A Large Sample Statistical Test for Equality of Percentile Profiles Across Multiple Populations

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ABSTRACT

In large sample studies where distributions may be skewed and not readily transformed to symmetry, it may be of greater interest to compare different distributions in terms of percentiles rather than means. For example, it may be more informative to compare two or more populations with respect to their within population body mass index (BMI) distributions by testing the hypothesis that their corresponding respective 90th percentiles for BMI are equal. The test statistic is asymptotically distributed as Chi-square with degrees of freedom dependent upon the number of percentiles tested and constraints of the null hypothesis. Results are presented from a simulation study of the empirical probability of Type I errors and the empirical power of the test under various alternative underlying parameter values. Numerical examples are provided to illustrate applications.

Keywords: Asymptotic Chi-square test, Equality of percentiles, Large sample test, Nonparametric test.

1. INTRODUCTION

Comparison of two or more populations commonly involves the comparison of population means. Analysis of Variance (ANOVA) is often used for this situation. However, in large sample studies where distributions may be skewed and not readily transformed to symmetry, it may be of greater interest to compare different distributions in terms of percentiles rather than means. For example, it may be more informative to compare two or more populations with respect to their within population body mass index (BMI) distributions by testing the hypothesis that their corresponding respective 90th percentiles for BMI are equal. More generally, it may be of interest to compare populations with respect to percentile profiles where a percentile profile is defined as a set of two or more percentiles for observations on a random variable from the same population. For example, in this paper, we present a large sample statistic for testing the hypothesis that percentile profiles in two or more populations are identical. We begin in Section 2 with an overview of the methods used for this study.

2. METHODS

Let Y denote a continuous random variable of interest and let $Q_1, Q_2, \dots, Q_p$ denote a set of p percentiles (quantiles) that in some sense characterize the distribution of the random variable across its range. Further let $y_1, y_2, \dots, y_n$ represent a random sample of observations and let $q_1, q_2, \dots, q_p$ represent, respectively, the usual sample estimates of

Q_1, Q_2, … , Q_p. Suppose random samples are available from each of K populations with percentiles $\boldsymbol{Q}_h = Q_1, Q_2, \ldots, Q_p$, h = 1, 2, … , K and there is interest in testing the hypothesis that the percentile profiles are identical across the K populations; that is interest is in testing

$$H_0\colon \boldsymbol{Q}_1 = \boldsymbol{Q}_2 = \cdots = \boldsymbol{Q}_K$$

The following approach is an extension of the median test.

1. Combine the K samples and obtain the usual estimates of the population percentiles for the corresponding combined populations.
2. Let y denote an arbitrary observation in the combined sample. Use the combined sample percentile estimates to define $p+1$ categories or bins denoted bin_1, bin_2, … , bin_$p+1$, where bin1 = min(y) ≤ all $y \leq q_1$, bin2 = q_1< all $y \leq q_2$, … , bin_$p+1$ = q_p < all y ≤ max(y) where min(y) and max(y), respectively, represent the minimum and maximum observations in the sample.
3. Construct a K by $p+1$ dimensional contingency table where the h^{th} row (h = 1, 2, … , K) of the table is obtained by determining separately for each sample the number and percentage of observations in each bin (column) defined in step 2. Note that, the percentile profiles are identical across the K populations if and only if the underlying population percentages are identical within each column of the table.
4. Use a Chi-square statistic with $p(K-1)$ degrees of freedom to test for homogeneity of the profile of row percentages.
5. If $K \geq 3$, and percentile profiles for the $k \geq 2$ population subsets are to be compared, repeat steps 1-4 for the subsets.

3. ILLUSTRATIVE EXAMPLE

We used data from the 2011-2012 National Health and Nutrition Examination Survey (NHANES). NHANES is a program of studies designed to assess the health and nutritional status of adults and children in the United States. The survey is unique in that it combines interviews and physical examinations. The participants of interest here are Non-Hispanic Black and White men and women ages 20-79 years. The random variable of interest is Body Mass Index (BMI). It is calculated using the formula BMI = weight (in kg)/[height (in meter)]^2. We wished to test the hypothesis that the respective 2.5$^{\text{th}}$, 5$^{\text{th}}$, 10$^{\text{th}}$, 25$^{\text{th}}$, 50$^{\text{th}}$, 75$^{\text{th}}$, 90$^{\text{th}}$, 95$^{\text{th}}$, and 97.5$^{\text{th}}$ percentiles were equal in the subpopulations (1) white men, (2) white women, (3) black men, and (4) black women. We used SAS 9.4 to calculate cut-off points for defining BMI bins related to percentiles min(BMI), $q_{2.5}$, q_5, q_{10}, q_{25}, q_{50}, q_{75}, q_{90}, q_{95}, $q_{97.5}$, and max(BMI). These were calculated for the combined sample of all white and black adults, ages 20-79 years. Further, we enumerated for each of the K populations the number of adults with BMIs in categories or bins, denoted bin1, bin2, … , bin10, where bin1 = min(BMI) ≤ number of BMIs ≤ $q_{2.5}$, bin2 = $q_{2.5}$< number of BMIs ≤q_5, … , bin10 = $q_{97.5}$ < number of BMI's ≤ max(BMI). We then constructed a 4×10 contingency table where rows 1-4 contained the number of adults in each of the 10 bins for (1) white men, (2) white women, (3) black men, and (4) black women, respectively. The Chi-square test with 27 degrees of freedom was used to test the global hypothesis of homogeneity of the row percentages across the four rows or, equivalently, homogeneity or the four percentile profiles.

Following rejection of the global hypothesis outlined above, we repeated the process by conducting four pair-wise comparisons using the Bonferroni method for multiple comparisons. Thus, for illustrative purposes, we tested for homogeneity of percentile profiles for (1) white men versus white women, (2) by calculating the 9 percentiles of interest for the combined sample of all white adults. These percentiles coupled with the minimum and maximum were used to define 10 new bins. We then constructed a 2×10 contingency table and used a Chi-square test with 9 degrees of freedom to test for homogeneity of percentile profiles for white men and white women. These same steps were followed for each of the remaining 3 pair-wise comparisons. Note that the global test plus the four pair-wise comparisons comprised a total of 5 analyses.

To perform the tests for homogeneity of BMI percentile profiles, estimates of the percentile values of interest were calculated for various groups of U.S. adults as shown in Table 1. These percentiles were used to define bins as described above. The number of adults with BMI in each of the 10 bins was enumerated for each group as required for testing the five hypotheses described above. The corresponding percentages are given in Tables 2-6 and displayed graphically in Figures 1-5. The Chi-square value, degrees of freedom and *p*-value are shown below each of Tables 2-6. All tests are statistically significant (all $p < 0.0002$) except that comparing homogeneity of profiles for white versus black males ($p = 0.1093$).

Table 1. Body Mass Index Percentile Profiles for Groups of U.S. Adults

Group	*n*	*P0*	*P2.5*	*P5*	*P10*	*P25*	*P50*	*P75*	*P90*	*P95*	*P97.5*	*P100*
Total Sample	3299	15.7	19.0	20.3	21.8	24.4	28.2	33.2	38.8	42.7	47.1	82.1
Men	1723	15.4	19.2	20.3	21.8	24.0	27.3	31.2	36.0	39.2	42.8	66.2
Women	1773	13.4	18.5	19.6	21.1	23.8	28.0	33.4	39.2	43.3	47.7	82.1
White Adults	2041	15.7	18.9	20.0	21.4	23.9	27.6	31.9	37.3	40.8	44.6	80.6
White Men	1025	15.7	19.6	20.7	22.2	24.4	27.7	31.3	36.1	39.4	43.4	66.2
White Women	1016	15.7	18.3	19.4	20.8	23.6	27.5	32.9	38.7	42.5	46.4	80.6
Black Adults	1455	15.8	19.2	20.6	22.3	25.1	29.3	34.4	40.4	45.0	50.0	82.1
Black Men	698	15.8	18.6	20.0	21.9	24.2	27.8	32.3	37.7	41.2	45.6	63.3
Black Women	757	17.4	20.2	21.5	22.8	26.2	31.0	36.0	42.5	47.8	52.7	82.1

Table 2. Adults across BMI-percentile Defined Bins by Gender and Race, NHANES, 2011-2012

	Bin									
Group	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
White Men	1.4	2.5	5.1	16.3	29.9	27.1	11.6	3.4	1.6	1.1
White Women	3.7	3.5	7.2	16.0	23.2	22.4	14.2	5.0	2.9	1.9
Black Men	3.2	2.7	3.3	17.2	26.9	25.1	14.1	3.8	1.7	2.2
Black Women	1.7	1.1	3.5	9.7	19.3	25.7	21.6	8.2	3.9	5.3

Chi-square = 176.6, df = 27,
$p < 0.0001$

Table 3. Females across BMI-percentile Defined Bins by Race, NHANES, 2011-2012

	Bin									
Group	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
White Women	3.3	3.6	6.8	17.9	26.2	22.9	12.2	4.0	2.0	0.6
Black Women	1.4	1.1	2.8	11.1	23.5	27.8	18.8	6.3	3.2	2.8

Chi-square = 78.92, df = 9,
$p < 0.0001$

Table 4. Males across BMI-percentile Defined Bins by Race, NHANES, 2011-2012

	Bin									
Group	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
White Men	1.6	2.4	5.4	15.3	25.8	26.2	14.5	4.8	2.0	2.0
Black Men	3.8	2.7	4.2	14.7	23.9	23.3	15.7	5.3	3.5	3.1

Chi-square = 16.95, df = 9,
$p = 0.1093$

Table 5. Blacks across BMI-percentile Defined Bins by Gender, NHANES, 2011-2012

	Bin									
Group	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Black Men	3.2	3.3	5.1	19.5	28.3	23.3	10.9	3.3	1.5	1.6
Black Women	1.7	1.8	4.9	10.9	22.1	26.6	18.8	6.5	3.6	3.2

Chi-square=62.20, df = 9,
$p < 0.0001$

Table 6. Whites across BMI-percentile Defined Bins by Gender, NHANES, 2011-2012

	Bin									
Group	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
White Men	1.4	2.0	4.3	15.0	27.1	28.2	14.4	4.2	1.8	1.7
White Women	3.5	3.0	5.8	15.1	22.9	21.8	15.8	5.9	3.1	3.3

Chi-square=35.18, df = 9,
$p = 0.0002$

Figure 1. Distribution of adults across BMI-percentile defined bins by gender and race, NHANES, 2011-2012

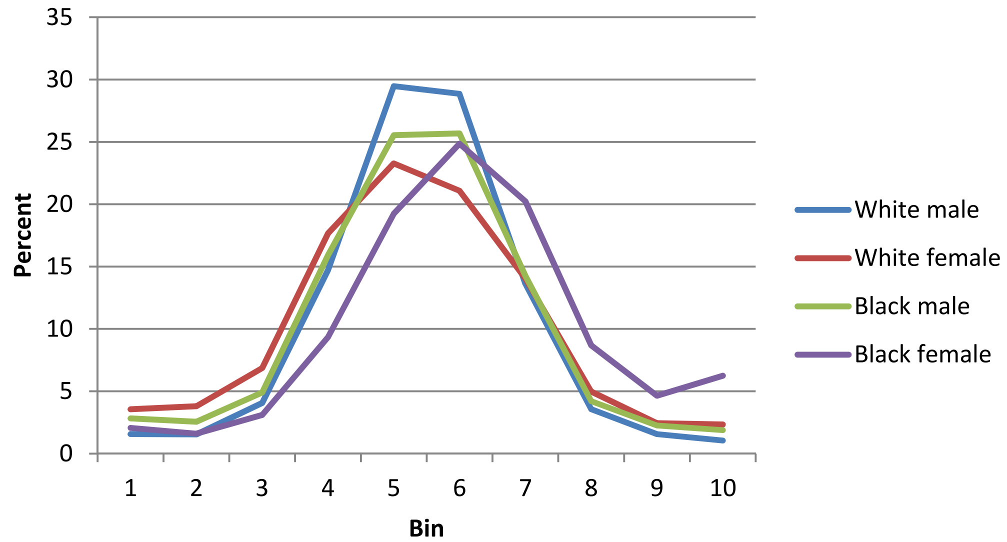
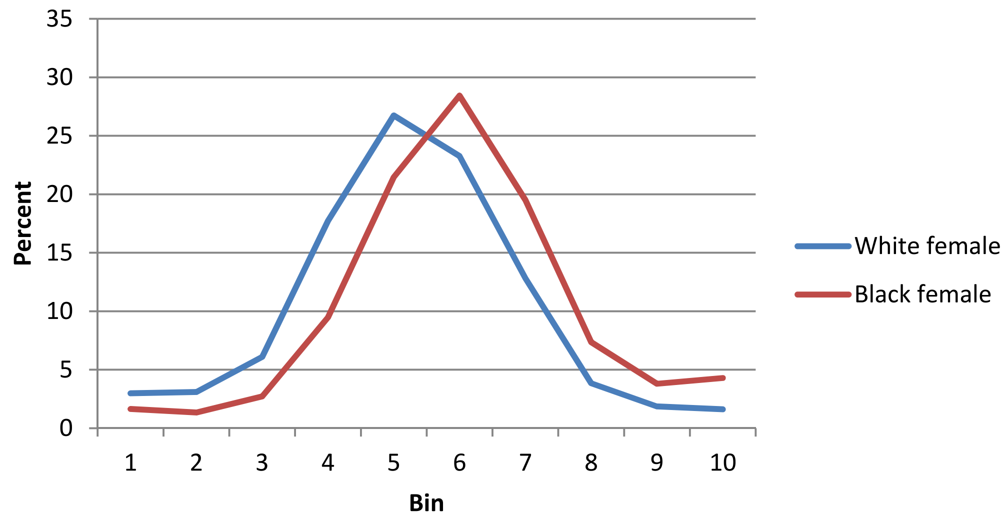


Figure 2. Distribution of women across BMI-percentile defined bins by race, NHANES, 2011-2012



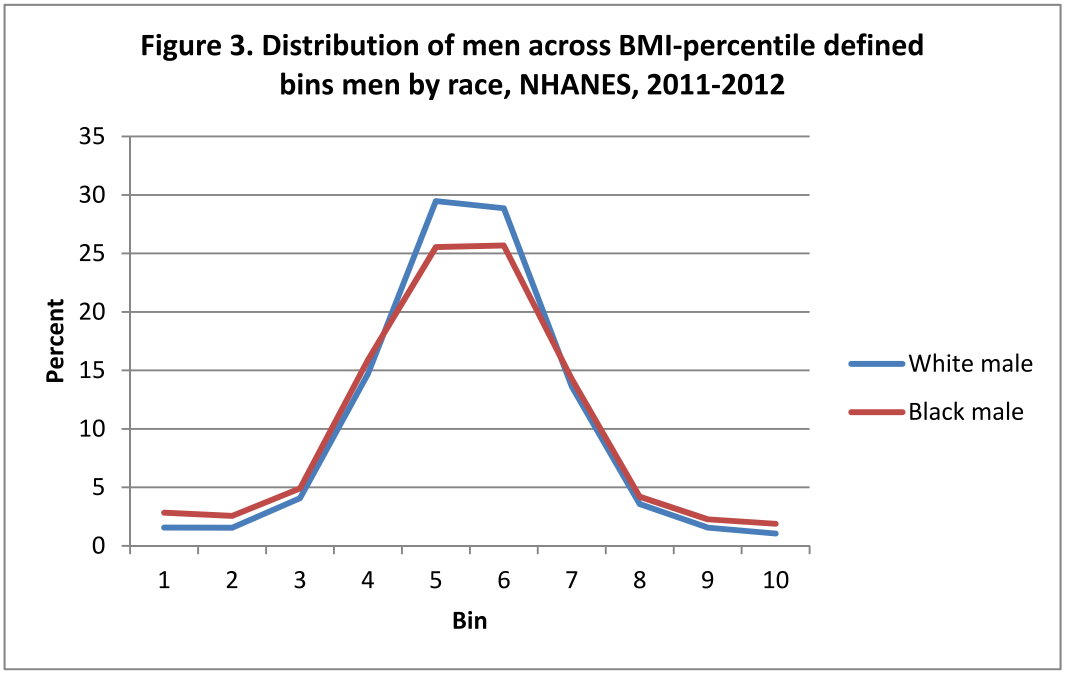


Figure 3. Distribution of men across BMI-percentile defined bins men by race, NHANES, 2011-2012
Percent
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Bin
White male
Black male

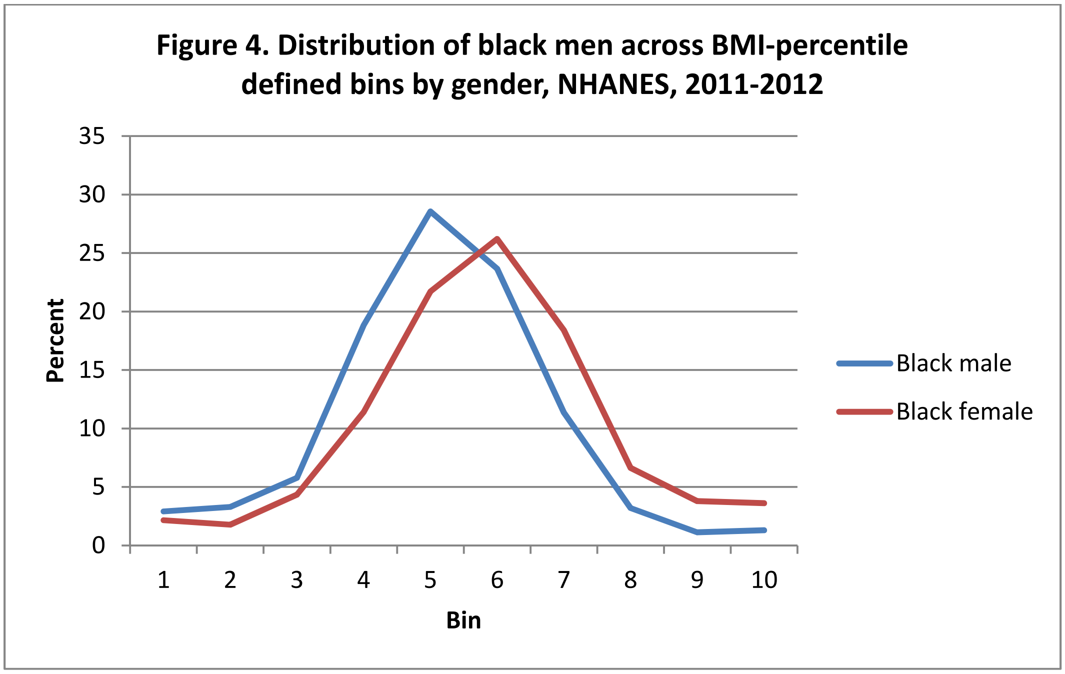
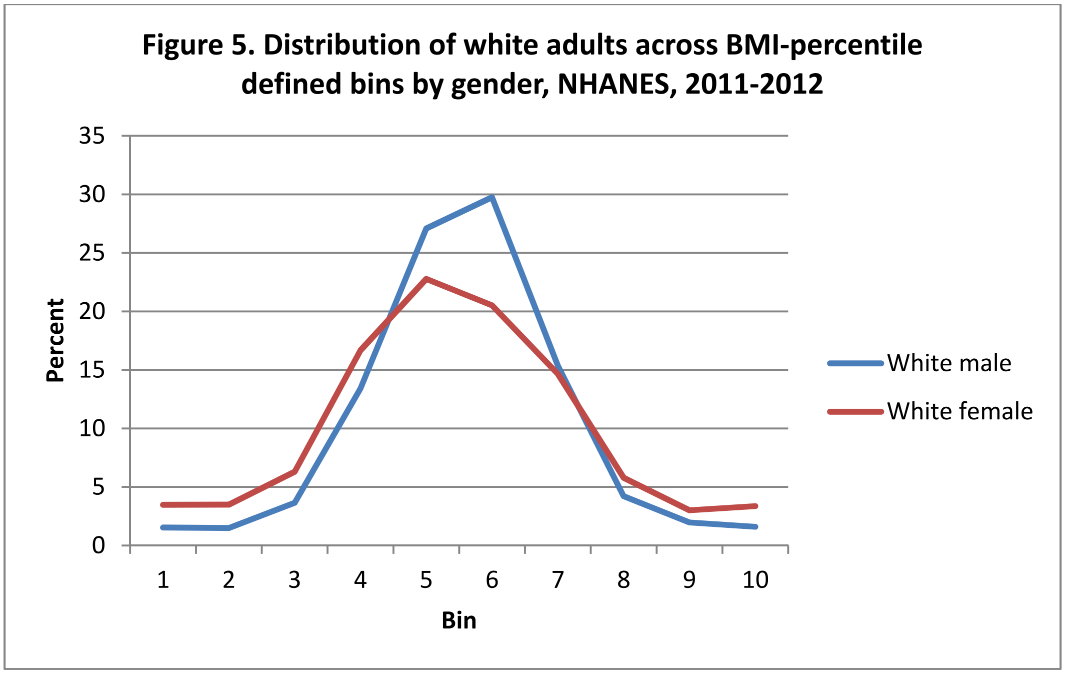


Figure 4. Distribution of black men across BMI-percentile defined bins by gender, NHANES, 2011-2012
Percent
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Bin
Black male
Black female



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