Establishing the Foundation of Hedge Fund Asset Allocation Decisions using Bayesian Modeling

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Abstract
This paper attempts to estimate the diversified fund-of-hedge-funds (FoHF) industry's aggregate hedge fund (HF) strategy allocations. Unlike long-only equity and fixed income indices that have published constituents and composition weights, such a benchmark does not exist for hedge fund investors and asset allocation decision makers. As a result, it's desirable yet difficult for a FoHF manager to asses whether the portfolio has significant strategy/style biases so performance attributions can be conducted. The author proposed several classic and Bayesian regression models to address this need. Hedge fund strategy allocations are model parameters; dependent variables are Diversified FoHF index and individual FoHF performance data; independent variables are major HF strategy index performance data. Investment industry experience provided guidance for setting Bayesian prior (ex-ante) parameter values; Markov Chain Monte Carlo simulations generated posterior (ex-post) allocation estimates. The author believes a Bayesian hierarchical model provides good balance between these objectives: (1) results that are consistent with industry experience and could be easily interpreted; (2) model parsimony and good fit to data. Future research opportunities such as capturing dynamic parameter behaviors are also discussed.

Key Words: Hedge Fund, Fund of Hedge Funds, Asset Allocation, Bayesian Statistics, Hierarchical Model, Markov Chain Monte Carlo

1 Introduction

1.1 Business Justification

Hedge funds (HFs) are private investment vehicles available only to accredited investors and qualified purchasers. Hedge funds deploy a wide variety of instruments and derivatives in various asset classes and regional markets to construct long and short positions in order to deliver superior risk-adjusted returns. Four major hedge fund categories are equity hedge (EH), event driven (ED), relative value (RV), and global macro (GM). Large pension funds, endowments, foundations, and institutions may have capabilities to perform in-house due diligence and asset allocation decisions in order to directly invest in hedge funds to form a multi-manager portfolio. Many investors access hedge funds via external fund-of-hedge-funds (FoHFs) to benefit from diversifications and lower volatilities. FoHF styles include diversified, growth-oriented, conservative, and hedge-oriented. A typical diversified FoHF tactically allocates to underlying hedge funds of various styles but the actual weights are unknown to the public. This work attempts to estimate the diversified FoHF industry's aggregate hedge fund strategy allocations.

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Traditional long-only equity investors usually track market-capitalization weighted regional market and sector indices to look for out-performance opportunities. Unlike popular equity and fixed income indices that have published constituents and composition weights, such a FoHF benchmark does not exist for hedge fund investors. This poses a real challenge to multi-manager hedge fund investors and many large institutions that deploy multi-asset investments including hedge funds.

During the iterative portfolio construction process, there are several ways of considering risk budgeting in a multi-manager hedge fund portfolio.

1. Nominal % strategy allocations, over/under weights (OW/UW) vs. a benchmark, and performance attributions (i.e. allocation and selection effects).
2. Realized performance contributions at the fund and strategy levels.
3. Pro forma (absolute and relative) marginal risk contributions at the fund and strategy levels.
4. Portfolio multi-factor sensitivities (betas) and excess returns (alpha) (e.g. Chang 2014).
5. Aggregated portfolio long and short market exposures from underlying hedge funds: by asset class, product type, region, credit rating, liquidity, trends in exposures, as well as exposure-based beta and volatility estimates.

The latter four approaches have been largely adopted by FoHFs and large institutions. *The first seemingly simple question remains unanswered in a consistent and rigorous way.* Asset-weighted HF strategy indices would have helped answer the question since it is reasonable to assume larger hedge funds have attracted more investments/allocations from FoHFs and large institutions. It is important yet difficult for asset allocation decision makers to asses the effect of HF strategy style tilts. A popular performance attribution method, Brinson Model (Brinson, Hood, Beebower 1995), requires "benchmark" weights to compare asset allocation and security/manager selection effects.

In addition to comparing risk-adjusted returns with indices and peers, hedge fund asset allocation decision makers may be interested in the following:

1. Are our hedge fund strategy allocations similar to the industry average? Where are the significant deviations (i.e. bets)? What drives the differences between our returns vs. the industry?
2. Which FoHFs are our peers? How similar are our allocations to those of peers? How large are the dispersions in the peer group allocations?

There are several challenges in estimating strategy allocations in the FoHF industry:

1. Lack of directly reported allocation data from FoHFs or large institutions.
2. There is not a standardized and consistent hedge fund style classification scheme.
3. FoHFs may invest in hedge funds that do not report to index data providers.
4. No index vendor offers both asset weighted FoHF and HF strategy index performance data. Two major HF data sources were considered.
    a) Hedge Fund Research, Inc.: HFRI[2] hedge fund strategy and FoHF index returns are all equal wighted and available by subscription only. HFRI index data include *published and unpublished* underlying monthly HF returns. It's difficult to compare realized FoHF returns with equal weighted HFRI FoHF index returns.

2 HFRI index website: www.hedgefundresearch.com/index.php?fuse=indices-new

b) Credit Suisse: HF strategy indices[3] are asset weighted, but lacks a FoHF index.

1.2 Methodology

Due to the four challenges mentioned above, the author is faced with estimating *unknown and unobservable* strategy allocations by regressions to fit HF strategy index returns to diversified FoHF returns. That is, the dependent variables (**Y**) are the Diversified FoHF index and individual FoHF returns; the independent variables (**X**) are major HF strategy index returns.

To answer the first question in Section 1.1, the author estimated the *industry average* strategy allocations by *constrained* regressions. In this sense, the *original equal weighted* FoHF index is fine as the dependent variable. If available, the asset-weighted FoHF index would have provided clues to how larger FoHFs allocate to HF strategies, but the average is actually more useful to asset allocators. It's assumed that FoHFs keep total allocations close to 100% most of the time. Most hedge funds provide restrictive liquidity terms and often demand lockups, redemption gates, quarterly or annual redemption schedules, and multi-month advance notice periods. FoHFs can only gradually adjust strategy allocations especially when cash levels are low as in recent years. In this study, it's assumed the aggregate industry strategy allocations are relative stable and the goal here is to find a model capable of estimating such (steady) allocations. The second set of questions in Section 1.1 could be answered using *multi-level/hierarchical* regression models with individual FoHF returns as dependent variables.

In this study, HFRI performance data in recent years, from January 2012 to April 2015, were used. First of all, as a baseline study, classic constrained regressions were set up as a quadratic optimization problem with equality and inequality constraints. The author tested both the original equal weighted (EW) and adjusted asset-weighted (AW) strategy index returns. The adjusted AW strategy returns are calculated by combing individual hedge funds' monthly AUM values (asset under management in USD) and returns. The caveat is that certain hedge funds choose not to publish monthly returns in the "public" database even though such returns are included in the EW strategy index returns. The lower and upper limits of each strategy allocation are set according to the industry experience–reasonable though may be subjective.

Next, Bayesian constrained regression models estimated strategy allocations at the FoHF industry level. The model parameters are allocations to four major strategies. Several probability density functions were tested for the model parameters.

The Bayesian hierarchical modeling (BHM) framework is well suited for answering the second set of questions by capturing the multi-level nature of the problem. Over 100 FoHFs, except very small funds, are classified into groups by size and volatility. Multi-level Bayesian regressions were conducted on these individual FoHFs to estimate *group-level and industry-level* strategy allocations.

Open-source R and JAGS (Just Another Gibbs Sampler) were used to perform classic statistical analysis and Markov Chain Monte Carlo (MCMC) simulations. Bayesian models are coded in the BUGS language (Bayesian inference Using Gibbs Sampling).

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3 CS HF index website: www.hedgeindex.com

2 Classic Constrained Regressions

The first step in this study was to estimate the diversified FoHF industry's average HF strategy allocations using classic (frequentist) constrained regressions. By using the Diversified FoHF Index as the dependent variable, it has the effect of *complete pooling* of individual FoHF returns. Independent variables are equal weighted (EW) and adjusted asset weighted (AW) HF strategy index returns in two baseline models.

The results from unconstrained linear regressions are not usable from asset allocation point of view: either the total weight is far from 100%, or certain strategy allocations do not make business sense. For example, using EW strategy indices as covariates, the allocation to the most easily accessible equity hedge strategy would have been 4.5%–too low to be realistic. Using AW strategy indices as covariates, the total strategy allocation would have been 81%–highly unlikely in the past 3-year period of rising risk appetite.

The constrained regression is equivalent to a quadratic optimization problem with equality and inequality constraints. Note that the classic statistical approach assumes that the allocations are unknown but fixed. The objective is to minimize the sum of squared errors between the dependent variables and the fitted values. The equality constraint means the total weight is 100%; the inequality constraints specify the lower and upper bounds of each strategy allocation. To achieve meaningful results, reasonable though somewhat subjective constraints are used. For example, a large number of hedge funds run equity long/short portfolios hence Equity Hedge is the most easily accessible and commonly invested strategy so its allocation limits are set higher.

The optimization problem is stated formally here. Let $\mathbf{Y} = \mathbf{X}\ \mathbf{w} + \boldsymbol{\varepsilon}$, where $\mathbf{w}$ is the unknown strategy allocation vector; $\mathbf{Y}$ is the dependent variable vector or matrix; $\mathbf{X}$ is the independent variable matrix; $\boldsymbol{\varepsilon}$ is the residual vector. The sum of squared errors is:

$$\boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon} = (\mathbf{Y} - \mathbf{X}\,\mathbf{w})^T (\mathbf{Y} - \mathbf{X}\,\mathbf{w}) = \mathbf{Y}^T\mathbf{Y} - 2\,(\mathbf{X}^T\mathbf{w})\,\mathbf{Y} + \mathbf{w}^T(\mathbf{X}^T\mathbf{X})\,\mathbf{w}$$

$$\Longrightarrow\ \tfrac{1}{2}\,(\boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon} - \mathbf{Y}^T\mathbf{Y}) = \tfrac{1}{2}\,\mathbf{w}^T(\mathbf{X}^T\mathbf{X})\,\mathbf{w} - (\mathbf{X}\,\mathbf{w})^T\mathbf{Y}$$

The objective function and constraints are:

$$\hat{\mathbf{w}} = \text{argmin}(\ \tfrac{1}{2}\,\mathbf{w}^T(\mathbf{X}^T\mathbf{X})\,\mathbf{w} - (\mathbf{X}\,\mathbf{w})^T\mathbf{Y}\ )$$

subject to $w_{\mathbf{EH}} + w_{\mathbf{ED}} + w_{\mathbf{RV}} + w_{\mathbf{GM}} = 1$,

and $0.15 \le w_{\mathbf{EH}} \le 0.35,\ \ 0.1 \le w_{\mathbf{ED}} \le 0.35,\ \ 0.1 \le w_{\mathbf{RV}} \le 0.35\ \ 0.1 \le w_{\mathbf{GM}} \le 0.25$

This type of regression is inherently difficult due to high correlations (see Table 1) between strategy index returns, though the issue is slightly mitigated using AW strategy indices. As a future research opportunity, HF sub-strategy index returns may provide more information when used as independent variables. A popular ridge regression package in R did not help since it only allows non-positive lower limits. The author also tested principal component regressions (PCR) by first running regressions vs. 4 principal components then converted the (allocation) results back to the original basis. However, the solved weights were similar to those by the constrained optimization. Regressions vs. 2 and 3 principal components yielded quite different results from those by 4 components. The author will not discuss these results in detail here.

Constrained regressions using adjusted AW HF strategy index returns provided lower tracking errors compared with models using original EW strategy index returns. The results are shown in Table 2. Though larger hedge funds have more capacities to take in more investments, the actual FoHF industry average allocations are likely somewhere between the two results in the table.

Table 1: Correlations between HF Strategy Indices (AW; EW results in parentheses)

Correlation	EH	ED	RV	GM
EH	1.00	0.84 (0.91)	0.72 (0.84)	0.29 (0.23)
ED	0.84 (0.91)	1.00	0.81 (0.87)	0.23 (0.16)
RV	0.72 (0.84)	0.81 (0.87)	1.00	0.34 (0.27)
GM	0.29 (0.23)	0.23 (0.16)	0.34 (0.27)	1.00

Table 2: Baseline Classic Constrained Regression Models

Solved Strategy Allocations (%)	EH	ED	RV	GM
AW HF Strategies	23.5	16.5	35.0	25.0
EW HF Strategies	15.0	31.6	28.4	25.0

3 Bayesian Regression Models

3.1 Bayesian Regressions with Constraints

The author constructed several baseline Bayesian regression models. The model parameters (as random variables) are allocations to the four major HF strategy indices; the dependent variable is the original EW FoHF index; the covariates are the adjusted AW strategy index returns. Probability density functions tested in the modeling process included Beta, Dirichlet, and Normal with additive log-ratio transformation. The latter approach, which guarantees total composition weight of 100%, is inspired by Aitchison's work on compositional data analysis (Aitchison, 1986). These three density functions provide different characteristics of correlations between the model parameters.

Two sets of prior mean (average) values were tested for each probability distribution: naive equal weights and empirical priors (i.e. results from classic constrained regressions). The last parameter, allocation to Global Macro, is the constraint, i.e., $w_{GM} = 1 - (w_{EH} + w_{ED} + w_{RV})$. To make business sense, the prior standard deviations of mean allocations are set to around 3% to 5%. No inequality constraints are imposed to strategy allocation parameters in the Bayesian regression models from this point onward.

The steps in a typical Bayesian analysis include:

1. Modeling: Model parameters of interest as random variables, specify likelihood functions and prior parameter values.
2. Updating: Update posterior (density) functions of parameters using data, priors, and likelihood.
3. Sampling: Generate samples from posterior (density) functions using Markov Chain Monte Carlo simulations.
4. Inference: Make point or interval inference on parameters from posterior samples. It is advisable to make sure posterior samples do not show significant autocorrelations.

The following simulation parameters are used throughout this study.

- Number of steps in adaptation: 1000
- Number of steps in burn-in: 2000
- Number of MCMC chains: 3
- Number of thinning steps: 4
- Number of posterior simulation steps per chain: 4000

It is important to note that the posterior probability is the weighted average of priors and observed data. When there are sufficient *good* data, the potential "bias" due to priors is generally not an issue.

The posterior mean strategy allocations of Bayesian regression models using Beta distributions with naive (equal) and empirical priors are summarized in Table 3. Values in parentheses are prior means. Deviance Information Criterion (DIC) values using the two sets of priors are -136.4 and -136.7 respectively. Lower DIC values indicate better model fit. The tracking error is also lower using empirical priors. However, the results are not satisfactory because posterior mean allocations to ED and RV are quite different from two sets of priors. In both cases, posteriors are very close to priors. This is likely because observed strategy and FoHF *index* performance data alone did not provide sufficient information to influence prior values.

Table 3: Bayesian Regression Models with Constraints (FoHF Index)

Posterior Mean Strategy Allocations (%)	**EH**	**ED**	**RV**	**GM**	**DIC**
Equal Priors	25.4	25.1	24.4	25.1	-136.4
Empirical Priors	23.4 (23.5)	16.3 (16.5)	35.5 (35.0)	24.8 (25.0)	-136.7

3.2 Bayesian Hierarchical Regression Models: Classified by Fund Size

The Bayesian hierarchical modeling (BHM) framework is well suited for answering the second set of questions in Section 1.1 by capturing the multi-level nature of the problem. Many examples and applications of BHM are described in (Gelman and Hill 2007).

In this section, over 100 diversified FoHFs are classified into three groups according to fund sizes (AUM values): large, medium, small. There are 13, 31, 67 FoHFs in the 3 groups respectively. The AUM thresholds are $2 billion and above for large FoHFs; less than 500 million but at least 100 million for small FoHFs; funds below 100 million are excluded as investment opportunities of such funds are limited hence not representative. One of the large FoHFs was also excluded due to its very different return characteristics compared with others in the group, so 100 FoHFs are used in hierarchical regressions.

3.2.1 Two-level Models with Group-level Parameters: Grouped By Fund Size

Two-level Bayesian hierarchical regressions with constraints were conducted. The model parameters are individual FoHF and group-level strategy allocations, specified as Beta or Dirichlet probability density functions. The dependent variables are the three size groups of individual FoHF returns. The covariates are the adjusted AW strategy index returns. Hierarchical models use more data, hence more information, compared with the previous model using only FoHF index data. The group-level *hyper-parameters* have *Bayesian shrinkage* effect, i.e. partial pooling instead of complete pooling.

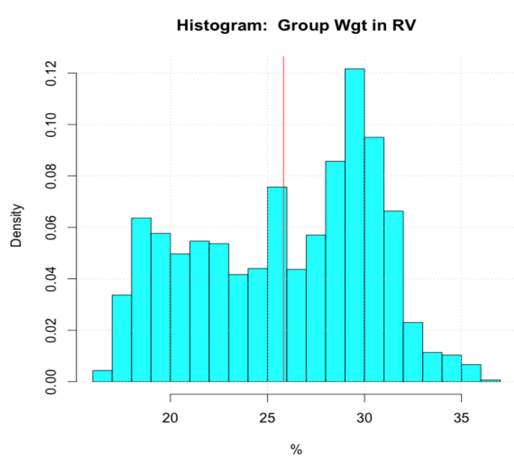
The posterior group mean strategy allocations using Beta distributions with naive (equal) and empirical priors are summarized in Table 4. Empirical prior means (displayed in parentheses) are estimated using classic constrained regressions. One surprising result is that, in most simulation cases, mean allocations to RV in all 3 groups are higher than 25% and also higher than prior values.

It appears that posterior allocation estimates to RV have the widest spreads between models using equal and empirical priors. The histogram of posterior group-level RV allocation revealed slight *bimodal* characteristics (Figure 1) when using equal priors. The author ran multiple MCMC simulations and found that mid-sized FoHF showed bimodal in ED, RV, or GM allocation histograms in some simulations. Small FoHFs also showed bimodal in ED, RV, or GM. It's not sufficient to simply look at posterior mean values.

Table 4: Two-level Bayesian Hierarchical Regression Models (110 FoHFs Classified by Fund Size)

Posterior Group Mean Strategy Allocations (%)		**EH**	**ED**	**RV**	**GM**	**DIC**
Equal Priors	Large FoHFs (12)	24.8	24.4	25.8	25.0	-554.7
	Medium (31)	22.6	26.3	27.6	23.5	-1390
	Small (67)	22.7	27.8	24.3	25.2	-2948
Empirical Priors	Large FoHFs (12)	18.6 (18)	23.4 (24)	36.0 (35)	22.0 (25)	-555.4
	Medium (31)	22.6 (22)	22.6 (24)	37.1 (35)	17.7 (19)	-1391
	Small (67)	20.4 (22)	19.6 (21.5)	33.3 (31.5)	26.7 (26)	-2949

Figure 1: Histogram of Large FoHF's Posterior Group Weight Samples (Equal Priors)



Some FoHFs prefer to consider multi-strategy hedge funds as the fifth HF strategy. To estimate allocations to five HF strategies, it's better to use HF sub-strategy index returns as independent variables to gain more granular information.

3.2.2 Three-level Models with Group-level and Industry-level Parameters: Grouped By Fund Size

Three-level Bayesian regressions with constraints were conducted. The model parameters are individual, group-level, and the industry-level strategy allocations. The dependent variables are the same three groups of individual FoHFs classified by AUM values. The covariates are the AW strategy index returns. Group-level parameters are specified as Beta distributions; industry-level parameters use Dirichlet distributions with Dirichlet priors.

Posterior group mean (average) allocations using Beta distributions are summarized in Table 5. The first set shows posterior mean strategy allocations using equal priors. The second set shows posterior allocation means using empirical priors (in parentheses). The industry-level's empirical *hyper-priors* are the averages of three group empirical priors. By adding a layer of *hyper-parameters*, the models have more shrinkage effect compared with the two-level models. It's evident from Table 5 that posterior group means are much closer to each other than the results in Table 4.

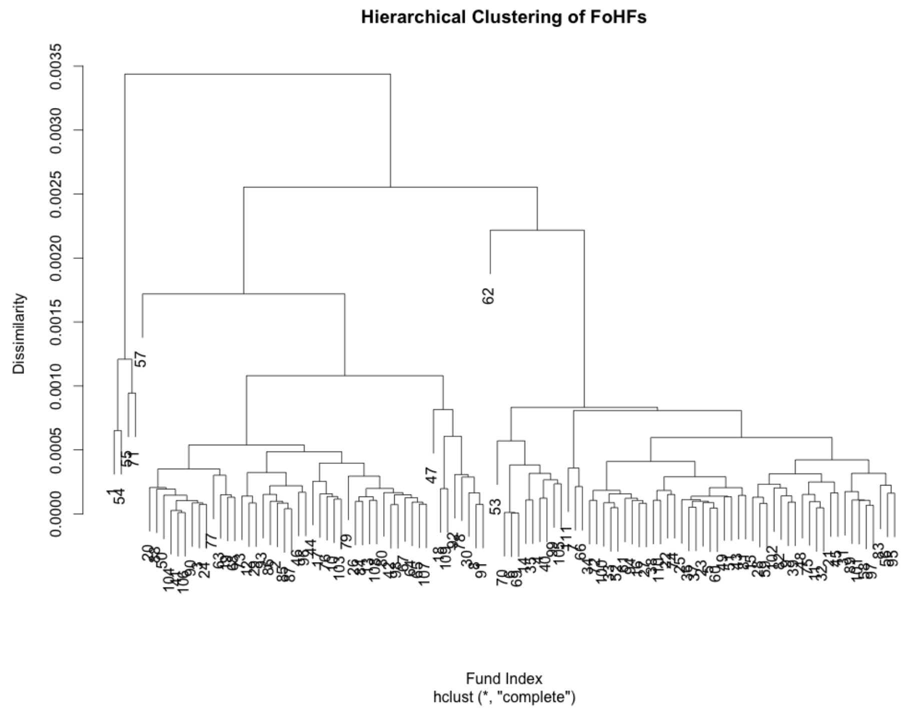
Table 5: Three-level Bayesian Hierarchical Regression Models (110 FoHFs Classified by Fund Size)

Posterior Group Mean Strategy Allocations (%)		**EH**	**ED**	**RV**	**GM**
Equal Priors DIC: -4893	Diversified FoHF Industry	24.0	25.9	26.5	23.6
	Large FoHFs (12)	23.7	28.7	25.9	21.7
	Medium (31)	24.8	25.4	27.8	22.0
	Small (67)	21.5	25.2	28.9	24.4
Empirical Priors DIC: -4891	Diversified FoHF Industry	19.4 (20.7)	24.7 (23.2)	33.7 (33.8)	22.3 (22.3)
	Large FoHFs (12)	20.2 (18.0)	25.7 (24.0)	31.8 (35.0)	22.3 (23.0)
	Medium (31)	17.9 (22.0)	23.8 (24.0)	35.4 (35.0)	22.9 (19.0)
	Small (67)	17.3 (22.0)	28.0 (21.5)	33.1 (31.5)	21.6 (25.0)

3.3 Bayesian Hierarchical Regression Models: Classified by Fund Volatility

This section considers return characteristics as classification criteria. A hierarchical clustering algorithm using covariance matrix as the distance (dissimilarity) measure first built a tree of the 111 FoHFs with similar funds closer together (Figure 2). A dissimilarity threshold was specified to break FoHFs into seven groups. Four groups having only one or two members are excluded from regressions. One such example is fund 62 in Figure 2. The three remaining groups have 8, 40, and 57 FoHFs respectively. The first group has the highest average annualized volatility (5.9%); the second group lies in between (3.6%); the third group has the lowest (2.3%).

Figure 2: Dendrogram of FoHF Clustering



3.3.1 Two-level Models with Group-level Parameters: Grouped By Fund Volatility

Two-level Bayesian hierarchical regressions with constraints were conducted. Model parameters are <u>individual</u> FoHF and <u>group-level</u> strategy allocations, specified as Beta or Dirichlet probability density functions; the dependent variables are the three groups of individual FoHF returns classified by volatility. Covariates are the adjusted AW strategy index returns.

Posterior mean strategy allocations using Beta probability density functions and Dirichlet priors are summarized in Table 6. In each group, the empirical priors are estimated from classic constrained regressions with the same inequality constraints as in Section 2 except the upper limit to GM is set higher to 30% (vs. 25% earlier) to accommodate the Lower Risk group's characteristics. It's interesting to note that the Higher Risk group showed higher allocations to the two risk-seeking strategies (EH and ED); while the Lower Risk group showed higher allocations to more hedged strategies (RV and GM) although the posterior mean (average) allocation to EH seems low using empirical priors. However, these results are largely consistent with intuition and industry experience.

The main differences in results between the two hierarchical modeling approaches in Section 3.2.1 and Section 3.3.1 are:

- Classified by fund size (see Table 4): RV received highest allocations, over 30% using empirical priors. EH and ED each received around 20% in most cases.
- Classified by volatility (see Table 6): ED received higher allocations than EH using empirical priors for all three groups. ED also received over 30% allocations (using empirical priors) except for the Lower Risk group. Allocations to EH and

ED are highest for the Higher Risk group and lowest for Lower Risk group.

Table 6: Two-level Bayesian Hierarchical Regression Models (104 FoHFs Classified by Volatility)

Posterior Group Mean Strategy Allocations (%)		**EH**	**ED**	**RV**	**GM**	**DIC**
Equal Priors	Higher Risk (8)	25.0	24.8	26.5	23.7	-367.9
	Medium Risk (40)	25.7	23.8	25.2	25.3	-1827
	Lower Risk (57)	21.0	24.6	27.7	26.7	-2479
Empirical Priors	Higher Risk (8)	32.0 (35)	39.1 (35)	19.4 (20)	9.5 (10)	-368.9
	Medium Risk (40)	28.6 (28.9)	34.4 (35)	21.4 (21.1)	15.6 (15)	-1828
	Lower Risk (57)	15.9 (15)	19.4 (20)	35.6 (35)	29.1 (30)	-2480

3.3.2 Three-level Models with Group-level and Industry-level Parameters: Grouped by Fund Volatility

The parameters of the three-level models are individual, group-level, and industry-level strategy allocations. The dependent variables are the three groups of individual FoHFs classified by volatility. The covariates are the AW strategy index returns. Group-level parameters are specified as Beta distributions; industry-level parameters use Dirichlet distributions with Dirichlet priors. Posterior mean allocations using Beta distributions are summarized in Table 7. The three-level model has clear shrinkage effects.

Table 7: Three-level Bayesian Hierarchical Regression Models (104 FoHFs Classified by Volatility)

Posterior Group Mean Strategy Allocations (%)		**EH**	**ED**	**RV**	**GM**
Equal Priors DIC: -4671	Diversified FoHF Industry	24.3	25.0	25.7	25.0
	Higher Risk (8)	21.8	26.6	27.0	24.6
	Medium Risk (40)	24.5	23.9	27.5	24.1
	Lower Risk (57)	25.5	24.1	24.1	26.3
Empirical Priors DIC: -4675	Diversified FoHF Industry	24.8 (26.3)	30.2 (30.0)	26.1 (25.4)	18.9 (18.3)
	Higher Risk (8)	28.5 (35.0)	27.0 (35.0)	25.3 (20.0)	19.2 (10.0)
	Medium Risk (40)	24.8 (28.9)	30.7 (35.0)	26.4 (21.1)	18.1 (15.0)
	Lower Risk (57)	18.5 (15.0)	33.1 (20.0)	28.3 (35.0)	20.1 (30.0)

4 Concluding Remarks

This paper started as an attempt to estimate the diversified fund-of-hedge-funds industry's aggregate hedge fund strategy allocations. The author showed that Bayesian hierarchical modeling technique is powerful in capturing aggregate group-level and industry-level asset allocations while allowing dispersions among individual FoHFs. It also allows using investment experience to set more meaningful ex-ante (prior) parameter probability distributions and find the desired Bayesian shrinkage (data pooling) effect.

The author believes the Bayesian hierarchical models in Section 3.3 (individual FoHFs classified by fund volatility) using empirical priors provide good balance between these objectives: (1) results that are consistent with industry experience and could be easily interpreted; (2) model parsimony and good fit to data. The two-level BHM using empirical priors in Section 3.3.1 has appropriate shrinkage effect so is more useful for asset allocation purpose. On the other hand, the three-level BHM provides a good way of estimating the overall industry average strategy allocations.

The models in Section 3.2 (FoHFs grouped by fund size) are also worth being considered as a reference since larger FoHFs tend to have different opportunity sets compared with smaller funds.

Learning from the results so far, it is logical to assess one's own FoHF (or multi-manager) portfolio from both volatility and AUM perspectives in order to pick the "right benchmark weights" for strategic asset allocation and performance attribution purposes.

The author expects to continue research on using Bayesian methods to model dynamic behaviors of FoHF allocations to help tactical asset allocation decisions. It is also worthwhile to test hedge fund sub-strategy index returns as covariates to gain more granular information.

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