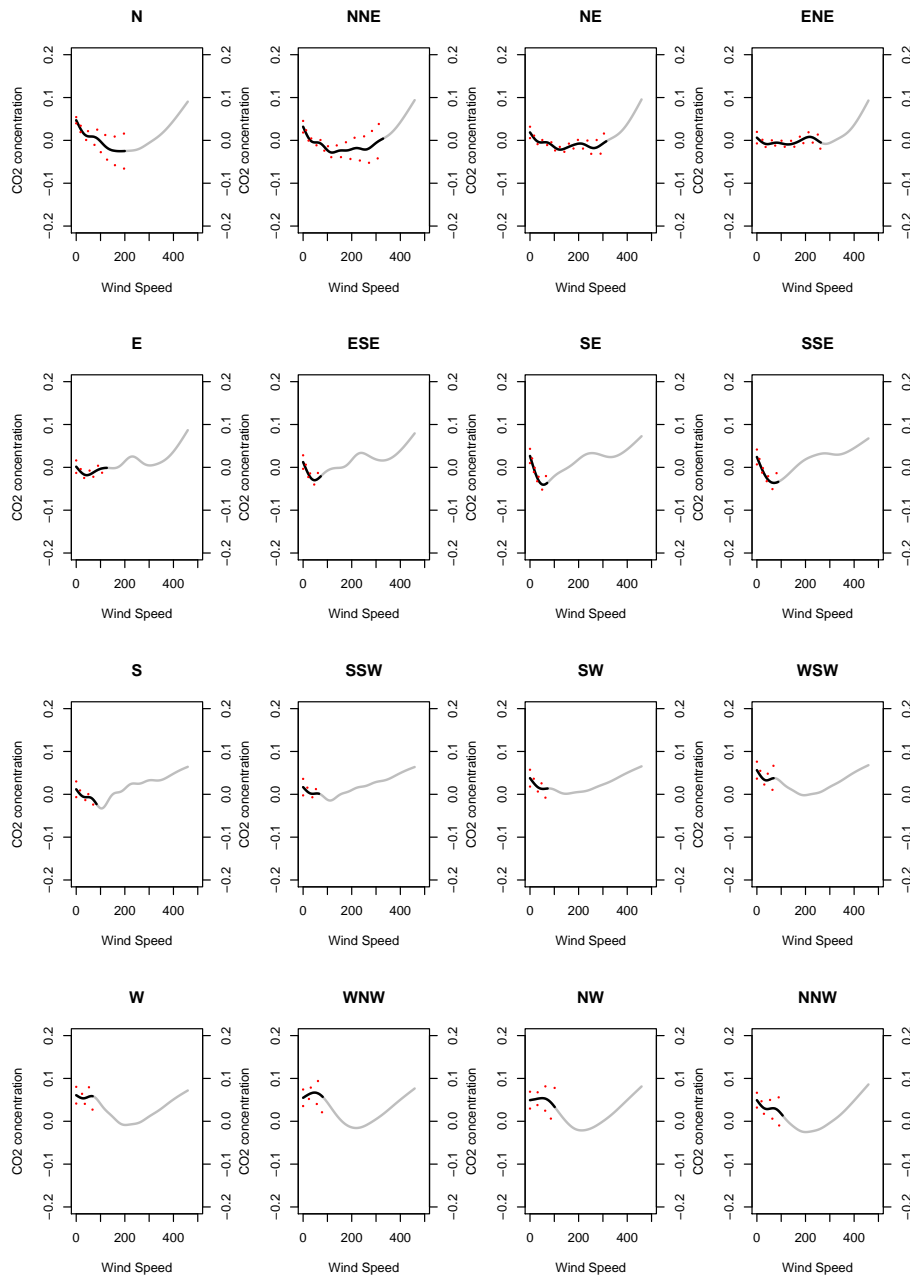


Figure 5: Estimation of CO2 concentration with wind speed on each directions: each figures show that horizontal axis is wind speed and vertical axis is CO2 concentration; estimated function with model2 is solid line(black), 99% confidence interval is broken line(red) and excluded function because of no data or unreliable based on confidence interval is solid line(gray); The top from left show north(N) to east-northeast(ENE) of 16 directions, the second from left east(E) to south-southeast(SSE) of 16 directions, the third from left south(S) to west-southwest(WSW) of 16 directions and the bottom from left west(W) to north-northwest(NNW) of 16 directions

reason why Syowa station is located on an island in a gulf and is surrounded with glacier so that air stays and it is given the influence of human activities that is not originally observed. And at east-southeasterly to north-northwesterly wind direction (5~16 of 16 directions) where strong wind speed hardly blows, predicted functions are high reliability only for less than wind speed 10m/s. So we don't know the trend of CO₂ concentration variation then. Therefore we cannot obtain the useful information from model2. The problem of model2 is that we fit not two functions distinguished it in a season but one function through the year to two variables smoothing function $f_{wd,ws}(D, S)$ with wind direction and speed. Wind direction and speed have seasonality, and if express one function $f_{wd,ws}(D, S)$, it treats the wind speed due to different factors together, for example, wind speed 10m/s can be attributed to katabatic winds in summer but to low atmospheric pressure closed from low latitudes in winter on 1 to 4 of 16 directions.

To solve this problem, we fit two functions distinguished it in summer and seasons except for summer to two variables smoothing function $f_{wd,ws}(D, S)$ with wind direction and speed. At function `gam(bam)` of `mgcv` package, we should choose 'by' in a function `te`. To figure CO₂ concentration variation with wind speed on each directions and seasons is Fig.6(summer) and Fig.7(seasons except for summer). A major characteristic is that the confidence intervals are wide both figures. Although a part of estimated functions(solid line: black) are excluded because confidence intervals are so wide, the estimated functions of Fig.6 and Fig.7 are wider than variation width of CO₂ concentration. Therefore this function predicted from model2 is low reliability. As the factor to be wide confidence interval, we consider that the data to fit $f_{wd,ws}(D, S)$ are not many by having distinguished it in summer and seasons except for summer so that error variance become excessively large. Seen the width of confidence interval and the variation width of CO₂ concentration, the function of Fig.6 and Fig.7 is also low reliability. According to the above results, we cannot obtain the useful information from model2 any cases. The problem is that in the case of expressing $f_{wd,ws}(D, S)$ one function we can't consider seasonality for wind speed and in the case of expressing $f_{wd,ws}(D, S)$ two functions we can't secure enough data.

2.4.3 Model 3: results and findings

Model3 is the model to solve the problem of model2. Wind direction has constancy except for almost calm of twelve o'clock to seventeen o'clock in summer. So the need to consider seasonality to wind direction is low. On the other hand, wind speed has seasonality. Thus we add the smoothing function $f_{ws,s}(S, t)$ of wind speed considered seasonality to model1 in place of $f_{wd,ws}(D, S)$. Adding this function is equivalent to consider directional constancy and almost calm in summer; If find wind speed and season(time), also find factors of the wind speed. That is, we can deal with the model2 problem that wind speed 10m/s can be attributed to katabatic winds in summer but to low atmospheric pressure closed from low latitudes in winter on 1 to 4 of 16 directions, and we can secure enough data because don't distinguish data.

The predicted values of daily pattern with model3 is almost same values with daily pattern of Fig.3, and this daily pattern with model3 is also significant in summer. Detailed comparison with daily pattern is in Section 4.4

Next figure out predicted values of CO₂ concentration variation with each months wind speed to Fig.9 Seen Fig.9, there are two Characteristics as follow:

1. CO₂ concentration increases around wind speed 20m/s in October to March,
2. CO₂ concentration increases around wind speed 5m/s in winter(June and July).

Consideration with atmosphere circulation system based these Characteristics is in Section 4.6.

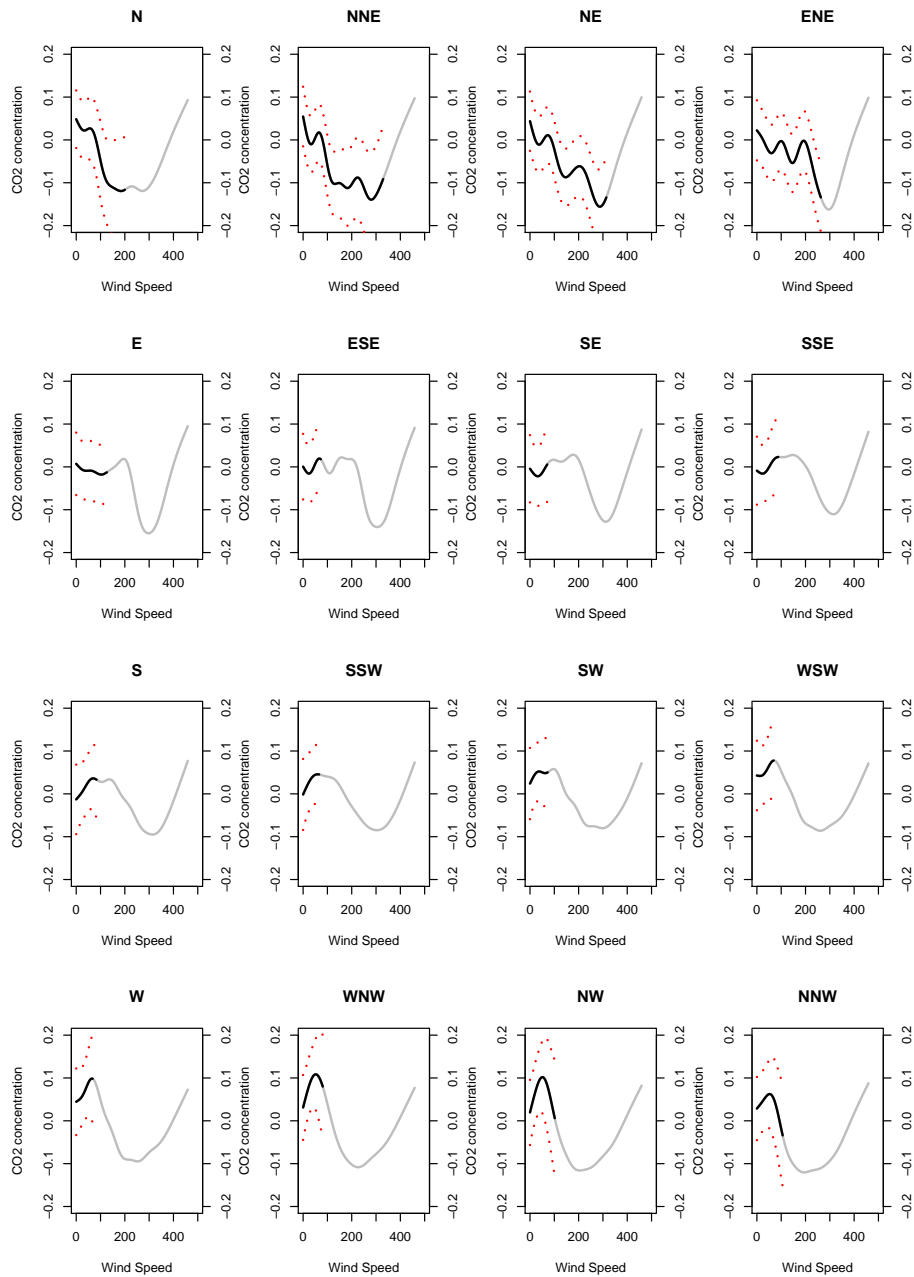


Figure 6: Estimation of CO2 concentration with wind speed on each directions in summer(December to February): each figures show that horizontal axis is wind speed and vertical axis is CO2 concentration; estimated function with model2 is solid line(black), 99% confidence interval is broken line(red) and excluded function because of no data or unreliable based on confidence interval is solid line(gray); The top from left show north(N) to east-northeast(ENE) of 16 directions, the second from left east(E) to south-southeast(SSE) of 16 directions, the third from left south(S) to west-southwest(WSW) of 16 directions and the bottom from left west(W) to north-northwest(NNW) of 16 directions

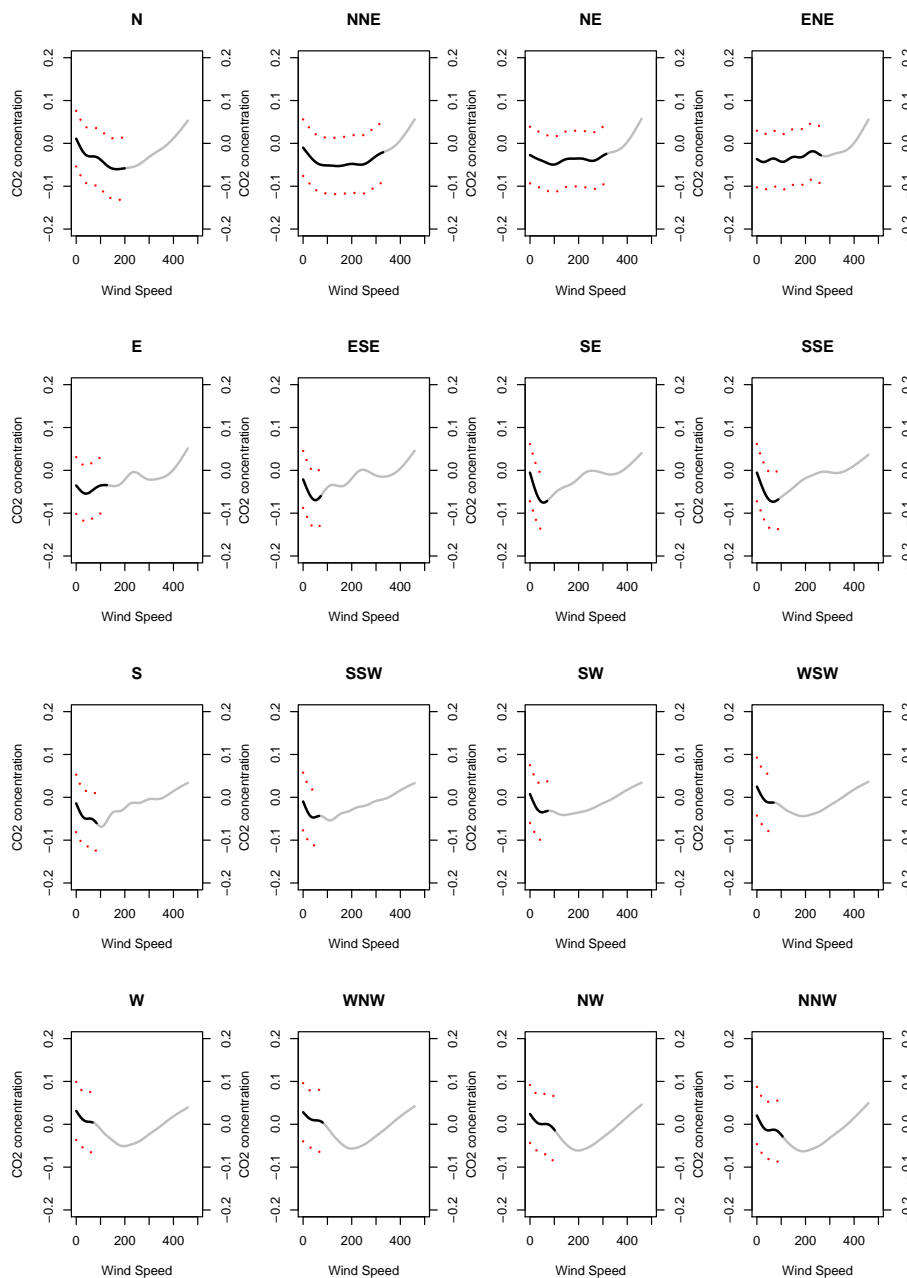


Figure 7: Estimation of CO2 concentration with wind speed on each directions in seasons except for summer: each figures show that horizontal axis is wind speed and vertical axis is CO2 concentration; estimated function with model2 is solid line(black), 99% confidence interval is broken line(red) and excluded function because of no data or unreliable based on confidence interval is solid line(gray); The top from left show north(N) to east-northeast(ENE) of 16 directions, the second from left east(E) to south-southeast(SSE) of 16 directions, the third from left south(S) to west-southwest(WSW) of 16 directions and the bottom from left west(W) to north-northwest(NNW) of 16 directions

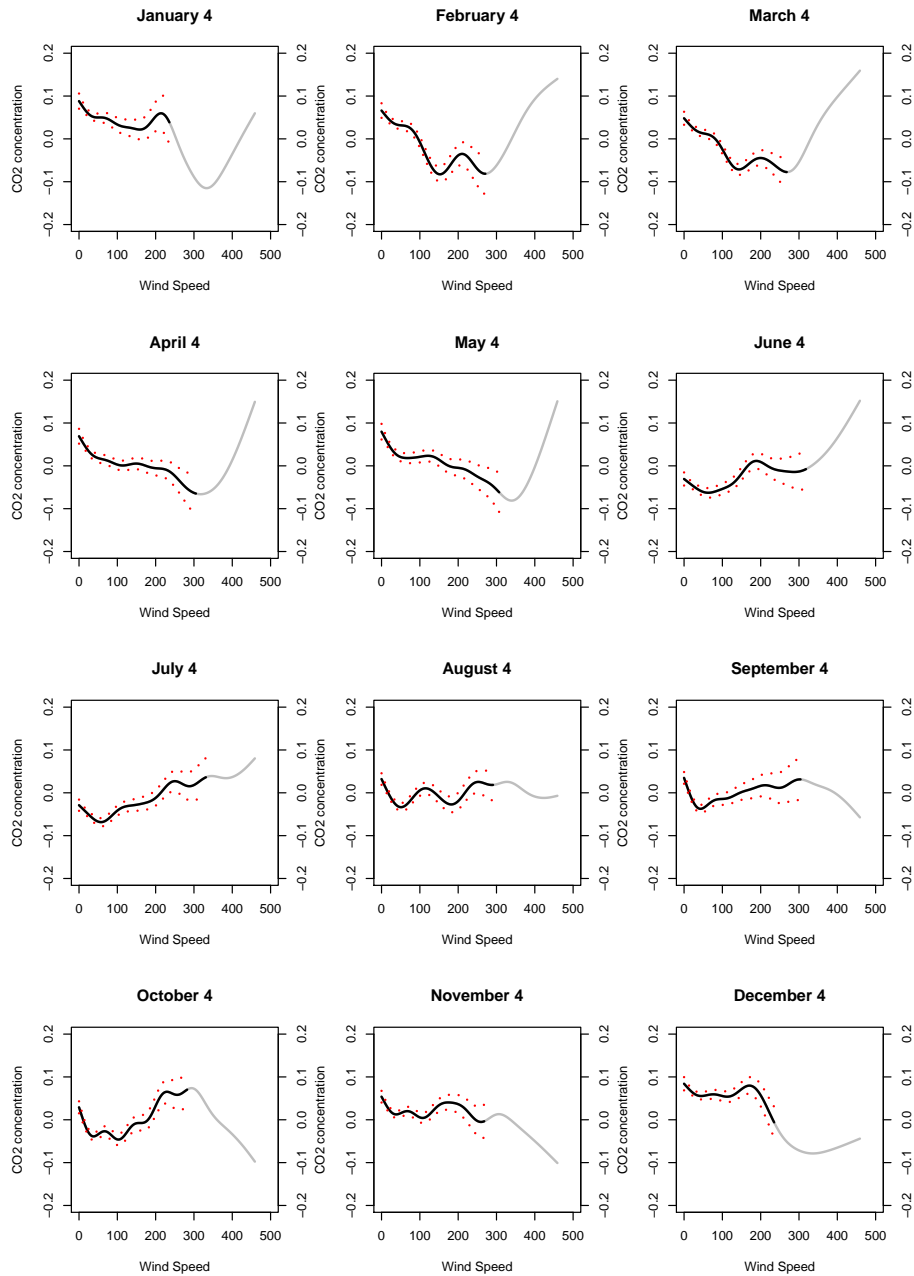


Figure 8: Estimation of CO2 concentration with wind speed at the 4th of each month: each figures show that horizontal axis is wind speed and vertical axis is CO2 concentration; estimated function with model2 is solid line(black), 99% confidence interval is broken line(red) and excluded function because of no data or unreliable based on confidence interval is solid line(gray); The top from left show January to March, the second from left April to June, the third from left July to September and the bottom from left October to December

Table 1: Comparison of AIC with models

Name of model	Model	AIC
model1	$y = f_{d,s}(t) + f_{tr}(t) + \epsilon$	19942.53
model2 (the case not distinguished seasons)	$y = f_{d,s}(t) + f_{tr}(t) + f_{wd,ws}(D, S) + \epsilon$	18725.62
model2 (the case distinguished seasons)	$y = f_{d,s}(t) + f_{tr}(t) + f_{wd,ws}(D, S) + \epsilon$	18100.00
model3	$y = f_{d,s}(t) + f_{tr}(t) + f_{ws,s}(S, t) + \epsilon$	16189.48

2.5 Consideration with daily pattern

It is said that CO₂ concentration doesn't have daily pattern at Syowa station so far, but we find CO₂ concentration at Syowa station has daily pattern in summer in Section 4.3. The cause of what are said CO₂ concentration doesn't have daily pattern at Syowa station is generally that there is not factors to influence on the CO₂ concentration by a unit one day. Factors to influence on the CO₂ concentration by a unit one day are, for example, human activities and respiration and so on. By the way, Syowa station is put not to influence human activities of antarctic expedition party when observe CO₂ concentration.

We consider that a significance factor of CO₂ concentration's daily pattern is exchange of CO₂ between sea level and atmosphere. It is only summer that daily pattern of CO₂ concentration is significance. This resembles characteristic of the island established Syowa station that is the state of island in summer only. The seawater includes much CO₂, and in fact 91%(except for amount of storage in sediment) of CO₂ in the earth surface is pooled in the deep sea water(Yanagi, 2001), and CO₂ is exchanged actively between sea level and atmosphere. Following this, the thickness of sea ice is also considered as a explanation factor of daily pattern: If it is some thickness, the sea ice can pass CO₂ from sea. However we don't have data of exchange of CO₂ at sea level or thickness of sea ice, so that cannot do the detailed consideration.

2.6 Model evaluation and comparison with daily pattern

Here do model evaluation and comparison with daily pattern. All smoothing functions in models can be expressed to use known basis functions, and the penalty can be expressed or approximated with quadratic form. Thus it is also easy to express penalized likelihood and possible to evaluate model with AIC criteria. Fig.1 shows AIC with three models. Model2 is low reliability, but shows to compare other models. We shows AIC for the case distinguished seasons and the case not distinguished seasons to fit two variables smoothing function $f_{wd,ws}(D, S)$ with wind direction and speed. Seen Table1, AIC of model3 is very lower approximately 2000-4000 than others. Therefore model3 is the highest reliability and the ideal model. As a result, we find the importance that wind speed considered seasonality is necessarily to model but wind direction is not.

Let consider the reason why model3 is the highest reliability. As shown in Section 4.1.1, wind direction has homeostasis but doesn't have seasonality. Hence the interpretability of wind direction for CO₂ concentration varied through the year is low, and if add $f_{wd,ws}(D, S)$ to model, the interpretability of model2 is down to treats the wind speed due to different factors together(cf. Section 4.3.2). On the other hand, adding wind speed considered seasonality to model, it can explain atmospheric flow of Syowa station and vicinity. So it seems that the interpretability of model3 is up. We say the details of interpretability for CO₂ concentration with wind speed in next section.

2.7 Consideration with atmosphere circulation system

In this section we show what two characteristics that said in Section 4.3.3 can understand from characteristics of the wind speed having seasonality at Syowa station, and refer to how can interpret it as atmosphere circulation system.

2.7.1 *About characteristic(a)*

The period of from January to March and from October to December is when CO₂ concentration lowers in annual pattern. On the other hand, CO₂ increased by human activities flows in from low latitude side with a low atmospheric pressure, and then wind speed is 20m/s. Furthermore this wind speed blows only when a low atmospheric pressure comes close from low latitude side. Thus, we can understand the reason why CO₂ concentration rises at 20m/s of characteristic(a) from the characteristic of wind speed with seasonality at Syowa station.

At the point of view of atmosphere circulation system, it is unknown how atmospheric lowered CO₂ concentration flow to Syowa station at the period of from January to March and from October to December. We found at a point of atmosphere circulation that the atmosphere directly flows in from low latitude and CO₂ concentration rises when wind speed is 20m/s. But we want to know how the atmosphere of CO₂ concentration that became low in the northern hemisphere flows into Syowa station at this time. The relation of CO₂ concentration decrease and wind speed do not have characteristics from Fig.9. So we cannot understand the atmosphere circulation mechanism. Therefore, it is unknown whether the atmosphere which have CO₂ concentration lowered in the northern hemisphere flows in low latitude side directly or whether it passes over Syowa station and flows into Syowa station from Antarctica with katabatic winds.

2.7.2 *About characteristic(b)*

Winter is when CO₂ concentration becomes higher in annual pattern. We want to know how atmospheric became higher CO₂ concentration flow to Syowa station in winter. The important points of characteristic(b) is that CO₂ concentration becomes the smallest at around 5m/s and rises as the wind speed (more than 5m/s) is strong. As the wind about 5m/s is influence by katabatic winds in winter, what CO₂ concentration is low at the time is equivalent to what CO₂ concentration of Antarctica center is low. Furthermore, blowing wind speed more than 5m/s is when a low pressure came close from low latitude side in winter, so that what CO₂ concentration rises if wind speed becomes strong, is regarded as what CO₂ concentration rises if the atmosphere flows in from low latitude side.

According to the above, we found that the atmosphere of CO₂ concentration became higher in the northern hemisphere flows in low latitude side directly with low pressure. The reason is that there is not a rise of CO₂ concentration not to include influence of human activities at less than wind speed 5m/s: Increasing CO₂ concentration at wind speed 0m/s is the reason why Syowa station is located on an island in a gulf and is surrounded with glacier so that air stays and it is given the influence of human activities that is not originally observed. However it is not appropriate that we say the all factors of CO₂ concentration risen at wind speed 5m/s is to inflow the atmospheric which CO₂ concentration risen in the northern hemisphere because Syowa station is affected by African continent. But we can interpret the atmosphere of CO₂ concentration became higher in the northern hemisphere flows in low latitude side directly because CO₂ concentration flowed into Syowa station from Antarctica with katabatic winds is low.

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