A Class of Advanced Probabilistic Models for Assessing Economic/Business Mobility

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ABSTRACT

The current paper suggests various types of models for computing and comparing the extent of Economic or Business Mobility. The suggested models include Markov Chain Models, Mixtured Probability Models, etc. A Class of Mixture Probabilistic Models for Various Economic/Business Records: For various input-output informatics, the study of pattern of the stratified record values of a national economy/business is important. Optimum mixture-distributions have been suggested to be appropriate in many cases. Advanced Heterogeneity Indices for Comparing Local or Global Economic/Business Mobility: New heterogeneity indices based on transition probability matrices have been suggested. These indices simultaneously display the individual, group-wise and overall discrepancy among several economic systems.

1. INTRODUCTION

For various input-output informatics the study of pattern of the stratified record values of a national economy/business is important. Earlier works on this topic are due to Houghton et al. (1990), Mahlman (1997), Ahsan et el (2008), Ali(1990, 2003), Quadir et el (2002), Gupta and Kundu (2001), Mudholkar and Srivastava (1993) and Nadarajah (2005). However Frigessiet el. (2002), Mendes and Lopes (2004), Behrens et el. (2004) have developed some mixture models. The drawback with all the aforementioned approaches is the prior specification of a parametric model for the buck of the distribution (and associated weight function where appropriate). A number of authors like Pearson (1894), Rider (1961), Blichke (1962, 1964), Chahine (1965), Roy et el (1992, 1993, 1998, 2005, 2006, 2007), Adnan (2009, 2010, 2011) worked on mixture distributions suggesting their theoretical properties. Tancredi et el. (2006) has proposed a semi-parametric mixture model, A. MacDonald et el proposed a flexible model which includes a non-parametric smooth kernel density estimator below some threshold accompanied with the PP model for the upper tail above the threshold. A mixture of hybrid-Pareto has been carried by Carreau and Bengio (2009). Patrizia Ciarliniet el (2004), Maurice Cox et el have introduced the use of a probabilistic tool, a mixture of probability distributions, to represent the overall population in a temperature comparison. This super-population is defined by combining the local populations in given proportions. The mixture density function identifies the total data variability and the key comparison reference value has a natural definition as the expectation value of this probability density. Mahmud, Adnan and Mia (2012) suggested an appropriate probability model for the extreme temperatures of the Jessor region of Bangladesh. In this paper, mixture distributions like inflation, life expectancy etc have been found. An economic process is a collection of random variables that represents the evolution of some economic process through the change of time, state or space. There are several (often infinitely many) directions in which the process may evolve. In case of discrete time, a stochastic process amounts to a sequence of random economic variables known as a time series (for example Markov chain). Random variables corresponding to various times (or points, in case of random fields) may be completely different. Although the random values of a stochastic process at different times may be independent random variables, in most commonly considered situations they exhibit complicated statistical correlations. Assessing these correlations can be evaluated by means of knowing transitions which express the changes of state of the system and the probabilities associated with various state-changes are called transition probabilities. Markov chain, due to Andrey Markov, is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states. Checking the discordance of two Markov Chains is a preliminary step of finding the mobility of any system over the change of time or place or another dimension(s). It is also a primary stage of comparing multiple Markov Chains. Unfortunately, the comparison among the economic processes is due to very few authors. Falay, B. (2007) described intergenerational income mobility by testing the equality of opportunity due to knowing the comparison of East and West Germany using a transition matrix having positive and negative elements. Altug, S. et el (2011) showed the cyclical dynamics of industrial production and employment over developed and developing countries by Tan et el and first passage time analysis. Muse et el (1992) proposed a likelihood ratio test for testing the equality of evolution rates. Tan et el (2002) developed a Markov-chain-test for time dependence and homogeneity using likelihood ratio test statistic. Dannemann et el (2007) proposed a method of testing the equality of transition parameters based on transition probabilities and likelihood ratio test statistic that simply gives the significant dissimilarity of the total transition but not that of the individual transition. Bartolucci, F. et al (2009) demonstrated the use of a multidimensional extension of the latent Markov model using a multidimensional two parameters logistic model where they developed likelihood ratio test based on log ratio of transition probabilities. Cho, J. S et al (2011) expresses a test of equality of two unknown positive definite matrices with an application of information matrix testing. Hillary. R. M. (2011) proposed a Bayesian method of estimation of the growth transition matrices. A new statistical method of pair-wise sequence alignment has been developed by Adnan et al (2011). However, there is no test for the equality of multiple economic transition probability matrices. The present study aims to improve the comparison method of multiple transition probability matrices considering the more analysis of economical or financial transition probabilities of the multiple sampled transition probability matrices. The author addresses new heterogeneity indices based on the difference among multiple economic transition frequency matrices which will ensure three advantages at least. These indices will accomplish not only an overall decision of the significant dissimilarity/similarity among economic processes but also that of all possible individual and group wise economic transitions that help the economist to quickly identify the portion of the total infrastructure of the entire economic mobility that is significantly differing from those of the other economic processes and detect the core fact(s) for possible differences among economic systems.

2. METHODS AND METHODOLOGY

The distribution of yearly information of an economic variable, $F(x)$, can be estimated by a mixture of single distributions. Mixture extreme value distribution can be formed with weights $(1-p)$ and p (where, p refers p-value). If we get the higher p-value for the goodness of fit test in case of the mixture extreme value distribution, and that p-value is greater than those of the other distributions, then we can say that the mixture extreme value distribution is the best probabilistic model for the observed data. So, the mixture model of the extreme value distribution with weights $(1-p)$ and p is given as of the following form

$$F(x) = (1-p) * F_1(x) + p * F_1(x)$$

where, $F(x)$ is the cumulative density function of the extreme value distribution with the estimated value of location parameter, $F_1(x)$ is the cumulative density function of the extreme value distribution with changing the value of location parameter.

Let the stochastic process is $\{X(t); t \in T\}$, then for each value of, $X(t)$ is a random variable. So, the process is a sequence of outcomes for discrete states and time space. These outcomes may be dependent on earlier ones in the sequence. A Markov chain is collection of random variables $\{X(t)\}$(where the index turns through 0, 1,…) having the property that, given the present, the future is conditionally independent of the past. So, the stochastic process $\{X_n, n \geq 0\}$ is called a Markov chain, if for $j, k, j_1, \ldots, j_{n-1} \in J$

$$Pr[X_n = k | X_{n-1} = j, X_{n-2} = j_1, \ldots, X_0 = j_{n-1}] = Pr[X_n = k | X_{n-1} = j] = P_{jk}$$

The outcomes are called the states of the Markov Chain; if X_n has the outcome j (i.e.,$X_n = j$) the process is said to be at state j at n^{th}trial. The conditional probability $Pr[X_{n+1} = j | X_n = i] = P_{ij}$ is known as transition probability referring the probability that the process is in state i and will be in state j in the next step and the transition probability P_{ij} satisfy the properties $(i) P_{ij} \geq 0 and\ (ii) \sum_j P_{ij} = 1$ for the transition probability matrix $P = [P_{ij}] \forall i, j = 1,2, \ldots, n$. Here two states i and j are said to be communicate state if each is accessible from the other, it is denoted by $i \leftrightarrow j$; then there exist integer m and n such that $P_{ij}{}^{(n)} > 0$ and $P_{ij}{}^{(m)} > 0$. If state $i$communicate with state j and state j communicate with state k then state i communicate with state k.

With an aim of developing a test procedure for the equality of two transition probability matrices from two Markov Chains or sequences of the realization of the economic growth factors, let us demonstrate our method assuming that we have a collection of $K-$ pairs of expression of the factor for economic growth of successive years from two populations (a total of k series of data of each paired successive years of the entire $k + 1$ years of data are collected) and want to test whether they come from same population or Markov Chains having r(r number of groups for each successive years) and let the hypothesis be

$$H_0: P = Q$$

$$H_0: \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} \end{pmatrix} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1r} \\ q_{21} & q_{22} & \cdots & q_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ q_{r1} & q_{r2} & \cdots & q_{rr} \end{pmatrix}$$

where P and Q are average transition probability matrices of a specific economic factor for two countries. After collecting $k-$pairs of yearly data of a specific factor of the economic growth from two countries, the maximum likelihood estimators of the transition probability matrices are obtained as $\hat{P}_{r\times r}, \hat{Q}_{r\times r}$ where $\hat{p}_{ij} = \frac{n_{ij}}{n_i}$ and $\hat{q}_{ij} = \frac{m_{ij}}{m_i}$whereas $n_{ij}$is the average frequency of the $(i, j)^{th}$ element of the 1st transition average of frequency matrix N constructed from $K-$pairs of data for K pairs of successive years for $K + 1$ years drawn from the 1stcountry and m_{ij} is the average frequency of the $(i, j)^{th}$ element of the 2nd transition average frequency matrix M constructed the same way from the 2nd country. Here $n_i = \sum_{j=1}^{r} n_{ij}, m_i = \sum_{j=1}^{r} m_{ij} \forall i, j = 1,2, \ldots, r$ where, r is the number of groups for an economic growth factor. Let, the difference matrix is D such that

$$\hat{D} = \hat{P}_{rr} - \hat{Q}_{rr}$$

$$= \begin{pmatrix} \widehat{p_{11}} - \widehat{q_{11}} & \widehat{p_{12}} - \widehat{q_{12}} & \cdots & \widehat{p_{1r}} - \widehat{q_{1r}} \\ \widehat{p_{21}} - \widehat{q_{21}} & \widehat{p_{22}} - \widehat{q_{22}} & \cdots & \widehat{p_{2r}} - \widehat{q_{2r}} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{p_{r1}} - \widehat{q_{r1}} & \widehat{p_{r2}} - \widehat{q_{r2}} & \cdots & \widehat{p_{rr}} - \widehat{q_{rr}} \end{pmatrix}$$

For large n, n_i, m, m_i; the asymptotic distribution of each element of transition probability matrices, according to the central limit theorem, are distributed as normal such that,

$$\widehat{p_{ij}} \xrightarrow{n_i \to \infty} N\left(p_{ij}, \frac{p_{ij}(1-p_{ij})}{kn_i}\right) \text{And} \widehat{q_{ij}} \xrightarrow{m_i \to \infty} N\left(q_{ij}, \frac{q_{ij}(1-q_{ij})}{km_i}\right)$$

$$\therefore \left(\widehat{p_{ij}} - \widehat{q_{ij}}\right) \xrightarrow{n_i, m_i \to \infty} N\left[\left(p_{ij} - q_{ij}\right), \frac{1}{k}\left(\frac{p_{ij}(1-p_{ij})}{n_i} + \frac{q_{ij}(1-q_{ij})}{m_i}\right)\right]$$

Therefore, $\begin{bmatrix} \widehat{p_{i1}} - \widehat{q_{i1}} \\ \vdots \\ \widehat{p_{ir}} - \widehat{q_{ir}} \end{bmatrix}$ is a multilevel (r level) multivariate (r variate) vector such that

$$\begin{bmatrix} \widehat{p_{i1}} - \widehat{q_{i1}} \\ \vdots \\ \widehat{p_{ir}} - \widehat{q_{ir}} \end{bmatrix} \sim N\left(, \begin{bmatrix} p_{i1} - q_{i1} \\ \vdots \\ p_{ir} - q_{ir} \end{bmatrix}, \frac{1}{k}\begin{bmatrix} \left(\frac{p_{i1}(1-p_{i1})}{n_i} + \frac{q_{i1}(1-q_{i1})}{m_i}\right) & \cdots & -\left(\frac{p_{i1}p_{ir}}{n_i} + \frac{q_{i1}q_{ir}}{m_i}\right) \\ \vdots & \ddots & \vdots \\ -\left(\frac{p_{i1}p_{ir}}{n_i} + \frac{q_{i1}q_{ir}}{m_i}\right) & \cdots & \left(\frac{p_{ir}(1-p_{ir})}{n_i} + \frac{q_{ir}(1-q_{ir})}{m_i}\right) \end{bmatrix}\right) \forall i = 1,2,\ldots,r$$

Although the concern proofs are very much trivial, are available from the author if required. However, after dividing each element of the difference matrix by their respective standard error, we obtain an element standardized matrix Z of the following form

$$Z = \frac{P_{ij} - Q_{ij}}{\sqrt{\frac{1}{k}\left(\frac{P_{ij}(1-P_{ij})}{n_i} + \frac{Q_{ij}(1-Q_{ij})}{m_i}\right)}}$$

$$Z = \begin{bmatrix} \frac{\widehat{p_{11}} - \widehat{q_{11}}}{\sqrt{\frac{1}{k}\left(\frac{p_{11}(1-p_{11})}{n_1}\right) + \left(\frac{q_{11}(1-q_{11})}{m_1}\right)}} & \cdots & \frac{\widehat{p_{1r}} - \widehat{q_{1r}}}{\sqrt{\frac{1}{k}\left(\frac{p_{1r}(1-p_{1r})}{n_1}\right) + \left(\frac{q_{1r}(1-q_{1r})}{m_1}\right)}} \\ \vdots & \ddots & \vdots \\ \frac{\widehat{p_{r1}} - \widehat{q_{r1}}}{\sqrt{\frac{1}{k}\left(\frac{p_{r1}(1-p_{r1})}{n_r}\right) + \left(\frac{q_{r1}(1-q_{r1})}{m_r}\right)}} & \cdots & \frac{\widehat{p_{rr}} - \widehat{q_{rr}}}{\sqrt{\frac{1}{k}\left(\frac{p_{rr}(1-p_{rr})}{n_1}\right) + \left(\frac{q_{rr}(1-q_{rr})}{m_r}\right)}} \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11} & \cdots & Z_{1r} \\ \vdots & \ddots & \vdots \\ Z_{r1} & \cdots & Z_{rr} \end{bmatrix}$$

Now, squaring each element of the Z matrix, a matrix χ^2 each of which is an individual chi-square of the following form is obtained the matrix of chi-squares,

$$\chi^2 = \begin{bmatrix} {Z_{11}}^2 & \cdots & {Z_{1r}}^2 \\ \vdots & \ddots & \vdots \\ {Z_{r1}}^2 & \cdots & {Z_{rr}}^2 \end{bmatrix} = \begin{bmatrix} {\chi_{11}}^2 & \cdots & {\chi_{1r}}^2 \\ \vdots & \ddots & \vdots \\ {\chi_{r1}}^2 & \cdots & {\chi_{rr}}^2 \end{bmatrix}$$

The above matrix of Chi-squares can also be called as element chi-square matrix. From this matrix we basically can test three types of hypothesis which are as follows:

(i) $H_0 : p_{ij} = q_{ij}$; Or the hypothesis of testing the equality of each population transition probabilities pair of the two population transition probability matrices $P$and Q.

(ii) $H_0 : (p_{i1} \quad p_{i2} \quad \cdots \quad p_{ir}) = (q_{i1} \quad q_{i2} \quad \cdots \quad q_{ir})$; Or, the hypothesis of checking the equality of the i^{th} row vector between the 1st and 2nd population transition probability matrix. Actually, it tests the equality of the frequentness of the transition of the randomness of two population or Markov Chain from each state to all states.

(iii) $H_0 : P = Q$; or the hypothesis of testing the equality of the total transitions between the two population Markov chain or sequence is significantly varying. It tests whether the two sample countries are drawn from same population Markov chain.

For the aforementioned tests the concern test statistics are given below respectively,

(i) Comparing each ${\chi_{ij}}^2$ with the tabulated ${\chi_{(1,\alpha)}}^2$ of 1 degrees of freedom,
(ii) Comparing each $\sum {\chi_{ij}}^2$ with the tabulated ${\chi_{(r-1,\alpha)}}^2$ of $(r-1)$ degree of freedom,
(iii) Comparing chi-square matrix sum=${\chi_{11}}^2 + \cdots + {\chi_{1r}}^2 + \cdots + {\chi_{r1}}^2 + \cdots + {\chi_{rr}}^2$ with the tabulated ${\chi_{(r(r-1),\alpha)}}^2$ of $r(r-1)$ degree of freedom.

3. DATA AND DATA ANALYSIS

For finding the probability models, Q-Q plot and P-P plot have been formed for various economic variables. The Q-Q plot on the basis of annual input/output data for the two economic variable's Life expectancy and Inflation distributions of Srilanka and Bangladesh consecutively have been found. The two Q-Q plot cannot conclude the origin of the data since for all cases the points fall approximately along with 45^0reference line [figure 1]

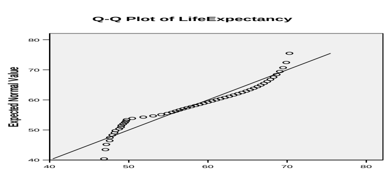

Figure 1 (a): Q-Q plot of Life expectancy of Bangladesh

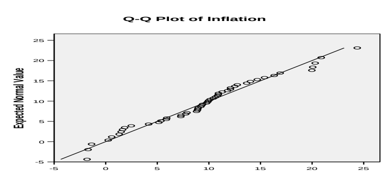

Figure 1 (b): Q-Q plot of Inflation of Srilanka

Letting the null and alternative hypothesis to be the data following a specific economic distribution the Kolmogorov-Smirnov test statistic is calculated as $\widehat{D_n} = \sup_x |F_n(x) - F(x)|$. Therefore, annual economic data are assumed to follow the specific distribution. The P-P plot of the sorted values (in ascending order) of the observed versus expected quintile's y_i determined by $y_i = \left(\frac{i-0.5}{n+1}\right)$ plotted. Figure 2 represents (for the distribution) the P-P plot on the basis of annual data where the plot shows that the points fall approximately along with 45^0reference line which means the data follows specific distribution.

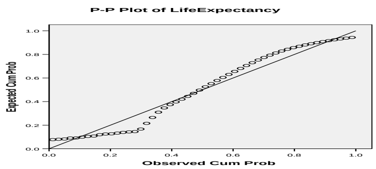

Figure 2 (a): P-P plot of Life expectancy of Bangladesh

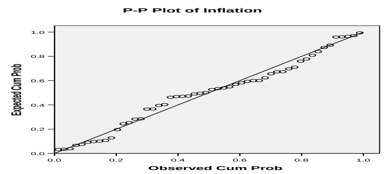

Figure 2 (b): P-P plot of Inflation of Srilanka

In the following figure 3, frequency curve also shows that data plots are approximately close to the original density which is an assurance that the data follows the specific mixture distribution.

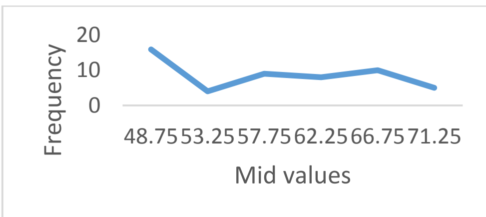

Figure 3 (a): Frequency curve of Life expectancy of Bangladesh

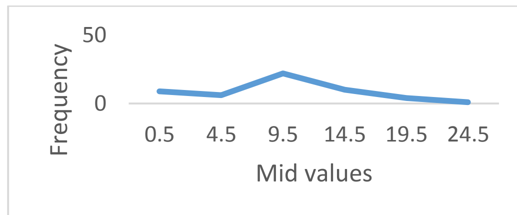

Figure 3 (b): Frequency curve of Inflation of Srilanka

Different plots of other economic variables are presented in appendix in figure 4, 5 and 6.
Since multimodality of the distribution of data indicates the possibility of the extreme of a mixture of two distributions, we have tried to fit an appropriate mixture of two distributions in this paper. As such, the cumulative density function of the mixture distribution is given by

$$F(x) = (1-p) * F_1(x) + p * F_1(x)$$

Now, letting the null hypothesis to be the data following the mixture Normal distribution against the alternative hypothesis not to be true, the value of chi-square test statistic on the basis of time of data under the postulated mixture model for different values of location and scale parameters have been observed. The p-value is found maximum for the values of location parameters and scale parameters for the mixture distribution. Therefore, the final mixture model for any economic data of a region like life expectancy of Bangladesh is as follows:

$$F(x) = (1-.0013121) * F_{Final\,1}(x) + .0013121 * F_{Final\,2}(x); x€[46.8792, 70.2949].$$

Mean and Variance of the data is 57.94600 *and* 60.35434. After classification we calculated the normal pdf and cdf of the data set and found the chi square 69.66935 with corresponding *value* $1.20071E-13$ *or* 0.0001. For better p value we calculated new pdf and cdf by changing the mean and variance to 63.5 *and* 60.3543398. Using this we got new *mixtured* normal $cdf = [(1 - new\ cdf) * p\ value] + [new\ cdf * p\ value]$, for this the chi-square value is 19.88725 with corresponding p *value* 0.0013121. And here our p *value* increased.

Similarly the p-value is observed maximum in case of economic variable like Inflation of Srilanka.

$$F(x) = (1-.0419525) * F_{Final\,1}(x) + .0419525 * F_{Final\,2}(x); x€[-1.8036, 24.3787]$$

Mean and Variance of the data is 9.374584 *and* 36.95724. Like the previous, after classification we found the chi square value 95.69147982 with corresponding *value* 0.0001. For better p *value* we calculated new pdf and cdf by changing the mean and variance to 20 *and* 229.1218. Using this new $mixtured\ cdf = [(1 - new\ cdf) * p\ value] + [new\ cdf * p\ value]$, for this the chi-square value is 11.52235 with corresponding $p\ value$ 0.041953. Here our $p\ value$ increased.

For analyzing economic mobility, different economic data indicator as variables like GDP per capita, GDP annual, Life expectancy at birth, Inflation, GNI and Population Growth data have been used to construct the transition probability matrices by collecting from the website of World Bank. By comparing the data's of different indicators of two countries like Srilanka and Bangladesh, summaries of our findings are given successively in Table 1 to 6. GDP per capita data of the two countries Srilanka and Bangladesh have been shown in current U.S. dollars from 1961 to 2012 [Table 1].

A test procedure for the equality of two transition probability matrices of Srilanka and Bangladesh from two sequences of the realization of the economic growth factors- GDP per capita, we considered a collection of $K = 51$ pairs of expression of the factor for economic growth of successive years from two populations (a total of 51 series of data of each paired successive years of the entire 52 years of data are collected) and want to test whether they come from same population or same Markov Chains having 5 (5 number of groups for each successive years depending on the values of data sets) and let the hypothesis be

$$H_0: GDP_S = GDP_B$$

where P means the data set for Srilanka and Q is for Bangladesh, i.e.,

$$H_0 = \begin{pmatrix} GDP_{S_{11}} & GDP_{S_{12}} & \dots & GDP_{S_{15}} \\ GDP_{S_{21}} & GDP_{S_{22}} & \cdots & GDP_{S_{25}} \\ \vdots & \vdots & \ddots & \vdots \\ GDP_{S_{51}} & GDP_{S_{52}} & \cdots & GDP_{S_{55}} \end{pmatrix}_{5\times5} = \begin{pmatrix} GDP_{B_{11}} & GDP_{B_{12}} & \dots & GDP_{B_{15}} \\ GDP_{B_{21}} & GDP_{B_{22}} & \cdots & GDP_{B_{25}} \\ \vdots & \vdots & \ddots & \vdots \\ GDP_{B_{51}} & GDP_{B_{52}} & \cdots & GDP_{B_{55}} \end{pmatrix}_{5\times5}$$

where GDP_S and GDP_B are average transition probability matrices of GDP per capita for - Srilanka and Bangladesh written as $GDP_{S_{ij}}$ and $GDP_{B_{ij}}$ sequentially. After collecting yearly data, the maximum likelihood estimators of the transition probability matrices are obtained as $\hat{P}_{5\times5}, \hat{Q}_{5\times5}$ where $\widehat{GDP}_{S_{ij}} = \frac{n_{ij}}{n_i}$ and $\widehat{GDP}_{B_{ij}} = \frac{m_{ij}}{m_i}$whereas $n_{ij}$is the average frequency of the $(i,j)^{th}$ element of the 1st transition average of frequency matrix N constructed from Srilanka and m_{ij} is the average frequency of the $(i,j)^{th}$ element of the 2nd transition average frequency matrix M constructed from Bangladesh. Here $n_i = \sum_{j=1}^{5} n_{ij}, m_i = \sum_{j=1}^{5} m_{ij} \forall i,j = 1,2,\dots,5$. Where, $r = 5$ is the number of groups for GDP per capita. Let, the difference matrix is D such that

$$\hat{D} = \widehat{GDP}_{S_{5\times5}} - \widehat{GDP}_{B_{5\times5}}$$

$$= \begin{pmatrix} \widehat{GDP}_{S_{11}} - \widehat{GDP}_{B_{11}} & \widehat{GDP}_{S_{12}} - \widehat{GDP}_{B_{12}} & \dots & \widehat{GDP}_{S_{15}} - \widehat{GDP}_{B_{15}} \\ \widehat{GDP}_{S_{21}} - \widehat{GDP}_{B_{21}} & \widehat{GDP}_{S_{22}} - \widehat{GDP}_{B_{22}} & \cdots & \widehat{GDP}_{S_{25}} - \widehat{GDP}_{B_{25}} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{GDP}_{S_{51}} - \widehat{GDP}_{B_{51}} & \widehat{GDP}_{S_{52}} - \widehat{GDP}_{B_{52}} & \cdots & \widehat{GDP}_{S_{55}} - \widehat{GDP}_{B_{55}} \end{pmatrix}$$

So, the transition probability matrices are [Table 1(c)-1(d)] as given as follows

Table 1 (c): Transition probability matrix of GDP per capita of Srilanka

Table 1 (d): Transition probability matrix of GDP per capita For Bangladesh

Class	<200	200-350	350-500	500-650	>650
<200	0.84615	0.15385	0	0	0
200-350	0.1	0.8	0.1	0	0
350-500	0	0	0.85714	0.14286	0
500-650	0	0	0	0.66667	0.33333
>650	0	0	0	0	1

Class	<200	200-350	350-500	500-650	>650
<200	0.84211	0.15789	0	0	0
200-350	0.09524	0.80952	0.09524	0	0
350-500	0	0.14286	0.71429	0.14286	0
500-650	0	0	0	0.5	0.5
>650	0	0	0	0	1

The Difference matrix, D=

$$\begin{pmatrix} 0.00405 & -0.0041 & 0 & 0 & 0 \\ 0.00476 & -0.0095 & 0.00476 & 0 & 0 \\ 0 & -0.1429 & 0.14286 & 0 & 0 \\ 0 & 0 & 0 & 0.16667 & -0.1667 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$Z =$

$$\begin{pmatrix} 0.29708 & -0.4452 & 0.29708 & 0 & 0 \\ 0 & -7.71362 & 4.72361 & 0 & 0 \\ 0 & 0 & 0 & 2.66764 & -2.6676 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\therefore \chi^2 = \begin{pmatrix} 0.049139 & 0.049139 & 0 & 0 & 0 \\ 0.088258 & 0.198172 & 0.088258 & 0 & 0 \\ 0 & 59.5 & 22.3125 & 0 & 0 \\ 0 & 0 & 0 & 7.116279 & 7.116279 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The above matrix of Chi-squares can also be called as element chi-square matrix. From this matrix we basically can test three types of hypothesis which are as follows:

(i) $H_0 : GDP_{S_{ij}} = GDP_{B_{ij}}$; the hypothesis of testing the equality of each GDP population transition probabilities pair of the matrices for Srilanka and Bangladesh respectively.

(ii) $H_0 : \left(GDP_{S_{i1}} \quad GDP_{S_{i2}} \quad \cdots \quad GDP_{S_{i5}}\right) = \left(GDP_{B_{i1}} \quad GDP_{B_{i2}} \quad \cdots \quad GDP_{B_{i5}}\right)$; the hypothesis of checking the equality of the i^{th} row vector or the frequentness of the transition probability matrix of two countries.

(iii) $H_0 : GDP_S = GDP_B$; the hypothesis of testing the equality of the total transitions for Srilanka and Bangladesh. It tests whether drawn from same population Markov chain.

For the aforementioned tests the concern test statistics are given below respectively,

(i) Comparing each ${\chi_{ij}}^2$ with the tabulated $\chi^2_{tab} = \chi^2_{(1,0.05)} = 3.841$ of 1 degrees of freedom,

(ii) Comparing each $\sum {\chi_{ij}}^2$ with the tabulated $\chi^2_{tab} = \chi^2_{(5-1,0.05)} = 9.488$ of $(5-1)$ degree of freedom,

(iii) Comparing chi-square matrix sum=${\chi_{11}}^2 + \cdots + {\chi_{15}}^2 + \cdots + {\chi_{51}}^2 + \cdots + {\chi_{55}}^2$ with the tabulated $\chi^2_{tab} = \chi^2_{(5(5-1),0.05)} = 31.41$ of $5(5-1)$ degree of freedom.

We will compare the individual matrix of different indicators of Srilanka and Bangladesh from the above χ^2matrix.

$$H_0: GDP_{B_{ij}} = GDP_{S_{ij}} \quad \forall i, j = 1,2, \dots ,5$$

For individual chi square at 5% level of significance when $\chi^2_{cal} > \chi^2_{tab}$, we will reject our null hypothesis of similarity. So we can form a individual decision matrix as [Table 1(e)].

Table 1 (e): Decision matrix of GDP per capita

$$\begin{pmatrix} S & S & S & S & S \\ S & S & S & S & S \\ S & DS & DS & S & S \\ S & S & S & DS & DS \\ S & S & S & S & S \end{pmatrix}$$

Now to compare row to decide whether there is any similarity between the GDP's of two countries, $H_0 = GDP_{B_{1i}} = GDP_{S_{1i}}, \dots, GDP_{B_{5i}} = GDP_{S_{5i}}$

The row total matrix of GDP per capita is $\begin{pmatrix} 0.098278 \\ 0.374688 \\ 81.8125 \\ 14.23256 \\ 0 \end{pmatrix}_{5\times1}$ So decision Matrix, $D = \begin{pmatrix} S \\ S \\ DS \\ DS \\ S \end{pmatrix}_{5\times1}$

For the total matrix, the calculated value of chi-square, $\chi^2_{cal} = \sum_{i=1}^{5}\sum_{j=1}^{5}\chi^2_{ij} = 96.51802$. At 5% level of significance here $\chi^2_{cal} > \chi^2_{5(5-1),0.05}$, so we will reject our null hypothesis of similarity. So there is dissimilarity between the GDP matrices of Srilanka and Bangladesh. Similarly for other Economic growth factors our findings are given in a chart followed by the contingency table showing all the six indicators of two countries.

Table 7: Contingency table of the Decision Matrix for the total factor

		BANGLADESH					
SRILANKA	FACTORS	GDP per capita	Life Expectancy at birth	Inflation	Population Growth	GDP	GNI
	GDP per capita	DS	DS	DS	DS	DS	DS
	Life Expectancy at birth	DS	DS	DS	DS	DS	DS
	Inflation	DS	DS	DS	DS	DS	DS
	Population Growth	DS	DS	DS	DS	DS	DS
	GDP	DS	DS	DS	DS	DS	DS
	GNI	DS	DS	DS	DS	DS	DS

Economic variable	Transition Probability Matrices	Difference Matrix and Z Matrix	χ^2Matrix	Decision Matrix

GDP per Capita

Transition Probability Matrices

Table 1 (c): Transition probability matrix of GDP per capita of Srilanka

Class	<200	200-350	350-500	500-650	>650
<200	0.846154	0.153846	0	0	0
200-350	0.1	0.8	0.1	0	0
350-500	0	0	0.857143	0.142857	0
500-650	0	0	0	0.666667	0.333333
>650	0	0	0	0	1

Table 1 (d): Transition probability matrix of GDP per capita For Bangladesh

Class	<200	200-350	350-500	500-650	>650
<200	0.842105	0.157895	0	0	0
200-350	0.095238	0.809524	0.095238	0	0
350-500	0	0.142857	0.714286	0.142857	0
500-650	0	0	0	0.5	0.5
>650	0	0	0	0	1

Difference Matrix and Z Matrix

D=

0.004049	-0.00405	0	0	0
0.004762	-0.00952	0.004762	0	0
0	-0.14286	0.142857	0	0
0	0	0	0.166667	-0.16667
0	0	0	0	0

$\therefore Z =$

0.221673	-0.22167	0	0	0
0.297082	-0.44516	0.297082	0	0
0	-7.71362	4.723611	0	0
0	0	0	2.667635	-2.66764
0	0	0	0	0

χ^2Matrix

For individual $\chi^2_{1,0.05} = 3.841$

0.049139	0.049139	0	0	0
0.088258	0.198172	0.088258	0	0
0	59.5	22.3125	0	0
0	0	0	7.116279	7.116279
0	0	0	0	0

For row $\chi^2_{4,0.05} = 9.488$

$$\begin{pmatrix} 0.098278 \\ 0.374688 \\ 81.8125 \\ 14.23256 \\ 0 \end{pmatrix}_{5\times1}$$

Total

$$\chi^2_{5(5-1),0.05} = 31.41$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4}\chi^2_{ij} = 96.51802$$

Decision Matrix

For individual

S	S	S	S	S
S	S	S	S	S
S	DS	DS	S	S
S	S	S	DS	DS
S	S	S	S	S

For row $\begin{pmatrix} S \\ S \\ DS \\ DS \\ S \end{pmatrix}_{5\times1}$

Total

(DS)

Life Expectancy

Transition Probability Matrices

Table 2 (c): Transition probability matrix of Life expectancy (at birth) total for Srilanka

Class	<61	61-65	66-70	>70
<61	0.666667	0.333333	0	0
61-65	0	0.909091	0.090909	0
66-70	0	0	0.96	0.04
>70	0	0	0	1

Table 2 (d): Transition probability matrix of Life expectancy (at birth) total for Bangladesh

Class	<61	61-65	66-70	>70
<61	0.967742	0.032258	0	0
61-65	0	0.9	0.1	0
66-70	0	0	1	0
>70	0	0	0	0

Difference Matrix and Z Matrix

D=

-0.30108	0.301075	0	0
0	0.009091	-0.00909	0
0	0	-0.04	0.04
0	0	0	0

Z=

-7.84684	7.846842	0	0
0	0.505216	-0.50522	0
0	0	-7.28869	7.28869
0	0	0	0

χ^2Matrix

For individual $\chi^2_{1,0.05} = 3.841$

61.57292	61.57292	0	0
0	0.255244	0.255244	0
0	0	53.125	53.125
0	0	0	0

For Row

$\chi^2_{3,0.05} = 7.815$

$$\begin{pmatrix} 123.1458 \\ 0.510487 \\ 106.25 \\ 0 \end{pmatrix}_{4\times1}$$

Total

$$\chi^2_{4(4-1),0.05} = 21.03$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4}\chi^2_{ij} = 229.9063$$

Decision Matrix

For individual

DS	DS	S	S
S	S	S	S
S	S	DS	DS
S	S	S	S

For row

DS
S
DS
S

Total (DS)

Inflation

Transition Probability Matrices

Table 3 (c): Transition probability matrix of Inflation for Srilanka

Class	<-1	-1-<6	6-<13	13-<20	<=20
<-1	0.333333	0.666667	0	0	0
-1-<6	0.083333	0.25	0.666667	0	0
6-<13	0	0.153846	0.538462	0.230769	0.076923
13-<20	0	0.142857	0.285714	0.285714	0.285714
<=20	0	0.333333	0.666667	0	0

Table 3 (d): Transition probability matrix of Inflation for Bangladesh

Class	<-1	-1-<6	6-<13	13-<20	<=20

Difference Matrix and Z Matrix

D=

0.083333	0.666667	-0.5	0	-0.25
0.030702	-0.53947	0.561404	0	-0.05263
0	-0.03663	-0.12821	0.087912	0.076923
-0.33333	0.142857	-0.38095	0.285714	0.285714
-0.25	0.333333	0.416667	0	-0.5

Z=

χ^2Matrix

For individual

$\chi^2_{1,0.05} = 3.841$

2.92823	306	204	0	68
5.347311	608.9874	684.708	0	53.83333
0	5.541118	41.62083	31.13786	110.5
76.5	59.5	71.69874	142.8	142.8
68	76.5	73.20574	0	204

For Row

$\chi^2_{4,0.05} = 9.488$

Decision Matrix

For individual

S	DS	DS	S	DS
DS	DS	DS	S	DS
S	DS	DS	DS	DS
DS	DS	DS	DS	DS
DS	DS	DS	S	DS

For row

	<-1	-1-<6	6-<13	13-<20	<=20
<-1	0.25	0	0.5	0	0.25
-1-<6	0.052632	0.789474	0.105263	0	0.052632
6-<13	0	0.190476	0.666667	0.142857	0
13-<20	0.333333	0	0.666667	0	0
<=20	0.25	0	0.25	0	0.5

1.711207	17.49286	-14.2829	0	-8.24621
2.312425	-24.6777	26.16693	0	-7.33712
0	-2.35396	-6.45142	5.58013	10.5119
-8.74643	7.713624	-8.46751	11.9499	11.9499
-8.24621	8.746428	8.556035	0	-14.2829

$$\begin{pmatrix} 580.9282 \\ 1352.876 \\ 188.7998 \\ 493.2987 \\ 421.7057 \end{pmatrix}_{5\times1}$$

Total

$$\chi^2_{5(5-1),0.05} = 31.41$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4}\chi^2_{ij} = 3037.609$$

$$\begin{pmatrix} DS \\ DS \\ DS \\ DS \\ DS \end{pmatrix}_{5\times1}$$

Total

(DS)

Population Growth

Table 4 (c): Transition probability matrix of Population Growth for Srilanka

Class	<2	>2
<2	0.95122	0.04878
>2	0.272727	0.727273

Table 4 (d): Transition probability matrix of Population Growth for Bangladesh

Class	<2	>2
<2	0.947368	0.052632
>2	0.058824	0.941176

D=

0.003851	-0.00385
0.213904	-0.2139

Z=

0.456145274	-0.456145274
11.00953926	-11.00951167

For individual $\chi^2_{1,0.05} = 3.841$

0.208068511	0.208068511
121.2099546	121.2093473

For row $\chi^2_{1,0.05} = 3.841$

$$\begin{pmatrix} 0.416137 \\ 242.4193 \end{pmatrix}_{2\times1}$$

For total

$$\chi^2_{2(2-1),0.05} = 5.991$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4}\chi^2_{ij} = 242.8354$$

For individual

S	S
DS	DS

For row

$$\begin{pmatrix} S \\ DS \end{pmatrix}_{2\times1}$$

Total (DS)

GDP Annual

Table 5 (c): Transition probability matrix of GDP annual for Srilanka

Class	<3	3-<4	4-<5	5-<6	>=6
<3	0.375	0.25	0	0.125	0.25
3-<4	0.375	0	0	0.25	0.375
4-<5	0.1	0.2	0.4	0.1	0.2
5-<6	0	0.181818	0.181818	0.272727	0.363636
>=6	0	0.214286	0.142857	0.285714	0.285714

Table 5 (d): Transition probability matrix of GDP annual for Bangladesh

Class	<3	3-<4	4-<5	5-<6	>=6
<3	0.307692	0.153846	0.076923	0.230769	0.230769
3-<4	0.4	0	0.2	0.2	0.2
4-<5	0.111111	0.111111	0.333333	0.444444	0
5-<6	0.230769	0.153846	0.230769	0.153846	0.230769
>=6	0.25	0	0.083333	0.25	0.416667

D=

0.067308	0.096154	-0.07692	-0.10577	0.019231
-0.025	0	-0.2	0.05	0.175
-0.0202	0.161616	0.030303	-0.35354	0.181818
-0.23077	0.027972	-0.04895	0.118881	0.132867
-0.25	0.214286	0.059524	0.035714	-0.13095

Z=

2.270857	3.791078	-7.50555	-4.61388	0.720038
-0.64843	0	-8.06226	1.531334	5.097069
-0.57891	4.01249	2.229151	-13.1612	11.95826
-14.2408	1.314762	-2.14116	5.119001	5.144058
-14.4222	14.09062	3.491645	1.481917	-5.05972

For individual

$$\chi^2_{1,0.05} = 3.841$$

5.156791	14.37228	56.33333	21.28788	0.518455
0.420457	0	65	2.344983	25.98012
0.335134	16.10008	4.969112	173.2162	143
202.8	1.728599	4.584579	26.20417	26.46133
208	198.5455	12.19158	2.196078	25.60078

For row

$$\chi^2_{4,0.05} = 9.488$$

$$\begin{pmatrix} 97.66873 \\ 93.74556 \\ 337.6205 \\ 261.7787 \\ 446.5339 \end{pmatrix}_{5\times1}$$

For total

$$\chi^2_{5(5-1),0.05} = 31.41$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4}\chi^2_{ij} = 1237.347$$

For individual

DS	DS	DS	DS	S
S	S	DS	S	DS
S	DS	DS	DS	DS
DS	S	DS	DS	DS
DS	DS	DS	S	DS

For row

$$\begin{pmatrix} DS \\ DS \\ DS \\ DS \\ DS \end{pmatrix}_{5\times1}$$

For total

(DS)

GNI

Table 6 (c): Transition probability matrix of GNI for Srilanka

Class	<2000	2000-<3700	>3700
<2000	0.97619	0.02381	0
2000-<3700	0	0.8	0.2
>3700	0	0	1

Table 6 (d): Transition probability matrix of GNI for Bangladesh

Class	<2000	2000-<3700	>3700
<2000	0.8	0.2	0
2000-<3700	0.083333	0.833333	0.083333
>3700	0	0	1

D=

0.17619	-0.17619	0
-0.08333	-0.03333	0.116667
0	0	0

Z=

8.692412	-8.69241	0
-6.60578	-1.10088	4.159789
0	0	0

For individual $\chi^2_{1,0.05} = 3.841$

75.55803	75.55803	0
43.63636	1.211941	17.30385
0	0	0

For row

$$\chi^2_{2,0.05} = 5.991$$

$$\begin{pmatrix} 151.1161 \\ 62.15215 \\ 0 \end{pmatrix}_{3\times1}$$

For total

$$\chi^2_{3(3-1),0.05} = 12.592$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4}\chi^2_{ij} = 213.2682$$

For individual

DS	DS	S
DS	S	DS
S	S	S

For row

$$\begin{pmatrix} DS \\ DS \\ S \end{pmatrix}_{3\times1}$$

For total

(DS)

CONCLUDING REMARKS

To develop and implement strategies for maximize the effects of economic knowledge, a more complete knowledge of how economic changes affect a country is needed. Therefore, from the statistical point of view, we have to find suitable probability models to explain the various economic patterns. Unlike previous studies on economic records, very simple mixture probabilistic models have been applied in case of various economic variables of local/entire region(s) of a country. Optimum distributions have been found to be relatively more appropriate in case (of the same regions). Economic transition probability matrices have been studied to find the discordance among themt. The proposed test ensembles the individual, group wise and overall pattern of the economic transition frequencies of one population economic system whether significantly differing from those of other population systems. Any inquiry and proof(s) of the mathematical development of the tests can be accessible from the authors.

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APPENDIX:

Table 1: Per Capita GDP

Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh
1961	143.0313	94.54856	**1974**	269.0896	176.5222	**1987**	408.1211	235.9797	**2000**	854.9267	355.9734
1962	139.6734	96.96754	**1975**	280.9201	269.1248	**1988**	420.4092	251.0324	**2001**	837.6988	348.7569
963	117.3113	98.67435	**1976**	261.8153	136.8201	**1989**	415.2908	264.7897	**2002**	903.8964	347.2186
1964	121.2364	97.04055	**1977**	294.3989	127.1895	**1990**	472.0865	283.9767	**2003**	984.8102	372.9805
1965	152.1246	103.2621	**1978**	192.6134	170.2537	**1991**	521.2465	281.5988	**2004**	1063.161	400.4725
1966	153.114	109.0921	**1979**	232.4911	193.9823	**1992**	556.8123	278.3381	**2005**	1242.404	421.1233
1967	158.8879	118.9762	**1980**	272.9112	219.8593	**1993**	585.8937	279.0643	**2006**	1423.477	427.2912
1968	150.1982	118.9601	**1981**	297.4233	232.5814	**1994**	654.9441	288.4317	**2007**	1614.411	467.1364
1969	160.4266	130.8871	**1982**	313.8171	207.3372	**1995**	718.4438	316.5086	**2008**	2013.911	537.6385
1970	183.5121	135.6187	**1983**	335.2088	192.2231	**1996**	757.9482	332.2363	**2009**	2057.114	597.7118
1971	186.7067	129.4118	**1984**	387.3277	213.4048	**1997**	812.7925	338.6986	**2010**	2400.016	664.0642
1972	198.5799	91.49192	**1985**	377.3804	229.2264	**1998**	840.8738	345.8759	**2011**	2835.992	731.8942
1973	219.6643	115.937	**1986**	397.1731	217.7529	**1999**	821.5965	351.5826	**2012**	2923.21	752.1561

Table 2: Life expectancy (at birth) total

Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh
1961	60.1268	47.6089	**1974**	65.84271	47.91995	**1987**	69.32173	58.43317	**2000**	71.15712	65.31973
1962	60.51159	48.20156	**1975**	66.20717	49.06163	**1988**	69.40317	58.96315	**2001**	71.90095	65.79034
1963	60.92051	48.77632	**1976**	66.5922	50.43424	**1989**	69.53434	59.48759	**2002**	72.57368	66.2399
1964	61.36351	49.29305	**1977**	67.00261	51.82485	**1990**	69.67949	60.00849	**2003**	73.09424	66.67093
1965	61.8409	49.64976	**1978**	67.43683	53.067	**1991**	69.7802	60.53183	**2004**	73.43402	67.08598
1966	62.345	49.73305	**1979**	67.88278	54.08868	**1992**	69.79641	61.06215	**2005**	73.60073	67.48956
1967	62.85405	49.50963	**1980**	68.30937	54.86893	**1993**	69.71673	61.60041	**2006**	73.63602	67.88727
1968	63.35141	49.01022	**1981**	68.678	55.4432	**1994**	69.56598	62.14517	**2007**	73.62002	68.28315
1969	63.82946	48.31085	**1982**	68.96232	55.91849	**1995**	69.42059	62.69388	**2008**	73.61995	68.68071
1970	64.28168	47.58451	**1983**	69.1491	56.38122	**1996**	69.38259	63.24207	**2009**	73.6612	69.08193
1971	64.7042	47.05068	**1984**	69.24415	56.85841	**1997**	69.52732	63.78376	**2010**	73.75527	69.4858
1972	65.10161	46.87924	**1985**	69.27595	57.36305	**1998**	69.88759	64.31344	**2011**	73.89929	69.8918
1973	65.47946	47.16617	**1986**	69.28754	57.89512	**1999**	70.45259	64.82659	**2012**	74.06805	70.29485

Table 3: Inflation for Srilanka

Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh
1961	-1.70879	6.257482	**1974**	24.37873	44.54272	**1987**	7.751492	10.88005	**2000**	7.277341	1.859661
1962	-1.36545	0.022224	**1975**	5.350308	80.56976	**1988**	10.11703	7.600678	**2001**	13.66475	1.585395
1963	1.637563	5.164059	**1976**	9.975571	-17.6304	**1989**	10.92316	8.500223	**2002**	11.81257	3.195375
1964	2.477616	-8.74322	**1977**	14.69107	-3.21016	**1990**	20.06327	6.335597	**2003**	5.149138	4.52763
1965	0.242008	7.931705	**1978**	10.91788	25.61889	**1991**	10.62401	6.596235	**2004**	8.801492	4.240429
1966	-1.80355	6.296189	**1979**	15.39731	12.56451	**1992**	9.403697	2.97637	**2005**	10.41873	5.074715
1967	1.838885	14.79196	**1980**	19.97751	17.55507	**1993**	9.884459	0.28697	**2006**	11.27703	5.172374
1968	12.0879	-5.76958	**1981**	20.88531	10.52793	**1994**	9.7705	3.771827	**2007**	14.02844	6.78645
1969	1.307945	11.82772	**1982**	12.10105	9.687499	**1995**	9.303528	7.345332	**2008**	16.32702	8.789101
1970	12.50847	0.510309	**1983**	16.90684	8.515266	**1996**	10.81742	4.234504	**2009**	5.879883	6.520954
1971	1.498457	2.963255	**1984**	20.30051	14.04688	**1997**	8.924575	3.090097	**2010**	7.298948	6.473623
1972	8.96686	4.40202	**1985**	0.583914	11.14966	**1998**	9.214064	5.274366	**2011**	7.875437	7.531911
1973	12.74861	61.40578	**1986**	5.917282	8.001182	**1999**	4.162763	4.655731	**2012**	8.887817	8.480115

Table 4: Population Growth

Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh
1960	2.776675	2.843069	1974	1.463533	1.610146	1988	1.370882	2.635168	2002	0.657513	1.675767
1961	2.71149	2.819068	1975	1.583304	1.930885	1989	1.352342	2.557865	2003	1.323062	1.578431
1962	2.668636	2.805497	1976	1.624259	2.297472	1990	1.122943	2.457173	2004	1.357252	1.461435
1963	1.322255	2.833328	1977	1.626993	2.583722	1991	1.470186	2.345716	2005	1.069638	1.336408
1964	2.988354	2.913769	1978	1.763162	2.770359	1992	0.916618	2.245386	2006	1.0835	1.203833
1965	2.365633	3.012165	1979	1.967826	2.81864	1993	1.254578	2.170139	2007	0.907343	1.09045
1966	2.433425	3.148174	1980	1.882392	2.773514	1994	1.378866	2.128621	2008	0.884346	1.0277
1967	2.281665	3.228313	1981	0.675815	2.709322	1995	1.360112	2.107735	2009	1.145905	1.03078
1968	2.439454	3.136685	1982	2.323441	2.673162	1996	1.096743	2.089733	2010	0.98777	1.079332
1969	2.144943	2.837679	1983	1.443856	2.651273	1997	1.257333	2.057447	2011	1.040422	1.142792
1970	2.115883	2.425393	1984	1.199241	2.654239	1998	1.156577	2.007175	2012		1.191944
1971	1.396626	1.969659	1985	1.520144	2.669165	1999	1.437657	1.93313	2013	0.759603	1.22048
1972	1.338519	1.616666	1986	1.783024	2.682795	2000	0.241103	1.842203			
1973	1.772549	1.47452	1987	1.513875	2.67474	2001	-1.60958	1.756785			

Table 5: Annual GDP for Srilanka

Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh
1961	4.226162	6.058161	1974	3.845844	9.591956	1987	1.725611	3.732264	2000	6	5.944507
1962	3.81802	5.453031	1975	6.126219	-4.08821	1988	2.472693	2.159202	2001	-1.54537	5.274014
1963	2.516749	-0.45589	1976	3.335108	5.661361	1989	2.299296	2.612352	2002	3.964656	4.415411
1964	3.906489	10.95279	1977	5.100603	2.673056	1990	6.4	5.941307	2003	5.940269	5.255994
1965	2.536964	1.606258	1978	5.653838	7.073838	1991	4.6	3.339356	2004	5.445061	6.270503
1966	5.023798	2.566812	1979	6.403541	4.801635	1992	4.4	5.039133	2005	6.241748	5.955478
1967	6.439015	-1.87586	1980	5.846033	0.819142	1993	6.9	4.574376	2006	7.668292	6.629337
1968	5.801105	9.489454	1981	5.699522	3.801998	1994	5.6	4.084704	2007	6.796826	6.427843
1969	7.716812	1.220858	1982	4.141507	2.376467	1995	5.5	4.925052	2008	5.950088	6.190432
1970	3.846634	5.619852	1983	4.813978	4.016088	1996	3.8	4.621968	2009	3.538912	5.741159
1971	1.306901	-5.47948	1984	5.099156	5.180672	1997	6.405416	5.387552	2010	8.015959	6.06934
1972	-0.41048	-13.9737	1985	4.999397	3.223281	1998	4.698397	5.227531	2011	8.24591	6.708097
1973	7.057387	3.32568	1986	4.355549	4.24857	1999	4.300573	4.86923	2012	6.341362	6.233705
									2013	7.250907	6.030243

Table 6: GNI (atlas method) for Srilanka

Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh	Year	Srilanka	Bangladesh
1962	1506145510		1975	377793 9480	146805 62833	1988	768184 0320	278628 53565	2001	156935 78136	506207 56153
1963	1415723035		1976	35834 17045	145614 26543	1989	759024 1601	289813 96415	2002	162870 04279	511045 82867
1964	1383923474		1977	367822 1312	127148 91854	1990	805239 1942	310438 01658	2003	182593 33205	55012 780201
1965	1464041933		1978	347799 5236	12442 29227	1991	86593 42299	324444 88895	2004	208629 99381	613151 47254
1966	1688218467		1979	390445 9700	150028 30794	1992	97303 80941	347255 67842	2005	236896 58659	667806 94137
1967	1928908338		1980	415540 4094	179552 91508	1993	10684653128	357708 81839	2006	268050 48366	705180 18371
1968	1956366310		1981	46268 77145	20876 647988	1994	115356 02749	366305 52969	2007	307787 53464	749259 59431
1969	2087943876		1982	48657 87890	20387 424324	1995	126397 01168	392147 56641	2008	357991 86220	834181 48915
1970	2073521864		1983	490172 3672	191930 49911	1996	136581 69095	416053 11674	2009	403851 84303	934053 19444
1971	2165171082		1984	530607 0161	19340 717075	1997	147324 91143	441496 43802	2010	467621 16586	1.0468 1E+11
1972	2386208619		1985	594965 0598	20376 889042	1998	151368 08960	451095 44563	2011	53832 014839	1.177 71E+11
1973	2801453628	839215 2948	1986	664634 2990	22909 422664	1999	15631 972883	469791 44005	2012	59247923108	1.2915 8E+11
1974	3346092710	10925 519289	1987	712407 9182	258774 04710	2000	164068 35123	497685 67666	2013	649664 69115	1.404 4E+11