The Statistical Reconciliation of Time Series of Accounts after a Benchmark Revision

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Abstract

In this study the 2003-2007 U.S. annual input-output accounts, GDP-by-industry accounts and expenditure-based GDP are reconciled with the 2002 and 2007 quinquennial benchmarks and all contemporaneous constraints of the input-output accounts for the in-between years. The series are adjusted according to statistical procedures able to deal with large systems of accounts subject to both temporal and contemporaneous constraints. Our objective is to adjust the preliminary levels of the series such that they (i) are consistent with the quinquennial benchmarks available, (ii) fulfill all the accounting relationships for any given year, and (iii) show movements that are as close as possible to the preliminary information. To this end we use a simultaneous least-squares procedure based on the proportional first difference (PFD) criterion, a movement preservation principle proposed by Denton (1971). According to our past experiences, we evaluate the possible adoption of (i) a pure proportional adjustment (PROP) for series with breaks and high volatility that deteriorate the meaningfulness of growth rates and (ii) a priori constraints for groups of variables according to their different reliability, where this can reasonably be assumed.

Key Words: Benchmarking, Reconciliation, Temporal and Contemporaneous Constraints, Input-output Accounts.

1. Introduction

Measurements of socio-economic phenomena are conducted at different frequencies, with different objectives. Monthly or quarterly information aims at providing a timely picture of the short-term movements. Annual data from sample surveys or administrative statistics from regulatory agencies rely on a large sample of units, and thus, they provide a more accurate indication of medium- and long-term trends than intra-annual data. The U.S. Economic Census collects the most comprehensive data available on business activities and provides a detailed and accurate portrait of the country's economy once every five years. Higher frequency measurements are generally required to match corresponding lower frequency benchmarks.

Theoretically, higher frequency measurements should be consistent with lower frequency benchmark. Of course, this is rarely the case in practice, and consistency must be imposed on the data. Furthermore, at each frequency, social-economic variables may be required to satisfy a number of aggregation and accounting relationships. A typical example is national accounts, where total aggregates of the economy must be consistent with the sum of detailed components (e.g. by industry or by commodity) and identities are established between flows of production, expenditure, and income. However, cross-sectional consistency among the observed variables is not automatically met and must be imposed on the data.

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In a system that uses both low and high frequency series, observed data need to be adjusted such that both temporal and contemporaneous constraints are satisfied. A reconciliation process aims at preserving as much as possible the content of the preliminary information available. Because the time series dimension of socio-economic variables is relevant, it is often necessary that the short term movements (or the growth rates) of the preliminary information are preserved in the best possible way.

In a recent study (Chen *et al.*, 2013), a specific problem of reconciling annual (preliminary) estimates of U.S. national accounts aggregates subject to quinquennial benchmarks available from detailed Input-Output (IO) tables was addressed. Given preliminary, revised, but not fully balanced annual IO accounts from 1998 to 2002 and two revised and fully balanced IO accounts for benchmark years of 1997 and 2002, annual IO accounts for the years 1998-2001 were fully balanced and revised, with the temporal profile of the preliminary aggregates preserved as much as possible. The objectives were to adjust the annual data such that they (i) were consistent with the quinquennial benchmarks available, (ii) fulfill all the IO accounting relationships for any given year, and (iii) show movements that are as close as possible to the preliminary information. A simultaneous least-squares procedure based on the proportional first difference (PFD) criterion, a movement preservation principle proposed by Denton (1971), was compared with a pure proportional (PROP) adjustment procedure.

The results showed that these objectives were best achieved through the least-squares procedure based on the PFD criterion, because this procedure was able to smooth the differences observed between the preliminary and the benchmark data, reducing the impact of the correction by distributing it over all the years. However, it was also noticed that a PFD adjustment provides unsatisfactory results for a small subset of series that present breaks and changes from positive to negative values. Because these movements are difficult to preserve, they were adjusted according to a pure proportional criterion. It was shown that a constrained optimization procedure which minimizes a combined PFD-PROP objective function improves the overall adjustment of the system, minimizing the impact on the year-to-year changes of the preliminary series.

The release of the 2007 benchmark IO tables in January 2014 makes it possible to perform a similar study for the 2003-2007 period. In this latest study, we apply a simultaneous constrained optimization procedure to reconcile two different sets of preliminary, unbalanced annual I–O tables for 2003 through 2007 with the fully balanced 2002 and 2007 benchmaartk IO tables. With the objectives outlined in the previous study, we wish to obtain revised and fully balanced IO accounts for the years 2003-2006, where the temporal profile of the preliminary aggregates is preserved as much as possible.

The paper is structured as follows. Section 2 describes the construction of the U.S. benchmark IO tables and the revision of previously published annual IO estimates. Section 3 briefly introduces benchmarking and reconciliation of economic time series. Section 4 presents and evaluates the results achieved using a least-squares procedure based on alternative objective functions. Finally, Section 5 concludes the paper.

2. Construction and Revision of U.S. Input-Output Accounts

The U.S. national accounts system measures gross domestic product (GDP) via the production, expenditure, and income approaches. For the system to be consistent, production-based GDP measured as total output less total inputs from the IO accounts, expenditure-based GDP measured as total final expenditures from the national income and product accounts (NIPAs), and gross domestic income (GDI) as measured in the NIPAs must all be reconciled.

The U.S. IO accounts consist of make and use tables that are classified into N industries and M commodities. This implies that, at a minimum, the IO accounts must satisfy N sets of industry and M sets of commodity cross-sectional accounting constraints for each period. Because data used to compile the IO accounts are obtained from a variety of sources, inconsistency often arises in the initial estimates due to differences in the definition and classification of source data items and due to measurement errors in the source data. Consequently, initial estimates rarely satisfy all of these cross-sectional accounting constraints, and some technique must be employed to impose consistency among the components of the system.

In addition to cross-sectional constraints, individual series in the IO accounts are also required to match certain temporal accounting constraints. For example, each component series of quarterly GDP-by-industry must average to a corresponding annual series, and each component series in the annual IO accounts must be consistent with a corresponding quinquennial benchmark level. The benchmark IO tables, based primarily on data from the Economic Census, contain more complete information and are typically more accurate than higher frequency series; however, these tables are far less timely as they are published only every five years and with a significant lag. Higher frequency source data used to estimate quarterly GDP-by-industry or annual IO tables are more timely, but they often contain incomplete information and are therefore less accurate. As a result, benchmarking or interpolation procedures must also be employed to align less-accurate, higher-frequency data with less-frequent, higher-accuracy data. The best way to achieve internal consistency within each period while minimizing the disruption to period-to-period movements in each series is to impose cross-sectional and temporal aggregation constraints simultaneously.

The benchmark IO tables from the U.S. industry economic accounts are constructed every five years using data compiled primarily from the quinquennial economic census conducted by the U.S. Census Bureau. The 2007 benchmark make table was constructed using data primarily from the 2007 Economic Census. Estimates in the make table were considered predetermined and were not adjusted during balancing of the benchmark IO tables.

The construction of the 2007 benchmark use table can be described in three main parts. First, initial use table estimates were prepared using data from a variety of sources. Initial estimates of intermediate inputs were prepared based primarily on business expense data from the Census Bureau; initial estimates on final expenditures were prepared based on data from the Census Bureau, the NIPAs, trade associations, private businesses, and other federal government agencies; and initial estimates of value added (VA) were estimated indirectly using a production-based approach by taking output from the make table less initial intermediate expenses from the use table. In addition, a separate set of income-based value added statistics was estimated directly using data from the NIPAs, the Bureau of Labor Statistics, and the IRS Statistics of Income program.

Second, the production-based VA estimates were reconciled with the income-based VA estimates. This reconciliation process imposed cross-sectional constraints only using a weighted least squares minimization technique.

Third, the use table was balanced using a bi-proportional, or RAS, balancing technique. The RAS procedure sequentially adjusts the columns and rows of the use table until the table has been forced to be consistent with a set of predetermined marginal and aggregate constraints. The predetermined marginal totals include gross output by industry and by commodity from the make table, final expenditures by category and by commodity, and reconciled VA estimates from the previous step in the process. The use table is also forced to satisfy the aggregation constraint that total VA across all industries is equal to total expenditures from the NIPAs.

A key feature of the 2007 comprehensive revision was that the 2007 benchmark IO tables were fully integrated with the time series of annual IO tables and with GDP from the NIPAs.

To integrate annual and benchmark IO tables, the 2002 and 2007 benchmark IO tables were used as temporal benchmarks in the revision of the previously published annual IO tables. The 2002 benchmark use table was balanced using a weighted least squares approach[1]. The 2002 benchmark IO tables were constructed according to the 2002 NAICS classification system and the 2007 benchmark IO tables were constructed according to the 2007 NAICS classification system. In order to have consistent elements in both temporal benchmark IO tables, the 2002 tables were adjusted to be consistent with the 2007 NAICS classification system.

The preparation of the annual time series of IO tables can be described in three main parts. First, the component series in the previously published annual make and use tables were adjusted from the 2002 NAICS classification system to the 2007 NAICS classification system. In addition, make and tables were adjusted to incorporate new information from the comprehensive revision of GDP.

Second, the component series in the make tables were benchmarked to the quinquennial benchmark make tables using the proportional first difference Denton method (Denton, 1971).

Third, each annual use table was balanced using the RAS balancing technique. During the annual balancing process, a set of pre-determined marginal controls were used similar to those used for the 2007 benchmark table. These included gross output from the make tables, final expenditures by category (but not by commodity), and VA controls prepared by extrapolating reconciled VA from the benchmark using income-based gross domestic income (GDI) as an indicator. For a detailed discussion of the methodologies used in preparing the 2007 benchmark table and annual IO time series, see Kim *et al.* (2014). For more details on the integration of the benchmark and annual IO accounts, see Strassner and Wasshausen (2013).

Note that when a benchmark revision occurs, both the levels and growth rates of the variables in the IO tables are affected. Figure 1 shows the percentage revision produced by the 2007 benchmark estimates on the level of GDP and the major final uses aggregates. The impact of the 2007 benchmark revision ranges from -0.32% for government consumption and gross investments to 18.24% for changes in business inventories, while the 2007 preliminary GDP level showed a 1.70% upward revision.

In this study, reconciliation is conducted at the level of detail of 65 industries, 67 commodities, 3 VA components and 11 final expenditure categories. The available preliminary data present temporal inconsistencies and accounting discrepancies (by industry, see Figure 2, and by commodity, see Figure 3). Discrepancies are in general small with a few outliers as seen in the figures.

At the chosen level of detail, the system of IO accounts consists of a total of 9,642 series, 4,355 from the make table and 5,287 from the use table. Of the 4,355 series from the make table, 801 are non-zero series, and of the 5,287 series from the use table, the non-zero series include 3,553 intermediate inputs, 193 VA and 294 final expenditures. In the next section, we discuss how to deal with all these issues in a consistent statistical framework.
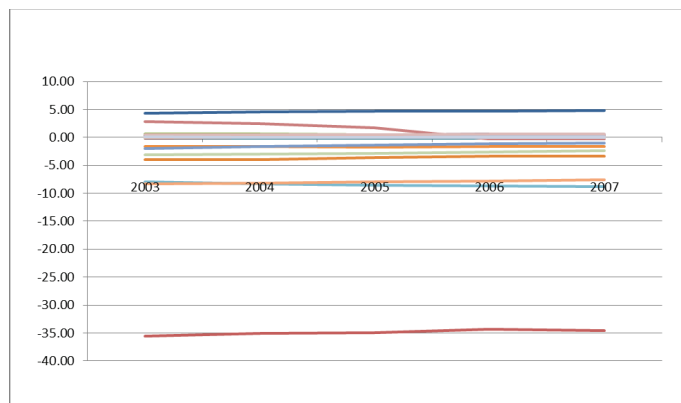
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[1]The 2002 benchmark use table was balanced using a weighted least squares approach with weights being the absolute value of the initial estimates. For details, see Rassier *et al.* (2007).
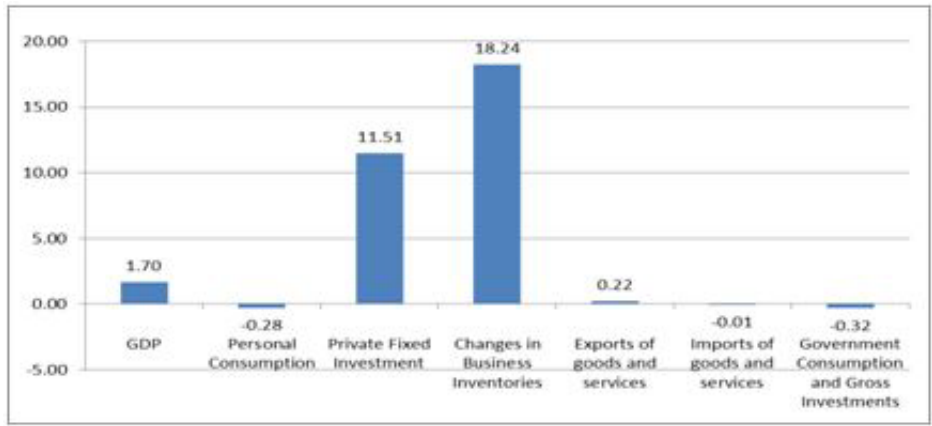
Figure 1: 2007 Benchmark Revisions of GDP and Final Uses by Major Category (% of preliminary 2007 values)
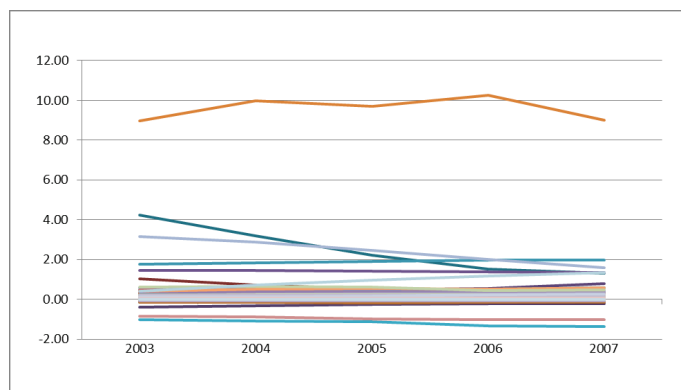
Figure 2: Discrepancies (%) by Industry

Figure 3: Discrepancies (%) by Commodity

3. Benchmarking and Reconciliation of Time Series

To impose temporal constraints in each component series, the modified Denton's PFD benchmarking method (Denton, 1971; Helfand et al., 1977; Cholette, 1984) has been implemented at the U.S. Bureau of Economic Analysis (BEA) since 2006. To impose contemporaneous constraints in the annual accounts, the usual reconciliation procedures use a RAS balancing technique to enforce accounting identities and reduce accounting discrepancies as much as possible. In a recent study (Chen, 2012), a generalized least-squares

(GLS) procedure was used to reconcile GDP estimates from IO, expenditures, and income accounts for a benchmark year according to the estimated reliabilities of initial source data items.

Consistency in the time series of the national account system requires that temporal and contemporaneous constraints be satisfied simultaneously. In recent years, two alternative reconciliation procedures have been introduced to restore temporal and contemporaneous constraints in a system of series (Quenneville and Rancourt, 2005; Di Fonzo and Marini, 2011). The two-step procedures consist of a univariate process to restore temporal constraints in each component series through benchmarking and a multivariate reconciliation process to restore contemporaneous constraints in the system in each period while preserving movements in each series. The two-step procedures are shown to be effective when low frequency benchmarks correspond to low frequency sums of the high frequency values (i.e. flow variables). However, each estimate in the quinquennial benchmark IO accounts pertains to the value of a variable of the benchmark year, not the quinquennial sum of the values of the variable. In this case, the two-step procedure may not be able to preserve the temporal movements in each component series during the reconciliation process. For this reason we decided to adopt a simultaneous approach.

The reconciliation problem can be formalized in a compact matrix form as follows. The U.S. annual IO accounts consist of make and use tables. The 65×67 make table matrix contains the gross output of 67 commodities from 65 industries. The use table consists of a 67×65 matrix of intermediate inputs, a 3×65 matrix of industry VA from industry income, and a 67×11 matrix of final uses.

Let $\mathbf{X}_t$, $\mathbf{Z}_t$, $\mathbf{V}_t$, and $\mathbf{Y}_t$ denote the matrices of preliminary estimates of gross output, intermediate inputs, VA and final uses, respectively, in the annual IO accounts for $t = 2003, \ldots, 2007$. Let $\bar{\mathbf{X}}_{2002}$, $\bar{\mathbf{Z}}_{2002}$, $\bar{\mathbf{V}}_{2002}$ and $\bar{\mathbf{Y}}_{2002}$ denote the corresponding matrices for benchmark year 2002, and $\bar{\mathbf{X}}_{2007}$, $\bar{\mathbf{Z}}_{2007}$, $\bar{\mathbf{V}}_{2007}$ and $\bar{\mathbf{Y}}_{2007}$ for benchmark year 2007.

The preliminary matrices can be conveniently rearranged into a one-dimensional vector of stacked time series. Let $\mathbf{x}_{t,i,j}$ denote the 6×1 column vector of the element (i, j) of the make table matrix $\mathbf{X}_t$, for $t = 2002, \ldots, 2007$[2]. We consider all (i, j) elements of the matrices even if they are zeros for all or for some years. There are 4,355 time series in the make table, which can be stacked into a single $26,130 \times 1$ vector as

$$\mathbf{x} = \left[\begin{array}{ccccccc} \mathbf{x}'_{2002,1,1} & \mathbf{x}'_{2002,1,2} & \cdots & \mathbf{x}'_{2002,67,65} & \cdots & \mathbf{x}'_{2007,67,63} & \mathbf{x}'_{2007,67,64} & \mathbf{x}'_{2007,67,65} \end{array} \right]'.$$

Vectors $\mathbf{z}$, $\mathbf{y}$, $\mathbf{v}$ can also be set up in the same fashion. Their row dimensions are 26,130, 4,422 and 1,170, respectively. The input vector of preliminary data of the problem is thus defined as

$$\mathbf{p} = \left[\begin{array}{cccc} \mathbf{x}' & \mathbf{z}' & \mathbf{y}' & \mathbf{v}' \end{array} \right]'$$

where $\mathbf{p}$ has row dimension of $57,852 \times 1$.

Let us now consider the constraints of the system. There are *exogenous* and *endogenous* constraints. The first type concerns the benchmark values for the years 2002 and 2007. Let $\mathbf{b}$ denote the vector of two-element time series from the benchmarked matrices previously defined, i.e.

$$\mathbf{b} = \left[\begin{array}{ccccccccc} \bar{x}^{1,1}_{2002} & \bar{x}^{1,1}_{2007} & \bar{x}^{1,2}_{2002} & \bar{x}^{1,2}_{2007} & \cdots & \bar{v}^{3,64}_{2002} & \bar{v}^{3,64}_{2007} & \bar{v}^{3,65}_{2002} & \bar{v}^{3,65}_{2007} \end{array} \right]',$$

with dimension of $19,284 \times 1$. Let $\mathbf{H}_1$ denote the $19,284 \times 57,852$ mapping matrix for the exogenous constraints specified in $\mathbf{b}$ for the benchmark years 2002 and 2007. Given

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[2]In order to link the reconciled series to the 2002 benchmarks, we consider the benchmark matrices of 2002 as part of the group of preliminary matrices as well.

that, as we have previously said, preliminary and benchmark 2007 values are different, it is $\mathbf{H}_1\mathbf{p} \neq \mathbf{b}$.

The endogenous constraints are defined by the set of accounting relationships defined by the IO tables. There are 67 row constraints (commodities) and 65 column constraints (industries) per year. The aggregation constraint of total GDP equals total VA is redundant and can be disregarded, as it follows from adding up the first 134 constraints. The contemporaneous constraints for 2002 are redundant, because benchmarked estimates are used as the preliminary estimates. In total, they add up to 792 constraints for $t = 2002, \ldots, 2007$. Let $\mathbf{H}_2$ denote the $792 \times 57,852$ matrix mapping the 57,852 elements in the preliminary vector $\mathbf{p}$ to the 792 accounting constraints. Clearly, it is $\mathbf{H}_2\mathbf{p} \neq \mathbf{0}_{792\times1}$.

In sum, we have

$$\left[\begin{array}{c} \mathbf{H}_1 \\ \mathbf{H}_2 \end{array} \right] \mathbf{p} \neq \left[\begin{array}{c} \mathbf{b} \\ \mathbf{0}_{792\times1} \end{array} \right], \tag{1}$$

and we wish to derive the $57,852 \times 1$ vector of reconciled values $\mathbf{r}$

$$\left[\begin{array}{c} \mathbf{H}_1 \\ \mathbf{H}_2 \end{array} \right] \mathbf{r} = \left[\begin{array}{c} \mathbf{b} \\ \mathbf{0}_{792\times1} \end{array} \right], \tag{2}$$

such that differences in the temporal dynamics between $\mathbf{r}$ and $\mathbf{p}$ are minimized.

To reconcile a system of time series, we use adjustment procedures based on the constrained optimization of two different objective functions:

- Proportional adjustment (PROP):

$$\sum_{i=1}^{n} \sum_{t=2003}^{2007} \frac{(r_{t,i} - p_{t,i})^2}{|p_{t,i}|} \tag{3}$$

- Proportional First Difference (PFD) adjustment, which is a multivariate extension of the univariate benchmarking solution proposed by Denton (1971) and modified by Cholette (1984):

$$\sum_{i=1}^{n} \sum_{t=2003}^{2007} \left(\frac{r_{t,i}}{p_{t,i}} - \frac{r_{t-1,i}}{p_{t-1,i}} \right)^2 \tag{4}$$

where n is the number of non-null variables of the system.

In both cases, the system is adjusted simultaneously (i.e. all variables and all years at the same time). However, the adjustment principles operate very differently. The PROP criterion distributes the differences proportionally to the levels of the variables. On the other hand, the PFD criterion preserves the year-to-year movements of the variables. Because our target is to preserve the changes in the preliminary variables, we expect that the PFD method provide more satisfactory results for this exercise.

We also define a combined objective function (see Bikker *et al.*, 2013):

$$\sum_{i \in S^{PFD}} \sum_{t=2003}^{2007} \left(\frac{r_{t,i}}{p_{t,i}} - \frac{r_{t-1,i}}{p_{t-1,i}} \right)^2 + \sum_{i \in S^{PROP}} \sum_{t=2003}^{2007} \frac{(r_{t,i} - p_{t,i})^2}{|p_{t,i}|} \tag{5}$$

where both the PFD criterion and the PROP criterion are utilized. The variables in the system are divided in two subsets (S^{PFD} and S^{PROP}, respectively). The PFD criterion is used for those series showing meaningful and interpretable movements over time (namely movements that we would like to preserve). The PROP[3] criterion is used for the remaining series with breaks in the movements. We call this procedure PFD-PROP.

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[3]In this exercise S^{PROP} refers to changes in business inventories and to all other series presenting negative and positive values. These series represent a small fraction of the series in the system (42 out of 4,355).

4. Results

In this study, we consider two sets of preliminary estimates in the reconciliation. The first set of preliminary estimates consists of the previously published annual IO tables from 2003 to 2007 with the adjustments necessary in order to be consistent with the 2007 NAICS classification system. The objective of this exercise is to evaluate the reconciled results from the three procedures based on alternative adjustment criteria described in equations (3), (4) and (5) in Section 3.

The second set of preliminary estimates is the revised (as described in the Section 2) and not yet balanced annual IO tables from 2003 to 2007. In theory, the previously published annual IO tables should be used directly as the preliminary estimates. However, in the actual production, the previously published IO tables could not be directly used as the preliminary estimates, because of the changes in the classification system and because new information from the benchmark revision of GDP needed to be incorporated. We consider the second set of preliminary estimates in order to be able to compare the reconciled results using the least squares procedure with the annual IO tables balanced using the RAS procedure during the recent benchmark revision.

In order to assess the global performance of the procedures, for each series we calculate the Mean Absolute Adjustment (MAA) and the Root Mean Squared Adjustment (RMSA) to the percentage levels:

$$MAA_i^L = 100 \times \frac{1}{5} \sum_{2003}^{2007} \left| \frac{\hat{r}_{t,i} - p_{t,i}}{p_{t,i}} \right|$$

$$RMSA_i^L = 100 \times \sqrt{\frac{1}{5} \sum_{2003}^{2007} \left(\frac{\hat{r}_{t,i} - p_{t,i}}{p_{t,i}} \right)^2}$$

and to the percentage growth rates:

$$MAA_i^R = 100 \times \frac{1}{5} \sum_{2003}^{2007} \left| \frac{\hat{r}_{t,i}}{\hat{r}_{t-1,i}} - \frac{p_{t,i}}{p_{t-1,i}} \right|$$

$$RMSA_i^R = 100 \times \sqrt{\frac{1}{5} \sum_{2003}^{2007} \left(\frac{\hat{r}_{t,i}}{\hat{r}_{t-1,i}} - \frac{p_{t,i}}{p_{t-1,i}} \right)^2}$$

for $i = 1, \ldots, n$, where n is the number of non-null series from the IO tables.

To compare the results, we focus on the 43 main aggregates of national accounts (gross domestic product (GDP), gross output, intermediate inputs and VA of 12 major industries, and 6 final expenditure categories). For the reconciliation using the first set of preliminary estimates, table 1 shows the averages of indices MAA and $RMSA$ calculated for the 43 main aggregates from the detailed reconciled series using the three alternative reconciliation procedures:

- Proportional adjustment (PROP), based on minimizing criterion (3);

- Proportional First Difference (PFD) adjustment, based on criterion (4);

- Combined PFD and PROP adjustment (PFD-PROP), as defined by criterion (5).

As expected, PROP minimizes the adjustment in terms of levels (both MAA^L and $RMSA^L$ are minimum). In agreement with our previous results (Chen *et al.*, 2013), PROP outperforms PFD in minimizing the adjustment in terms of growth rates. The PFD criterion

Table 1: Reconciliation of the previously published annual IO tables. Summary measures of adjustment

Criterion	Levels		Growth Rates	
	MAA^L	$RMSA^L$	MAA^R	$RMSA^R$
PROP	1.395	3.892	1.440	4.336
PFD	5.217	14.782	3.646	11.570
PFD-PROP	3.256	5.760	1.536	2.597

is penalized by series in the system that present changes from positive to negative values (e.g. changes in business inventories). To overcome this difficulty, the PFD-PROP procedure adjusts all these series according to PROP while it maintains the PFD approach for the rest of the series. As it is noticed in Table 1, the PFD-PROP procedure achieves the minimum values for $RMSA^R$, while its MAA^R is slightly larger than PROP.

Figure 4 displays the boxplots of $RMSA_i^L$ (top chart) and $RMSA_i^R$ (bottom chart) for the 43 aggregates (the absolute distance metric of MAA gives a less pronounced difference between the performance of the three procedures and it is not shown). The visual inspection of the boxplots confirms that PFD-PROP produces the smallest adjustment of the growth rates, while PROP provides the best results in preserving the original levels. As for the growth rates, this conclusion is evident looking at the $RMSA$ statistics.
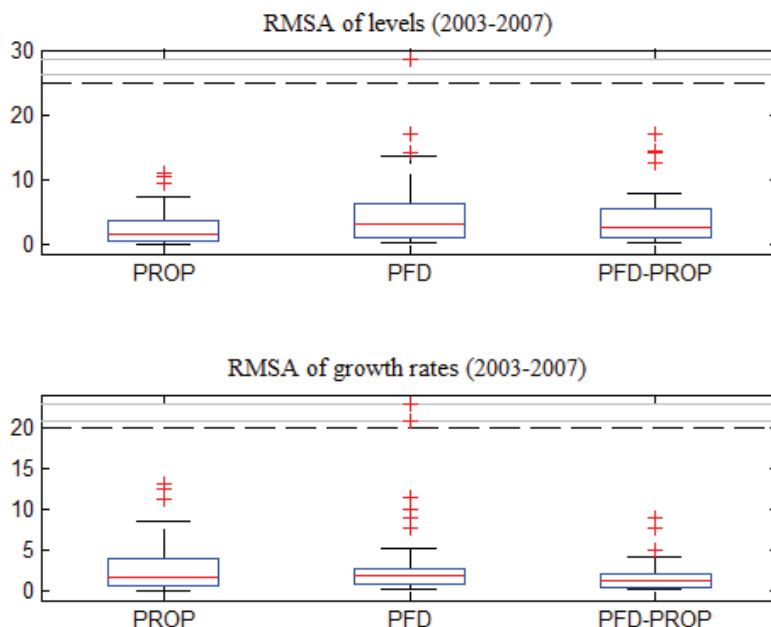
Figure 4: Boxplot of RMSA statistics

To understand the different type of adjustment conducted by PROP and PFD-PROP, it is useful to look at the treatment of some aggregate series, like GDP (figure 5). The left-

hand charts refer to the levels, the right-hand charts to the growth rates, and the adjustments to both levels and growth rates are shown in the bottom charts. It clearly appears that the adjustment to GDP under the PROP criterion all occurs in the year 2007, while the PFD-PROP criterion produces (growing) adjustments to the levels across the entire period. This last feature results in smoothed estimates of the growth rates rather than the abrupt jumps produced by the PROP criterion, with a large positive correction of the preliminary 2007 growth rates.
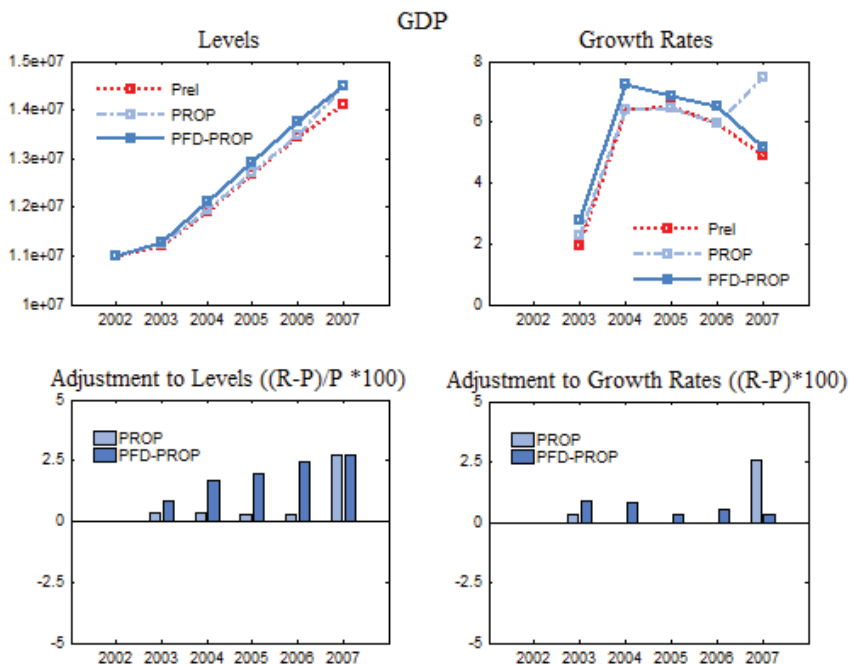
Figure 5: Gross Domestic Product: Adjustments to Levels and Growth Rates

Results from reconciliation using the second set of preliminary estimates are shown in Table 2. As discussed in Section 2, certain variables were set as pre-determined in the final balancing of the IO tables during the benchmark revision. In order to conduct a comparable evaluation, we set the same variables to be pre-determined in the reconciliation using the least squares procedure. Pre-determined variables are controlled by setting alterability coefficients to 0.01. The pre-determined variables included gross output from the make tables, final uses by category, and two components of value added - compensation of employees and net taxes on production. The free variables were intermediate inputs and the third value added component - gross operating surplus.

In order to assess the different impact of the adjustment procedures, we keep the pre-determined aggregates distinct from the free ones. We find that RAS with pre-determined variables out-performs pure PFD-PROP (with no pre-determined variables) in adjusting the pre-determined variables, but it greatly over-adjusts the free variables. Pure PFD-PROP corrects the predetermined variables more than RAS both in levels and growth rates. However, it best preserves the dynamics of the 'free' variables. In turn, PFD-PROP with pre-determined variables outperforms pure PFD-PROP and RAS in adjusting the pre-determined variables and gives better results than RAS with pre-determined variables for the free aggregates (MAA and $RMSA$ are lower for both levels and growth rates).

Table 2: Reconciliation of the revised unbalanced IO tables. Summary measures of adjustment

Criterion	Pre-determined aggregates*				Free aggregates**			
	MAA^L	$RMSA^L$	MAA^R	$RMSA^R$	MAA^L	$RMSA^L$	MAA^R	$RMSA^R$
RAS with pre-determined variables	0.07	0.51	0.09	0.55	8.91	193.34	9.47	162.20
PFD-PROP with pre-determined variables	0.01	0.04	0.01	0.04	1.94	5.01	3.01	7.24
PFD-PROP no pre-determined variables	1.28	3.16	1.78	3.44	0.70	1.66	1.09	2.30

* Include 206 pre-determined aggregates: output (65 industries), final uses (11 categories), compensation of employees (65 industries) and net taxes on production (65 industries).
** Include 130 free aggregates: intermediate consumption (65 industries) and gross operating surplus (65 industries).

5. Conclusions

In this paper we have shown how to reconcile annual preliminary series of national accounts with quinquennial benchmarks available from IO tables. Our objective was to minimize the impact of the adjustment on the movements in the preliminary series. In general, we have found that this objective is best achieved through a constrained optimization procedure based on a movement preservation principle, in our case the PFD criterion proposed by Denton (1971) and modified by Cholette (1984). Looking at the temporal dynamics of the data, the PFD-based procedure is able to smooth the differences observed between the preliminary and the benchmark data of 2007, reducing the impact of the correction by distributing it over all the years.

However, we have noticed that a PFD adjustment provides unsatisfactory results for series that present breaks and changes from positive to negative values. Because these movements are more difficult to preserve, these series should be adjusted according to a pure proportional criterion. We have shown that a constrained optimization procedure that minimizes a combined PFD-PROP objective function improves the overall adjustment of the system, minimizing the impact on the year-to-year changes of the preliminary series. Moreover, we have also shown the flexibility of the constrained optimization procedure to set pre-determined variables in reconciliation via alterability coefficients according to production requirements.

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