Temporal aggregation effects on a structural mean-change of time series

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Abstract
In this article we investigate the effects of temporal aggregation on a mean change of time series, through the two statistical tests—the likelihood ratio (LR) test and the cumulative sum (CUSUM) test to detect the mean change. We propose a modified LR test statistic when aggregate data are used for testing. Also we show that the CUSUM test statistic is free from the temporal aggregation effect. The Monte Carlo simulations verify the theoretical results.

Key Words: Temporal Aggregation, Mean Change, Likelihood Ratio Test, Cumulative Sum Test

1. Introduction

A time series can be influenced by an interruptive event and so a structural break in mean may occur before and after the event. This discordance is said to be a mean change (or a mean shift) of the time series. It is known that we cannot directly use traditional statistical tests of independent samples, such as the t-test, for detecting a mean change because the observations are almost certainly dependent and no possibility for randomization exists (Box and Tiao, 1965). Therefore various alternative approaches have been proposed and developed in literature. When testing a mean change is of interest, the issue has been discussed within two frameworks of the likelihood ratio (LR) test (see Hinkley, 1970; Chang et al., 1988; Tsay, 1988; Chen and Tiao, 1990; Balke, 1993; Chen and Liu, 1993; Tsay et al., 2000; Sánchez and Peña, 2003; Galeano et al., 2006) and the cumulative sum (CUSUM) test (see Page, 1955; Hinkley, 1971; Brown et al., 1975; Krämer et al., 1988; Ploberger and Krämer, 1992; Bai, 1994; Chu et al., 1995; Lobato, 2001; Aue et al., 2008; Juhl and Xiao, 2009; Shao and Zhang, 2010).

Another point of interest is temporal aggregation of a time series process. Although theoretically many different time units, like second, minute, hour, day, week, month, quarter, and year, can be used for observing and recording, the available time series from publications are often temporally aggregated. In literature, it is known that the aggregation has substantial effects on the statistical properties of the process. That is, an ARIMA model structure transforms due to the temporal aggregation and consequent changes of model parameters (Amemiya and Wu, 1972; Brewer, 1973; Abraham, 1982; Weiss, 1984; Stram and Wei, 1986). The model converges to an IMA limiting model as the aggregation order goes to infinity (Tiao, 1972; Wei, 1978a). As the order of aggregation is higher, the information loss in parameter estimation becomes more serious (Tiao and Wei, 1976; Wei, 1978b). Lütkepohl (1984, 1986) analyzes the temporal aggregation for VARMA models and investigates its impact on the efficiency of the multivariate forecasts. Temporal aggregation affects the linearity test (Granger and Lee, 1999; Teles and Wei, 2000), the normality test (Teles and Wei, 2002), and the unit-root test (Teles et al., 2008). Their studies show the aggregation strengthens the linearity, induce the normality, and reduce the unit-root characteristic, respectively. Hotta et al. (2004) investigate the aggregation effects of some discordance due to additive and innovative outliers on forecasting values.

In this paper, we study the temporal aggregation effects on testing a mean change. To include both parametric and nonparametric tests in our study and their comparisons, we concentrate on the case of a single mean change. The paper is organized as follows. In **Section 2**, we review the two commonly used tests, i.e., the LR test and the CUSUM test. **Section 3** presents aggregation effects on the model parameters and the error variance. In **Section 4**, we investigate effects on the LR test and the CUSUM test when temporally aggregated data are used. **Section 5** shows some Monte Carlo simulation results on the effects of aggregation. Also, some concluding remarks are given in **Section 6**.

2. Commonly Used Tests on a Time Series Mean Change

The problem of interest is to identify a mean change in a time series. It can be reworded as testing the null hypothesis of a constant mean, i.e.,

$$H_0 : \mu_1 = \cdots = \mu_n \equiv \mu$$

against the alternative of a mean shift starting at a time point *k*, i.e.,

$$H_a : \mu_1 = \cdots = \mu_k \neq \mu_{k+1} = \cdots = \mu_n,$$

for $1 < k \leq n$ and $k \in \mathbb{Z}$, where $\mathbb{Z}$ denotes the set of integers and μ_i is an expected value of the series at time *i* (see Sen and Srivastava, 1975; James et al., 1987; Aue and Horváth, 2013).

2.1 A Likelihood Ratio Test

Consider two time series processes:

1. A stationary process $\{X_t^{(0)}, t = 1, \ldots, n\}$, which follows an ARMA(*p*,*q*) model of

$$\phi_p(B) X_t^{(0)} = \theta_q(B) a_t, \tag{2.1}$$

   where a_t is a Gaussian white noise of mean zero and variance σ_a^2, $\phi_p(B) = (1 - \phi_1 B - \cdots - \phi_p B^p)$ and $\theta_q(B) = (1 - \theta_1 B - \cdots \theta_q B^q)$ are polynomials of B, and B is the backshift operator such that $B^j X_t^{(0)} = X_{t-j}^{(0)}$, $j \in \mathbb{Z}$. Here, all the roots of $\phi_p(B)$ and $\theta_q(B)$ are assumed to be outside of the unit circle and share no common roots.
2. A discordant process $\{X_t, t = 1, \ldots, n\}$ with a mean change starting at a time point *k*, which can be modeled as

$$X_t = X_t^{(0)} + w_k (1 + B + B^2 + \cdots) I_t(k) = X_t^{(0)} + \frac{w_k}{(1-B)} I_t(k), \tag{2.2}$$

   where w_k is a shift-magnitude and $I_t(k)$ is an indicator function of

$$I_t(k) = \begin{cases} 1 & \text{if } t = k \\ 0 & \text{otherwise.} \end{cases}$$

Define the contaminated residual e_t as $e_t = \pi(B)X_t$ for $t = 1,\ldots,n$. From (2.1) and (2.2), we have

$$e_t = \pi(B)X_t = a_t + w_k \frac{\pi(B)}{(1-B)} I_t(k), \qquad (2.3)$$

where the polynomial $\pi(B) = (1 - \pi_1 B - \pi_2 B^2 - \cdots) = \phi_p(B)/\theta_q(B)$. Equation (2.3) can be rewritten as a linear form of

$$e_t = w_k y_t + a_t, \qquad (2.4)$$

where

$$y_t = \frac{\pi(B)}{(1-B)} I_t(k) = \begin{cases} 0 & \text{for } t < k \\ 1 & \text{for } t = k \\ 1 - \sum_{j=1}^{t-k} \pi_j & \text{for } t > k. \end{cases} \qquad (2.5)$$

Then the shift-magnitude w_k is estimated by the OLS estimator,

$$\hat{w}_k = \frac{\sum_{t=k}^{n} e_t y_t}{\sum_{t=k}^{n} y_t^2} = \frac{e_k + \sum_{t=k+1}^{n} e_t \left(1 - \sum_{j=1}^{t-k} \pi_j\right)}{1 + \sum_{t=k+1}^{n} \left(1 - \sum_{j=1}^{t-k} \pi_j\right)^2} \qquad (2.6)$$

and the standard deviation of the OLS estimator $\hat{w}_k$ is

$$\sigma_{\hat{w}_k} = \frac{\sigma_a}{\sqrt{\sum_{t=k}^{n} y_t^2}} = \frac{\sigma_a}{\sqrt{1 + \sum_{t=k+1}^{n} \left(1 - \sum_{j=1}^{t-k} \pi_j\right)^2}}. \qquad (2.7)$$

To test for the mean change at a known time point *k*, Chang et al. (1988) and Tsay (1988) propose a likelihood ratio (LR) test statistic, which they show to be

$$\lambda_k = \hat{w}_k \big/ \sigma_{\hat{w}_k}, \qquad (2.8)$$

and it can be rewritten as

$$\lambda_k = \frac{e_k + \sum_{t=k+1}^{n} e_t \left(1 - \sum_{j=1}^{t-k} \pi_j\right)}{\sigma_a \sqrt{1 + \sum_{t=k+1}^{n} \left(1 - \sum_{j=1}^{t-k} \pi_j\right)^2}}. \qquad (2.9)$$

We note that the LR statistic λ_k in (2.9) is dependent on the model parameters.

Let us consider the general AR(*p*) model, which has been very widely used in practice. If the stationary series $X_t^{(0)}$ follows an AR(*p*) process of $(1 - \phi_1 B - \cdots - \phi_p B^p) X_t^{(0)} = a_t$ where $a_t \overset{iid}{\sim} N(0, \sigma_a^2)$, then the LR statistic λ_k to test for a mean change becomes

$$\lambda_k = \frac{e_k + \sum_{t=k+1}^{n} e_t \left(1 - \sum_{j=1}^{t-k} \phi_j\right)}{\sigma_a \sqrt{1 + \sum_{t=k+1}^{n} \left(1 - \sum_{j=1}^{t-k} \phi_j\right)^2}}, \qquad (2.10)$$

where $\phi_{p+1} = \phi_{p+2} = \cdots = \phi_{n-k} = 0$ if $n - k > p$. As a special case, if the series $X_t^{(0)}$ follows an AR(1) process of $(1 - \phi B) X_t = a_t$, the LR test statistic becomes

$$\lambda_k = \frac{e_k + (1-\phi)\sum_{t=k+1}^{n} e_t}{\sigma_a \sqrt{1+(n-k)(1-\phi)^2}}, \tag{2.11}$$

which is associated with the AR parameter ϕ.

When the time point k of the mean change is unknown, we use $\sup_{k=2,\ldots,n} |\lambda_k|$ as the test statistic, i.e.,

$$\sup_{k=2,\ldots,n} |\lambda_k| = |\lambda_s|, \tag{2.12}$$

where a time point $s \in \{2,\ldots,n\}$. If the supremum exceeds a predetermined critical value $L > 0$, then we reject the null hypothesis (For more discussions, see Chang et al., 1988; Tsay, 1988; Balke, 1993; Chen and Liu, 1993; Tsay et al., 2000; Galeano et al., 2006).

2.2 A Cumulative Sum Test

Suppose that a series X_t for $t = 1,\ldots,n$ is contaminated by a mean change starting at a time point *k*, as described in **Section 2.1**. To test for the mean change at a known time point *k*, the CUSUM test statistic (Brown et al., 1975) is given by

$$c_k = \frac{1}{\sigma_X \sqrt{n}} \sum_{T=K}^{N} (X_t - \bar{X}_n), \tag{2.13}$$

where σ_X is the standard deviation of X_t, which satisfies the long-run variance of

$$\sigma_X^2 = \lim_{n\to\infty} \frac{1}{n} E\left[\sum_{t=1}^{n} (X_t - \mu_t)^2\right] = \lim_{n\to\infty} nVar(\bar{X}_n), \tag{2.14}$$

$\mu_t = E(X_t)$, and $\bar{X}_n = \sum_{t=1}^{n} X_t / n$.

Similarly to the LR test procedure, when the time point *k* is unknown, we use $\sup_{k=2,\ldots,n} |c_k|$ as the test statistic, i.e.,

$$\sup_{k=2,\ldots,n} |c_k| = |c_s|, \tag{2.15}$$

where a time point $s \in \{2,\ldots,n\}$. If the supremum exceeds a predetermined critical value $Q > 0$, then we reject the null hypothesis of no mean shift (For more discussion, see Brown et al., 1975; Bai, 1994; Lobato, 2001; Shao and Zhang, 2010).

3. Temporal Aggregation Effects on AR Models

Because of its easy interpretations, an AR model has often been used to describe the process of a time series. In this section, we investigate the effects of aggregation on its model form, parameters, and error variance.

Consider the *m*-period nonoverlapping (or simply, the *m*th order) aggregate series Z_T of the discordant series X_t, which is defined as

$$Z_T = \sum_{t=m(T-1)+1}^{mT} X_t = (1 + B + \cdots + B^{m-1}) X_{mT}, \tag{3.1}$$

where the aggregation order *m* is a positive integer for $m < n$ and the aggregate time unit $T = 1,\ldots,N$ for $N = n/m$ (Tiao, 1972; Wei, 2006, p.508). Similarly, the *m*th order aggregate series $Z_T^{(0)}$ of the stationary series $X_t^{(0)}$ is given by $Z_T^{(0)} = (1 + B + \cdots + B^{m-1})$ $X_{mT}^{(0)}$.

It has been known that if the stationary series $X_t^{(0)}$ follows an AR(*p*) process of $(1-\phi_1 B - \cdots - \phi_p B^p)X_t^{(0)} = a_t$ for $p > 0$ and $p \in \mathbb{Z}$, then the aggregate series $Z_T^{(0)}$ is also stationary and follows an ARMA(*P*,*Q*) process of

$$(1-\Phi_1\mathcal{B} - \cdots - \Phi_P\mathcal{B}^P)Z_T^{(0)} = (1-\Theta_1\mathcal{B} - \cdots - \Theta_Q\mathcal{B}^Q)A_T\,, \qquad (3.2)$$

where A_T is a Gaussian white noises of mean zero and variance σ_A^2 and $\mathcal{B}$ is the aggregate backshift operator defined as $\mathcal{B} = B^m$. The orders *P* and *Q* are determined by the roots of $(1-\phi_1 B - \cdots - \phi_p B^p)$. For the details and proofs, we refers readers to Amemiya and Wu (1972), Brewer (1973), and Stram and Wei (1986).

It follows that an AR(1) process of $(1-\phi B)X_t^{(0)} = a_t$ transforms upon the *m*th order temporal aggregation to an ARMA(1,1) process of $(1-\Phi\mathcal{B})Z_T^{(0)} = (1-\Theta\mathcal{B})A_T$ for $m > 1$. For this aggregate transformation, Ahsanullah and Wei (1984) derive the autocovariance function of the aggregate series in terms of the AR(1) parameter ϕ and the aggregation order *m*. Teles et al. (2008) also derive the exact aggregate model parameters when the nonaggregate series is an AR(1) process with $\phi = 1$. However, the exact parameter expressions of the aggregate model for the general nonaggregate AR(*p*) process have never been developed. We now derive the results and summarize them in the following **Theorem 3.1**.

Theorem 3.1. Assume that the nonaggregate series $X_t^{(0)}$ follows an AR(*p*) process. Then the *m*th order aggregate series $Z_T^{(0)}$ is known to follow an ARMA(*P*,*Q*) process and the model parameters can be expressed as functions α_i and β_j with respect to the AR(*p*) parameters and the aggregation order *m*, i.e.,

$$\Phi_i = \alpha_i(\phi_1,\ldots,\phi_p,m), \quad i = 1,\ldots,P \qquad (3.3)$$

and

$$\Theta_j = \beta_j(\phi_1,\ldots,\phi_p,m), \quad j = 1,\ldots,Q\,. \qquad (3.4)$$

Also the error variance σ_A^2 can be written as $\sigma_A^2 = (\rho\sigma_a)^2$ with

$$\rho = \left(\frac{1 + f_1^2 + f_2^2 + \cdots + f_{(P+1)m-(p+1)}^2}{1 + \Theta_1^2 + \cdots + \Theta_Q^2}\right)^{1/2} \qquad (3.5a)$$

or, equivalently,

$$\rho = \left(\frac{f_m + f_1 f_{m+1} + f_2 f_{m+2} + \cdots + f_{Pm-(p+1)} f_{(P+1)m-(p+1)}}{-\Theta_1 + \Theta_1\Theta_2 + \cdots + \Theta_{Q-1}\Theta_Q}\right)^{1/2} \qquad (3.5b)$$

where f_h is a function of $\phi_1,\ldots,\phi_p$ and *m*, for $h = 1,\ldots,[(P+1)m-(p+1)]$.

Proof. $m \geq 2$ and $m \in \mathbb{Z}$ (aggregation):

When multiplying $(1-\phi_1 B-\cdots-\phi_p B^p)X_t^{(0)}=a_t$ by $\frac{(1-\Phi_1 B^m-\cdots-\Phi_P B^{Pm})(1+B+\cdots+B^{m-1})}{(1-\Theta_1\mathcal{B}-\cdots-\Theta_Q\mathcal{B}^Q)}$, we obtain

$$(1-\Phi_1 B^m-\cdots-\Phi_P B^{Pm})(1+B+\cdots+B^{m-1})X_t^{(0)}=\left[\frac{(1-\Phi_1 B^m-\cdots-\Phi_P B^{Pm})(1+B+\cdots+B^{m-1})}{1-\phi_1 B-\cdots-\phi_p B^p}\right]a_t. \quad (3.6)$$

Let $W_t=(1-\Phi_1 B^m-\cdots-\Phi_P B^{Pm})(1+B+\cdots+B^{m-1})X_t^{(0)}$. Since the highest degree of the MA polynomial in (3.6) is $(P+1)m-(p+1)$, W_t is expressed as the MA form of

$$W_t=(1+\psi_1 B+\cdots+\psi_{(P+1)m-(p+1)}B^{(P+1)m-(p+1)})a_t. \quad (3.7)$$

From Equations (3.6) and (3.7),

$$\begin{aligned}&(1-\Phi_1 B^m-\cdots-\Phi_P B^{Pm})(1+B+\cdots+B^{m-1})\\&=(1-\phi_1 B-\cdots-\phi_p B^p)(1+\psi_1 B+\cdots+\psi_{(P+1)m-(p+1)}B^{(P+1)m-(p+1)}).\end{aligned} \quad (3.8)$$

By distributing and collecting like terms in (3.8), the parameters, Φ_i for $i=1,\ldots,P$ and ψ_h for $h=1,\ldots,(P+1)m-(p+1)$, are sequentially associated with $\phi_1,\ldots,\phi_p$ and m. Specifically, the ith parameter Φ_i can be expressed as $\Phi_i=\alpha_i(\phi_1,\ldots,\phi_p,m)$, which is a function of $\phi_1,\ldots,\phi_p$, and m; similarly, the hth parameter ψ_h can be expressed as $\psi_h = f_h(\phi_1,\ldots,\phi_p,m)$ or simply f_h. Now the MA form of (3.7) can be rewritten as

$$W_t=(1+f_1 B+\cdots+f_{(P+1)m-(p+1)}B^{(P+1)m-(p+1)})a_t. \quad (3.9)$$

When $t=mT$,

$$W_{mT}=a_{mT}+f_1 a_{mT-1}+f_2 a_{mT-2}+\cdots+f_{(P+1)m-(p+1)}a_{mT-(P+1)m+(p+1)}. \quad (3.10)$$

Then the variance of W_{mT} is

$$Var(W_{mT})=\sigma_a^2(1+f_1^2+\cdots+f_{(P+1)m-(p+1)}^2), \quad (3.11)$$

and the covariance between W_{mT} and W_{mT+m} is

$$Cov(W_{mT},W_{mT+m})=\sigma_a^2(f_m+f_1 f_{m+1}+f_2 f_{m+2}+\cdots+f_{Pm-(p+1)}f_{(P+1)m-(p+1)}), \quad (3.12)$$

By the definition of (3.1) and the equation of (3.2), W_{mT} can be rewritten as

$$\begin{aligned}W_{mT}&=(1-\Phi_1 B^m-\cdots-\Phi_P B^{Pm})(1+B+\cdots+B^{m-1})X_{mT}^{(0)}\\&=(1-\Phi_1\mathcal{B}-\cdots-\Phi_P\mathcal{B}^P)Z_T^{(0)}=(1-\Theta_1\mathcal{B}-\cdots-\Theta_Q\mathcal{B}^Q)A_T,\end{aligned} \quad (3.13)$$

where $\mathcal{B}=B^m$, $Q=\left\lfloor P+1-\frac{P+1}{m}\right\rfloor\le P$, and $\lfloor x\rfloor$ indicates the largest integer not greater than a real number x. Using the MA form of (3.13), the variance of W_{mT} is

$$Var(W_{mT})=\sigma_A^2(1+\Theta_1^2+\cdots+\Theta_Q^2), \quad (3.14)$$

and the covariance between W_{mT} and W_{mT+m} is

$$Cov(W_{mT},W_{mT+m})=\sigma_A^2(-\Theta_1+\Theta_1\Theta_2+\cdots+\Theta_{Q-1}\Theta_Q). \quad (3.15)$$

Using the formulas of (3.11), (3.12), (3.14), and (3.15), we obtain the quotient of the variance and the covariance, i.e.,

$$\begin{aligned}\frac{Var(W_{mT})}{Cov(W_{mT},W_{mT+m})}&=\frac{\sigma_a^2(1+f_1^2+\cdots+f_{(P+1)m-(p+1)}^2)}{\sigma_a^2(f_m+f_1 f_{m+1}+\cdots+f_{Pm-(p+1)}f_{(P+1)m-(p+1)})}\\&=\frac{\sigma_A^2(1+\Theta_1^2+\cdots+\Theta_Q^2)}{\sigma_A^2(-\Theta_1+\Theta_1\Theta_2+\cdots+\Theta_{Q-1}\Theta_Q)}.\end{aligned} \quad (3.16)$$

Consider a compound function

$$g(\phi_1,\cdots,\phi_p,m)=\left(\frac{1+f_1^2+\cdots+f_{(P+1)m-(p+1)}^2}{f_m+f_1f_{m+1}+\cdots+f_{Pm-(p+1)}f_{(P+1)m-(p+1)}}\right) \tag{3.17}$$

or simply g. Then the quotient (3.16) has an equation form

$$(1+\Theta_1^2+\cdots+\Theta_Q^2)+g(\Theta_1-\Theta_1\Theta_2-\cdots-\Theta_{Q-1}\Theta_Q)=0 \tag{3.18}$$

with respect to $\Theta_j \quad (j=1,\ldots,Q)$. In Equation (3.18), the real solutions for Θ_j are associated with g. Thus the solutions can be expressed as a function of $\phi_1,\ldots,\phi_p$ and m, i.e., $\Theta_j=\beta_j(\phi_1,\ldots,\phi_p,m)$. Also we derive the error variance of

$$\sigma_A^2=\sigma_a^2\left(\frac{1+f_1^2+f_2^2+\cdots+f_{(P+1)m-(p+1)}^2}{1+\Theta_1^2+\cdots+\Theta_Q^2}\right)$$

or, equivalently,

$$\sigma_A^2=\sigma_a^2\left(\frac{f_m+f_1f_{m+1}+f_2f_{m+2}+\cdots+f_{Pm-(p+1)}f_{(P+1)m-(p+1)}}{-\Theta_1+\Theta_1\Theta_2+\cdots+\Theta_{Q-1}\Theta_Q}\right)$$

which follows from Equations (3.11), (3.12), (3.14), and (3.15).

□

Because of the need for the later analysis, we now derive the exact parameter expressions for the aggregate model when the nonaggregate model is AR(1). The results are summarized in **Lemma 3.1**.

Lemma 3.1. Suppose that the series $X_t^{(0)}$ follows an AR(1) process. Then the *m*th order aggregate series $Z_T^{(0)}$ follows an ARMA(1,1) process and the model parameters are expressed as

$$\Phi=\phi^m \tag{3.19}$$

and

$$\Theta=-(\eta/2)\pm\sqrt{(\eta^2/4)-1} \tag{3.20}$$

where

$$\eta=\frac{\sum_{j=1}^{m}\left(\sum_{i=1}^{j}\phi^{i-1}\right)^2+\sum_{j=1}^{m-1}\left(\sum_{i=1}^{m}\phi^{i-1}-\sum_{i=1}^{j}\phi^{i-1}\right)^2}{\sum_{j=1}^{m-1}\left(\sum_{i=1}^{j}\phi^{i-1}\right)\left(\sum_{i=1}^{m}\phi^{i-1}-\sum_{i=1}^{j}\phi^{i-1}\right)}. \tag{3.21}$$

Also the error variance σ_A^2 is written as $\sigma_A^2=(\rho\sigma_a)^2$ where

$$\rho=\sqrt{\frac{1}{(1+\Theta^2)}\left[\sum_{j=1}^{m}\left(\sum_{i=1}^{j}\phi^{i-1}\right)^2+\sum_{j=1}^{m-1}\left(\sum_{i=1}^{m}\phi^{i-1}-\sum_{i=1}^{j}\phi^{i-1}\right)^2\right]} \tag{3.22a}$$

or, equivalently,

$$\rho=\sqrt{-\frac{1}{\Theta}\left[\sum_{j=1}^{m-1}\left(\sum_{i=1}^{j}\phi^{i-1}\right)\left(\sum_{i=1}^{m}\phi^{i-1}-\sum_{i=1}^{j}\phi^{i-1}\right)\right]}. \tag{3.22b}$$

□

We note that **Theorem 2.1** of Teles et al. (2008) is a special case of our **Lemma 3.1** with $\phi=1$.

In **Lemma 3.1**, we remark that the aggregate model of $Z_T^{(0)}$ is stationary and invertible if the nonaggregate model of $X_t^{(0)}$ is stationary. This follows because $0<|\Phi|=\left|\phi^m\right|<1$ for $m>1$ and $m\in\mathbb{Z}$, and the MA parameter Θ is chosen to be

$$\Theta=\begin{cases} -\frac{\eta}{2}+\sqrt{\frac{\eta^2}{4}-1} & \text{for } 0<\phi<1 \\ -\frac{\eta}{2}-\sqrt{\frac{\eta^2}{4}-1} & \text{for } -1<\phi<0 \end{cases} \tag{3.23}$$

and so $|\Theta|<1$.

4. Effects On the Test Statistics When an Aggregate Series is Used

4.1 Aggregation Effects on the LR Statistic

Let K be the shift point of the aggregate discordant series Z_T for $1<K<N$ and $K\in\mathbb{Z}$. Then, in the same manner as (2.9), the LR statistic to test for a mean change is

$$\Lambda_K=\frac{\mathcal{E}_K+\sum_{T=K+1}^{N}\mathcal{E}_T\left(1-\sum_{h=1}^{T-K}\Pi_h\right)}{\sigma_A\sqrt{1+\sum_{T=K+1}^{N}\left(1-\sum_{h=1}^{T-K}\Pi_h\right)^2}}, \tag{4.1}$$

where $\mathcal{E}_T=\Pi(\mathcal{B})Z_T$ and $\Pi(\mathcal{B})=1-\Pi_1\mathcal{B}-\Pi_2\mathcal{B}^2-\cdots=\frac{1-\Phi_1\mathcal{B}-\cdots-\Phi_P\mathcal{B}^P}{1-\Theta_1\mathcal{B}-\cdots-\Theta_Q\mathcal{B}^Q}$.

When the time point K is unknown, we use $\sup_{K=2,\ldots,N}|\Lambda_K|$ as the test statistic, i.e.,

$$\sup_{K=2,\ldots,N}|\Lambda_K|=|\Lambda_S|, \tag{4.2}$$

where a time point $S\in\{2,\ldots,N\}$.

Here we note that Λ_K of (4.1) is associated with the AR parameter Φ's and the MA parameter Θ's. In **Theorem 4.1**, we clarify the association and propose the modified LR statistic when aggregate data are used.

Theorem 4.1. Assume that the nonaggregate stationary series $X_t^{(0)}$ follows an AR(p) process. Then the LR statistic to test a mean change for the aggregate series Z_T is given by $\sup_{K=2,\ldots,N}|\Lambda_K|$, where

$$\Lambda_K=\frac{\mathcal{E}_K+\sum_{T=K+1}^{N}\mathcal{E}_T\left(1-\sum_{i=1}^{T-K}\Phi_i\right)+G}{\sigma_A\sqrt{1+\sum_{T=K+1}^{N}\left(1-\sum_{i=1}^{T-K}\Phi_i\right)^2+F}}, \tag{4.3}$$

Here F and G are functions for their variables of $\Phi_1,\ldots,\Phi_P$, $\Theta_1,\ldots,\Theta_Q$, and $\mathcal{E}_{K+1},\ldots,\mathcal{E}_N$. We note that $\Phi_{P+1}=\Phi_{P+2}=\cdots=\Phi_{N-K}=0$ if $N-K>P$.

Proof. When multiplying both sides of $1-\Pi_1\mathcal{B}-\Pi_2\mathcal{B}^2-\cdots=\frac{1-\Phi_1\mathcal{B}-\cdots-\Phi_P\mathcal{B}^P}{1-\Theta_1\mathcal{B}-\cdots-\Theta_Q\mathcal{B}^Q}$ by the polynomial $(1-\Theta_1\mathcal{B}-\cdots-\Theta_Q\mathcal{B}^Q)$ and collecting like terms, the parameters Π_h

$(h=1,2,\ldots)$ are sequentially associated with Φ_i $(i=1,\ldots,P)$ and Θ_j $(j=1,\ldots,Q)$ where $Q \le P$. Then

$$1+\sum_{T=K+1}^{N}\left(1-\sum_{h=1}^{T-K}\Pi_h\right)^2 = 1+\sum_{T=K+1}^{N}\left(1-\sum_{i=1}^{T-K}\Phi_i\right)^2 + F\,, \tag{4.4}$$

and

$$\mathcal{E}_K + \sum_{T=K+1}^{N}\mathcal{E}_T\left(1-\sum_{h=1}^{T-K}\Pi_h\right) = \mathcal{E}_K + \sum_{T=K+1}^{N}\mathcal{E}_T\left(1-\sum_{i=1}^{T-K}\Phi_i\right) + G\,, \tag{4.5}$$

where F and G are functions for their variables of $\Phi_1,\ldots,\Phi_P$, $\Theta_1,\ldots,\Theta_Q$, and $\mathcal{E}_{K+1},\ldots,\mathcal{E}_N$. We note that $\Phi_{P+1}=\Phi_{P+2}=\cdots=\Phi_{N-K}=0$ if $N-K>P$. When plugging Equations (4.4) and (4.5) into Equation (4.1), we obtain the expression

$$\Lambda_K = \frac{\mathcal{E}_K + \sum_{T=K+1}^{N}\mathcal{E}_T\left(1-\sum_{i=1}^{T-K}\Phi_i\right)+G}{\sigma_A\sqrt{1+\sum_{T=K+1}^{N}\left(1-\sum_{i=1}^{T-K}\Phi_i\right)^2+F}}\,.$$

□

We note that Λ_K of (4.3) is a function of the AR parameters $\phi_1,\ldots,\phi_p$ of $X_t^{(0)}$ and the error standard deviation σ_a^2 because of the expressions of Φ_i , Θ_j , and σ_A^2 in **Theorem 3.1**.

Comparing the two expressions of λ_k in (2.10) and Λ_K in (4.3), Λ_K includes three additional parameters—F, G, and ρ , where $\rho=\sigma_A/\sigma_a$ given in either (3.5a) or (3.5b). Therefore we may not expect that the null distribution of $\sup_{K=2,\ldots,N}|\Lambda_K|$ is identical to the null distribution of $\sup_{k=2,\ldots,n}|\lambda_k|$ when $m>1$. However, Λ_K reduces to λ_k when $m=1$ with $F=0$, $G=0$, and $\rho=1$. We demonstrate the location and scale changes of the null distribution through the Monte Carlo studies in **Section 5**.

In **Lemma 4.1**, for the later illustration and analysis, we derive the LR test statistic for the aggregate model when the nonaggregate series follows an AR(1) model.

Lemma 4.1. Assume that the nonaggregate stationary series $X_t^{(0)}$ follows an AR(1) process. Then the LR statistic to test a mean change for the aggregate series Z_T is given by $\sup_{K=2,\ldots,N}|\Lambda_K|$, where

$$\Lambda_K = \frac{\mathcal{E}_K + (1-\Phi)\sum_{T=K+1}^{N}\mathcal{E}_T + G}{\sigma_A\sqrt{1+(N-K)(1-\Phi)^2+F}}\,. \tag{4.6}$$

Here

$$\begin{aligned} F = {} & 2(1-\Phi)\sum_{T=K+1}^{N}\left[(1-\Phi)(\Theta+\cdots+\Theta^{T-K-1})+\Theta^{T-K}\right] \\ & + \sum_{T=K+1}^{N}\left[(1-\Phi)(\Theta+\cdots+\Theta^{T-K-1})+\Theta^{T-K}\right]^2, \end{aligned} \tag{4.7}$$

and

$$G = \mathcal{E}_{K-1}\Theta + \sum_{T=K+2}^{N} \mathcal{E}_T \left[(1-\Phi)(\Theta + \cdots + \Theta^{T-K-1}) + \Theta^{T-K} \right]. \tag{4.8}$$

□

4.2 Aggregation Effects on the CUSUM Statistic

In like manner as (2.13), using the aggregate series Z_T is

$$C_K = \frac{1}{\sigma_Z \sqrt{N}} \sum_{T=K}^{N} (Z_T - \bar{Z}_N), \tag{4.9}$$

where σ_Z is the standard deviation of Z_T, which satisfies the long-run variance of

$$\sigma_Z^2 = \lim_{N\to\infty} \frac{1}{N} E\left[\sum_{T=1}^{N} (Z_T - \mu_T)^2 \right] = \lim_{N\to\infty} N \cdot Var(\bar{Z}_N), \tag{4.10}$$

$\mu_T = E(Z_T)$, and $\bar{Z}_N = \sum_{T=1}^{N} Z_T / N$.

When the time point K is unknown, we use $\sup_{K=2,\ldots,N} |C_K|$ as the test statistic, i.e.,

$$\sup_{K=2,\ldots,N} |C_K| = |C_S|, \tag{4.11}$$

where a time point $S \in \{2, \ldots, N\}$.

We note that C_K of (4.9) is free from model parameters when compared to Λ_K of (4.3). It implies that there does not exist a modified CUSUM test statistic which is expressed in terms of aggregate model parameters. Now we investigate temporal aggregation effects on C_K.

Assume that $N = n/m$ and $K = \lceil k/m \rceil$ where $\lceil x \rceil$ indicates the smallest integer not less than a real number *x*. Then we have

$$\bar{Z}_N = \frac{1}{N} \sum_{T=1}^{N} Z_T = \frac{m}{n} \sum_{t=1}^{n} X_t = m\bar{X}_n \tag{4.12}$$

and so

$$\frac{1}{\sqrt{N}} \sum_{T=K}^{N} (Z_T - \bar{Z}_N) = \frac{\sqrt{m}}{\sqrt{n}} \left[\sum_{T=\lceil k/m \rceil}^{n/m} \sum_{t=m(T-1)+1}^{mT} (X_t - \bar{X}_n) \right] = \frac{\sqrt{m}}{\sqrt{n}} \left[\sum_{t=k}^{n} (X_t - \bar{X}_n) + \Delta_m \right] \tag{4.13}$$

where

$$\Delta_m = \begin{cases} 0 & \text{if } k = m(K-1)+1 \\ \sum_{t=m(K-1)+1}^{k-1} (X_t - \bar{X}_n) & \text{if } m(K-1) < k \le mK. \end{cases} \tag{4.14}$$

Using the long-run variance properties of (2.14) and (4.10), σ_Z becomes

$$\sigma_Z = \lim_{N\to\infty} \sqrt{N \cdot Var(\bar{Z}_N)} = \sqrt{m} \lim_{n\to\infty} \sqrt{n Var(\bar{X}_n)} = \sqrt{m}\sigma_X. \tag{4.15}$$

Therefore, Equation (4.9) becomes

$$C_K = \frac{1}{\sigma_X \sqrt{n}} \sum_{t=k}^{n} (X_t - \bar{X}_n) + \frac{\Delta_m}{\sigma_X \sqrt{n}} = c_k + \frac{\Delta_m}{\sigma_X \sqrt{n}}. \tag{4.16}$$

In Equation (4.16), we note that the aggregation effects on C_K are only from the extra term $\Delta_m / (\sigma_X \sqrt{n})$. However, in general, the effect from Δ_m is too small to affect C_K. Thus C_K in (4.9) is approximately equal to c_k in (2.13). It also implies that the null distribution of $\sup_{K=2,\ldots,N} |C_K|$ is approximately equal to the null distribution of $\sup_{k=2,\ldots,n} |c_k|$. We illustrate the null distribution through the Monte Carlo studies in **Section 5**.

5. Simulation Studies of the Aggregation Effects

In this section, we obtain percentiles of the empirical null distributions of the LR test and the CUSUM test through the Monte Carlo simulations.

To demonstrate the empirical properties, we consider the cases in which the nonaggregate stationary series $X_t^{(0)}$ follows an AR(1) process of $(1-\phi B)X_t^{(0)} = a_t$ with $\phi = -0.5$, 0.3, 0.5, 0.8, and 0.95, assuming $\sigma_a = 1$. So the aggregate stationary series $Z_T^{(0)}$ becomes an ARMA(1,1) model as shown in **Lemma 3.1**. Under the null hypothesis of no mean change, we generate 10,000 different series of size $n = 1200$ for every ϕ. Also we consider the *m*th order temporal aggregation of the simulated series for $m = 3$, 6, and 12.

For the LR test, all the model parameters and the error standard deviation are assumed to be known. We compute the original test statistic $\sup_{k=2,\ldots,n} |\lambda_k|$ using λ_k in (2.11) for the nonaggregate and the modified statistic of $\sup_{K=2,\ldots,N} |\Lambda_K|$ using Λ_K in (4.6) for the aggregate series, where $N = n/m$. Through searching the supremum in every series, we obtain 10,000 suprema for the given ϕ and *m*. Then we construct the distribution of the 10,000 values as the empirical null distribution.

The results are listed in **Table 5.1** for all the combinations of ϕ and *m*. We note that the corresponding values to higher percentiles, for example, 90%, 95%, or 99%, can be employed as the critical value *L* at significance level of 10%, 5%, or 1%, respectively.

The distributions are also drawn in **Figure 5.1**. Through the plots, we notice the null distribution move its location and change its scale, depending on its choice of the aggregation order *m* and the model parameter ϕ. In general, the null distribution moves to left as *m* increases and this leftward location shift gets intense as ϕ increases. Also for given ϕ, the height of parabola is lower and the width is wider as *m* increases.

For the CUSUM test, the CUSUM test statistic is free from model parameters and there does not exist a modified CUSUM test statistic which reflects the aggregation model transformation shown in **Theorem 4.1**. So we can apply the CUSUM test statistic $\sup_{k=2,\ldots,n} |c_k|$ using c_k in (2.13) to the nonaggregate series and the aggregate series. First, we estimate the standard deviation of the series, σ_X, using the self-normalized estimator which is a better estimator as discussed in Shao and Zhang (2010). Then, we find the CUSUM test statistic in the simulated series of $m = 1$ and their aggregation of $m > 1$. We obtain 10,000 suprema under different conditions of ϕ and *m* and draw their distribution as the empirical null distribution of the CUSUM test statistic. The empirical distributions for all the choices of ϕ and m are illustrated in **Table 5.2** and **Figure 5.2**.

Table 5.1: Percentiles of the empirical null distribution for the LR test

ϕ	m	N	25%	50%	75%	90%	95%	99%
-0.50	1	1200	1.634	1.980	2.389	2.790	3.045	3.569
	2	400	1.550	1.893	2.304	2.712	2.973	3.490
	6	200	1.491	1.825	2.246	2.661	2.920	3.463
	12	100	1.413	1.750	2.171	2.608	2.861	3.385
0.30	1	1200	1.730	2.066	2.453	2.866	3.128	3.683
	2	400	1.601	1.938	2.340	2.755	3.025	3.601
	6	200	1.521	1.860	2.264	2.687	2.977	3.544
	12	100	1.433	1.774	2.180	2.617	2.904	3.483
0.50	1	1200	1.813	2.148	2.524	2.913	3.151	3.695
	2	400	1.632	1.975	2.367	2.770	3.016	3.548
	6	200	1.541	1.884	2.282	2.690	2.934	3.457
	12	100	1.444	1.787	2.190	2.619	2.863	3.402
0.80	1	1200	2.144	2.446	2.812	3.200	3.440	3.947
	2	400	1.809	2.132	2.516	2.929	3.174	3.734
	6	200	1.616	1.952	2.347	2.776	3.049	3.597
	12	100	1.470	1.807	2.223	2.657	2.926	3.510
0.95	1	1200	2.786	3.036	3.342	3.672	3.892	4.334
	2	400	2.353	2.625	2.968	3.325	3.544	4.013
	6	200	2.027	2.332	2.699	3.076	3.323	3.826
	12	100	1.725	2.049	2.435	2.846	3.109	3.615

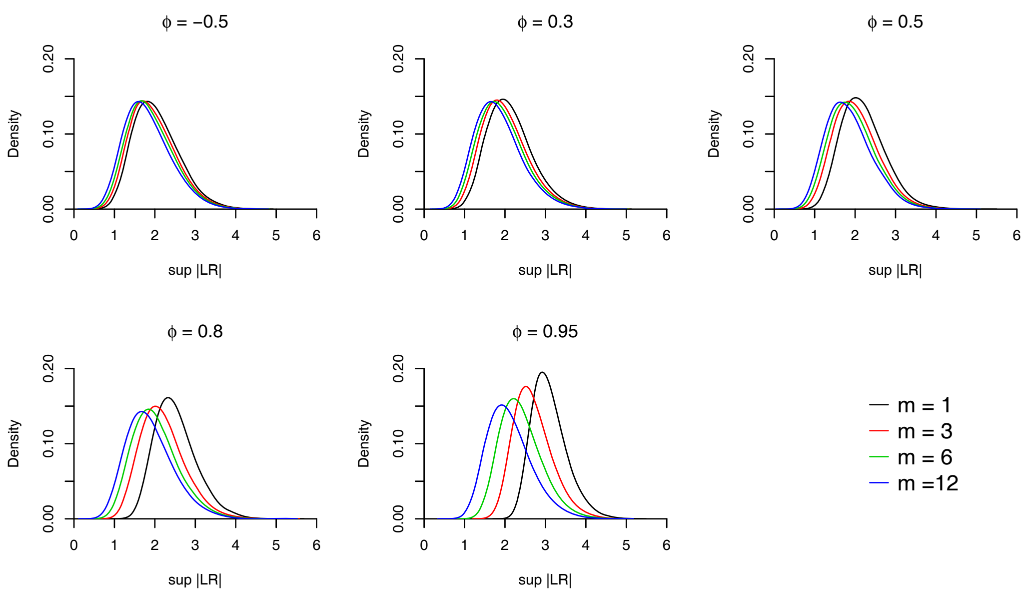

Figure 5.1: Empirical null distributions for the LR test

Table 5.2: Percentiles of the empirical null distribution for the CUSUM test

ϕ	m	N	25%	50%	75%	90%	95%	99%
-0.50	1	1200	2.323	3.141	4.387	5.774	6.673	8.716
	2	400	2.314	3.137	4.372	5.718	6.641	8.601
	6	200	2.340	3.164	4.369	5.720	6.590	8.504
	12	100	2.362	3.198	4.351	5.655	6.524	8.411
0.30	1	1200	2.149	2.928	4.183	5.656	6.620	8.831
	2	400	2.243	3.049	4.277	5.717	6.688	8.816
	6	200	2.317	3.130	4.343	5.735	6.695	8.787
	12	100	2.378	3.186	4.371	5.747	6.612	8.752
0.50	1	1200	2.096	2.883	4.158	5.629	6.608	8.809
	2	400	2.189	3.012	4.268	5.699	6.676	8.828
	6	200	2.272	3.111	4.336	5.761	6.723	8.812
	12	100	2.348	3.183	4.348	5.727	6.674	8.715
0.80	1	1200	2.061	2.844	4.104	5.574	6.618	8.824
	2	400	2.119	2.931	4.192	5.643	6.683	8.895
	6	200	2.198	3.026	4.288	5.697	6.723	8.968
	12	100	2.300	3.135	4.364	5.734	6.730	8.860
0.95	1	1200	2.052	2.931	4.338	5.941	7.166	9.785
	2	400	2.080	2.963	4.363	5.957	7.174	9.778
	6	200	2.118	3.012	4.411	5.987	7.186	9.840
	12	100	2.194	3.100	4.477	6.019	7.223	9.850

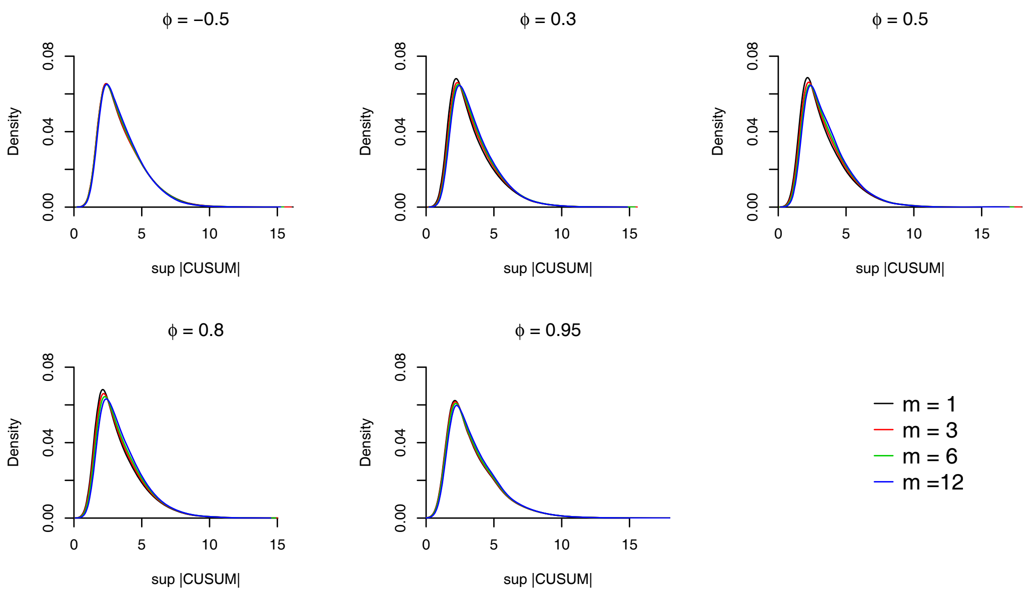

Figure 5.2: Empirical null distributions for the CUSUM test

In **Table 5.2**, we see that the percentiles change as *m* increases in given ϕ, which implies that the null distribution shifts as *m* increases. However the shift is relatively small. The decreasing rates of percentiles in **Table 5.2** are approximately 2.23% for $\phi = -0.5$, 0.12% for $\phi = 0.3$, -1.00% for $\phi = 0.5$, -1.69% for $\phi = 0.8$, and -0.80% for $\phi = 0.95$, when $m = 1$

increases to $m = 12$. In addition, the location changes are too small to be identified through plots in **Figure 5.2**. This result can be explained by Δ_m in (4.16). As we discussed in **Section 4.2**, Δ_m is too small to have effects on the changes. Thus the null distribution of the CUSUM test statistic almost keeps its location and scale even though *m* increases.

6. Concluding Remarks

In this paper, we analyze the temporal aggregation effects on a mean change of a time series. For the LR test, we propose a modified LR test statistic when aggregate data are used for testing. We show that the temporal aggregation leads the null distribution of the LR test statistic shifted to the left. In accordance with the distribution change, the test powers increase as the aggregation order *m* increases. Therefore we conclude that the temporal aggregation strengthens the LR test for a mean change in time series. However, to get this consistent result, our proposed modified LR test statistic needs to be used. For the CUSUM test, we show that it is free from the temporal aggregation effects. As a result, the CUSUM test may not get the benefit of the magnified mean change from temporal aggregation.

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