Performance of Some Tests Compares for Equality of Means in Skewed Distribution Data

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Abstract

Right skewed distributed data are frequently used in many scientific research especially lognormal, Weibull and gamma distributions. It is important to compare different in means between the several groups in skewed data. The purpose of this study was to evaluate some tests applied for ANOVA when data followed a lognormal, Weibull or gamma distribution. In this study, performance of these tests compared and a simulation study performed to compare these tests according to type one error and powers in different combinations of parameters and various sample size.

Key Words: Right skewed distribution, generalized p-value, parametric bootstrap, fiducial based approach

1. Introduction

Classical ANOVA test is conducted to compare means of groups under the following hypothesis:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k = \mu \qquad H_1: \exists\, \mu_i \neq \mu_j,\ 1 \leq i \leq j \leq k \tag{1}$$

where k is the number of the populations with normal distribution, μ_i is the mean of i.th population, and μ is the general mean. Classical one factor analysis of variance model is given as follows;

$$x_{ij} = \mu + \mu_i + \varepsilon_{ij} \qquad i = 1, \dots, k \qquad j = 1, \dots, n_i \tag{2}$$

where ε_{ij} denote the error term.

One of the assumptions to conduct analysis of variance is normality. There are numerous tests in order to conduct analysis of variance in the literature such as Brown- Forsythe test, Welch test, Bootstrap method, generalized *F*-test, fiducial *p*-value etc. However, for most applications the data are not normally distributed. In this study the performance of the tests are examined for lognormal, gamma and Weibull distributed data sets and simulation study results are interpreted.

2. Tests considered

Statistical tests available in the literature can be roughly divided into either exact methods or approximation methods. In this section, we will propose two approximation tests and three exact tests for ANOVA.

2.1 Welch test

Welch (W) test is commonly used by the researchers for unequal variance situations since it is quite practical.

$$W = \frac{\sum_{i=1}^{k} w_i[(\bar{X}_i - \bar{X})^2/(k-1)]}{1 + \frac{2(k-2)}{k^2-1}\sum_{i=1}^{k}\frac{1}{n_i-1}\left(1-\frac{w_i}{\sum w_j}\right)^2} \tag{3}$$

where $w_i = \frac{n_i}{S_i^2}$. The denominator of Equation 3 is given by Equation 4:

$$f = \frac{1}{\frac{3}{k^2-1}\sum_{i=1}^{k}\frac{1}{n_i-1}\left(1-\frac{w_i}{\sum w_j}\right)^2} \tag{4}$$

The W value calculated by Equation 3 has F distribution with $k-1$ and f degrees of freedoms.

If $P(F_{k-1,f} > w) < \alpha$,then the null hypothesis is rejected (Welch 1951).

2.2 Brown-Forsythe test

Another test for unequal variance case is to test null hypothesis of equality of means by Brown and Forsythe (BF) test:

$$B = \frac{\sum_{i=1}^{k} n_i(\bar{X}_i - \bar{X})^2}{\sum_{i=1}^{k}\left(1-\frac{n_i}{n}\right)S_i^2} \tag{5}$$

B statistic has the F distribution with $k-1$ and v degrees of freedoms where v is:

$$v = \frac{\left[\sum_{i=1}^{k}\left(1-\frac{n_i}{n}\right)S_i^2\right]^2}{\sum_{i=1}^{k}\frac{\left(1-\frac{n_i}{n}\right)^2 S_i^4}{n_i-1}} \tag{6}$$

If $P(F_{k-1,v} > b) < \alpha$ holds then the null hypothesis is rejected (Brown-Forsythe 1974).

2.3 Weerahandi's generalized *F* -test

While the right tail region is used to analyze the test statistic in classical F test, right tail sample space region is used for Generalized F (GF) test approach. Instead of $S_i^2 = \frac{1}{n_i-1}\sum_{i=1}^{n_i}\left(X_{ij}-\bar{X}_i\right)^2$Weerahandi proposed the use of $S_i^2 = \frac{1}{n_i}\sum_{i=1}^{n_i}\left(X_{ij}-\bar{X}_i\right)^2$and test statistics β_j is defined as the function of $\frac{n_i S_i^2}{\sigma_i^2}$ statistic as given in Equation 7

$$B_j = \frac{\left(\sum_{i=1}^{j} \frac{n_i S_i^2}{\sigma_i^2}\right)}{\sum_{i=1}^{j+1} \frac{n_i S_i^2}{\sigma_i^2}} \qquad j = 1, \dots, k-1 \tag{7}$$

Hence β_j has the Beta distribution;

$$B_j \sim beta\left[\sum_{i}^{j} \frac{(n_i - 1)}{2}, \frac{(n_{j+1} - 1)}{2}\right]$$

The random variable's value for i^{th} sample $\frac{n_i S_i^2}{\sigma_i^2}$ is obtained as using $\tilde{S}_e$ and β_j in Equation 8

$$\frac{n_i S_i^2}{\sigma_i^2} = \tilde{S}_e (1 - B_{i-1}) B_2 \dots B_{k-1}, i = 2, \dots, k-1 \tag{8}$$

The discrimination for $\frac{n_i S_i^2}{\sigma_i^2}$ is not dependent to any unknown parameter. So it is not affected by the acceptance of the

$$1 - E\left(H_{k-1,n-k}\left\{\frac{n-k}{k-1}\tilde{s}_b\left[\frac{n_1 s_1^2}{B_1 B_2 \dots B_{k-1}}, \dots, \frac{n_k s_k^2}{(1 - B_{k-1})}\right]\right\}\right) \tag{9}$$

Here the obtained function is the cumulative distribution function of F distribution with $k-1$ and $n-k$ degrees of freedoms. Generalized p-value is the expected value of that distribution. if $p < \alpha$ then the null hypothesis is rejected. (Weerahandi 1995).

2.4 The parametric bootstrap test

The parametric bootstrap (PB) involves sampling from the estimated models. That is, samples or sample statistics are generated from parametric models with the parameters replaced by their estimates. Recall that under $H_0: \mu_1 = \dots = \mu_k$ all X_i's have the same mean. As the test statistic T_N in (4) is location invariant, without loss of generality, we can take this common mean to be zero. Using these facts, the parametric bootstrap *pivot variable* can be developed as follows.

Let $\bar{X}_{Bi} \sim N\left(0, \frac{S_i^2}{n_i}\right)$ and $S_{Bi}^2 \sim \frac{\chi_{n_i - 1}^2}{(n_i - 1)}$, $i = 1, \dots, k$. Then the PB pivot variable based on the test statistic is given by

$$\tilde{S}_b = \tilde{S}_b(\sigma_1^2, \dots, \sigma_k^2) = \sum_{i=1}^{k} \frac{n_i \bar{X}_i^2}{S_i^2} - \frac{\left(\sum_{i=1}^{k} \frac{n_i \bar{X}_i^2}{S_i^2}\right)^2}{\sum_{i=1}^{k} \frac{n_i}{S_i^2}} \tag{10}$$

Noticing the fact that X_{Bi} is distributed as $Z_i\left(S_i / \sqrt{n_i}\right)$, where Z_i is a standard normal random variable, it can be easily verified that the PB pivot variable in (10) is distributed as

$$\tilde{S}_{bB}\left(Z_i, \chi^2_{n_i-1}; S_i^2\right) = \sum_{i=1}^{k} \frac{Z_i^2(n_i-1)}{\chi^2_{n_i-1}} - \frac{\left[\sum_{i=1}^{k} \frac{\sqrt{n_i} Z_i (n_i-1)}{S_i^2 \chi^2_{n_i-1}}\right]^2}{\sum_{i=1}^{k} \frac{n_i(n_i-1)}{S_i^2 \chi^2_{n_i-1}}} \quad (11)$$

For a given $\left(s_1^2, \ldots, s_k^2\right)$ of $\left(S_1^2, \ldots, S_k^2\right)$ and level α, the PB test rejects H_0 in (1) when

$$P\left\{\tilde{S}_{bB}\left(Z_i, \chi^2_{n_i-1}; s_i^2\right) > \tilde{s}_b\right\} \quad (12)$$

where T_{No} is an observed value of T_N (Krishnamoorthy, Lu ve Mathew 2006).

2.5 Fiducial approach based on generalized *p*- value

Li, Wang and Liang (2011) developed new test for (1) by using the concept of fiducial and generalized *p*-value approach (FG).

Let $U_{1i} \sim N(0,1)$, $U_{2i} \sim \chi^2_{n_i-1}$, $i = 1,2, \ldots, k$ and be mutually independent. Note that $\bar{X}_i \sim N\left(\mu_i, \frac{\sigma_i^2}{n_i}\right)$, $(n_i - 1)\, S_i^2 \sim \chi^2_{n_i-1} \sigma_i^2$ for $i = 1,2, \ldots, k$ and these statistics are all mutually independent. We therefore can express $\bar{X}_i$ and $(n_i - 1)S_i^2$ as functions of U_{1i} and U_{2i}; i.e.,

$\bar{X}_i = \mu_i + \frac{\sigma_i^2}{n_i} U_{1i}$ and $(n_i - 1)S_i^2 = \sigma_i^2 U_{2i}$, $i = 1,2, \ldots, k$

Given an observation $\left(\bar{x}_i, s_i^2\right)$ and (u_{1i}, u_{2i}), the equations of $\bar{x}_i = \mu_i + \frac{\sigma_i^2}{n_i} u_{1i}$ and $(n_i - 1)s_i^2 = \sigma_i^2 u_{2i}$ have the unique solutions

$$\mu_i = \bar{x}_i - \frac{u_{1i}}{\sqrt{u_{2i}/(n_i-1)}} \sqrt{\frac{s_i^2}{n_i}} \; and \; \sigma_i^2 = \frac{(n_i-1)s_i^2}{u_{2i}}$$

hence for given $\left(\bar{x}_i, s_i^2\right)$, the fiducial distribution of μ_i is the same as that of

$$T_{\mu_i} = \bar{x}_i - t_i \sqrt{s_i^2/n_i} \quad i = 1,2, \ldots, k$$

here $t_i \sim t(n_i - 1)$, $i = 1,2, \ldots, k$ and they are mutually independent. Then the fiducial distribution could be derived by

$$T_F(t; s^2) = \sum_{i=1}^{k} t_i^2 - \frac{\left(\sum_{i=1}^{k} \frac{\sqrt{n_i}}{s_i} t_i\right)^2}{\sum_{i=1}^{k} \frac{n_i}{s_i^2}} \quad (13)$$

Here $t = (t_1, \ldots, t_k)$. Because $T(x; s^2)$ is the observed value of T_F under the null hypothesis, the *p*-value for (1) is given by

$$p = Pr\{T_F \geq T(x; s^2)\}$$

Accordingly, we reject the null hypothesis when $p < \alpha$ for a given level α (Li, Wang and Liang 2011).

3. Simulation studies

In this section, the type I error rates and powers of the tests are estimated using Monte Carlo simulation. To estimate the Type I error rates of the Welch test and BF test, we used simulation consisting of 100,000 runs for each of the sample size and parameter configurations. That is, for a given $(\bar{x}_1, \ldots, \bar{x}_k; s_1^2, \ldots, s_k^2)$, we generated 100,000 W's given in (3), and estimated the Type I error rates by the proportion of times W exceeded $F_{k-1,f,\alpha}$, where $F_{a,b,\alpha}$ denotes the upper αth quantile of an F distribution with degrees of freedoms a and b. The type I error rates of the BF tests are similarly estimated.

For comparison between the powers of the tests each combination of n_i and σ_i^2 the rejection rate of each testing procedure is calculated and compared with the nominal level 0.05 when the means are not all equal. In this section we use 50000 runs for each of the sample sizes and parameter configurations to calculate the powers of the W, BF, GF, PB and FG tests. For k=3 and k=5 we provide the powers of these tests. In both cases of equal and unequal variances for k=3 and k=5 simulated type I error probabilities and power are given in Tables 1, 2, 3, 4, 5 and 6.

When $k = 3$ and data followed lognormal distribution the W, GF and FG have similar Type I error rates but the BF and PB tests seem to be very conservative for small samples. When $k = 5$ and data followed lognormal distribution the BF, FG and W tests exceed the nominal level but the GF and PB tests control the Type I errors satisfactorily.

When the sample is Gamma distributed, for $k = 3$ and for the small samples, while the BF, W, FG and PB tests' type I errors are below the nominal levels (conservative), for the different sample sizes the W and BP tests' type I errors are quite above the nominal levels. For $k = 5$ all the type I errors are below the nominal level and for the different sample sizes the W, BF and FG tests' type I errors are above the nominal level.

For the Weibull distributed data, for $k = 3$ and for the small samples, type I error rates of the BF and PB are below the nominal level (conservative), the results of GF type I error rates are satisfactory for both small and large sample sizes. For small samples and $k = 5$ type I error rates of the W test are quite above the nominal level. Also type I error rates of the GF are satisfactory for both small and big sample sizes.

Table 1a: Type I error rates of the proposed for k=3 and $\alpha = 0.05$ when data followed lognormal distribution.

m_i	σ_i^2	n_i	BF	W	GF	PB	FG
(0,0,0)	(0.3,0.3,0.3)	(10,10,10)	0.038	0.049	0.047	0.038	0.047
		(50,50,50)	0.048	0.051	0.049	0.051	0.048
		(100,100,100)	0.049	0.050	0.049	0.051	0.049
		(100,150,200)	0.050	0.052	0.050	0.051	0.052
		(250,250,250)	0.050	0.050	0.050	0.050	0.051
		(200,250,300)	0.050	0.050	0.050	0.051	0.051
		(500,500,500)	0.050	0.050	0.050	0.050	0.050
(0,0,0)	(0.5,0.5,0.5)	(10,10,10)	0.033	0.047	0.044	0.042	0.045
		(50,50,50)	0.047	0.052	0.048	0.049	0.048
		(100,100,100)	0.047	0.051	0.049	0.049	0.048
		(100,150,200)	0.048	0.053	0.050	0.052	0.053
		(250,250,250)	0.050	0.050	0.050	0.050	0.052
		(200,250,300)	0.050	0.050	0.050	0.050	0.051
		(500,500,500)	0.050	0.050	0.050	0.050	0.050
(0,0,0)	(1,1,1)	(10,10,10)	0.025	0.039	0.038	0.036	0.034
		(50,50,50)	0.042	0.051	0.043	0.052	0.051
		(100,100,100)	0.044	0.051	0.045	0.052	0.051
		(100,150,200)	0.047	0.055	0.046	0.053	0.053
		(250,250,250)	0.048	0.051	0.048	0.051	0.051
		(200,250,300)	0.048	0.051	0.049	0.051	0.051
		(500,500,500)	0.049	0.050	0.050	0.050	0.050

Table 1b: Type I error rates of the proposed for k=4 and $\alpha = 0.05$ when data followed lognormal distribution.

m_i	σ_i^2	n_i	BF	W	GF	PB	FG
(0,0,0,0)	(0.3,0.3,0.3,0.3)	(10,10,10,10)	0.061	0.060	0.045	0.051	0.055
		(50,50,50,50)	0.055	0.056	0.048	0.049	0.053
		(100,100,100,100)	0.052	0.052	0.049	0.051	0.052
		(100,150,200,250)	0.051	0.053	0.050	0.052	0.052
		(250,250,250,250)	0.051	0.051	0.050	0.049	0.051
		(200,250,300,350)	0.051	0.051	0.050	0.049	0.051
		(500,500,500,500)	0.050	0.050	0.050	0.050	0.050
(0,0,0,0)	(0.5,0.5,0.5,0.5)	(10,10,10,10)	0.067	0.059	0.042	0.041	0.035
		(50,50,50,50)	0.058	0.057	0.045	0.052	0.040
		(100,100,100,100)	0.053	0.055	0.048	0.052	0.045
		(100,150,200,250)	0.051	0.055	0.049	0.053	0.052
		(250,250,250,250)	0.050	0.053	0.049	0.051	0.052
		(200,250,300,350)	0.051	0.053	0.050	0.050	0.051
		(500,500,500,500)	0.050	0.052	0.050	0.050	0.051
(0,0,0,0)	(1,1,1,1)	(10,10,10,10)	0.079	0.055	0.034	0.042	0.030
		(50,50,50,50)	0.069	0.061	0.041	0.060	0.032
		(100,100,100,100)	0.063	0.059	0.044	0.056	0.042
		(100,150,200,250)	0.056	0.063	0.046	0.056	0.056
		(250,250,250,250)	0.057	0.055	0.047	0.055	0.054
		(200,250,300,350)	0.056	0.053	0.046	0.054	0.053
		(500,500,500,500)	0.055	0.052	0.048	0.052	0.052

Table 2a: Powers of the proposed for k=3 and $\alpha = 0.05$ when data followed lognormal distribution.

m_i	σ_i^2	n_i	BF	W	GF	PB	FG
(0.2,0.5,0.3)	(0.5,0.3,0.8)	(10,10,10)	0.06	0.10	0.14	0.10	0.11
		(50,50,50)	0.22	0.30	0.31	0.30	0.30
		(100,100,100)	0.44	0.52	0.53	0.51	0.52
		(100,150,200)	0.57	0.61	0.62	0.61	0.60
		(200,250,300)	0.83	0.85	0.86	0.85	0.86
		(250,250,250)	0.86	0.89	0.89	0.88	0.88
		(500,500,500)	0.99	0.99	0.99	0.99	0.99
(0.5,0.3,0.2)	(0.8,0.5,0.3)	(10,10,10)	0.10	0.11	0.16	0.09	0.11
		(50,50,50)	0.76	0.75	0.75	0.75	0.75
		(100,100,100)	0.98	0.98	0.98	0.98	0.98
		(100,150,200)	0.99	0.99	0.99	0.99	0.99
		(200,250,300)	0.99	1.00	0.99	1.00	1.00
		(250,250,250)	0.99	1.00	0.99	0.99	1.00
		(500,500,500)	1.00	1.00	1.00	1.00	1.00
(1,2,3)	(0.5,1,1.5)	(10,10,10)	0.59	0.82	0.80	0.79	0.83
		(50,50,50)	0.99	0.99	0.99	1.00	1.00
		(100,100,100)	1.00	1.00	1.00	1.00	1.00
		(100,150,200)	1.00	1.00	1.00	1.00	1.00
		(200,250,300)	1.00	1.00	1.00	1.00	1.00
		(250,250,250)	1.00	1.00	1.00	1.00	1.00
		(500,500,500)	1.00	1.00	1.00	1.00	1.00

Table 2b: Powers of the proposed for k=4 and $\alpha = 0.05$ when data followed lognormal distribution.

m_i	σ_i^2	n_i	BF	W	GF	BP	FG
(0.2,0.5,0.3,0.2)	(0.5,0.3,0.8,0.3)	(10,10,10,10)	0.07	0.15	0.11	0.13	0.15
		(50,50,50,50)	0.32	0.65	0.30	0.63	0.65
		(100,100,100,100)	0.68	0.94	0.65	0.94	0.93
		(100,150,200,250)	0.98	0.99	0.99	0.98	1.00
		(250,250,250,250)	0.99	0.99	0.99	0.99	1.00
		(200,250,300,350)	1.00	1.00	1.00	1.00	1.00
		(500,500,500,500)	1.00	1.00	1.00	1.00	1.00
(0.5,0.3,0.2,0.3)	(0.8,0.5,0.3,0.8)	(10,10,10,10)	0.22	0.15	0.42	0.10	0.12
		(50,50,50,50)	0.84	0.65	0.87	0.75	0.77
		(100,100,100,100)	0.99	0.94	0.99	0.99	0.98
		(100,150,200,250)	0.99	0.99	1.00	1.00	1.00
		(250,250,250,250)	1.00	0.99	1.00	1.00	1.00
		(200,250,300,350)	1.00	1.00	1.00	1.00	1.00
		(500,500,500,500)	1.00	1.00	1.00	1.00	1.00
(1,2,3,4)	(0.5,1,1.5,2)	(10,10,10,10)	0.91	0.95	0.94	0.92	0.95
		(50,50,50,50)	0.99	1.00	1.00	1.00	1.00
		(100,100,100,100)	1.00	1.00	1.00	1.00	1.00
		(100,150,200,250)	1.00	1.00	1.00	1.00	1.00
		(250,250,250,250)	1.00	1.00	1.00	1.00	1.00
		(200,250,300,350)	1.00	1.00	1.00	1.00	1.00
		(500,500,500,500)	1.00	1.00	1.00	1.00	1.00

Table 3a: Type I error rates of the proposed for k=3 and $\alpha = 0.05$ when data followed gamma distribution.

m_i	σ_i^2	γ_i (shape)	λ_i (scale)	n_i	BF	W	GF	BP	FG
(0.2,0.2,0.2)	(0.3,0.3, 0.3)	(0.13,0.13, 0.13)	(1.49,1.49, 1.49)	(10,10,10)	0.011	0.018	0.055	0.014	0.030
				(50,50,50)	0.036	0.057	0.054	0.055	0.062
				(100,100,100)	0.043	0.057	0.052	0.053	0.060
				(100,150,200)	0.047	0.062	0.052	0.057	0.058
				(250,250,250)	0.047	0.053	0.051	0.051	0.053
				(200,250,300)	0.048	0.055	0.051	0.052	0.052
				(500,500,500)	0.050	0.051	0.051	0.050	0.050
(0.2,0.2,0.2)	(0.5,0.5, 0.5)	(0.08,0.08, 0.08)	(2.5,2.5, 2.5)	(10,10,10)	0.007	0.007	0.051	0.040	0.030
				(50,50,50)	0.031	0.054	0.062	0.058	0.058
				(100,100,100)	0.039	0.057	0.059	0.057	0.056
				(100,150,200)	0.045	0.066	0.058	0.065	0.055
				(250,250,250)	0.047	0.055	0.057	0.057	0.055
				(200,250,300)	0.048	0.059	0.059	0.057	0.054
				(500,500,500)	0.049	0.053	0.053	0.051	0.051
(1,1,1)	(0.5,0.5, 0.5)	(2,2,2)	(0.5,0.5, 0.5)	(10,10,10)	0.040	0.047	0.055	0.047	0.045
				(50,50,50)	0.049	0.048	0.053	0.051	0.048
				(100,100,100)	0.050	0.050	0.052	0.051	0.049
				(100,150,200)	0.050	0.051	0.052	0.047	0.048
				(250,250,250)	0.050	0.050	0.051	0.049	0.050
				(200,250,300)	0.050	0.050	0.051	0.052	0.051
				(500,500,500)	0.050	0.050	0.050	0.050	0.050

Table 3b: Type I error rates of the proposed for k=4 and $\alpha = 0.05$ when data followed gamma distribution

m_i	σ_i^2	γ_i (shape)	λ_i (scale)	n_i	BF	W	GF	BP	FG
(0.2,0.2, 0.2,0.2)	(0.3,0.3, 0.3,0.3)	(0.13,0.13, .13,0.13)	(1.49,1.49, .49,1.49)	(10,10,10,10)	0.012	0.031	0.026	0.016	0.030
				(50,50,50,50)	0.038	0.076	0.041	0.071	0.071
				(100,100,100,100)	0.044	0.070	0.045	0.068	0.066
				(100,150,200,250)	0.047	0.071	0.045	0.072	0.073
				(250,250,250,250)	0.048	0.059	0.048	0.055	0.053
				(200,250,300,350)	0.047	0.062	0.048	0.056	0.053
				(500,500,500,500)	0.049	0.054	0.049	0.053	0.052
(0.2,0.2,0.2,0.2)	(0.5,0.5, 0.5,0.5)	(0.08,0.08, .08,0.08)	(2.5,2.5, 2.5,2.5)	(10,10,10,10)	0.008	0.011	0.016	0.009	0.012
				(50,50,50,50)	0.035	0.079	0.035	0.075	0.078
				(100,100,100,100)	0.040	0.075	0.040	0.072	0.075
				(100,150,200,250)	0.046	0.084	0.043	0.067	0.073
				(250,250,250,250)	0.045	0.065	0.045	0.059	0.057
				(200,250,300,350)	0.047	0.068	0.046	0.060	0.055
				(500,500,500,500)	0.048	0.059	0.048	0.054	0.053
(1,1,1,1)	(0.5,0.5, 0.5,0.5)	(2,2, 2,2)	(0.5,0.5, 0.5,0.5)	(10,10,10,10)	0.041	0.064	0.055	0.057	0.057
				(50,50,50,50)	0.048	0.055	0.051	0.052	0.052
				(100,100,100,100)	0.049	0.053	0.050	0.048	0.051
				(100,150,200,250)	0.050	0.053	0.051	0.055	0.049
				(250,250,250,250)	0.049	0.050	0.050	0.053	0.050
				(200,250,300,350)	0.050	0.051	0.050	0.052	0.051
				(500,500,500,500)	0.050	0.050	0.050	0.050	0.050

Table 4a: Powers of the proposed for k=3 and $\alpha = 0.05$ when data followed gamma distribution**.**

m_i	σ_i^2	γ_i (shape)	λ_i (scale)	n_i	BF	W	GF	BT	FG
(0.2,0.5, 0.3)	(0.5,0.3, 0.8)	(0.08,0.83, 0.11)	(2.5,0.6, 2.67)	(10,10,10)	0.26	0.42	0.46	0.40	0.44
				(50,50,50)	0.54	0.72	0.72	0.73	0.72
				(100,100,100)	0.78	0.89	0.88	0.88	0.88
				(100,150,200)	0.86	0.94	0.93	0.93	0.93
				(250,250,250)	0.98	0.99	0.99	0.99	1.00
				(200,250,300)	0.99	0.99	0.99	1.00	0.99
				(500,500,500)	1.00	1.00	1.00	1.00	1.00
(0.5,0.3, 0.2)	(0.8,0.5, 0.3)	(0.31,0.18, 0.13)	(1.6,0.67, 1.49)	(10,10,10)	0.07	0.10	0.17	0.08	0.10
				(50,50,50)	0.45	0.48	0.49	0.46	0.51
				(100,100,100)	0.77	0.77	0.77	0.78	0.78
				(100,150,200)	0.87	0.86	0.86	0.85	0.85
				(250,250,250)	0.99	0.99	0.99	0.99	0.99
				(200,250,300)	0.99	0.99	0.99	0.99	0.99
				(500,500,500)	1.00	1.00	1.00	1.00	1.00
(0.2,0.3, 0.5)	(0.8,0.5, 0.3)	(0.05,0.18, 0.83)	(0.25,0.6, 1.67)	(10,10,10)	0.28	0.47	0.51	0.44	0.47
				(50,50,50)	0.57	0.70	0.70	0.70	0.69
				(100,100,100)	0.74	0.85	0.82	0.87	0.85
				(100,150,200)	0.84	0.93	0.85	0.93	0.93
				(250,250,250)	0.97	0.99	0.99	0.99	0.99
				(200,250,300)	0.97	0.99	0.99	0.99	0.99
				(500,500,500)	1.00	1.00	1.00	1.00	1.00

Table 4b: Powers of the proposed for k=4 and $\alpha = 0.05$ when data followed gamma distribution**.**

m_i	σ_i^2	γ_i (shape)	λ_i (scale)	n_i	BF	W	GF	BP	FG
(0.2,0.5, 0.3,0.2)	(0.5,0.3,0.8,0.3)	(0.08,0.83,0.11,0.13)	(2.5,0.83, 0.11,0.13)	(10,10,10,10)	0.22	0.45	0.52	0.40	0.44
				(50,50,50,50)	0.60	0.85	0.85	0.84	0.86
				(100,100,100,100)	0.87	0.98	0.97	0.98	0.97
				(100,150,200,250)	0.98	1.00	1.00	1.00	1.00
				(250,250,250,250)	0.99	1.00	1.00	1.00	1.00
				(200,250,300,350)	1.00	1.00	1.00	1.00	1.00
				(500,500,500,500)	1.00	1.00	1.00	1.00	1.00
(0.5,0.3, 0.2,0.3)	(0.8,0.5,0.3,0.8)	(0.31,0.18,0.13,0.11)	(0.31,0.18, 0.13,0.11)	(10,10,10,10)	0.06	0.45	0.21	0.08	0.10
				(50,50,50,50)	0.37	0.85	0.46	0.44	0.45
				(100,100,100,100)	0.66	0.97	0.73	0.72	0.73
				(100,150,200,250)	0.78	0.81	0.80	0.81	0.80
				(250,250,250,250)	0.98	1.00	0.99	0.99	0.99
				(200,250,300,350)	0.97	0.98	0.98	0.98	0.98
				(500,500,500,500)	1.00	1.00	1.00	1.00	1.00

(0.2,0.3, 0.5,0.2)	(0.8,0.5,0.3,0.8)	(0.05,0.18,0.83,0.05)	(0.25,0.6, 1.67)	(10,10,10,10)	0.26	0.58	0.62	0.53	0.58
				(50,50,50,50)	0.55	0.81	0.82	0.81	0.82
				(100,100,100,100)	0.76	0.93	0.93	0.93	0.94
				(100,150,200,250)	0.93	0.99	0.99	0.99	0.99
				(250,250,250,250)	0.98	1.00	1.00	1.00	1.00
				(200,250,300,350)	0.99	1.00	1.00	1.00	1.00
				(500,500,500,500)	1.00	1.00	1.00	1.00	1.00

Table 5a: Type I error rates of the proposed for k=3 and $\alpha = 0.05$ when data followed Weibull distribution.

a_i (shape)	b_i (scale)	n_i	BF	W	GF	BP	FG
(1.75,1.75,1.75)	(1.50,1.50,1.50)	(10,10,10)	0.046	0.054	0.052	0.048	0.039
		(50,50,50)	0.049	0.052	0.051	0.049	0.053
		(100,100,100)	0.049	0.051	0.049	0.051	0.051
		(100,150,200)	0.049	0.051	0.050	0.050	0.050
		(250,250,250)	0.050	0.050	0.050	0.049	0.050
		(200,250,300)	0.050	0.051	0.050	0.050	0.050
		(500,500,500)	0.050	0.050	0.050	0.050	0.050
(1.25,1.25,1.25)	(1,1,1)	(10,10,10)	0.041	0.053	0.050	0.047	0.046
		(50,50,50)	0.047	0.053	0.050	0.052	0.050
		(100,100,100)	0.049	0.052	0.050	0.051	0.050
		(100,150,200)	0.049	0.051	0.050	0.051	0.050
		(250,250,250)	0.050	0.050	0.050	0.050	0.050
		(200,250,300)	0.050	0.051	0.050	0.050	0.050
		(500,500,500)	0.050	0.050	0.050	0.050	0.050
(1.75,1.75,1.75)	(1,1,1)	(10,10,10)	0.046	0.055	0.051	0.048	0.046
		(50,50,50)	0.048	0.051	0.051	0.049	0.052
		(100,100,100)	0.049	0.050	0.051	0.049	0.049
		(100,150,200)	0.049	0.051	0.051	0.050	0.050
		(250,250,250)	0.050	0.051	0.050	0.050	0.050
		(200,250,300)	0.050	0.050	0.050	0.050	0.050
		(500,500,500)	0.050	0.050	0.050	0.050	0.050

Table 5b: Type I error rates of the proposed for k=4 and $\alpha = 0.05$ when data followed Weibull distribution.

a_i (shape)	b_i (scale)	n_i	BF	W	GF	BP	FG
(1.75,1.75,1.75,1.75)	(1.5,1.5,1.5,1.5)	(10,10,10,10)	0.046	0.061	0.050	0.048	0.049
		(50,50,50,50)	0.050	0.052	0.049	0.049	0.054
		(100,100,100,100)	0.050	0.052	0.051	0.051	0.053
		(100,150,200,250)	0.050	0.052	0.050	0.049	0.051
		(250,250,250,250)	0.050	0.051	0.050	0.050	0.050
		(200,250,300,350)	0.050	0.050	0.050	0.050	0.050
		(500,500,500,500)	0.050	0.050	0.050	0.050	0.050

(1.25,1.25,1.25,1.25)	(1,1,1,1)	(10,10,10,10)	0.040	0.065	0.048	0.048	0.047
		(50,50,50,50)	0.048	0.055	0.049	0.048	0.048
		(100,100,100,100)	0.049	0.053	0.049	0.052	0.052
		(100,150,200,250)	0.050	0.052	0.050	0.051	0.051
		(250,250,250,250)	0.050	0.052	0.050	0.050	0.050
		(200,250,300,350)	0.050	0.050	0.050	0.050	0.050
		(500,500,500,500)	0.050	0.050	0.050	0.050	0.050
(1.75,1.75,1.75,1.75)	(1,1,1,1)	(10,10,10,10)	0.045	0.062	0.051	0.047	0.051
		(50,50,50,50)	0.049	0.055	0.050	0.052	0.049
		(100,100,100,100)	0.051	0.052	0.050	0.051	0.049
		(100,150,200,250)	0.050	0.051	0.050	0.050	0.050
		(250,250,250,250)	0.050	0.050	0.050	0.050	0.050
		(200,250,300,350)	0.050	0.050	0.050	0.050	0.050
		(500,500,500,500)	0.050	0.050	0.050	0.050	0.050

Table 6a: Powers of the proposed for k=3 and $\alpha = 0.05$ when data followed Weibull distribution**.**

a_i (shape)	b_i (scale)	n_i	BF	W	GF	BP	FG
(1.25,1.75,1)	(1.50,2.15,2)	(10,10,10)	0.09	0.15	0.22	0.15	0.07
		(50,50,50)	0.49	0.59	0.61	0.59	0.40
		(100,100,100)	0.84	0.89	0.89	0.89	0.70
		(100,150,200)	0.92	0.94	0.94	0.94	0.88
		(250,250,250)	1.00	1.00	1.00	1.00	0.97
		(200,250,300)	1.00	1.00	1.00	1.00	0.97
		(500,500,500)	1.00	1.00	1.00	1.00	1.00
(1,1.25,1.75)	(2,1.75,1.50)	(10,10,10)	0.07	0.08	0.14	0.07	0.08
		(50,50,50)	0.49	0.53	0.55	0.52	0.28
		(100,100,100)	0.84	0.88	0.88	0.87	0.47
		(100,150,200)	0.93	1.00	0.95	0.95	0.70
		(250,250,250)	1.00	1.00	1.00	1.00	0.81
		(200,250,300)	1.00	1.00	1.00	1.00	0.85
		(500,500,500)	1.00	1.00	1.00	1.00	1.00
(1,1.25,1.50)	(1.75,2,2.50)	(10,10,10)	0.10	0.12	0.18	0.12	0.06
		(50,50,50)	0.31	0.32	0.34	0.35	0.31
		(100,100,100)	0.54	0.57	0.58	0.58	0.57
		(100,150,200)	0.74	0.77	0.77	0.76	0.80
		(250,250,250)	0.92	0.93	0.93	0.93	0.94
		(200,250,300)	0.94	0.94	0.94	0.94	0.95
		(500,500,500)	1.00	1.00	1.00	1.00	1.00

Table 6b: Powers of the proposed for k=4 and $\alpha = 0.05$ when data followed Weibull distribution**.**

a_i (shape)	b_i (scale)	n_i	BF	W	GF	BP	FG
(1.25,1.75,1,1.75)	(1.5,2.15,1.75,1.50)	(10,10,10,10)	0.12	0.19	0.28	0.17	0.12
		(50,50,50,50)	0.63	0.77	0.78	0.76	0.62
		(100,100,100,100)	0.94	0.98	0.98	0.98	0.87
		(100,150,200,250)	0.99	0.99	1.00	1.00	0.98
		(250,250,250,250)	1.00	1.00	1.00	1.00	1.00
		(200,250,300,350)	1.00	1.00	1.00	1.00	1.00
		(500,500,500,500)	1.00	1.00	1.00	1.00	1.00
(1,1.25,1.75,1)	(2,1.75,1.50,2)	(10,10,10,10)	0.07	0.10	0.19	0.08	0.10
		(50,50,50,50)	0.51	0.66	0.68	0.65	0.33
		(100,100,100,100)	0.89	0.96	0.96	0.96	0.53
		(100,150,200,250)	1.00	1.00	1.00	1.00	0.79
		(250,250,250,250)	1.00	1.00	1.00	1.00	0.86
		(200,250,300,350)	1.00	1.00	1.00	1.00	0.89
		(500,500,500,500)	1.00	1.00	1.00	1.00	0.98
(1,1.25,1.75,1)	(1.75,2,2.5,1.5)	(10,10,10,10)	0.13	0.21	0.30	0.22	0.09
		(50,50,50,50)	0.52	0.62	0.63	0.61	0.59
		(100,100,100,100)	0.84	0.89	0.90	0.90	0.88
		(100,150,200,250)	1.00	0.99	1.00	1.00	0.98
		(250,250,250,250)	1.00	1.00	1.00	1.00	1.00
		(200,250,300,350)	1.00	1.00	1.00	1.00	1.00
		(500,500,500,500)	1.00	1.00	1.00	1.00	1.00

4. Conclusion

One of the main assumptions in ANOVA is that samples are selected from the normal distribution. However in real life samples may come from skewed distributions such as lognormal, Weibull, and gamma.

In this study, we evaluated performance of Welch, Brown-Forsyth, generalized *F*- test, the parametric bootstrap test and the method based fiducial *p*-value for the right skewed distributions. Simulation results indicate that performances of tests for ANOVA when the sample size increases (more than 100) type I error is approaching to the nominal level and power takes satisfactory levels.

Acknowledgement

This study is supported by the Research Foundation of Anadolu University. (Project Number: 1202F38).

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