Statistical Error-based Controller Design of Hybrid Systems with Mixed Time Delays

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Abstract
The authors have recently shown that an error-based controller design method can be used to successfully synchronize general chaotic systems with no inherent time delays. This approach employs interval estimates obtained from initial error distributions of the response state variables to formulate an interval representation of the original response system that provided stability requirements for our controller design. In specific cases where the number of controller parameters equals the number of state variables, a system of linear constraint equations arose that lead to a direct solution for the required controller gains. In the present investigation, we extend this robust approach to hybrid systems where the design goal is to synchronize inherently delayed and non-time delayed systems. Such mixed designs are encountered in many physical and biological systems. Surprisingly, most past studies on coupled oscillator systems have avoided hybrid model synchronization. Here it is shown that our statistical error-based controller design method can be successfully extended to such time delay systems.

Key Words: Statistical error design, controls, synchronization, hybrid systems

1. Overview

This paper focuses on the extension of the statistical error-based controller approach to time delay hybrid systems that pair a functional system with a chaotic oscillator. The former system arises in many practical settings where there is a dependence on past state information. Two classical examples of delay systems are the Mackey-Glass (1977) and Marcus and Westervelt (1989) equations. Specifically, the aim here is to force a classical Lorenz system to track a time delay linear time invariant (LTI) system first proposed by Olgac and Sipahi (2002). The global stability of this LTI system was assessed via a Rekasius (1980) based methodology that isolated localized pockets (regions) of stability that depended upon the time delay value. Unlike our prior study where global stability of each chaotic oscillator persists over the entire phase space, it is localized in this functional system. Hence, for an arbitrary time delay, local or global stability is not a certainty. Care must be taken to identify those stable zones prior to implementing the statistical error-based controller design (SEBCD) strategy. An advantage of SEBCD in such cases is that the controller structure is only affected by the response system stability .i.e., even if the drive is unstable a stable controller can still be devised for a stable response system. The success of present inquiry only requires performing independent and separate stability analyzes of the respective drive and response systems. For the

Olgac and Sipahi system equation below, these stability pockets were found to lie in the intervals $0 < \tau < 0.1624$ and $0.1859 < \tau < 0.2222$.

$$\frac{dx}{dt} = \mathbf{A}\,x + \mathbf{B}\,x\,(t - \tau)$$

$$A = \begin{bmatrix} -1 & 13.5 & -1 \\ -3 & -1 & -2 \\ -2 & -1 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} -5.9 & 7.1 & -70.3 \\ 2 & -1 & 5 \\ 2 & 0 & 6 \end{bmatrix}$$

Any cases with delays outside these intervals produce unstable drive dynamics. Nevertheless, synchronization is achieved in spite of the fact that the response system tracks the unstable drive to oblivion. Since the same response model is employed here as in Morgan and Morgan (2013) that statistical error-based controller structure is applicable. The present controller study highlights the impact of embedded time delays of functional equations on gain specifications and the importance of decoupling the controller design via the SEBDC approach.

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2. Approach and Preliminary Findings

This new method is based on the premise that an interval approximation model (crisp fuzzy system) representation of the system of ordinary differential equations that describes an individual chaotic (response) oscillator can be used to determine arbitrary 'controller parameters' for a general hybrid system. The initial step in this process involves constructing interval approximations for each nonlinear and/or positive linear term in our chaotic model from descriptive statistics of the unsynchronized response system. Thus, armed with these interval approximations, (local) stability of each term in the model is assessed and used to determine the global stability requirements for a given drive-response combination. Under this paradigm, the stability of the drive system is not necessary for establishing overall system synchronization, only the response. The drive stability can however affect overall closed-loop stability as demonstrated in this study.
Figure 1 shows the original uncoupled dynamic response of our hybrid system prior to any synchronization. The drive for this system is the Olgac and Sipahi LTI that is linked dynamically to a classical Lorenz oscillator (response). Figure 2 highlights a synchronized state with a high degree of fidelity between drive and response dynamics. Note the high level of compression that occurs with synchronization and the speed at which it is achieved.

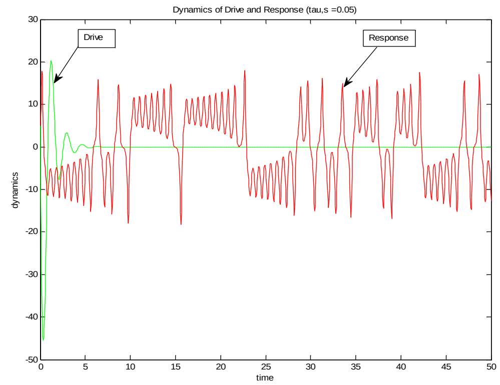


Figure 1: Dynamic Response of an Unsynchronized System

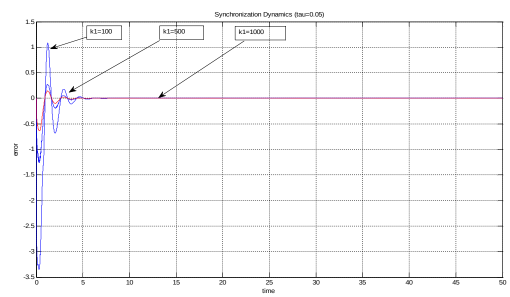


Figure 2: Dynamic Response of a Synchronized System

Again, the general SEBCD procedure, as outlined in Morgan and Morgan (2013), is employed. Unlike our prior effort, Morgan and Morgan (2012) that incorporated a design procedure suggested by Bhiwani and Patre (2011) for a classical proportional-integral-derivative (PID) controller, the current approach eliminates a cumbersome optimization step encountered with the former. The constraints imposed in the present design are that all error-based moment gains must be positive and all generated errors are bounded by the initial state errors between the original drive and response systems. Three design cases are possible with this approach: under specified, uniquely specified and over specified. Here, only the uniquely specified case where the number of controller parameters matches the number of state equations was considered. The basic design philosophy has been outlined in detail in our prior papers, Morgan and Morgan (2012, 2013). Those approaches removed local nonlinearities via construction of fuzzy intervals for each nonlinear term appearing in the response system equations. Thus, a system of ordinary differential equations are converted, as shown below in Table 1, into a system of linear interval equations that can be used to estimate error-based moment controller gains and yield a gain-error characteristic polynomial that addresses local and global system stability.

Table 1. Conversion of Lorenz System to Fuzzy Model	
Original Model	**Extended Fuzzy Model**
$\frac{dx}{dt} = 10\,(y - x)$	$\frac{dx}{dt} = \left[10 - \sum_{i=1}^{3} k_{xi}\ \varepsilon_{xi}^{i-1}\right] y - 10x$
$\frac{dy}{dt} = -xz + 28\,x - y$	$\frac{dy}{dt} = \left[28 - \sum_{i=1}^{3} k_{yi}\ \varepsilon_{xyi}^{i-1}\ - z_s\right] x - y$
$\frac{dz}{dt} = xy - 2.67\,z$	$\frac{dz}{dt} = \left[y_s - \sum_{i=1}^{3} k_{zi}\ \varepsilon_{zi}^{i-1}\right] x - 2.67z$

The gain-error characteristic equation for the Lorenz system takes the following form

$$p\,(\varepsilon) = (k_1 - C_{max}) + k_2\ \epsilon + k_3\ \epsilon^2$$

The C_{max} term appearing in that equation is the maximum value observed in the bracketed terms of the interval description of the original Lorenz system. Two distinct solutions are possible based upon the sign of the discriminant associated with the gain-error characteristic polynomial. Interestingly enough, the sign of the discriminant also dictates the type of image produced. A negative value of this quantity generates overlapped images while a positive one produces displaced images. It was also observed that the k_1 gain controlled the error level between synchronized states (Figure 3), as reported in Morgan and Morgan (2012), and that the regression model (Figure 4) developed in that study was valid for the current investigation as well. There the relationship between controller gain and the correlation coefficient revealed the presence of two distinct zones (unstable and stable regions) separated by a critical gain value. A single regression model was adequate for describing the general dependency of synchronization fidelity to controller gain for a wide class of chaotic oscillators. The size of the instability region was found to be bounded by the length of the maximum fuzzy interval while the minimum fuzzy interval enclosed the un-entangled (critical) point. The histograms of Figure 3 show the decrease in the associated synchronization error with increasing k_1 gain. Although not readily apparent in Figure 3, in most cases the error distributions were invariant to a change in applied controller gain, k_1 .

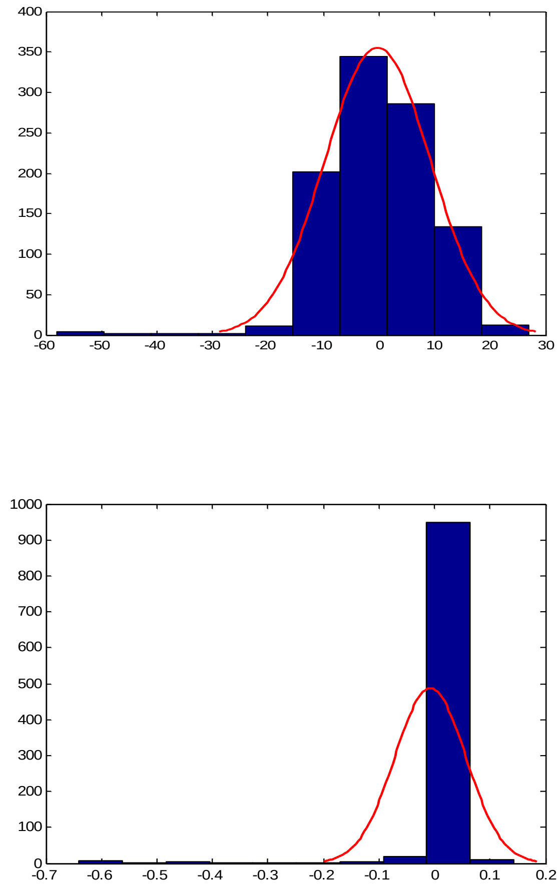


Figure 3: Histograms of Errors for a Time Delay of 0.05

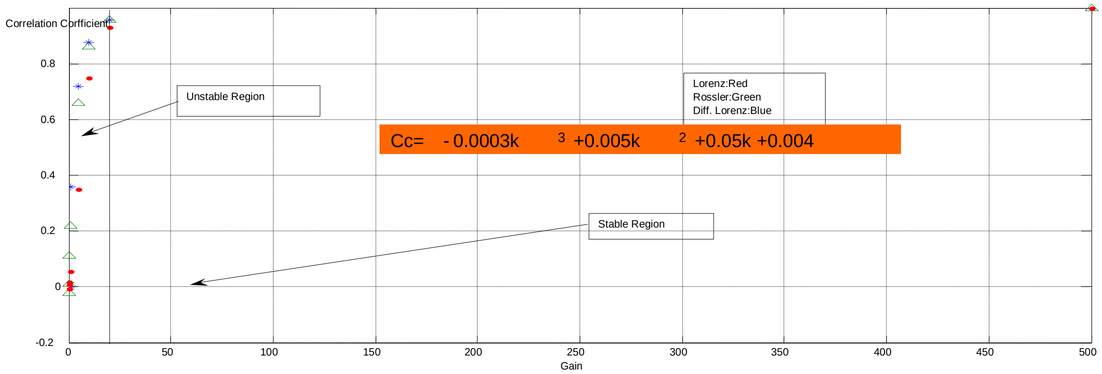
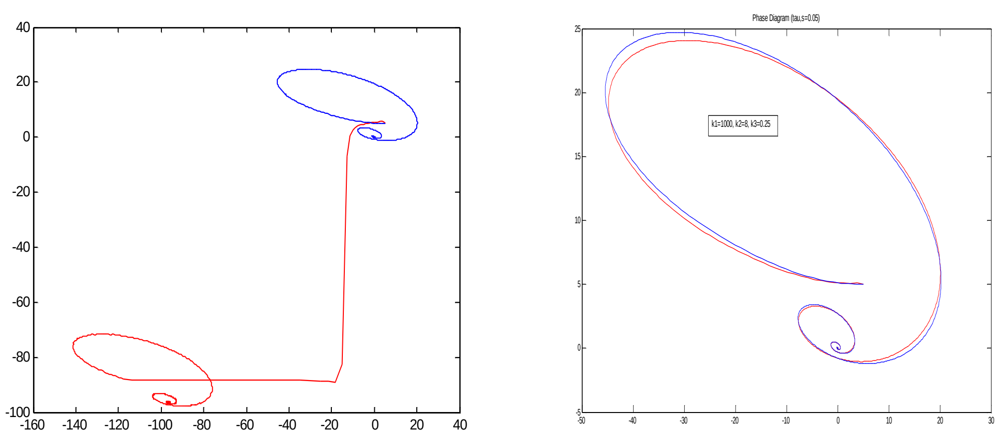


Figure 4: Correlation Coefficient versus Controller Gain, k

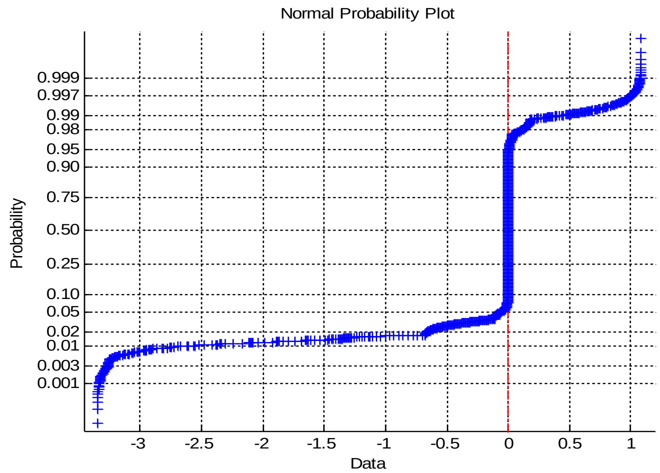
The normal probability plots of these errors also capture those features of the data (Figures 5 and 6). The other two gains (k_2 and k_3) only affected convergence rates and image displacements. Table 2 highlights the effect of controller gains on the degree of synchronization. The values recorded in this table in parentheses are correlation coefficients obtained after the removal of the mean horizontal and vertical displacement distances of the stereo images. The high coefficient values in these cases indicate that the stereo images are essentially identical even though they are spatially displaced.



a. Displaced Images

b. Overlapped Images

Figure 5: Images Produced by the Discriminant Effect



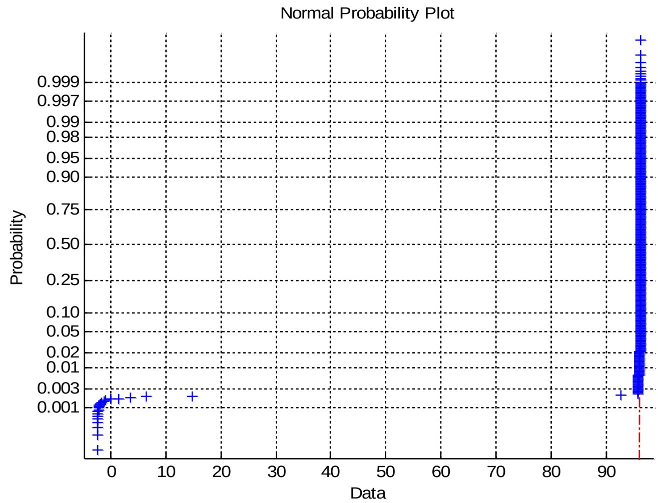


Figure 6: Normal Probability Plots of Discriminant Produced Images

Gains	**K1 = 91**	**K1 = 154**
K2 = 8 (overlapped images)	0.9996	0.9997
K2 = 15 (stereo images)	-0.1786 (0.9593)	0.0266 (0.9642)
K2 = 25 (stereo images)	0.6377 (0.9831)	0.5258 (0.9777)

Table 2: Effect of Controller Gains on Degree of Synchronization

Conclusions

An error-based controller design method is proven to be successful for synchronizing hybrid, as well as, arbitrary chaotic systems. This method uses interval estimates of the response state variables to formulate an interval representation of the original response system. For the case where the controller parameters and state variables are matched, a system of linear constraint equations are directly solvable for the controller gains. A unique characteristic equation provides stability requirements for the controller gains that can produce two uniquely different solutions that depend upon the sign of the discriminant embedded in the gain error characteristic equation. A simple regression model devised for predicting the effect of proportional control gains on synchronization fidelity in Morgan and Morgan (2012) is also valid for the functional drive system.
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