The Current State of Bayesian Image Analysis

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Abstract

Data: The Visual Field Map (VFM) is obtained by activating the visual cortex in the brain with a dynamic target presented to the subject. Changes in the visual system can be simulated with images that have different size wedges (0, 18, 27, 36, 45 and 90 degrees) removed from a circular disk which is presented to the subject's eye. The output of the visual system is a set of activated voxels (from 295 to 619) in the visual cortex determined by functional MRI, which then is used to induce a figure on a virtual circular retina using an (r, theta) representation for the location. The virtual retina will have more points in the center as does the real retina.

Methods: A Bayesian non-parametric spatial model, a spatial Dirichlet Process model, is used to model the ratio of two different images induced by the two different angular wedges. Gelfand, Kottas and MachEachern (JASA, 2005) introduced a Dirichlet Process as a prior mixing distribution on the family of densities DP(v G). Under the null hypothesis of no difference, the ratio of the two densities will have a constant posterior density. Deviations from this will be used to indicate the probability of a perturbed visual system.

Key Words: Dirichlet; Spatial Statistics

1. Bayesian Inference for Spatial Processes

There are two types of inference for spatial fMRI data: estimation of shapes and hypothesis testing of differences in the shapes under different conditions. Since the shapes are often irregular a spatial non-parametric model with some potential smoothing is needed. The most flexible non-parametric model is the Dirichlet Process mixture model.

This generalized non-parametric spatial model is a Bayesian non-parametric spatial model where the number of components can vary as the number of points. Gelfand, Kottas and MacEachern (2005) Introduced a Dirichlet Process as a prior mixing distribution on a family of densities $DP(\nu G_0)$.

Central to the DP is the notion of a random probability measure on the space of distribution functions defined on the space Θ (with σ field B).

The family $DP(\nu\ G_0)$ is indexed by $\nu > 0$, a scalar precision parameter that controls the amount of clustering in the spatial measure on Θ and G_0 which is the specified base distribution. We will assume that G_0, the base density, is bivariate normal, with constant variance. Although DP(ν G0) is almost certainly discrete on the set of observed points, $Y(s_1) \ldots Y(s_N)$, it can be countably or uncountably infinite because of the order ($\aleph_1$) of the space we are modeling.

This gives the representation (it is almost surely discrete) of

$$\sum_{i=1}^{\infty} \omega_i \delta_i\,(\theta_i)$$

Where θ_i is the location of the point mass. Because this gives a model that is too bumpy, a DDP, a Dependent Dirichet Process, is actually used to smooth the process. Instead of having i represent each of the points in {Y(s)}, it represents the index value, or the center of the bivariate normal distribution, G_0.

2. Example: Visual Field Maps

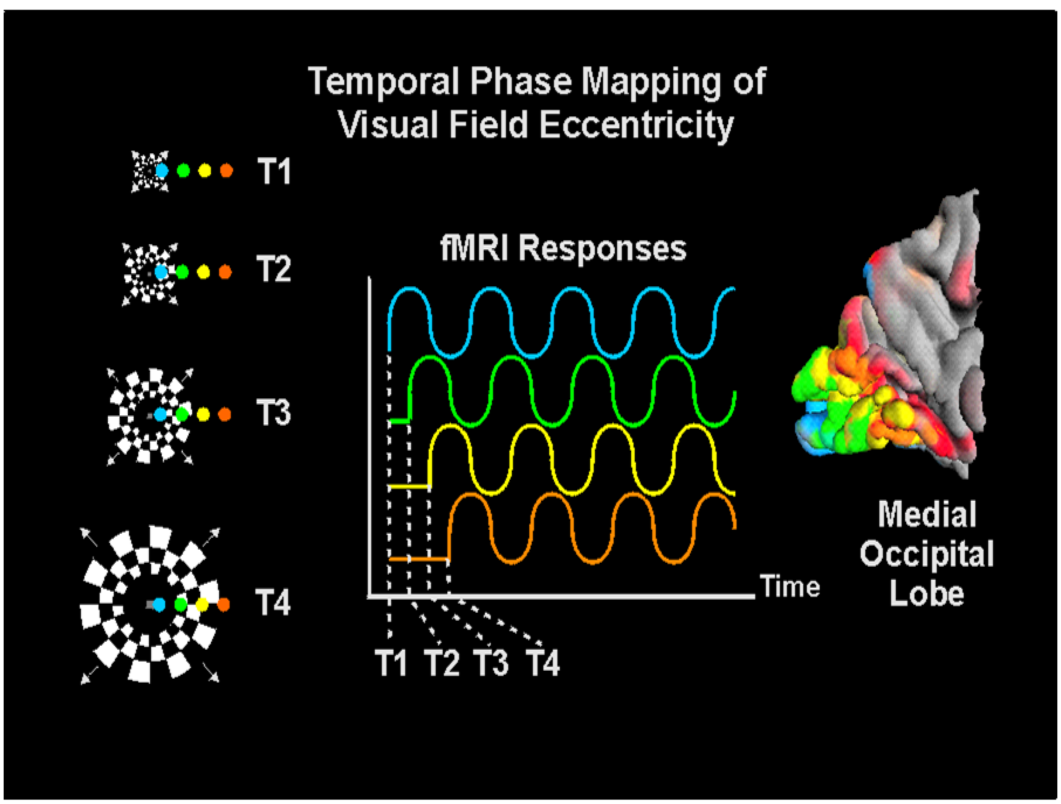


Figure 1: Constructing a visual field map (VFM) – step 1

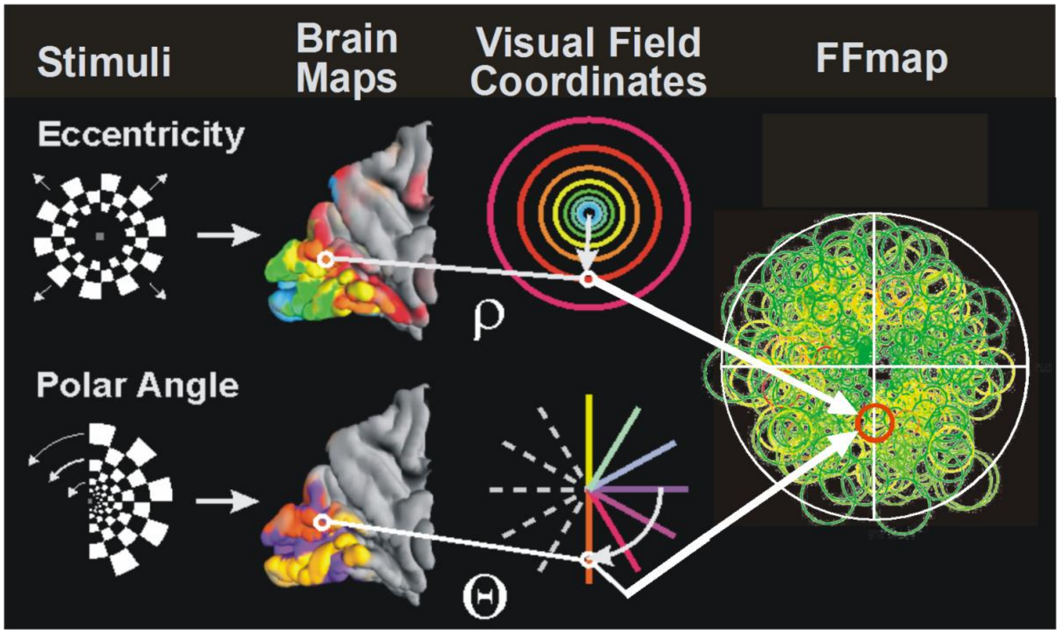


Figure: 2 constructing the visual field map – step 2.

Wedges were used to mask part of the visual field when the data was collected from a subject. The wedges were of sizes 0 degrees, 18 degrees, 27 degrees, 36 degrees, 45 degrees and 90 degrees. The points below from the VFM in figure 3 are induced by the effect of the stimulus on the retina and then the area of activation in the cortex.

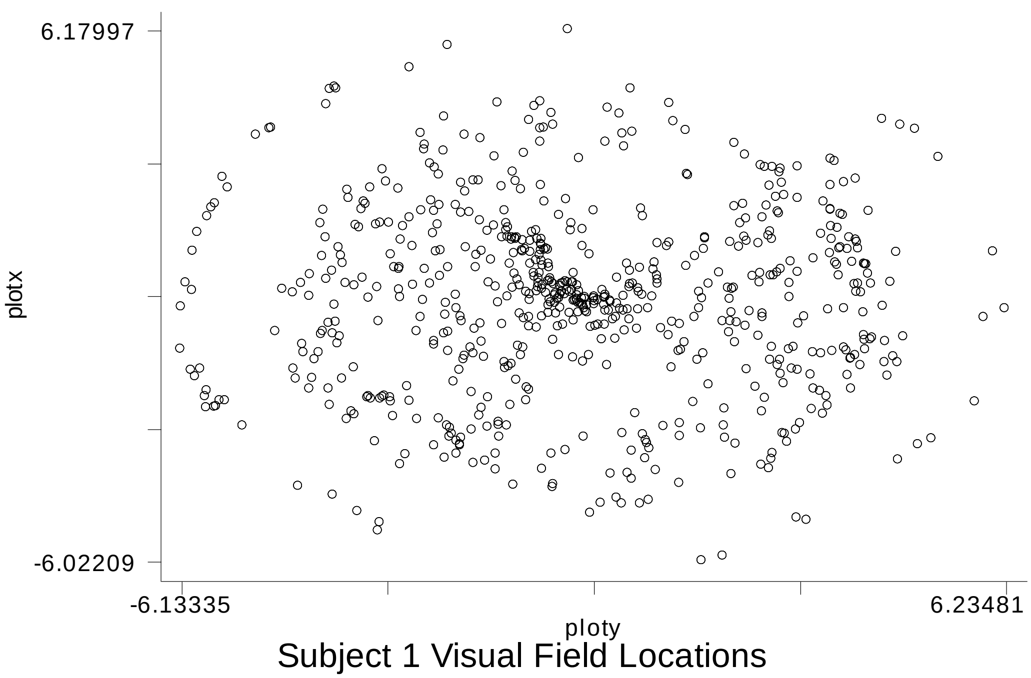


Figure 3: Points from a visual field map (VFM). Note the points in the plane are not uniform. They are clustered towards the middle just like the points of the retina.

The actual set of points with different masks (0 degrees, 45 degrees and 90 degrees as displayed below. Because of the (i) the "**winner take all"** rule for assigning activation, (ii) the voxel may span two different regions and (iii) the noise involved in a real fMRI signal some of the points in figure 4 actually appear to be in the gap where no data is found.

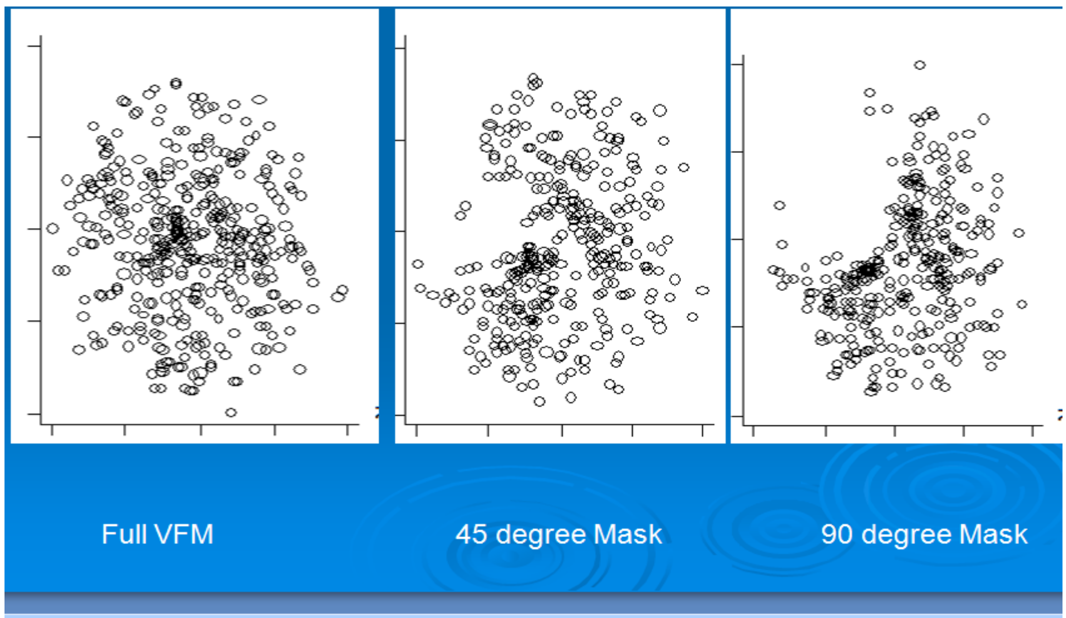
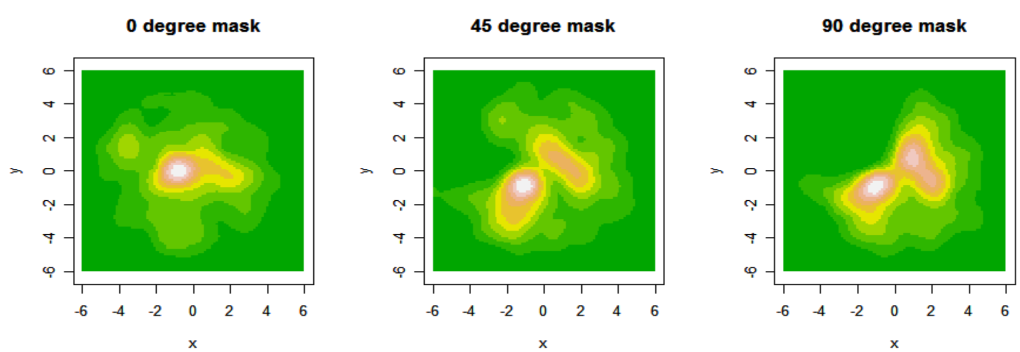


Figure 4. The effect of different masks on the same person.

3. Estimation

The intensity is not monotone decreasing from the center of the retina; because of the visual features like the blind spot, as well as the individual pattern of the retinal response. The DPM (Dirichilet Process Mixture Model) will smooth the Dirichlet process which is very irregular. The advantage of the DPM is that it provides a smooth transition without ringing when there is a sharp change in the surroundings - like a mask.



4. Hypothesis Testing

Bayesian hypothesis testing uses Bayes Factors to test whether two images (models) produce the same inference. Namely is

$$f_0(y \mid \theta_0, 0) = K\, f_1(y \mid \theta_1, 1)$$

In other words, are the posterior distributions the same. Using more complex models - with the appropriate combination of the Bayes Factors , more than just two images can be compared.

5. Conclusions

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