A DYNAMIC CORRELATION ANALYSIS OF CPI SUBCOMPONENTS USING CONTINUOUS WAVELET TRANSFORMS

David J. Doorn[1]

[1]West Chester University of Pennsylvania, 725 S Church Street, West Chester, PA 19383

Abstract

The continuous wavelet transform allows for the estimation of the spectral characteristics of a time series as a function of time. We make use of this to employ cross wavelet transforms and wavelet coherence analysis in investigating commonalities in the time-frequency behavior of the eight subcomponents of the CPI. Cross wavelet transforms allow for assessment of common power structures across series, while coherence analysis is in effect a localized correlation measure that can reveal the strength of co-movements of the series over time and frequency and any changes in that strength. The goal is to provide a foundation for the development of a weighting scheme across the subcomponent series for an alternative measure of core inflation. Rather than dropping some of the subcomponents completely to arrive at a core measure, the goal is to extract the common frequency components of all of the subcomponent series to develop an index based on actual co-movements in all prices over particular time scales. Initial results indicate strong correlations between some subcomponents at certain time scales and frequencies and less for others, along with significant changes in these over time.

Key Words: CPI, core inflation, wavelet, coherence

1. Introduction

Inflation in the general level of prices in the economy is an important concern of economists, as well as of policymakers and the general public. Accurately measuring price inflation in a manner useful for tracking the state of the economy and developing policy reactions to it is a daunting task and a variety of alternative measures have been developed over time in an effort to more closely track trend, or "core", inflation. These measures seek to reduce transitory volatility in prices in order to isolate an underlying signal for generating more useful forecasts for determining potential policy actions to be undertaken. Core measures of price inflation have come in a variety of forms in the literature and have included exclusionary methods, such as leaving the volatile food and energy components out of the Consumer Price Index (CPI) – the most well-known measure of core inflation – as well as a number of other techniques. These methods have included trimmed mean and weighted median measures, as well as measures that make use of variance-weighting the index components or using weights based on regression coefficients.[1]

In this paper we discuss the preliminary work being done in development of another alternative measure that, while similar in context to some of the component smoothing measures that have been discussed in the past,[2] intends to take quite a different approach using wavelet methods. The goal of this research is to provide a foundation for the development of a weighting scheme across the subcomponent series for an alternative measure of core inflation. As the current study developed out of previous work, it is useful to first review and discuss that work before getting into the present strand of research.

2. Background to current study

The current study is a result of previous work the author has done with Jane Binner and Richard Anderson[3] on a wavelet based measure of core inflation, which was in itself an extension of the work done in Anderson *et al* (2007). The goal of that work was to derive an alternative measure of core inflation using wavelet methods as a filtering/denoising mechanism. Our extension was intended to wavelet filter the CPI subcomponent indices and then reconstruct the All Items Index from these filtered subcomponents. In doing this the goal was to develop a core inflation measure that would avoid excluding any of the series that go into constructing the overall CPI, thus extracting the underlying signal retaining all of the sub-index components. While working on this, we became aware of two papers making use of wavelet filtering in their own attempts at developing alternative

[1] See Detmeister (2011) and Detmeister (2012) for good discussion of and comparison of the effectiveness for forecasting of many different core inflation measures. See also Dolmas and Wynne (2008) for discussion of a joint Federal Reserve Bank of Dallas Federal Reserve Bank of Cleveland research conference entitled "Price Measurement for Monetary Policy", as well as references to the papers presented at the conference.
[2] Ibid.
[3] Jane Binner is currently Chair of Finance in the Department of Accounting and Finance at the University of Birmingham and Richard G. Anderson is recently retired from his position as Vice President and Economist at the Federal Reserve Bank of St. Louis.

core inflation measures. These were Dowd *et al* (2011) and Baqaee (2010). As these papers used wavelet methods in filtering only the overall price indexes they studied,[4] we were undeterred in our efforts to filter the CPI subcomponents in an effort to improve upon the results given in the first of these papers. Before considering our preliminary results in this phase of the study, it is worth taking a detour to briefly discuss wavelet methods in general.

2.1. Wavelet Analysis

This discussion follows closely the development in Gencay et al. (2002), chapter 4. The familiar Fourier analysis allows the decomposition of a time series into its component frequencies using sine and cosine as basis functions. Wavelet analysis is somewhat analogous to this, but expands the set of basis functions that are permissible. In addition, wavelet analysis allows for decomposition of a time series across both frequency and scale, unlike the Fourier transform which only generates frequency components. In this way, we are allowed a much richer view of a time series' dynamics than is obtainable using Fourier methods.

A wavelet, $\psi(t)$, is a function of time that satisfies the admissibility condition

$$C_\psi = \int_0^\infty \frac{\Psi(f)}{f} df < \infty \text{ and } \int_{-\infty}^\infty \psi(t) dt = 0$$

with $\Psi(f)$ being the Fourier transform of $\psi(t)$. A wavelet is also required to have unit energy

$$\int_{-\infty}^\infty \left|\psi(t)dt\right|^2 = 1.$$

This means that a wavelet must have non-zero entries, but these cancel each other out over the length of the function**. Figure 1** gives an example of two wavelet basis functions from the same family, known as the "least asymmetric" (LA) family of wavelets.[5] The wavelets shown here are the LA(4) and LA(8) versions, with the number indicating the approximate length in time (support) of the non-zero elements of the basis functions. Note again that these elements must average to zero for all wavelets.

[4] While the first of these papers sought to develop a core inflation measure for the U.S. based on the CPI, the latter paper focused on New Zealand price level measures in its efforts.

[5] This family is also known as the "symlet" family of wavelet basis functions, although they are clearly not perfectly symmetric.

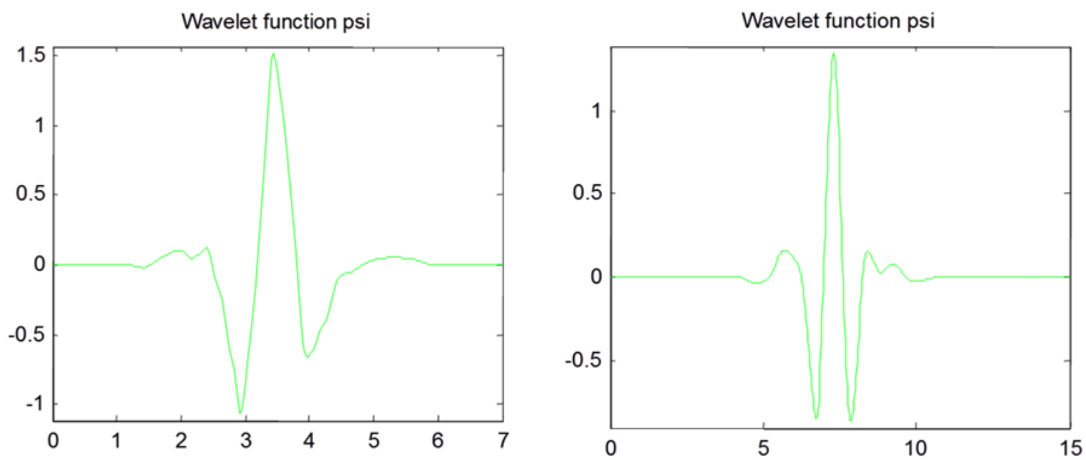

Figure 1: Two Wavelet Functions from the Least Assymetric Family, LA(4) and LA(8)

2.1.1 Wavelet Transforms

A continuous wavelet transform (CWT) of a time series $x(t)$ is a projection of that series onto a wavelet function

$$W(u,s)=\int_{-\infty}^{\infty} x(t)\psi_{u,s}(t)dt$$

with $\psi_{u,s}(t)=s^{-\frac{1}{2}}\cdot\psi\left(\frac{t-u}{s}\right)$ denoting a wavelet basis function, such as the LA(8), that is stretched or compressed by the dilation parameter s and shifted over t = 1, ..., T by the shift parameter u. The projection then generates wavelet coefficients that are functions of s, representing the scale of the analysis, and u, representing location. A large wavelet coefficient indicates high correlation between the shape of the wavelet function and the time series at that scale of analysis and particular point in time. The CWT, then, provides a complete representation of these coefficients for all values of u and s. This is called a *decomposition* or *analysis* of the series. The series can then be *reconstructed* or *synthesized* from the wavelet coefficients by using an inverse transform.

Because the CWT is continuous and retains information on the parameters and coefficients at every scale and location, there is a large amount of information generated from its application. As it turns out, we can downsample the wavelet coefficients from the CWT by using a *discrete wavelet transform* (DWT). This collects only the minimum number of coefficients necessary to exactly reconstruct the initial series, or signal. This number will vary by scale of analysis with a *critical sampling* requiring $s=2^{-j}$ and $u=k\cdot 2^{-j}$, where j and k index the dilations and translations used in the analysis.

The DWT can be expressed in matrix form as an application of the filter and using filtering terminology from signal processing. Let $\mathbf{x}$ be a vector of length $T=2^J$ (dyadic length). Then we can apply the wavelet filter W to obtain $\omega=W\mathbf{x}$, where $W\mathbf{x}$ is the convolution of the matrix DWT filter, *W*, and the vector time series.[6] The resulting ω is the matrix of wavelet coefficients at each scale $j = 1, ..., J$, that is

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \ldots & \omega_J & \nu_J \end{bmatrix}^T$$

Each ω_j is thus a vector of wavelet coefficients associated with scales of length $\lambda_j = 2^{j-1}$ [7] and ν_j is a vector of scaling coefficients which just gives the averages over scale length 2^J, the length of the series.

2.1.2 Multi-Resolution Analysis

A *multi-resolution analysis* (MRA) is an additive decomposition of a time series using the DWT to generate the coefficients for different values of the parameters *s* and *u* from a critical sampling decomposition across time and scale. The wavelet coefficients are then used to reconstruct the elements of the signal at each of the different scales of analysis. This can be represented

$$x_t = \sum_{j=1}^{J+1} d_{j,t} \qquad t = 0, ..., T-1$$

where d_j is known as the wavelet detail at scale j and J represents the maximum scale available given the particular time series being analyzed. Note that d_{J+1} is the detail associated with ν_J in the matrix of wavelet coefficients above and is just equal to the sample mean of **x**. This means that each observation x_t is a linear combination of wavelet detail coefficients across all scales of analysis.

Table 1: Monthly Frequency Resolution by Detail Level

Level (j)	Monthly Frequency Resolution
1	1 - 2
2	2 - 4
3	4 - 8
4	8 - 16
5	16 - 32
6	32 - 64

It is not necessary to do a complete decomposition across all scales. A partial wavelet transform can be obtained up to scale J_P, s.t. $J_P < J$, which just decreases the number of wavelet coefficients in the matrix above and increases the number of scaling coefficients, ν_{J_P}. This idea extends to the MRA as well. We can choose to reconstruct only the details we are interested in, leaving something known as a wavelet *smooth*, or *approximation*, to represent the rest. This is represented as

[6] The details of this convolution and the pyramid scheme used to implement it are beyond the scope of this article. For references see Gencay et al. (2002), Percival and Walden (2000), and Mallat (1989).

[7] This means that for j = 1, the scale is of length $2^0 = 1$ and when j = 2, the scale is of length $2^1 = 2$, and so on ...

$$s_j = \sum_{k=j+1}^{J+1} d_k$$

so while d_j represents the variability of a time series at each scale λ_j, s_j is an accumulation over the remaining scales that becomes smoother as $j \to J$. This leads also to the definition of a wavelet *rough*,

$$r_j = \sum_{k=1}^{j} d_k$$

which sums the lower scale, or high-frequency, details of the time series. Thus we can represent **x** as the decomposition $\mathbf{x} = s_j + r_j$, although the typical MRA gives a number of the low scale details in addition to the wavelet smooth. In fact, in the example that follows we will report a partial wavelet transform of the series of interest up to level J = 6. Instead of the wavelet rough, we report the details at each scale, along with the wavelet smooth s_6. In other words, our MRA will be the decomposition

$$\mathbf{x} = s_6 + d_1 + d_2 + d_3 + d_4 + d_5 + d_6.$$

Recall that the detail at each level indicates the contribution to the overall variability of the series from components with scales of length 2^{j-1}. **Figure 2** indicates this frequency resolution for each detail level for the monthly data utilized. The first panel, labeled **s** is the original signal that is then broken out into its time/frequency components. Levels 1 through 3 correspond with the lower time-scale high-frequency components of the signal, from bi-monthly (d_1) through roughly semi-annual contributions (d_3). The higher scales can be thought of as representing lower-frequency contributions over periods from roughly annual (d_4) through 5-years (d_6). The wavelet smooth (a_6) is then the sum of those components representing variability in the series over scales longer than 2^6 = 64 months. Note that this approximation essentially captures the low frequency time trend of the series.

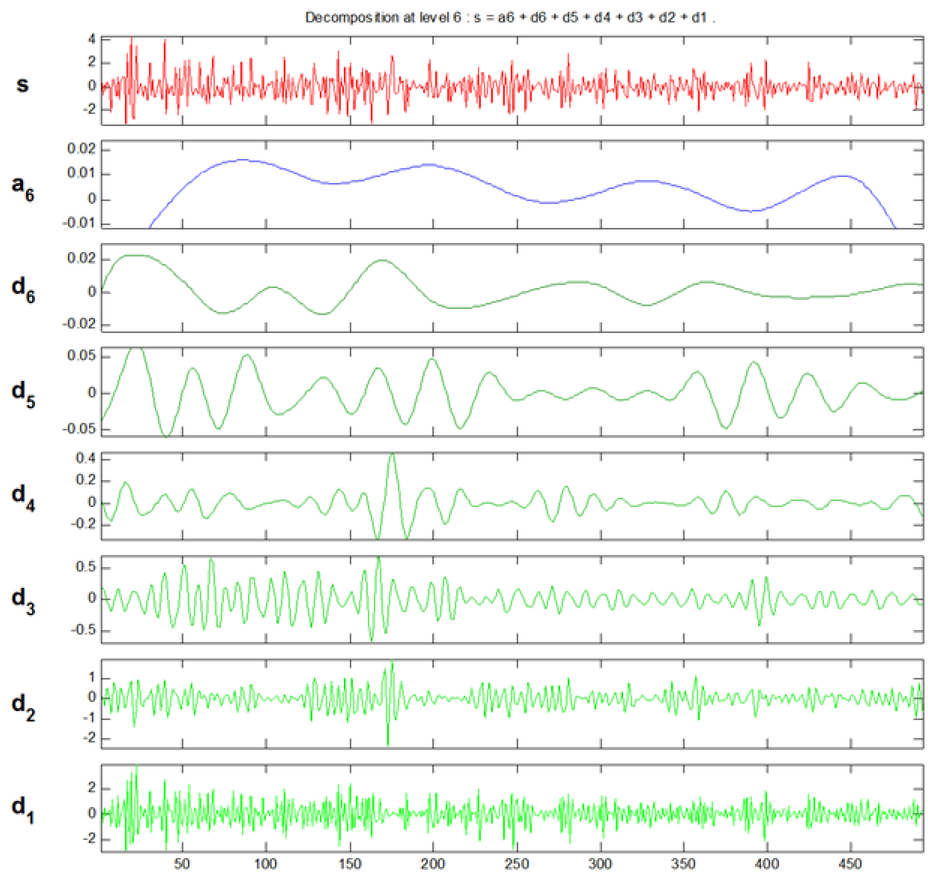

Figure 2: Multi-Resolution Analysis Example

2.1.3 The MODWT

While the DWT has many useful properties as is, it also has some limitations that make it not always the most desirable form for use in analyzing time series. Two of the main drawbacks are the requirement of a dyadic length (power of two) analysis series, necessary for the critical sampling to take place, and also the fact that the DWT is not *shift invariant*, essentially meaning that features present in the original time series may not line up exactly in the MRA details at every scale.

The Maximal Overlap Discrete Wavelet Transform (MODWT) is a variant of the DWT that avoids these drawbacks, although at some cost that we won't get into here.[8] The main difference with the MODWT is that it does not perform just a critical sampling of the CWT wavelet coefficients, but instead retains much more information than is absolutely necessary to reconstruct the signal (although not as much as with the CWT). This allows us to analyze any length time series. Unlike the DWT, the MODWT is shift invariant as well, meaning that features in the original series will be represented at the correct point in time in the details of an MRA across all scales of analysis. This makes it very useful in detecting breakpoints, since certain types of breaks will show up across all

[8] The interested reader is encouraged to investigate these more technical properties of wavelets in Gencay et al. (2002) and Percival and Walden (2000).

scales of analysis and this can help to pinpoint the timing and perhaps nature of a particular break. The MODWT is what is used in our initial filtering of the CPI and its subcomponent indices.

2.2. Wavelet Filtering of the CPI Subcomponents

The Dowd *et al* (2011) paper mentioned above used a version of the MODWT to filter the CPI All Items index and found that, depending on the particular basis function used, level 4 or level 5 wavelet approximations performed as well or better than alternative measures of core inflation, according to testing criteria defined in their paper. Our preliminary results can be viewed as a natural extension of their analysis, as we end up also using a level 4 approximation to reconstruct the All Items index.

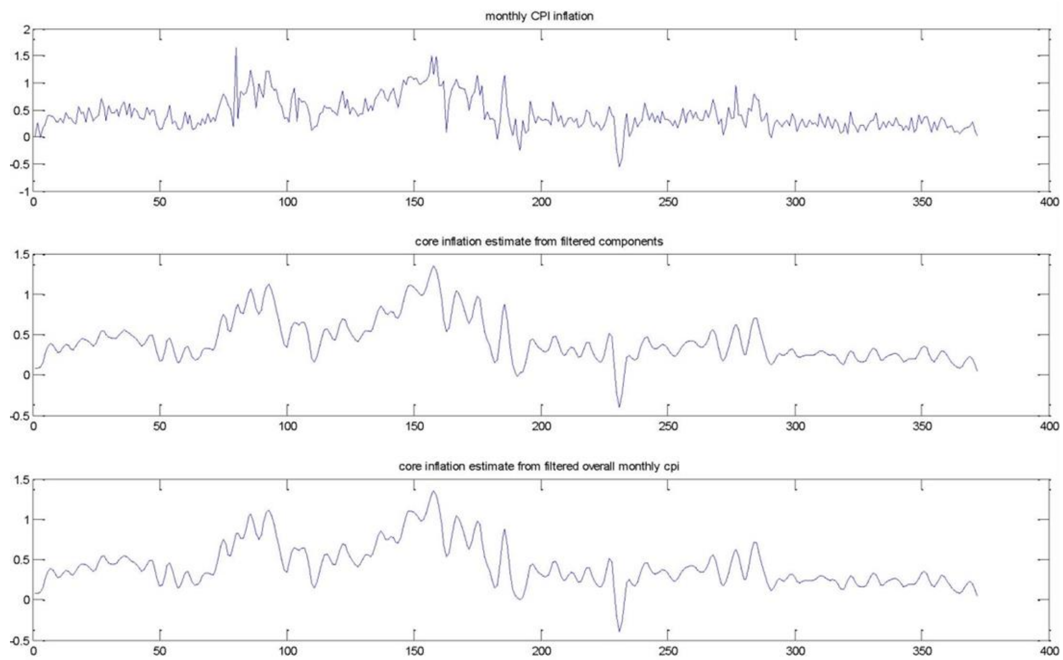

Figure 3: Trial Estimates with 36 Filtered Subcomponent Indexes

In our reconstruction of the All Items index using the filtered subcomponents, we use the same relative importance multipliers (expenditure weights) as BLS does in their original construction of the All Items index. In fact, in our study the version of the All Items Index we use we construct initially using the unfiltered subcomponents and their relative importance measures. This allows for a more accurate basis of comparison for the filtered versions than just relying on the published overall All Items Index. Below we first give the results from filtering 36 subcomponents of the CPI using the MODWT. For this we used CPI data spanning from January 1967 to December 1997. **Figure 3** provides the resulting series from our reconstruction of the All Items Index from these filtered subcomponents. We then repeat the exercise using an even finer breakdown by filtering 190 of the subcomponent indexes before reconstructing the All Items Index. The finer level subcomponent data was not available from earlier years, so we used data spanning from 1998 to 2008 for this portion of the study. These results are given in **Figure 4**. Note that the first panel in each set of graphs contains the monthly CPI inflation rate using the All Items index. The second panel gives our resulting inflation series from reconstructing the All Items index after filtering the subcomponent indexes using the MODWT. The third panel indicates the inflation series that results from filtering only the All Items index, as done in Dowd *et al* (2011).

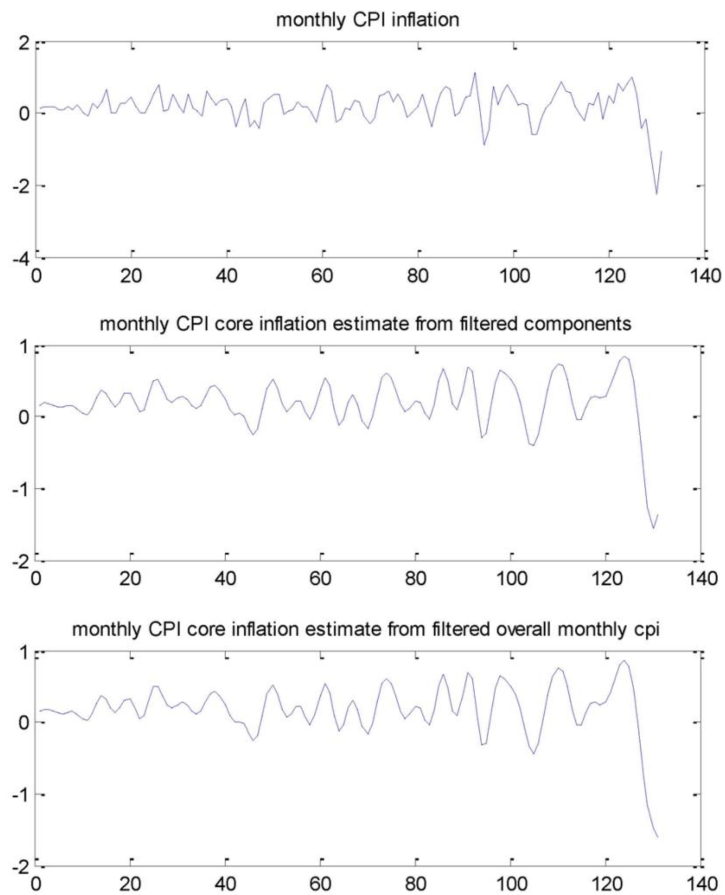

Figure 4: Trial Estimates with 190 Filtered Subcomponent Indexes

While slightly smoother series resulted from filtering the subcomponent series and then reconstructing the All Items index, in neither case were they sufficiently different from the Dowd *et al* (2011) results to merit pursuing further. Consideration of other approaches led to thie current study, intended to determine similarities in movements across subcomponent series and construct a core CPI index based on co-movements across the subcomponents. While we are still in the initial stages of this alternative approach, it is worth sharing some preliminary results as they indicate this may be a fruitful avenue of research.

3. Wavelet Coherency Analysis

The continuous wavelet transform discussed above allows for the estimation of the spectral characteristics of a time series as a function of time.[9] We make use of this to employ c**ross wavelet transforms** and **wavelet coherence** analysis in investigating commonalities in the time-frequency behavior of the eight subcomponents of the CPI.
Cross wavelet transforms allow for assessment of common power structures across series.[10] Coherency is in effect a localized correlation measure that can reveal the strength of co-movements of the series over time and frequency and any changes in that strength.

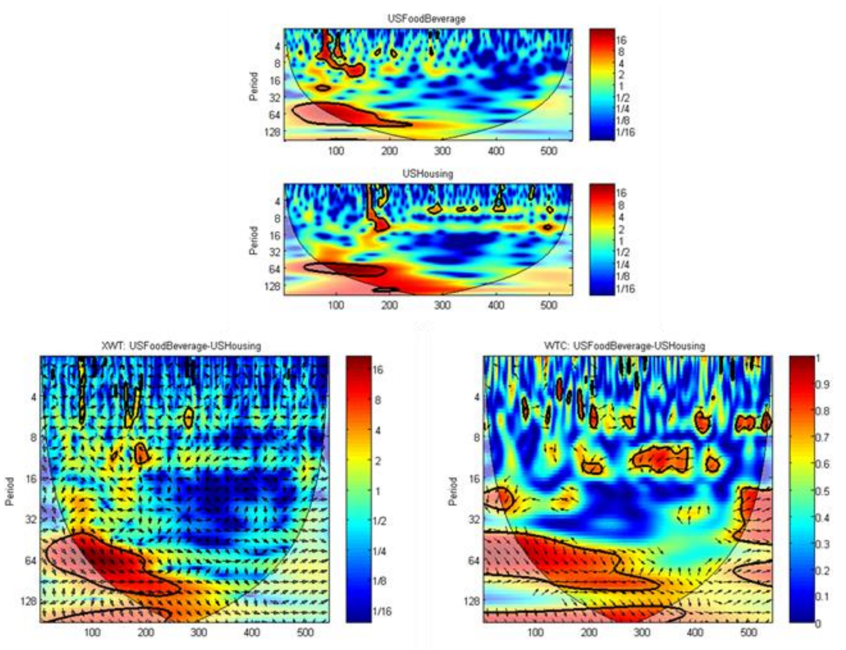

Figure 5: Cross Wavelet Transform and Wavelet Coherency Graphics – All Items vs ??? Subcomponent

The goal is to provide a foundation for the development of a weighting scheme across the subcomponent series for an alternative measure of core inflation. **Figure 5** and **Figure 6** give examples of the cross wavelet transforms and wavelet coherence between some of the CPI components analyzed. The top two graphics in each figure show the power spectrum of each series individually, indicating the energy of each series and how it changes across time and frequency. The cross wavelet transforms exposes regions with high common power and further reveals information about the phase relationship between the series, with the red areas indicating the highest common power at those

[9] See Aguiar and Soares (2011) for an excellent introduction to continuous wavelet transforms and the use of the Cross Wavelet Transform and Wavelet Coherence in the analysis of time series data. Also see Aguiar-Conrariaa *et al* (2008), and Aguiar *et al* (2012) for interesting applications of these approaches. Grinsted *et al* (2004) presents a widely cited early application of these methods as well. Some additional applications of note include Sanderson and Fryzlewicz (2007) and Rua and Nunes (2009).

[10] In the interest of space we refer the reader to the articles cited above for background details on these measures, in particular Aguiar and Soares (2011) and the references cited within.

times and frequencies (note the scale to the right of each graph).. The final panel indicates the coherency, again with the higher levels of correlation indicated by red.

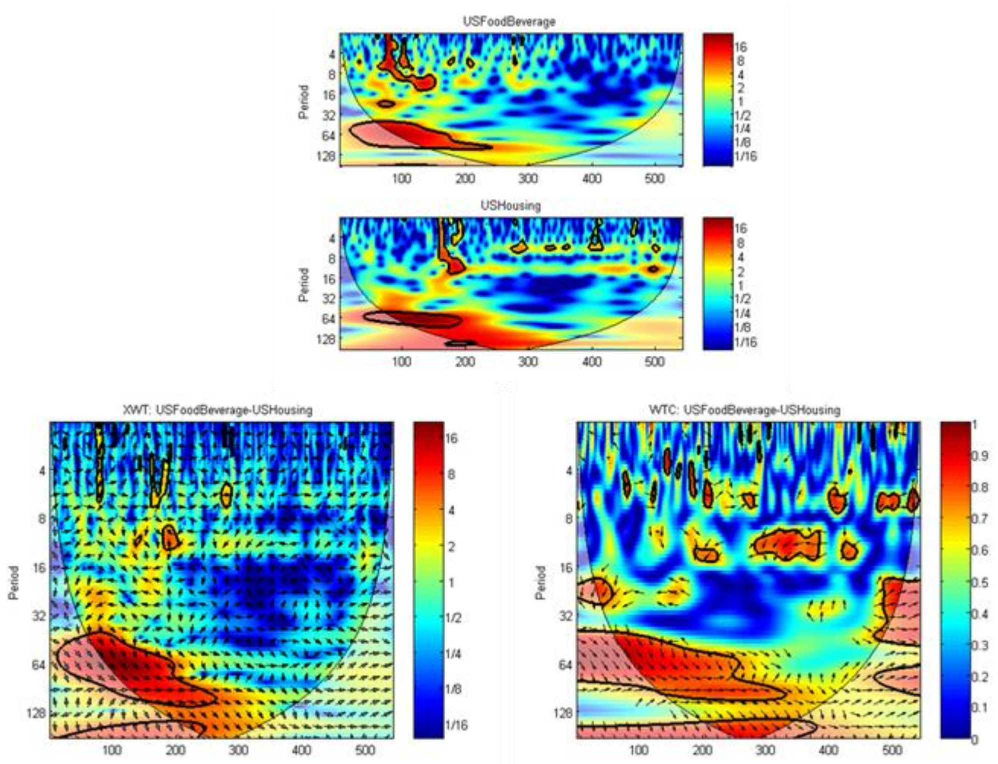

Figure 6: Cross Wavelet Transform and Wavelet Coherency Graphics – Food and Beverages component vs Housing Component

4. Conclusion

Rather than dropping some of the subcomponents completely to arrive at a core measure, the goal is to extract the **common frequency components of all of the subcomponent series** to develop an index based on actual co-movements in all prices over particular time scales. Initial results indicate strong correlations between some subcomponents at certain time scales and frequencies and less for others, along with significant changes in these over time.

References

Aguiar-Conrariaa, L., N. Azevedo and M.J. Soares. 2008. Using wavelets to decompose the time–frequency effects of monetary policy. *Physica A* 387: 2863–2878.

Aguiar, L.F., and M.J. Soares. 2011. The Continuous Wavelet Transform: A Primer. *NIPE Working Papers*, NIPE - Universidade do Minho, http://EconPapers.repec.org/RePEc:nip:nipewp:16/2011.

Aguiar, L.F., M.M.F. Martins, M.J. Soares. 2012. The yield curve and the macroeconomy across time and frequencies. *Journal of Economic Dynamics and Control* 36:12, 1950-1970.

Anderson, R.G., F.N.G. Andersson, J. Binner, and T. Elger. 2007. Core Inflation as Idiosyncratic Persistence: A Wavelet Approach to Measuring Core Inflation. *Working paper presented at "Price Measurement for Monetary Policy" conference*, Dallas, May 24–25, 2007.

David Baqaee. 2010. Using wavelets to measure core inflation: The case of New Zealand. *North American Journal of Economics and Finance* 21:241–255.

Detmeister, Alan. 2011. The Usefulness of Core PCE Inflation Measures. *Finance and Economics Discussion Series*, Federal Reserve Board, Washington, D.C. Working Paper Number 2011-56.

Detmeister, A. 2012. What Should Core Inflation Exclude? *Finance and Economics Discussion Series*, Federal Reserve Board, Washington, D.C. Working Paper Number 2012-43.

Dolmas, J. and M.A. Wynne. 2008. "Measuring core inflation: notes from a 2007 Dallas Fed conference," *Staff Papers*, Federal Reserve Bank of Dallas, Federal Reserve Bank of Dallas, issue May.

Dowd, K., J. Cotter, and L. Loh. 2011. U.S. Core Inflation: A Wavelet Analysis. *Macroeconomic Dynamics* 15: 513–536.

Gencay, R., F. Selcuk, and B. Whitcher. 2002. An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. Academic Press, 525 B Street, Suite 1900, San Diego, CA 92101.

Grinsted, A. , J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11: 561–566.

Rua, A. and L.C. Nunes. International comovement of stock market returns: A wavelet analysis. *Journal of Empirical Finance* 16 (2009) 632–639.

Sanderson, J. and P. Fryzlewicz. 2007. Locally stationary wavelet coherence with application to neuroscience. In: *Proceedings of the 56th session of the International Statistical Institute*, 22-29 August 2007, Lisbon, Portugal. LSE Research Online: September 2009.