The use of composite estimators for estimating forest biomass and growth from permanent sample plots established by the angle count method

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Abstract

The private corporate forest landowners of the United States have long abandoned the practice on relying upon permanent sample plots (PSPs) to measure the components of forest growth. Many of the PSPs were inexpensively established using the Bitterlich angle count method, however they furnished estimates of growth with high variance or with non-additive properties of biomass/volume ($V_2 \neq [V_1 + \Delta V]$). McTague (2010) proposed a new unbiased variable basal area factor that is weakly correlated, in some forest conditions, to the conventional Bitterlich estimator. Gains in precision are possible by using composite estimates that weighted inversely by variance between the new and conventional estimators. This paper will demonstrate that the Bitterlich estimator is a special case of a composite estimate between the new variable basal area factor and its antithetic counterpart. When the new estimator is extended to the application of estimating change in forest stock or standing inventory from remeasured forest plots, it is possible to obtain unbiased, efficient, additive, and inexpensive estimates of growth.

Keywords: Angle count sampling, composite estimators, forest growth and change estimates, permanent sample plots

1. Introduction

Numerous authors have bemoaned the demise of the use of permanent sample plots on private forest timberlands in North America, based on an angle count (point sampling) estimator, for measuring forest stock and growth. Assertive and sometimes humorous accounts have vigorously claimed that point sampling estimators are very appropriate for measuring growth (Furnival 1979, Husch et al. 1982). The ubiquitous installation of permanent point sample plots in the 1960's and 1970's was based on the low cost, ease of measurement, and efficiency of basal area and volume estimates. The disfavor of permanent point sample points became evident later when analyzing growth from remeasurements. Additive or compatible estimates, such as those reported by Van Deusen et al. (1986) typically displayed high variance for the attribute of growth, while the efficient change estimator of Grosenbaugh (1958) was non-additive for biomass and or volume ($V_2 \neq [V_1 + \Delta V]$).

Galik et al. (2013) have recently described the huge potential role of private timberlands in carbon sequestration and exchange of carbon credits. Participation rules for commercialization and program design of exchange frequently involve monitoring requirements and some level of verification of the change in biomass and carbon stocks. Permanent sample plots, based on point or angle count samples, offer an attractive methodology for direct observation of change in forest biomass stocks. This paper explores the use of a new point sampling estimator introduced by McTague (2010) and its properties in the estimation of change of forest biomass/volume. Comparisons are made to other estimators of change using the traditional *angle count* constant for basal area factor (BAF_{AC}) created by Bitterlich (1948), and newer variable basal area factors proposed by Flewelling (1981), Iles and Carter (2007), and McTague (2010).

2. Previous studies

The work presented here builds upon some empirical comparisons among growth estimators from permanent point samples presented by Hradetzky (1995) and Thérien (2011). Unfortunately, a definitive answer regarding the most efficient change estimator for measuring forest growth is not possible; the results are still partially driven by the data, duration between measurement occasions, and the specific component of forest growth that is analyzed. The problem of estimating growth from permanent point samples, as described by Hradetzky (1995), is of varying sample composition from one measurement occasion to the next. Excluding the consideration of trees that die, or saplings that grow into merchantable size during the remeasurement interval, the sample plot on each remeasurement occasion will normally contain an augmented list of 'in' trees. Despite the baffling name of nongrowth trees that was introduced by Martin (1982) to denote the list of new 'in' trees, there is nothing mysterious about the updated list with new trees included 'in' the sample at the time of remeasurement. Irrespective of the location of the permanent point (plot) sample center, the probability of inclusion in the sample increases as trees grow larger in diameter. What differs among the various growth estimators examined in this paper is the computation of the basal area and expansion factors for the new 'in' trees at the time of remeasurement. The expansion factor (BAF/g) is used to compute the representative trees per hectare of a sample 'in' tree and it is expressed as the quotient of the basal area factor (BAF) and the individual sample tree basal area (g).

Efficiency was apparently not compromised when Van Deusen et al. (1986) and Nakajima et al. (1996) elected to compute basal area identically for both the new 'in' trees on the plot at the time of remeasurement, and for the original list of 'in' trees that were measured on both occasions. The original list of 'in' trees is generally referred to as survivor trees, since they are measured 'in' and are alive on both measurement occasions. Grosenbaugh (1958) and Beers and Miller (1964) preferred to allocate a basal area and expansion factor of zero to the new 'in' trees. The preponderance of literature suggests that the growth estimator of Grosenbaugh is more efficient than the Van Deusen estimator for most forest conditions and remeasurement intervals (Gregoire 1993, Hradetzky 1995, and Thérien 2011). The drawback of the Grosenbaugh estimator is that it is non-additive (incompatible). Bitterlich (1984) suggested that the Van Deusen type estimator was of little practical value since it measured the increase rather than the increment of sample trees. Roesch et al. (1989) introduced a novel and efficient approach to the computation growth from permanent point samples at the time of remeasurement that maintains the property of additivity. All 'in' trees at the time of remeasurement are assigned the expansion factor from measurements of the current diameter (d at time 2). The same expansion factor is then retroactively applied to the original 'in' trees of the previous measurement.

Flewelling (1981) derived an unbiased variable basal area factor estimator that is a function of the tree diameter (d) and distance from the point center to the sample 'in' tree. The expansion factor, computed with the BAF of Flewelling, has a large value for big 'in' trees located near the point center and a reduced value for small 'in' trees located away from the point center. This type of basal area and expansion factor is ideally suited for estimating growth from remeasured point samples, since newly recruited ‘in’ trees at the time of remeasurement have a diminished impact on change statistics. In order to remain fully additive however, the biomass/volume statistics of V_1 and V_2 must also be computed with the variable BAF factor when computing the change statistic of ΔV. The variable BAF factors derived by Iles and Carter (2007) and McTague (2010) have similar properties to the Flewelling estimator. The same set of trees included ‘in’ the sample and trees located further from the point center have less impact on computation of stock or change statistics.

3. Methods

Comparisons among the unbiased estimators presented in this article are made by analyzing the variances associated with repeated sampling at the stand ages of 12 and 16 using simulation. The simulation procedure is repeated at each measurement occasion 500 times using a 40 hectare forest stand and a sampling intensity of one point (plot) per hectare. The inclusion of ‘in’ trees is determined with a $BAF_{AC} = 4$ using the same point location at ages 12 and 16. The population consists of 500 *Pinus taeda* plantation trees in Santa Catarina, Brazil generated from the growth and yield model described by McTague and Bailey (1987). Their approach relies on stand level equations to predict the attributes of dominant height, basal area, and survival. Individual tree diameters are predicted from a diameter distribution function with recovered parameters from the stand level equations. The growth period of 4 years is the typical interval between harvest activities in pine plantations of southern Brazil and it is uncommon to witness competition induced mortality after repeated thinning. Age 16 approximates the point of forest stand development that corresponds to the culmination of mean annual increment.

The modeled population is derived from a thinned age 12 stand, grown to age 16, with a site index of 20 m (base age 15). Table 1 contains the parameters of the population generated from the growth and yield model. Using prediction equations for the 10^{th} and 63^{rd} diameter percentiles of the McTague and Bailey (1987) growth model, Weibull parameters are recovered for the stand that are consistent with the stand level estimate of basal area and the user-stipulated 500 trees per hectare. The diameters are then assigned to the 500 trees with a procedure described by Clutter and Allison (1974), using varying diameter class widths to maintain an equal proportion of trees in each class. In this case, each diameter class contains a proportion of 1/500 or the proportion of 0.002 of the total trees, and the diameter class assignment is computed with the following formula

$$d_i = a + b\left[-\ln(1.001 - 0.002i)\right]^{1/c}$$

where i = 1 to 500, d_i = diameter breast height of diameter class i, and a, b, and c, are Weibull distribution parameters. Individual tree heights are predicted using a methodology reported by McTague (1985) by first computing the spread between the heights of trees corresponding to the maximum diameter and minimum diameter class ($h_{dmax} - h_{dmin}$). Individual tree height is then predicted as

$$h_i = h_{d\min} + F(x)(h_{d\max} - h_{d\min})$$

where h_i is individual tree height for diameter class i and *F(x)* is the value of the cumulative density function of the Weibull distribution for diameter class i, as defined in Table 1. It is assumed that the percentile of each tree in the population remains unchanged during the growth interval.

Table 1: Population parameters of a thinned plantation of *Pinus taeda* with 500 trees per hectare (N) with productivity of site index 20, grown from age 12 to age 16.

	Age 12	*Age 16*	*growth*
Forest stand basal area (G)	28.8 m^2/ha	39.1 m^2/ha	10.3 m^2/ha
Total green weight outside bark (W)	195.2 t/ha	335.5 t/ha	140.3 t/ha
Mean top height (H)	17.0 m	20.9 m	3.9 m
Quadratic mean diameter (d_g)	27.1 cm	31.6 cm	4.5 cm
Weibull location (a) parameter	19.13	22.11	
Weibull scale (b) parameter	8.709	10.257	
Weibull shape (c) parameter	2.241	2.015	

Note: the 3-parameter Weibull cumulative density function is expressed as $F(x) = 1-\exp(-((x-a)/b)^c)$, where x ($x \geq a$) is the random variable representing tree diameter (d).

Individual tree volume and green weight from the stump to the top of the tree is computed with the two steps of Equation (1)[1]

$$v = 0.0000289198 d^{1.764357} h^{1.356831}$$
$$w = 0.9672764 v^{1.01834} (d^2 h)^{-0.004977} \quad (1)$$

where

v = total stem volume of a tree outside bark in m^3
w = total stem green weight outside bark in Kg
d = individual tree diameter in cm determined at breast height of 1.3 m
h = total tree height in m

Each of the 40 hectares in the simulation at ages 12 and 16 contains the same population of 500 trees. The trees were randomly sorted for each hectare and assigned once, at age 12, to grid coordinates. The spacing between rows was maintained at 2.5 m while the distance between trees in the row was randomly pertubated. Since the simulated stand has been thinned with a combination of row and low thinning, every fourth row was completely removed. Multiple random starts are used in the systematic inventory of 40 hectares. A small random perturbation, associated with the point (plot) location, is also simulated. The edge-effect and the potential bias associated with point locations close to a property boundary are not investigated here. The point locations in this study are at least 12.5 m away from the property boundary.

4. Results

McTague (2010) presented a new unbiased basal area estimator that is function of conventional Bitterlich *angle count* basal area factor (BAF_{AC}) and the *borderline factor* (BLF), which is determined by the tree diameter (d) and distance (R) from the sample point to center of the 'in' tree. Basal area is computed with the *variable* basal area factor estimator (BAF_V) as

[1] Carbon (C) content in Kg is computed as C = 0.1737w (Mello et al. 2008). Tons of carbon dioxide equivalent (tCO_2e) equals C(44/12)/1000.

$$G = \sum BAF_V = 2BAF_{AC} \sum_{i=1}^{m} \left(1 - \frac{BAF_{AC}}{BLF_i} \right) \tag{2}$$

where

m = the number of ‘in’ trees at the point (plot)
G = the forest stand basal area in m^2/ha
BLF_i = the borderline factor for tree i, or $(d_i/R_i)^2/4$

Noting a weak correlation between the conventional estimator BAF_{AC} and the variable estimator BAF_V, McTague (2010) suggested that gains in efficiency could be obtained by creating a weighted composite estimator using BAF_{AC} and BAF_V. The conventional *angle count* estimator presented by Bitterlich (1948) is represented as

$$G = \sum_{i=1}^{m} BAF_{AC} \tag{3}$$

It is apparent however, that additional gains in efficiency are possible. A generalized composite model that includes an antithetic variate of BAF_V, guarantees an improvement over the conventional BAF_{AC} if the basal area composite weight (w_G) differs from the value of 0.5. The generalized composite model utilizes an *antithetic variate* basal area factor (BAF_{AV}), which is expressed as

$$G = \sum BAF_{AV} = 2BAF_{AC}^2 \sum_{i=1}^{m} \left(\frac{1}{BLF_i} \right) \tag{4}$$

The unbiased BAF_{AV} factor behaves as the mirror opposite to BAF_V for sample ‘in’ trees. The basal area value for borderline ‘in’ trees approaches $2BAF_{AC}$ while trees close to the point center have a basal area approaching a value of zero with the antithetic variate estimator. As demonstrated with Equation (6), the conventional Bitterlich angle count basal area estimator, Equation (3), is a special case of the generalized composite estimator, Equation (5), when the basal area composite weight (w_G) equals 0.5.

Generalized composite $$Y = w_G \sum BAF_V + (1 - w_G) \sum BAF_{AV} \qquad w_G \neq 0.5 \tag{5}$$

Special case $$Y = BAF_{AC} = 0.5 \sum BAF_V + 0.5 \sum BAF_{AV} \qquad w_G = 0.5 \tag{6}$$

The BAF_{AV} estimator by itself is counter-intuitive and it can display rather large values of variance; its real value is inherent with its weak or negative correlation with BAF_V. The Bitterlich BAF_{AC} is indeed a very special case of the generalized composite estimator, since it does not require any measurement of the borderline factor (BLF) of sample ‘in’ trees. The optimal *stand attribute* composite weight (w_{sa}) for computing stand basal area per hectare (G), green weight per hectare (W), trees per hectare (N) with the generalized composite estimator is computed as

$$w_{sa} = \frac{\sigma_{AV}^2 - \sigma_V \sigma_{AV} \rho_{AVV}}{\sigma_V^2 + \sigma_{AV}^2 - 2\sigma_V \sigma_{AV} \rho_{AVV}} \tag{7}$$

where σ_V^2 = variance of the *variable* BAF estimate, σ_{AV}^2 = variance of the *antithetic variate* BAF estimate, and ρ_{AVV} = the correlation between the *antithetic variate* and *variable* BAF estimate.

As demonstrated later, a different approach is proposed to estimate the optimal stand attribute composite weight for change statistics for basal area and green weight per hectare. How does the generalized composite estimator Equation (5) compare against the special case (conventional Bitterlich angle count) estimator Equation (3)? This forest sampling question is addressed using simulation with the 40 hectare forest described in the Methods section for ages 12 and 16. Trees are included 'in' the sample based on a BAF_{AC} = 4 m^2/ha. Comparisons are also made to the variable BAF estimators of Flewelling (1981) and Iles and Carter (2007). In this example, the *variable Flewelling* estimator ($BAF_{V\text{-}Fl}$) is expressed as

$$G = \sum BAF_{V-Fl} = \begin{cases} 0.312771\sum_{i=1}^{m-t} BLF_i & , BLF < 36 \\ 11.25977 \times t & , BLF \geq 36 \end{cases} \tag{8}$$

where m = number of sample 'in' trees at a point (plot), and t = number of trees at the point that subtend an angle larger than 6°52.78' which corresponds to a *borderline factor* (BLF) of 36.

The *variable Iles and Carter* estimator ($BAF_{V\text{-}IC}$) is expressed as

$$G = \sum BAF_{V-IC} = 3BAF_{AC}\sum_{i=1}^{m}\left(1-\frac{R_i}{R_c}\right) \tag{9}$$

where R_i is the the distance from the sample point (plot) to center of the 'in' tree i, R_c is the critical or limiting distance of the plot radii for tree i with diameter breast height d_i and is computed as $R_c = 0.5d_i \big/ \sqrt{BAF_{AC}}$

The trees per hectare (N) represented by each 'in' tree is computed with the expansion factor. Irrespective of whether a constant or variable basal area factor is employed, the individual tree expansion factor is expressed as the quotient of BAF and the individual tree basal area g, or (BAF/g_i). The green weight (W) per hectare represented by each 'in' tree is computed as the product of the individual tree green weight of Equation (1) and the expansion factor, or (w_i x BAF/g_i). In the example presented here, the variances, covariance, and composite estimator weight of Equation (7) were estimated from simulation using a sampling intensity of one point (plot) per hectare for a sample size of 40. The computation for the optimal stand attribute composite weight w_{sa} and other estimates were repeated 500 times. Comparisons among the estimators for the attributes of basal area, green weight per hectare, and trees per hectare are presented in Table 2. The variances reported in Table 2 are made from analysis-of-variance-type calculations of the between repetitions component. From Table 2 it is possible to derive the gain in relative efficiency from using the composite estimators over that of the conventional estimators for basal area, trees per hectare, and green weight per hectare. The relative efficiency estimates at age 16, defined as $\frac{\hat{\sigma}^2_{Conventional}}{\hat{\sigma}^2_{Composite}}$, are 1.67, 1.86, and 1.61 for the attributes of basal area, trees per hectare, and green weight per hectare respectively for the generalized composite estimator.

Table 2: Estimates and efficiencies of the point sampling estimators obtained from repeated sampling using a BAF_{AC} = 4 for basal area, trees per hectare, and green weight per hectare.

		Basal area in m²/ha (G) $\sum BAF$		*Trees per hectare (N)* $\sum BAF\frac{1}{g_i}$		*Green weight in t/ha (W)* $\sum BAF\frac{w_i}{g_i}$	
Age and estimator	*BAF Eq.*	*Mean*	*Variance of mean*	*Mean*	*Variance of mean*	*Mean*	*Variance of mean*
Age 12							
Conventional	(3)	28.9	0.607	500.9	189.6	195.9	29.2
Flewelling	(8)	28.9	0.856	500.2	298.2	196.0	40.8
Iles and Carter	(9)	28.9	0.637	500.1	223.4	195.9	30.8
Generalized	(6)	28.9	0.414	500.6	132.5	195.8	20.5
composite	w_{sa}	$\overline{w}_G =$	0.80	$\overline{w}_N =$	0.78	$\overline{w}_W =$	0.80
Age 16							
Conventional	(3)	39.1	0.838	500.2	158.3	335.9	63.4
Flewelling	(8)	39.2	0.942	500.8	172.3	336.3	72.4
Iles and Carter	(9)	39.2	0.669	500.7	121.5	336.3	52.1
Generalized	(6)	39.2	0.501	500.5	85.0	336.4	39.3
composite	w_{sa}	$\overline{w}_G =$	0.87	$\overline{w}_N =$	0.85	$\overline{w}_W =$	0.87

Note: g_i = individual tree basal area in m²/ha computed as $(\frac{\pi}{40000})d_i^2$, and w_i = individual tree green weight in Kg/1000. The reported variance is the square of the standard error of the mean. It is computed using the analysis-of-variance-type formulas as $v(\bar{y}_{sy}) = \frac{SSB}{n(k-1)}$, where n = 40, k = 500, and SSB is the between repetitions sums of squares.

Composite weights (w_{sa}) were computed for the stand attributes of basal area per hectare, trees per hectare, and green weight per hectare for each repetition. While 0.80 represents the average composite weight for basal area per hectare (w_G) at age 12, the range over 500 repetitions varied from 0.57 – 0.99. In the case at age 12 where $\overline{w}_G = 0.80$, the average correlation between Equation (2) and Equation (4) equals $\hat{\rho}_{AVV} = -0.05$.

4.1 Efficiency of permanent sample plot growth estimators

This paper reports the statistical properties of growth (change) estimators of an older, repeatedly thinned plantation. It excludes consideration of tree mortality or ingrowth (saplings that grow into merchantable size during the remeasurement interval). Neither mortality nor ingrowth are considered as the components of growth that are responsible for the high variance of the additive (compatible) estimator. The high variance is normally attributed to the list of new 'in' trees at the time of remeasurement. Martin (1982) denotes these new 'in' trees as nongrowth trees and Hradetzky (1995) states that they are additional sample trees of the subsequent inventory. Expanding upon the notation of Hradetzky (1995), unbiased change estimators for green weight per hectare of a pure panel design are presented below

$$Z_1 = BAF_{AC}\left[\sum_{i=1}^{m_2}\frac{w_{i2}}{g_{i2}} - \sum_{i=1}^{m_1}\frac{w_{i1}}{g_{i1}}\right] \tag{10}$$

where

- Z_1 = change in green weight (t/ha) using the additive Van Deusen et al. (1986) estimator
- w_{ij} = total stem green weight for tree i at time j in Kg/1000. j = 1 for age 12 and j = 2 for age 16
- g_{ij} = individual tree basal area in m^2/ha for tree i at time j
- m_j = number of sample 'in' trees at a point (plot) at time j. The number of new 'in' trees at the time of remeasurement equals $(m_2 - m_1)$

The Z_1 estimator is preferred by Eastaugh and Hasenauer (2013) because it is resistant to measurement error. Long growth periods, high growth rates, or cases where growth exceeds the initial stand basal area, G_1, also favor the use of the Z_1 estimator (Flewelling and Thomas 1984, Hradetzky 1995). The most widely used estimator for computing change in biomass/volume from point samples is the non-additive Grosenbaugh (1958) estimator which essentially ignores the contribution of the new 'in' trees at the time of remeasurement into the calculation of growth:

$$Z_2 = BAF_{AC}\left[\sum_{i=1}^{m_1}\frac{(w_{i2} - w_{i1})}{g_{i1}}\right] \tag{11}$$

where Z_2 = change in green weight (t/ha) using the non-additive Grosenbaugh (1958) estimator.

The Roesch et al. (1989) estimator is similar to Equation (11) in that only one expansion factor is used. Rather than employing an expansion factor computed at time 1 and employing it to the original m_1 trees, the Roesch et al. (1989) utilizes an expansion factor computed at time 2 and applies it to the entire list (m_2) of 'in' trees at time 2.

$$Z_3 = BAF_{AC}\left[\sum_{i=1}^{m_2}\frac{(w_{i2} - w_{i1})}{g_{i2}}\right] \tag{12}$$

where Z_3 = change in green weight (t/ha) using the conditionally additive Roesch et al. (1989) estimator.

The drawback to the Z_3 estimator is that it requires measurement the of attribute w_{i1} at time 1, for the $(m_2 - m_1)$ trees that 'out' at time 1 but 'in' at time 2. In the simulation example employed in this paper, this information is readily available, however for operational inventories, it is perhaps fanciful to believe that w_{i1} of trees excluded of the sample at time 1 would be observed . The Z_3 estimator is additive when adjustments are made to the green weight per hectare (W_1) at time 1, based upon measurement of green weight per hectare (W_2) at time 2.

The remainder of the change estimators presented here are variable; the BAF is different from tree to tree in the sample and it changes over time for a specific tree of interest. The Flewelling (1981) estimator for change in green weight per hectare is expressed as

$$Z_4 = \sum_{i=1}^{m_2} BAF_{V-Fl(j=2)} \frac{w_{i2}}{g_{i2}} - \sum_{i=1}^{m_1} BAF_{V-Fl(j=1)} \frac{w_{i1}}{g_{i1}} \quad (13)$$

where Z_4 = change in green weight (t/ha) using the additive Flewelling (1981) estimator and $\sum BAF_{V-Fl(j)}$ is calculated at time j using Equation (8).

The Iles and Carter (2007) estimator for change in green weight per hectare is

$$Z_5 = \sum_{i=1}^{m_2} BAF_{V-IC(j=2)} \frac{w_{i2}}{g_{i2}} - \sum_{i=1}^{m_1} BAF_{V-IC(j=1)} \frac{w_{i1}}{g_{i1}} \quad (14)$$

where Z_5 = change in green weight (t/ha) using the additive Iles and Carter (2007) estimator and $\sum BAF_{V-IC(j)}$ is computed at time j using Equation (9). Over time, the value of $BAF_{V-IC(j)}$ increases as the subject tree of interest grows in diameter, and the critical or limiting distance R_c increases.

Given the results presented in Table 2 and the success of the generalized composite estimator in efficiently predicting stand basal area, trees per hectare, and green weight per hectare, a natural progression is to test its capabilities in estimating change. In this case, the generalized composite growth estimator for green weight per hectare is expressed as

$$Z_6 = w_{W2} \sum_{i=1}^{m_2} BAF_{V(j=2)} \frac{w_{i2}}{g_{i2}} + (1 - w_{W2}) \sum_{i=1}^{m_2} BAF_{AV(j=2)} \frac{w_{i2}}{g_{i2}} \quad (15)$$

$$- w_{W1} \sum_{i=1}^{m_1} BAF_{V(j=1)} \frac{w_{i1}}{g_{i1}} - (1 - w_{W1}) \sum_{i=1}^{m_1} BAF_{AV(j=1)} \frac{w_{i1}}{g_{i1}}$$

where Z_6 = change in green weight (t/ha) using the additive generalized composite estimator, $\sum BAF_{V(j)}$ is calculated at time j using Equation (2), and , $\sum BAF_{AV(j)}$ is calculated at time j using Equation (4). The composite weights (w_{Wj}) at time j are computed for each of the 500 repetitions using Equation 7). As previously stated, the average green weight per hectare composite weights at age 12 (time 1) and age 16 (time 2) are $w_{W1} = 0.80$ and $w_{W2} = 0.87$ respectively.

The composite weights (w_{Wj}) that were inserted in Equation (15) provide efficient estimates of standing green weight per hectare (W) at ages 12 and 16. By no means however, are they the best for estimating change or growth of biomass/volume. If the focus of the forest inventory is strictly aimed at estimating efficient estimates of change, alternative methods can be employed to maximize the correlation between the successive panel survey estimates of green weight per hectare for the permanent sample plots. In the case of forests managed to maximize carbon sequestration or change in carbon stocks, the path to efficient estimates is attained by improving the correlation between successive measurements. Generalized least squares estimation represents a common approach to assigning appropriate weights to panel surveys (Legg et al. 2005 and Baltagi 1998). Under the assumption that there is little authority or capability of the forest inventory administrator to change the estimates Y_1 by altering the composite weight for green weight per hectare, (w_{W1}) at time 1, the focus remains on how to estimate composite weight at time 2 for Y_2 that will maximize the correlation between time periods. The following restricted linear model can be used to estimate the optimal composite weight (w_{W2}) for estimating change of green weight per hectare

$$Y_1 = b_0 + b_1 X_1 + (1 - b_1) X_2 \qquad (16)$$

where

$Y_1 = w_{W1} \sum_{i=1}^{m_1} BAF_{V(j=1)} \frac{w_{i1}}{g_{i1}} + (1 - w_{W1}) \sum_{i=1}^{m_1} BAF_{AV(j=1)} \frac{w_{i1}}{g_{i1}}$, the previously determined optimal composite estimate for standing green weight per hectare at time 1

$X_1 = \sum_{i=1}^{m_2} BAF_{V(j=2)} \frac{w_{i2}}{g_{i2}}$

$X_2 = \sum_{i=1}^{m_2} BAF_{AV(j=2)} \frac{w_{i2}}{g_{i2}}$

b_0 = negative estimate of change or growth in t/ha

b_1 = regression estimate of the composite weight for green weight per hectare at time 2 (w_{W2})

In the example presented here, the b_1 parameter was estimated from fitting Equation (16) using a sampling intensity of one point (plot) per hectare for a sample size of 40. The computation for the regression composite weight for basal area per hectare and green weight per hectare were repeated 500 times. The *regression composite* estimator for estimating the change in green tons per hectare (W) is presented below

$$Z_7 = Y_2 - Y_1 \qquad (17)$$

where $Y_2 = b_1X_1 + (1 - b_1)X_2$ and the terms b_1, X_1, X_2, and Y_1 are previously explained.

Comparisons among the seven change estimators for the attributes of basal area, and green weight per hectare are presented in Table 3. Since there is no change in the population value of trees per hectare between ages 12 and 16 of a repeatedly thinned stand, no attempt was made to analyze the behavior of the change estimators for stand density. The variances reported in Table 3 are made from analysis-of-variance-type calculations of the between repetitions component. Altering the average composite weight for green weight per hectare at age 16 from 0.87 in Equation (15) to 1.008 in Equation (17) results in a considerable improvement in the efficiency for the estimation of growth. Essentially the new weight extends the range of the estimate of green weight per hectare at age 16 and it improves the correlation between age 12 and 16 stand attributes. This effect of changing the composite weight on the correlation of Age 16 vs Age 12 values is displayed in Figure 1.

Table 3: Estimates and efficiencies of the point sampling change estimators obtained for basal area and green weight per hectare.

			Change in green weight per hectare, ΔW in t/ha.		*Change in basal area per hectare, ΔG in m²/ha.*	
Estimator	*Source*	*Eq.*	*Mean*	*Variance of Mean*	*Mean*	*Variance of Mean*
Z_1	Van Deusen et al.	(10)	140.0	52.9	10.3	0.81
Z_2	Grosenbaugh	(11)	140.5	16.8	10.3	0.10
Z_3	Roesch et al.	(12)	140.3	11.7	10.3	0.07
Z_4	Flewelling	(13)	140.3	13.7	10.3	0.16
Z_5	Iles and Carter	(14)	140.4	10.1	10.3	0.13
Z_6	Generalized composite	(15)	140.3	14.5	10.3	0.19
Z_7	Regression composite	(17)	140.4	9.8	10.3	0.09
		w_{sa}	$\overline{w}_{W2} =$	1.008	$\overline{w}_{G2} =$	1.045

1. The computation of basal area per hectare employs Equations (10), (13)-(15), and (17), however the term of (w_{ij}/g_{ij}) is omitted. In the case of the Grosenbaugh and Roesch estimators for basal area, the term in parenthesis in Equations (11) and (12) $(w_{i2} - w_{i1})$ should be replaced with $(g_{i2} - g_{i1})$.
2. The reported variance growth or change is the square of the standard error of the mean. It is computed using the analysis-of-variance-type formulas as $v\left(\bar{y}_{sy}\right) = \frac{SSB}{n(k-1)}$, where n = 40, k = 500, and SSB is the between repetitions sums of squares.

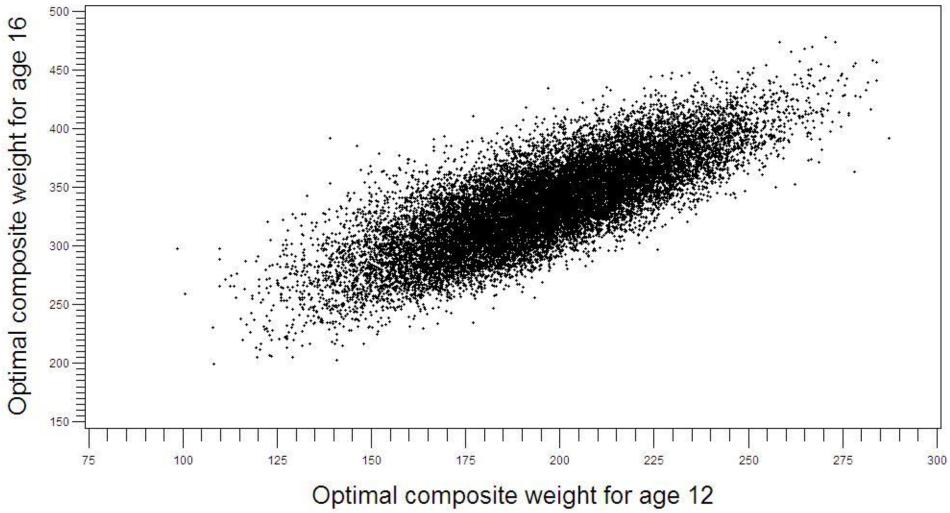
5. Discussion

Additivity and efficiency are possible when permanent sample plots are established with a point sampling design. If tree diameter and distance from the sample point to the subject 'in' tree are measured during the forest inventory, it is possible to compute the borderline factor (BLF) and the variable basal area factors BAF_V and BAF_{AV} of Equation (2) and (4). Both BAF_V and BAF_{AV} are used to compute the generalized composite estimator, Equation (5), which will always be equal or more efficient than the conventional estimator represented by either Equation (3) or Equation (6). Plot size clearly has an effect on efficiency of the estimate for change statistics and for stand attributes any given age. Based on the quadratic mean diameters at age 12 and age 16 presented in Table 1, the average plot size increased by 51.9 m^2 at time 2. This explains why the variance for the estimate of trees per hectare (N) decreased in Table 2 for Age 16. Increased plot size and the increase of $(m_2 - m_1)$ sample 'in' trees also explains why the Roesch et al. (1989) change estimator is superior to the Grosenbaugh (1958) in Table 3.

In the example provided here, the drawback attributed to using the additive (compatible) Z_1 change estimator of Van Deusen et al. (1986) is also manifested in terms of high variance. All six other alternatives ($Z_2 - Z_7$) are superior in providing efficient change estimates for basal area and green weight per hectare. If the goal is to monitor forest stands that are commercialized for carbon sequestration credits, the variable and additive

BAF estimators (Z_4 –Z_7) offer an attractive alternative to the common non-additive estimator of Z_2. While efficient, the new Z_7 composite estimator proposed in this example, represents a drastic procedure for dealing with borderline 'in' trees at time 2, since it is possible to assign a negative value for a stand attribute. It may prove more reasonable to constrain the value of estimated b_1 coefficient of Equation (16) to $0 \leq b_1 \leq 1$.

1a.



1b.

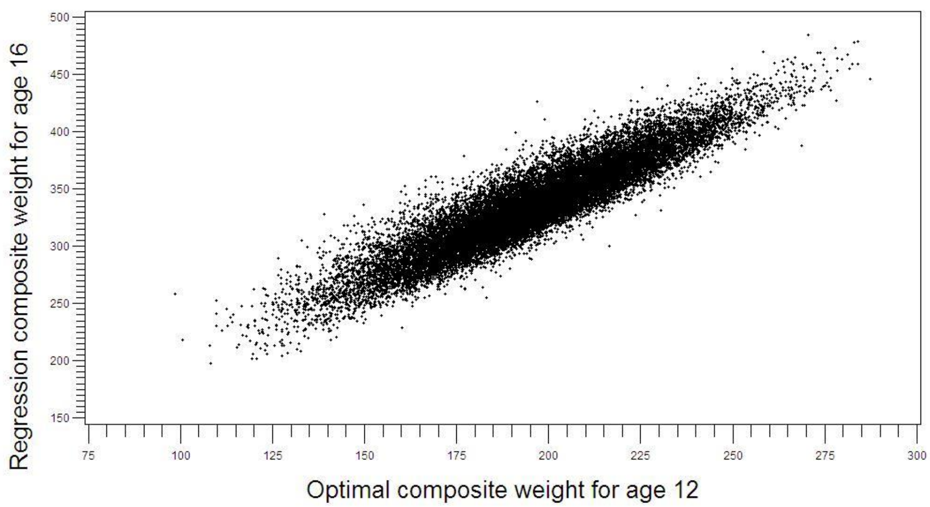


Figure 1: The top panel displays the Age 16 vs Age 12 green weight per hectare values used the in estimator Z_6 of Equation (15). The average correlation of Fig. 1a is $\hat{\rho} = 0.79$. The bottom panel displays the Age 16 vs Age 12 green weight per hectare values used in estimator Z_7 of Equation (17). The average correlation of Fig. 1b is $\hat{\rho} = 0.92$. Derived from the sample size of 40 and 500 repetitions, each graph contains 20,000 paired observations.

6. References

Baltagi, B.H. 1998. Panel data methods. Handbook of Applied Economic Statistics. Ed. A. Ullah and D.E. A. Giles. Marcel Dekker, Inc. New York.

Beers, T.W., and C.I. Miller. 1964. Point sampling: Research results theory and applications. Purdue Univ. Ag. Exp. Stn. Res. Bull. No. 786.

Bitterlich, W. 1948. Die Winkelzählprobe. Allemeine Forst- und Holzwirtschaftliche Zeitung. 59(1/2):4-5.

Bitterlich, W. 1984. The relascope idea: Relative measurements in forestry. Commonwealth Agricultural Bureaux, Farnham Royal, UK pp. 242.

Clutter, J.L., and B.J. Allison. 1974. A growth and yield model for *Pinus radiata* in New Zealand. In *Proceedings* Growth models for tree and stand simulation. Royal Coll. For., Res. Notes No. 30, Stockholm.

Eastaugh, C.S., and H. Hasenauer. 2013. Biases in volume increment estimates derived from successive angle count sampling. For. Sci. 59(1):1-14.

Flewelling, J.W. 1981. Compatible estimates of basal area and basal area growth for remeasured point samples. For. Sci. 27(1):191-203.

Flewelling, J.W., and C.E. Thomas. 1984. An improved estimator for merchantable basal area growth based on point samples. For. Sci. 30(3):813-821

Furnival, G. 1979. Forest sampling – past performance and future expectations. In *Forest Resource Inventories Workshop Proceedings*. Vol. 1. Frayer, W.E., ed,. pp. 320-326. Colorado State University, Fort Collins, CO.

Galik, C.S., B.C. Murray, and D.E. Mercer, .2013. Where is the carbon? Carbon sequestration potential from private forestland in the southern United States. J. For. 111(1): 17-25.

Gregoire, T.G. 1993. Estimation of forest growth from successive surveys. For. Ecol. Manage. 56: 267-278.

Grosenbaugh, L.R. 1958. Point-sampling and line-sampling: probability theory, geometric implications, synthesis. USDA For. Serv. South. For. Exp. Stn. Occasional Paper 160.

Hradetzky, J. 1995. Concerning the precision of growth estimation using permanent horizontal point samples. For. Ecol. Manage., 71: 203-210.

Husch, B., C.I. Miller, and T.W. Beers. (1982) *Forest Mensuration*. 3rd ed. John Wiley & Sons, New York. 402 pp.

Iles, K., and D.H.H. Carter. 2007. “Distance-variable” estimators for sampling and change measurement. Can. J. For. Res. 37: 1669-1674.

Legg, J.C., W.A. Fuller, and S.M. Nusser. (2005) Estimation for longitudinal surveys with repeated panels of observations. In *Proceedings of the Survey Research Methods Section*, pp. 3300-3306. American Statistical Association.

Martin, G.L. .1982. A method for estimating ingrowth on permanent sample points. For. Sci. 28(1): 281-293.

McTague, J.P. .2010. New and composite point sampling estimates. Can. J. For. Res. 40: 2234-2242.

McTague, J.P., and R.L. Bailey. 1987. Compatible basal area and diameter distribution models for thinned loblolly pine plantations in Santa Catarina, Brazil. For. Sci. 33(1):43-51.

McTague, J.P. 1985. Growth and yield of slash and loblolly pine in the state of Santa Catarina, Brazil. Ph.D. diss., Univ. Ga. 219 p.

Mello, A.A. de, C. Carnieri, J.E. Arce, C.R. Sanquetta, and K.S. Weber. 2008. Planejamento do suprimento de matéria-prima em uma indústria florestal utilizando programação em metas e considerando o estoque de carbono. Cerne. 14(4):341-350.
Nakajima, N.Y., S. Yoshida, and M. Imanaga. 1996. Comparison of change estimation between four ground-survey methods for use in a continuous forest inventory system. J. For. Plann. 2: 145-150.
Roesch, F.A., E.J. Green, and C.T. Scott. 1989. New compatible estimators for survivor growth and ingrowth from remeasured horizontal point samples. For. Sci. 35(2): 281-293.
Thérien, G. 2011. Relative efficiency of point sampling change estimators. Math. Comput. For. Nat. -Res. Sci. 3(2):64-72.
Van Deusen, P.C., T.R. Dell, and C.E. Thomas. 1986. Volume growth estimation for permanent horizontal points. For. Sci. 32(2): 415-422.