Sieve Bootstrap-Based Prediction Intervals for Autoregressive Processes with GARCH Innovations

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Abstract

A sieve bootstrap-based method for obtaining prediction intervals for autoregressive processes with innovations following a *GRACH* volatility structure is proposed. Re-sampling is done on residuals obtained after a two-stage process which fits an *AR* model first and *GARCH* parameters estimated from the residuals of the *AR* model. Both the orders of the *AR* and *GARCH* processes are considered unknown and are estimated using the AIC and AICC criterion, respectively. This is in contrast to an existing method for *ARMA-GARCH* processes that assumes both the *ARMA* and *GARCH* orders. The proposed method produces intervals that are conditional on the observed data and the interval width is allowed to vary with the conditional variance predicted for the forecast period. A Monte-Carlo simulation study shows that the proposed method produces intervals with coverage probabilities reasonably close to the nominal level.

Key Words: Conditional Heteroskedasticity, Forecast Intervals, Residual Bootstrap, *ARMA-GARCH* Models, Volatility

1. Introduction

Empirical time series that exhibit conditional heteroskedasticity is quite common in areas such as finance. Such behaviour is commonly modelled using the well-known *ARCH* and *GARCH* models introduced by Engle (1982) and Bollerslev (1986), respectively. *GARCH* processes in particular have drawn the attention of many researchers and diverse modified versions of these processes have been introduced. Exponential *GARCH* (Nelson, 1991), Non-linear *GARCH* (Engle and Ng, 1993), and Integrated *GARCH* (Baillie et *al*, 1996) are some of the widely available modified versions of the regular *GARCH* process.

For practitioners, one important aspect of modelling empirical time series is obtaining point or interval forecasts. Studies which address the problem of point forecasting are prevalent in time series literature (for example Baillie and Bollerslev (1992), Anderson et *al*. (2001), Engle and Patton (2001), and Poon (2005)). However, there is relatively less work in the area of interval forecasting. This is especially true in the case of processes with conditionally varying volatility. Pioneers in developing prediction intervals for heteroskedastic processes are Pascual, Romob, and Ruiz (2005). They proposed a bootstrap procedure to compute prediction intervals for both returns and volatility of

GARCH(1,1) processes. More recently, Chen et *al*. (2011) developed a computationally faster method to generate prediction intervals for pure *GARCH* processes. While the above processes only looked at pure *GARCH* processes, Goncalves and Kilian (2007) established the asymptotic validity of the bootstrap method to *AR*(∞) processes with conditional volatility when the re-sampling is done without modeling the heteroskedasticity.

The objective of this study is to propose prediction intervals for autoregressive processes with *GARCH* errors, with the property that the interval width changes with the conditional heteroskedasticity found at the time of prediction. Specifically, we would like to obtain prediction intervals that are wider when the conditional heteroskedasticity is high and narrower when the conditional heteroskedasticity is low. Some work along these lines has already been done. For example Shimizu (2013) introduced bootstrap-based prediction intervals for *ARMA-GARCH* processes and the reader is referred to Shimizu (2010) for the theoretical justification for this method.

Our proposed procedure differs from Shimizu's method in two important ways. First, he assumes the orders p, q of the $ARMA(p, q)$ process whereas our method uses the AIC criterion to estimate the unknown order of the *AR* portion of an *AR-GARCH* model. Second, we also estimate the orders r, s of the $GARCH(r, s)$ formulation using the AICC criterion. In contrast, Shimizu assumes that r and s are known. Thus, the proposed procedure is more general as well as practical compared to Shimizu's method. Moreover, Shimizu reports simulation results for a limited number of *AR-ARCH* processes while the results presented herein cover a larger set of *AR-GARCH* models.

The rest of this paper is organized as follows. The sieve bootstrap procedure is introduced in Section 2. In Section 3 results of a Mote-Carlo simulation are presented. An application of the proposed method is presented in Section 4 and we conclude this paper in Section 5 with a discussion of the results.

1.1 The Sieve Bootstrap (SB) Procedure

This procedure was first introduced by Buhlmann (1997). It re-samples the residuals obtained by fitting an $AR(p)$ model, and assumes that the order, p, of the process is obtained using some criterion such as AIC from among models with $p = 1, 2, ..., p_{\max}$ where $p_{\max} \to \infty$ as $n \to \infty$. Alonso adopted this technique in a sequence of publications appearing in 2002, 2003, and 2004 to obtain prediction intervals for *ARMA* processes. Mukhapadhyay et *al.* (2010) used a modified version of this procedure to generate bootstrap-based prediction intervals for invertible time series. Furthermore, Rupasinghe and Samaranayake (2012) extended the application of SB prediction intervals to *FARIMA* processes. The primary advantage of the SB method is that it does not require knowledge of the orders associated with the underlying process because it always fits an $AR(p)$ model with p estimated using some criterion.

1.2 Model

The autoregressive model with *GARCH* innovations we consider in this study is as follows:

$$\phi(B)x_t = u_t,$$

$$\text{where } \phi(z) = 1 - \phi_1 z - \ldots - \phi_p z^p \text{ and} \tag{1}$$

$$u_t = \sigma_t \varepsilon_t,\ t \in Z,$$
$$\sigma_t^2 = a_0 + \sum_{i=1}^{r} a_i u_{t-i}^2 + \sum_{j=1}^{s} b_j \sigma_{t-j}^2,$$

where $\{\varepsilon_t\}_{t=1}^{n}$ is a sequence of independent, identically distributed $(i.i.d)$, random variables with zero mean, unit variance, with $E(\varepsilon_t^3) = 0,$ and $E(\varepsilon_t^4) = k < \infty$. Additionally we assume that $a_0, a_i,$ and b_j are unknown parameters satisfying $a_0 > 0,\ a_i \geq 0,$ and $b_j \geq 0,$ for $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, s.$ Moreover, assume that the weakly stationary property of $GARCH(r, s)$, namely $\sum_{i=1}^{m}(a_i + b_i) < 1$, presented by Tsay (2002), is satisfied and that $\phi(z) = 0$ implies $|z| > 1.$

2. The Proposed Procedure

Step 1. Following Alonso (2003), select a maximum order $p_{\max} = c\left((n/\log(n))^{1/(2r+2)}\right)$ where $r > 2,$ for some $c > 0.$ Note that this is a large sample order and c must be chosen sufficiently large $(\sim 15 \text{ to } 20)$ for the method to work. We used $p_{\max} = 20,\ 27$ for $n = 300,\ 1{,}000$respectively. Then, find the optimal order, $\hat{p},$ using the AIC criterion among the values $p = 1, 2, ..., p_{\max}$.

Step 2. Using Yule-Walker or Least-Squares method, estimate the coefficients $\hat{\phi}_1, ..., \hat{\phi}_{\hat{p}}$ of the $AR(\hat{p})$ process. The Yule-Walker method was employed in this study following the example of Alonso et *al*. (2002, 2003, and 2004) and Rupasinghe and Samaranayake (2012).

Step 3. Compute the $(n - \hat{p})$ residuals using $\tilde{\varepsilon}_t = \sum_{j=1}^{\hat{p}} \hat{\phi}_j (X_{t-j} - \bar{X}),$
where $\hat{\phi}_0 = 1,\ t \in (\hat{p}, ..., n)$, and $\bar{X}$ is the mean of the process $\{X_t\}_{t=1}^{n}$.

Step 4. Center the residuals if using the Yule-Walker method (Thombs and Schucany, 1990). Denote the centered residuals by $\hat{\varepsilon}_t;\ t \in (\hat{p}, \ldots, n),$ so $\hat{\varepsilon}_t = \tilde{\varepsilon}_t - \bar{\tilde{\varepsilon}}_t,$ where $\bar{\tilde{\varepsilon}}_t = (n - \hat{p})^{-1} \sum_{t=\hat{p}}^{n} \tilde{\varepsilon}_t$.

Step 5. Using the centered residuals and the AICC criterion, estimate the orders $\hat{r}$ and $\hat{s}$ of the $GARCH(r,s)$ model.

Step 6. Determine the Maximum Likelihood Estimates of the *GARCH* coefficients $\hat{\alpha}_0$, $\hat{\alpha}_1,...,\hat{\alpha}_{\hat{r}}$, and $\hat{\beta}_1,...,\hat{\beta}_{\hat{s}}$ using the residual process $\{\hat{\varepsilon}_t\}$.

Step 7. Generate the error process $\{v_t\}$ by first creating a conditional variance process $\{\hat{\sigma}_t^2\}$, using the estimated *GARCH* coefficients, $\hat{\alpha}_0$, $\hat{\alpha}_1,...,\hat{\alpha}_{\hat{r}}$, and $\hat{\beta}_1,...,\hat{\beta}_{\hat{s}}$, and the relationship $v_t = \hat{\varepsilon}_t / \hat{\sigma}_t$; for $t \in \{\hat{m}, \ldots, n\}$, where $\hat{m} = \max(\hat{r}, \hat{s})$.

Step 8. Center the v_t's to obtain

$$\hat{v}_t = v_t - \bar{v}_t;\ t \in \{\hat{m},...,n\},\ \text{where}\ \bar{v}_t = (n-\hat{m})^{-1}\sum_{t=\hat{m}}^{n} v_t .$$

Step 9. Denote the empirical distribution of the centered error process $\{\hat{v}_t\}_{t=\hat{m}}^{n}$ by

$$\hat{F}_{v,n}(\hat{\varepsilon}) = \sum_{t=\hat{m}+1}^{n} 1_{\{\hat{v}_t \le \hat{\varepsilon}\}}.$$

Step 10. Obtain a bootstrap *GARCH* error process $\{\varepsilon_t^*\}_{t=1}^{n+150}$ by first creating a bootstrap conditional variance process $\{\hat{\sigma}_t^{*2}\}$ using the estimated *GARCH* coefficients in Step 6 and then using $\varepsilon_t^* = \hat{v}_t^* \sqrt{\hat{\sigma}_t^{*2}}$; $t = 1:n+150$. Here, $\{\hat{v}_t^*\}_{t=1}^{n+150}$ is obtained from $\{\hat{v}_t\}_{t=\hat{m}}^{n}$ by sampling with replacement.

Step 11. Estimate the *GARCH* parameters $\hat{\alpha}_0^*, \hat{\alpha}_1^*, ..., \hat{\alpha}_{\hat{r}}^*$, and $\hat{\beta}_1^*,...,\hat{\beta}_{\hat{s}}^*$ of the process $\{\varepsilon_t^*\}$ using Maximum Likelihood Estimation.

Step 12. Using the estimates $\hat{\alpha}_0^*, \hat{\alpha}_1^*, ..., \hat{\alpha}_{\hat{r}}^*$, and $\hat{\beta}_1^*,...,\hat{\beta}_{\hat{s}}^*$ obtained in Step 11 and employing a method similar to that in Step 10, create a bootstrap *GARCH* error process and denote it by $\{\varepsilon_t^{**}\}$.

Step 13. Re-create an autoregressive (*AR*) process with *GARCH* errors, $\{\varepsilon_t^{**}\}$, created in Step 12 and using the relationship

$$X_t^* - \bar{X} = -\sum_{j=1}^{\hat{p}} \hat{\phi}_j \left(X_{t-j}^* - \bar{X}\right) + \varepsilon_t^{**},\ \text{where}\ X_t^* = \bar{X}\ \text{for}\ t = 1,2,...,\hat{p}.$$

Note that we generate $n+150$ ε_t^{**} and X_t^* values and discard the first 150 to minimize the effect of the initial conditions.

Step 14. Fit an $AR(\hat{p})$ model to $\{X_t^*\}_{t=1}^n$ using Yule-Walker method and denote the estimated AR coefficients by $\hat{\phi}_1^*, \ldots, \hat{\phi}_{\hat{p}}^*$.

Step 15. Using the *GARCH* coefficients $\hat{\alpha}_0^*, \hat{\alpha}_1^*, \ldots, \hat{\alpha}_{\hat{r}}^*$, and $\hat{\beta}_1^*, \ldots, \hat{\beta}_{\hat{s}}^*$ estimated in Step 11 create *k*-step ahead bootstrap *GARCH* error process $\{\varepsilon_{n+k}^{**}\}$, and then using the estimated coefficients $\hat{\phi}_1^*, \ldots, \hat{\phi}_{\hat{p}}^*$, generate *k*-step ahead bootstrap observations by recursion as follows:

$X_{n+k}^* - \bar{X} = -\sum_{j=1}^{\hat{p}} \hat{\phi}_j^* \left(X_{n+k-j}^* - \bar{X}\right) + \varepsilon_{n+k}^{**}$, where $k > 0$ and $X_t^* = X_t ; t \le n$.

Note that the bootstrap distribution of X_{n+k} should be conditioned on the originally observed data rather than on the bootstrap data. Thus, set $X_t^* = X_t$ for $t \le n$, as suggested by Cao et *al*. (1997) and Alonso et *al*. (2002, 2004).

Step 16. Obtain the bootstap distribution of X_{n+k}, denoted by $F_{\hat{X}_{n+k}^*}^*(.)$, by repeating Steps 10 to 15 *B* times, where *B* = 1,000.

Step 17. A $100(1-\alpha)\%$ prediction interval for X_{n+k} is then given by:

$\left[Q^*(\alpha/2), Q^*(1-\alpha/2)\right]$, where $Q^*(k) = \hat{F}_{\hat{X}_{n+k}^*}^{*-1}(k)$ is the k^{th} percentile of the estimated bootstrap distribution of $\{X_t\}$.

3. Monte-Carlo Simulation

A Monte-Carlo simulation study was carried out with two different error distributions ($N(0,1)$ and *t* with 5 degrees of freedom) for sample sizes 300 and 1,000. Following models were used in creating the heteroskedastic error structure for *GARCH* errors:

Model I:

$$\sigma_t^2 = 0.1 + 0.4\varepsilon_{t-1}^2,$$

Model II: (2)

$$\sigma_t^2 = 0.05 + 0.1\varepsilon_{t-1}^2 + 0.85\sigma_{t-1}^2,$$

Model III:

$$\sigma_t^2 = 0.05 + 0.3\varepsilon_{t-1}^2 + 0.15\varepsilon_{t-1}^2 + 0.25\sigma_{t-1}^2 + 0.1\sigma_{t-1}^2.$$

Note that the first two of the above models were employed by Chen et *al.* (2011) in their simulation study.

We considered *AR*(1) and *AR*(2) processes in the simulation study with error structures given by Models I, II and III in (2).

The AR models considered were:

$$x_t = \phi_1 x_{t-1} + \varepsilon_t,$$
$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \varepsilon_t,$$

with $\phi_1 \in \{0, 0.4, 0.6, 0.9, 1.11\}$ and $\phi_2 \in \{0, 0.2, 0.4, 0.6\}$.

For each combination of model, sample size, nominal coverage, and the error distribution, N = 1,000 independent time series were generated, and for each series Steps 1 through 16 were carried out.

When computing the coverage probabilities of these N = 1,000 simulations, R = 1,000 future observations $\{X_{n+k}\}$ per each simulation were generated.

The coverage for the i^{th} simulation run is given by:

$$C(i) = R^{-1} \sum_{r=1}^{R} I_A\left[x_{n+k}^r(i)\right], \text{ where } A = \left[Q^*(\alpha/2), Q^*(1-\alpha/2)\right].$$

The bootstrap length and the theoretical length for the i^{th} simulation run are given by:

$$L_B(i) = Q^*(1-\alpha/2) - Q^*(\alpha/2) \text{ and } L_T(i) = x_{n+k}^r(1-\alpha/2) - x_{n+k}^r(\alpha/2),$$

respectively.

Also, the following equations were used for appropriate calculations:

Mean Coverage: $\overline{C} = N^{-1} \sum_{i=1}^{N} C(i)$,

Standard Error of Mean Coverage: $SE_{\overline{C}} = \left\{\left[N(N-1)\right]^{-1} \sum_{i=1}^{N} \left[C(i) - \overline{C}\right]^2\right\}$,

Mean Bootstrap Length: $\overline{L}_B = N^{-1} \sum_{1=1}^{N} L_B(i)$,

Standard Error of Mean Length: $SE_{\overline{L}_B} = \left\{\left[N(N-1)\right]^{-1} \sum_{i=1}^{N} \left[L_B(i) - \overline{L}_B\right]^2\right\}$,

Mean Theoretical Length: $\overline{L}_T = N^{-1} \sum_{1=1}^{N} L_T(i)$.

The simulation results show that in most cases the proposed method provides nominal or near nominal coverage, with mean interval length close to the theoretical length. The coverage probabilities are usually slightly below nominal for one-step-ahead prediction for sample size 300 but the coverage improves for sample size 1,000. In fact, the coverage is approximately 95% when rounded to two decimal places for lead lengths greater than 1 or for sample size 1,000. When one of the roots of the autoregressive polynomial is close to unity, however, the coverage falls well below nominal for all lead lengths when the sample size is 300 (see Table 5). The coverage does improve when the sample size increases to 1,000.

Following tables present a subset of the simulations results we obtained in this study:

Table 1: Coverage of 95% intervals for $(1-0.4B)x_t = u_t$ with Model I and normal errors. *AR* Root : 2.5

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	1.5492	0.9434 (0.0013)	1.5466 (0.0250)
	1000	1.5343	0.9469 (0.0005)	1.5302 (0.0144)
10	300	1.6904	0.9482 (0.0034)	1.7689 (0.0094)
	1000	1.6941	0.9486 (0.0019)	1.7630 (0.0036)
20	300	1.6910	0.9480 (0.0037)	1.7698 (0.0096)
	1000	1.6799	0.9509 (0.0018)	1.7609 (0.0036)

Table 2: Coverage of 95% intervals for $(1-0.9B)x_t = u_t$ with Model I and normal errors. *AR* Root : 1.11

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	1.5559	0.9416 (0.0014)	1.5504 (0.0251)
	1000	1.5449	0.9464 (0.0005)	1.5440 (0.0189)
10	300	3.3802	0.9371 (0.0027)	3.4032 (0.0251)
	1000	3.3746	0.9466 (0.0012)	3.4455 (0.0091)
20	300	3.5976	0.9346 (0.0037)	3.6200 (0.0303)
	1000	3.5882	0.9462 (0.0012)	3.6514 (0.0103)

Table 3: Coverage of 95% intervals for $(1-0.2B)x_t = u_t$ with Model II and normal errors. *AR* Root : 5

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	3.8189	0.9418 (0.0015)	3.7924 (0.0451)
	1,000	3.8164	0.9459 (0.0005)	3.7980 (0.0270)
10	300	3.8938	0.9458 (0.0026)	3.9883 (0.0355)
	1,000	3.8848	0.9487 (0.0013)	3.9821 (0.0196)
20	300	3.8867	0.9457 (0.0030)	4.0302 (0.0339)
	1,000	3.9149	0.9471 (0.0016)	4.0249 (0.0150)

Table 4: Coverage of 95% intervals for $(1-0.4B-0.2B^2)x_t = u_t$ with Model III and normal errors. *AR* Roots: -3.449, 1.449

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	1.8017	0.9407 (0.0020)	1.7966 (0.0359)
	1,000	1.7970	0.9467 (0.0006)	1.8023 (0.0246)
10	300	2.1442	0.9438 (0.0047)	2.3145 (0.0240)
	1,000	2.1337	0.9494 (0.0023)	2.3291 (0.139)
20	300	2.1332	0.9455 (0.0049)	2.3553 (0.0229)
	1,000	2.1619	0.9484 (0.0025)	2.3387 (0.0025)

Table 5: Coverage of 95% intervals for $(1-1.11B+0.18B^2)x_t = u_t$ with Model II and normal errors. *AR* Roots: 1.096, 5.071

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	3.8297	0.9396 (0.0018)	3.8134 (0.0289)
	1,000	3.8393	0.9459 (0.0005)	3.8355 (0.0289)
10	300	10.4372	0.9258 (0.0029)	10.0980 (0.1105)
	1,000	10.1446	0.9444 (0.0011)	10.4961 (0.0632)
20	300	11.3133	0.9248 (0.0034)	11.0979 (0.1167)
	1,000	11.3447	0.9431 (0.0014)	11.5197 (0.0548)

Table 6: Coverage of 95% intervals for $(1-0.4B)x_t = u_t$ with Model I and t distributed errors. *AR* Root : 2.5

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	1.5584	0.9437 (0.0014)	1.5593 (0.0312)
	1,000	1.5195	0.9476 (0.0006)	1.5305 (0.0152)
10	300	1.6760	0.9491 (0.0034)	1.7856 (0.0154)
	1,000	1.6761	0.9507 (0.0017)	1.7658 (0.0049)
20	300	1.6653	0.9497 (0.0031)	1.7880 (0.0153)
	1,000	1.6696	0.9509 (0.0017)	1.7605 (0.0050)

Table 7: Coverage of 95% intervals for $(1-0.2B)x_t = u_t$ with Model II and t distributed errors. *AR* Root : 5

Lead Length	Sample Size	Theoretical Length	Mean Coverage (SE)	Mean Length (SE)
1	300	3.8589	0.9412 (0.0019)	3.8684 (0.0844)
	1,000	3.7716	0.9468 (0.0006)	3.7928 (0.0375)
10	300	3.9667	0.9431 (0.0032)	4.1218 (0.0702)
	1,000	3.8838	0.09476 (0.0016)	4.0075 (0.0264)
20	300	3.8988	0.9445 (0.0037)	4.1695 (0.0648)
	1,000	3.9115	0.9463 (0.0017)	4.0578 (0.0220)

Table 8: Coverage of 95% intervals for $(1-0.4B-0.2B^2)x_t = u_t$ with Model III and t distributed errors. *AR* Roots : -3.449, 1.449

Lead Length	Sample Size	Theoretical Length (SE)	Mean Coverage (SE)	Mean Length (SE)
1	300	1.8015	0.9414 (0.0020)	1.8029 (0.0551)
	1,000	1.7095	0.9483 (0.0006)	1.7359 (0.0221)
10	300	2.1216	0.9381 (0.0051)	2.2745 (0.0417)
	1,000	2.0811	0.9467 (0.0025)	2.2433 (0.0116)
20	300	2.0705	0.9450 (0.0052)	2.2956 (0.0351)
	1,000	2.1126	0.9472 (0.0027)	2.2731 (0.0098)

When the underlining error obeys a *t*-distribution with five degrees of freedom, the coverage probabilities do not show much difference from what is obtained under standard normal errors (for example see Tables 3 and 7). This shows some evidence that the proposed method works well under heavy-tail error distributions. This is validated further by simulation results not reported in this paper.

4. Application to S&P 500 Data

The proposed method was applied to a real data set obtained from S&P 500 index. S&P 500 is a popular stock market index among econometricians and statisticians as it provides a broad snapshot of the overall U.S. equity market. In fact, this index tracks over 70% of all U.S. equity market. For our study we used S&P index data from 2010 to 2014. Note that the log returns of the closing prices were used for generating prediction intervals.

The following figure shows one-step-ahead prediction intervals we generated for this S&P data using the proposed method:

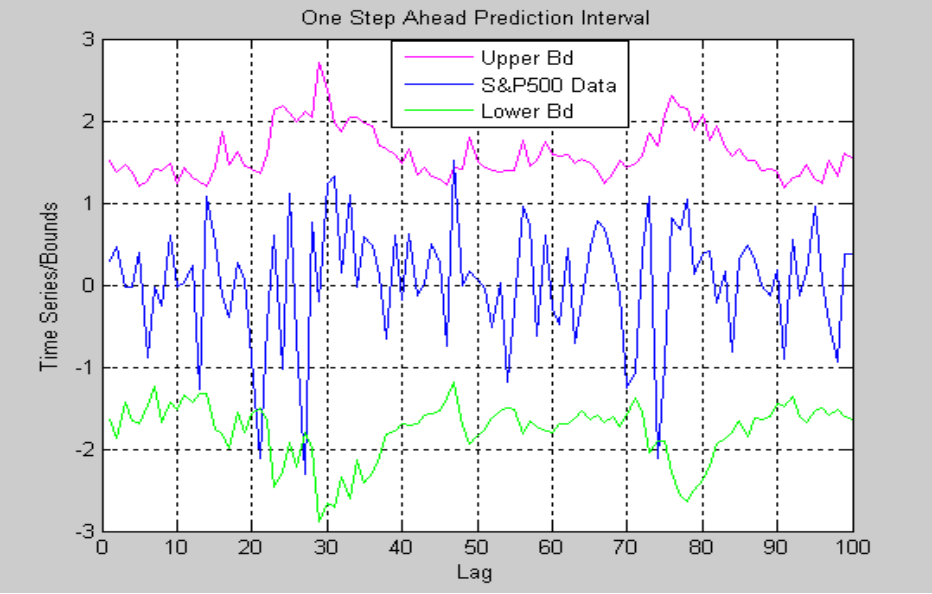

Figure 1: One-step-ahead prediction intervals for S&P 500 data

Clearly, the intervals change width based on existing volatility. For example the method yields wider intervals at lag 30 when predicted volatility is large but provides relatively narrower intervals at lags 10 and 60 when the volatility is low.

5. Conclusion

A bootstrap-based method for obtaining prediction intervals for *AR-GARCH* models was presented and its finite sample properties were investigated using a Monte-Carlo simulation study. In contrast to an existing method, the proposed method does not assume the knowledge of the orders of the *AR* or the *GARCH* portions of the model. Simulation results show that the model performs well under most situations providing coverage probabilities close to the nominal level. Further extensions to this procedure are possible, such as using finite Autoregressive approximations to obtain prediction intervals for ARMA-GARCH processes using the sieve-bootstrap.

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