Combination of Uniform Binomial (CUB) Models: An Application to the Evaluation of Food Packaging

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Abstract

CUB models have been developed in order to take into account the latent process of evaluating an object (product, service, etc.). Feeling and uncertainty are supposed to be involved in the choice process. Several model extensions have been developed since their introduction and application studies have proven to be a very useful approach.

In a survey on food packaging, respondents had to evaluate their grade of attention paid to some grocery product characteristics and their satisfaction towards packaging attributes. Thanks to CUB models interesting results can be drawn. For instance respondents are very interested in "provenance" and "seasonality" of products with some group differences and they are satisfied towards food preservation again with interesting differences among groups.

Key Words: CUB models, customer satisfaction, food packaging

1. Introduction

In the context of preference evaluation, CUB (Combination of a Uniform and a shifted Binomial random variables) models are a promising method proposed by D'Elia and Piccolo (2005) for the first time. Within the framework of preference study, revealed and stated preferences are the two main approaches (Louviere *et al*., 2000; Alriksson and Öberg, 2008), where CUB models are mainly concerned with the stated preferences (Iannario and Piccolo, 2011). The discrete choice process of an item can be interpreted as the interaction of two subjective components, the feeling and the uncertainty toward an item. A probabilistic structure has been proposed (D'Elia and Piccolo, 2005) in order to take into account the psychological process of evaluation. In survey respondents are usually asked to rate several items (products, services, etc.) from a m-point scale or to rank m items. The model has been initially proposed for ranks data where the respondents are asked to rank m items from the best preferred to the worst. In this way we obtain a vector of length i, i=1,..,n, for object j, j=1,..,m. Using an ordinal scale to rate objects we obtain the same result. Ratings are more easily adopted and sometimes preferred by respondents (Piccolo and D'Elia, 2008).

In a context of food preference evaluation, CUB models have been successfully adopted. For example Piccolo and D'Elia (2008) analyzed data where consumers were asked to assign a score from 1 (dislike extremely) to 9 (like extremely) to 5 smoked salmons. CUB models have been applied to preference evaluation data and customer satisfaction data in order to help to take decisions on marketing (Kennet and Salini, 2011; Iannario *et al*., 2012; Corduas *et al*., 2013). The possibility to introduce object and subject covariates improve the ability of CUB models to detect product or service attributes that are valuable to the customer. For example Corduas *et al*. (2013) investigate which wine attributes are important for Italian consumers.

The aim here is to present and apply CUB models to a customer satisfaction questionnaire evaluating food packaging.

2. The CUB models

2.1 The mixture model

The CUB model assumes the involvement of two latent variables during the evaluation process, the *feeling* and the *uncertainty* towards the item. The choice process of an item can be represented by the mixture of two components that are the subjective liking/disliking of an object and the uncertainty (D'Elia and Piccolo, 2005). The two components are well described by a mixture of a shifted Binomial random variable (*feeling*) and a discrete Uniform random variable (*uncertainty*). Motivations for the choice of a shifted Binomial and a discrete Uniform distributions are describe in Corduas *et al*. (2009). What they call "feeling" represents the subject's motivation, awareness and full understanding of the problem. The respondent that is facing a task, i.e. evaluation, preference, level of attention, etc., has to choose a grade that summarizes what he/she feels. A *shifted Binomial* random variable seems to mimic the behaviour of selecting a grade by a pairwise comparison (D'Elia, 2000).

The second component is the *uncertainty* of the choice process. Respondents may have a limited time to evaluate an item, or may have a poor understanding of the problem. This is not the case of a random answer but the case of a response where other factors contribute to the final choice. A *discrete Uniform* random variable describes the uncertainty behaviour when no grade prevails over the others, that is the case of maximum uncertainty (Corduas *et al.* 2009).

The uncertainty and the feeling components are combined in a mixture model (D'Elia, 2004; D'Elia and Piccolo, 2005; Corduas *et al.* 2009) so that the realization of a random variable Y is explained by the following model:

$$\Pr(Y = y) = \pi\left[\binom{m-1}{y-1}(1-\xi)^{y-1}\xi^{m-y}\right] + (1-\pi)\left[\frac{1}{m}\right], \quad y = 1,2,\ldots,m\,,$$

with Y varies from 1 to m, $\xi \in [0,1]$, $\pi \in (0,1]$ and Iannario (2010) proves that the mixture distribution is identifiable when *m* is greater than 3. Moreover it is a very flexible distribution that is able to assume very different shapes (Piccolo, 2003a; D'Elia and Piccolo, 2005).

The weights π and (1-π) are considering the uncertainty aspect of the choice process, where (1-π) is considered a *measure of uncertainty* and (1- π)/*m* is a measure of the *uncertainty share*. The interpretation of parameter ξ depends on the initial coding of the measurement scale. When *m*=1 is coded as the minimum, the feeling/agreement tends to the minimum when ξ tends to 1. In this case (1-ξ) is a measure of feeling/agreement. An E-M algorithm has been adopted and maximum likelihood parameter estimation is effectively obtained by D'Elia (2003) and Piccolo (2003b).

2.2 Model extensions

D'Elia (2003) and Piccolo (2003b) provide a formal description of a CUB model with covariates. Rater's covariates are linked to uncertainty (*p* covariates) and feeling (*q* covariates) components by two logistic functions.

In the model extension

$$\Pr(Y_i = y_i) = \pi_i \left[\binom{m-1}{y_i - 1} (1-\xi_i)^{y_i - 1} \xi_i^{m-y_i} \right] + (1-\pi_i)\left[\frac{1}{m}\right], \quad y = 1,2,...,m$$

parameters π_i and ξ_i are explained by two covariate vectors x_i=(1,x_{i1},…,x_{ip}) and w_i=(1,w_{i1},…,w_{iq}) linked thanks to the logistic functions

$$\pi_i = \frac{1}{1+e^{-\beta_0-\beta_1 x_{i1}-\ldots-\beta_p x_{ip}}} = \frac{1}{1+e^{-\beta_0 - x_i\beta}}$$

and

$$\xi_i = \frac{1}{1+e^{-\gamma_0-\gamma_1 w_{i1}-\ldots-\gamma_q w_{iq}}} = \frac{1}{1+e^{-\gamma_0 - w_i\gamma}} .$$

A CUB(p,q) model is intended to link parameters π and ξ, related to uncertainty and feeling respectively, to some covariates. In this regard the model is effective in detecting important subjects' or objects' attributes that are playing a significant role in the choice process. Piccolo (2003b) and D'Elia (2004) showed empirical evidences about CUB models with covariates. Since then others have applied the CUB model with covariates to real data set (Piccolo and D'Elia, 2008; Cicia *et al*., 2010; Corduas *et al*., 2013).

Among the CUB model extensions, one of them is searching for explaining a *shelter choice* behaviour (Iannario, 2012). As the estimated probabilities of CUB model do not fit well to the observed one, the explanation of such behavior could be an "over selection" of some scores in order to simplify a response. When there are reasons to suppose that Y=*c* has been over selected, *c* is said a *shelter choice*. For example, lazy persons could prefer median choice or a basic choice like "satisfy" is less demanding when raters are asked to choose among "satisfy", "very satisfy", "extremely satisfy" (Iannario, 2012). In order to take into account the "shelter behaviour" the following model extension has been developed to catch the *shelter effect*.

$$\Pr(\mathrm{Y} = y) = \pi_1 \left[\binom{m-1}{y-1} (1-\xi)^{y-1} \xi^{m-y} \right] + \pi_2 \left[\frac{1}{m}\right] + (1-\pi_1+\pi_2) D_y^{(c)}$$

where $\theta = (\pi_1, \pi_2, \xi)$ is the parameter vector and $D_y^{(c)}$ is a degenerate random variable with $D_y^{(c)} = 1$, if $y = c$ or $D_y^{(c)} = 0$, if $y \neq c$. The quantity $\delta = 1\text{-}\pi_1,\text{-}\ \pi_2$ characterizes the relative contribution of the shelter effect at Y=*c*. Iannario (2012) provides a detailed explanation of M-L estimate and inference for the CUB model with shelter effects.

Among the several extensions of CUB model (Iannario, 2013) we mention a Beta-binomial introduction instead of the shifted Binomial (called CUBE model) in order to grasp uncertainty, feeling and *overdispersion* (Iannario, 2014).

2.3 Fitting measures for CUB models

CUB models estimate probabilities given the parameter vector $\theta=(\pi, \xi)$. A good model should fit estimated probabilities $p_y(Y=y \mid \theta)$ to observed relative frequencies f_y. The absolute distance between estimated and observed probabilities is called *normalized dissimilarity index (Diss)* and it is considered a measure of the goodness-of-fit (Corduas *et al.*, 2009; Iannario, 2009).

$$Diss = 0.5\sum_{y=1}^{m}\left| f_y - p_y(\theta)\right|$$

We have a satisfactory fitting when $Diss \leq 0.1$ (Iannario, 2009) and an acceptable *Diss* index when $0.08 \leq Diss \leq 0.12$. The index indicates the proportion of respondents that should modify their choices in order to reach a perfect fitting (Corduas *et al.*, 2009). *Diss* index cannot be extended and it is not provided for CUB model with covariates. In this case the log-likelihoods are good candidates in order to compare CUB model without and with covariates (Piccolo, 2003b; Corduas *et al.*, 2009).

Log-likelihood differences are compared with the quintile of the χ^2 with degree of freedom as reported in Table 1.

Table 1: CUB model comparisons

CUB models	*Δ Log-likelihood*	*Degree of freedom*
CUB(p,0) vs CUB(0,0)	$2(\ell_{10}-\ell_{00})$	p
CUB(0,q) vs CUB(0,0)	$2(\ell_{01}-\ell_{00})$	q
CUB(p,q) vs CUB(0,0)	$2(\ell_{11}-\ell_{00})$	p+q

Among fitting measures Iannario (2009) considers also those based on *saturated* log-likelihood (ℓ_{sat}) and uninformative log-likelihood (ℓ_0) for saturated and null model respectively.

3. Case study: description and results

3.1 The survey on food packaging

The basic question behind the questionnaire on food packaging was to have an overview on satisfaction and pitfalls customers encounter when are facing with packaged food and when are buying foods at the grocery store.

The study has been conducted in Italy and in Austria and the main questions concerned some packaging attributes like the ability to preserve food from waste and the reseal-ability and easy peel of the packaging. About the buying behaviour, we asked to rate the attention paid to some aspects like brand, packaging, price, etc. Table 2 shows the main variables and the measurement scale coding.

Table 2: Variables and measurement scales

Variables	*Attributes considered*	*Coding*
Attention paid to some aspects at the grocery store	Nutrition facts, no GMO food, region of provenance, seasonality, brand, price, discounted price, innovation, advertisement, packaging.	1=minimum attention; 6= maximum attention
Satisfaction about some packaging attributes	Ability to preserve the food, reseal-ability and easy peel.	1=minimum satisfaction; 10=maximum satisfaction
Opinions about packaging reliability	Preservatives are the main responsible for the freshness, packaging is the main responsible for the freshness.	1=no at all; 10=definitely yes

The respondents were 209 in total. We asked demographic information and habit information, some of them introduced as covariates (Table 3) to improve the CUB model fitting.

Table 3: Covariates for CUB model

Covariates	*Description*	*Coding*
Sex	36% males; 64% females	0= male;1= female
Nationality	68% Italy; 32% Austria	0= Italy;1= Austria
Age	Min: 20; Max: 82	Continuous variable
Educational level	9.3% elementary; 28.1% intermediate; 43.4% high school; 19.2% graduate	1= elementary; 2= intermediate; 3= high school; 4= graduate
Income (monthly in Euros)	26.3% <800; 54.1% 800-1700; 13.9% 1800-2900; 5.7% >2900	1= <800; 2= 800-1700; 3= 1800-2900; 4= >2900
Purchase frequency of packaged fresh food	54% Rarely; 46% frequently	0= rarely; 1= frequently
Attention paid to: biodegradable packaging	66% yes; 34% no	0= yes; 1= no
Reseal-able packaging	66.4% yes; 33.6% no	0= yes; 0= no
Easy peel	55.6% yes; 44.4% no	0= yes; 0= no

3.2 Results

3.2.1 Variables related to salient aspects of grocery products

Respondents were asked to express their attention level about some aspects related to products bought at the supermarket. CUB models were applied to the attention variables and the results are reported in Table 4.

Table 4: CUB model estimates for attention variables

Coding	*Variable name*	π *(s.e.)*	ξ *(s.e.)*	*Diss*	$\ell_{(00)}$
1	al values	.157(.045)	.99(.054)	.078	-321.810
2	No GMO food	.254(.094)	.820(0.071)	.109	-326.081
3	Provenance	.387(.069)	.082(.030)	.057	-306.455
4	Seasonality	.345(.073)	.075(.037)	.093	-310.359
5	Brand	.327(.101)	.485(.054)	.045	-326.487
6	Price	.779(.056)	.146(.017)	.113	-265.980
7	Discounted price	.610(.061)	.060(.019)	.106	-260.895
8	Innovation	.053(.066)	.99(.231)	.087	-330.258
9	Advertisement	.489(.099)	.864(.044)	.166	-305.048
10	Packaging	.177(.098)	.339(.09)	.064	-329.843

Table 4 reports parameter estimates, dissimilarity index and log-likelihood. Dissimilarity indexes are lower than .12 except for "Advertisement". The advantage of CUB model is twofold: the first one is a parsimonious parameterization, the second one is the possibility to visualize parameters into two dimensional space (Figure 1).

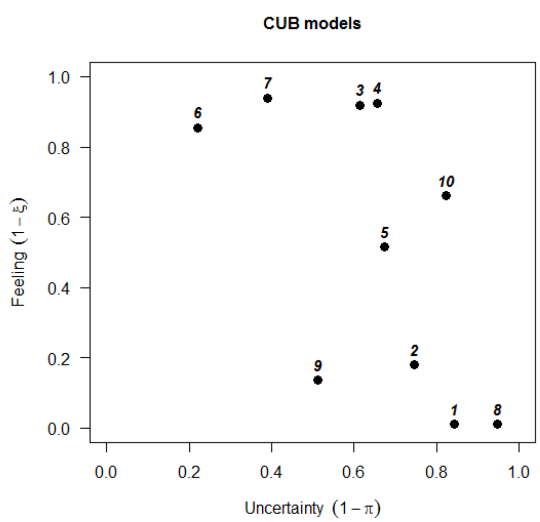

Figure 1: Attention variables as coded in Table 4 with increasing attention (feeling) and uncertainty when parameters tend to 1

Figure 1 has a great explanatory power so that we see the variable with the higher attention and the lowest uncertainty (6 = price) and the one in the opposite position (8 = innovation). Price and discounted price receive the highest attention and the lowest uncertainty. On the other hand respondents don't pay attention to nutrition facts and innovation but there are very different behaviours among respondents (high uncertainty). About the packaging attribute, they pay attention to it but the uncertainty is high: respondents display very different attention levels.

Table 5: Significant covariates for CUB models

Coding	*Variable name*	π	ξ	$2(\ell_{pq}\text{-}\ell_{00})$	*Df, p-value*
1	Nutrition facts	Gender	Education	14.074	2, < .0001
2	No GMO food	-	Nationality	4.174	1, < .05
3	Provenance	-	Age	27.560	1, < .0001
4	Seasonality	-	Age	29.586	1, <.0001
5	Brand	-	Gender	4.582	1, <.05
6	Price	-	Income	34.522	1, <.0001
7	Discounted price	-	Income	32.574	1, <.0001
8	Innovation	-	Age	19.560	1, <.0001

Log-likelihoods of CUB models with and without covariates are compared by Chi-square tests. *P*-values (Table 5) show that the log-likelihoods for CUB models with covariates increase. Significant covariates displayed in Table 5 are useful to understand how respondents differ when they pay attention to attributes at the supermarket. In order to have an indication of how covariates modify the parameter estimate, Table 6 describes the direction taken by $1\text{-}\xi$ (attention).

Table 6: Significant covariates for CUB models

Variable name	*Covariate*	*Coding*	*Attention*
No GMO food	Nationality	0;1	Increase
Provenance	Age	20-82	Increase
Seasonality	Age	20-82	Increase
Brand	Gender	0;1	Decrease
Price	Income	1;4	Decrease
Discounted price	Income	1;4	Decrease
Innovation	Age	20-82	Decrease

From Table 6 we see that older respondents paid more attention to the provenance of the food than younger respondents and that males (coded as 0) paid more attention to brand than females, or again that Italians paid less attention to no GMO food than Austrian.
CUB models with covariates that are significant for “nutrition facts” capture the behaviour of subgroups obtained by crossing the categorical variables “gender” and “education”. Thanks to the logistic functions reported in section 2.2 we estimate parameters π and ξ for each subgroup and ultimately we can also derive the estimated rating distributions.

Table 7: Gender and Education covariates for nutrition facts

Gender-Education	$1\text{-}\pi$	$1\text{-}\xi$	$E(Y)$
Male-Elementary	.604	0	2.51
Female-Elementary	.844	0	3.11

Male-Intermediate	.604	.0007	2.51
Female-Intermediate	.844	.0007	3.11
Male-High school	.604	.047	2.61
Female-High school	.844	.047	3.15
Male-Graduate	.604	.788	4.07
Female-Graduate	.844	.788	3.73

The expected value *E(Y)* in Table 7 can be derived as follows (Corduas *et al.*, 2009):

$$E(Y) = \pi(m-1)\left(\frac{1}{2}-\xi\right)+\frac{(m+1)}{2}\ .$$

Male responses are less spread than the female ones (lower uncertainty) and educational level seems to affect the attention to the nutrition facts. The expected values are quite similar for females whereas males display a clear gap between "high school" (*E(Y)*=2.61) and "graduate" (*E(Y)*=4.07) conditions.

Estimated probability distributions conditioned to the covariates "gender" and "educational level" are displayed in Figure 2.

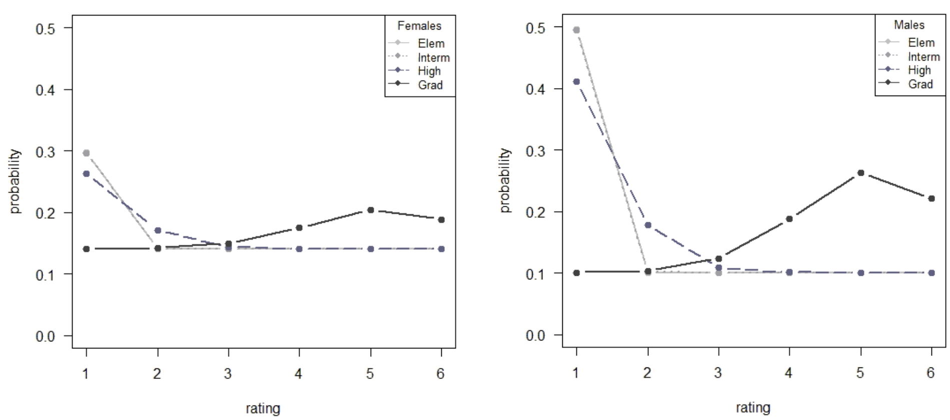

Figure 2: Estimated probability distributions of responses to "nutrition facts" for females (left panel) and males (right panel)

The female distributions are flatter than the male ones indicating a higher uncertainty, whereas for both groups we cannot discriminate between "elementary" and "intermediate" because the distributions are overlapped and "graduate" clearly is connected with a higher level of attention to the nutrition facts.

CUB models have the capacity to grasp a large variety of choice behaviours. In order to give a demonstration of such power, the response to the variable "advertisement" seems to be a good candidate. CUB model applied to that variable does not fit well (*Diss*= .166) and when we look at the observed relative frequencies we see without a doubt a Mode at y=1. The score "1" receives an upward number of choices that could indicate a shelter choice at c= 1. So we test the hypothesis that some respondents have chosen the lowest score in order to simplify the task applying the CUB model with shelter choice at c= 1 (Table 8).

Table 8: CUB model without and with shelter for "advertisement"

Model	*π (s.e.)*	*ξ (s.e.)*	*Dissimilarity*	*Log-likelihood*
CUB	π = .489(.099)	ξ = .864(.044)	.166	-305.048
CUB+shelter	π_1 = .504(.075)	ξ = .621(.037)	.021	-293.032
	π_2 = .209(.082)			
	π^* = .707(.109)			

The shelter effect δ = .285(.044) is significant and the fitting improves a lot (Dissimilarity index decreases from .166 to .021). With a shelter effect explaining for an over-selection of score "1", both ξ and π exhibit a considerable displacement towards higher attention (from .135 to .378) and lower uncertainty (from .511 to .292).

3.2.2 The satisfaction and opinion variables

Respondents were asked also to assess their satisfaction concerning some packaging characteristics such as the ability to preserve the food, the reseal-ability and easy peel and to express their opinion about two factors (packaging and preservatives) that are involved in the food preservation. Opinion variables need further elucidation: we investigate the belief respondents have on packaging and preservatives regarding food preservation.

CUB model applications to satisfaction and opinion variables (Table 9) show acceptable fitting indexes for "easy peel" and "preservatives".

Table 9: CUB model estimates for satisfaction and opinion variables

Coding	*Variable name*	*π (s.e.)*	*ξ (s.e.)*	*Diss*	$\ell_{(00)}$
1	Preservation	.695(.065)	.366(.018)	.123	-384.280
2	Reseal-ability	.668(.066)	.333(.019)	.122	-383.851
3	Easy peel	.628(.072)	.418(.021)	.085	-393.884
4	Preservatives	.536(.071)	.293(.023)	.104	-399.914
5	Packaging	.553(.074)	.322(.025)	.164	-400.412

The two dimensional space (Figure 3) suggests two clusters, the satisfaction variables (1, 2, 3) and the opinion variables (4, 5) with the second ones in the upper right corner of the space. The upper right corner indicates high uncertainty and high feeling.

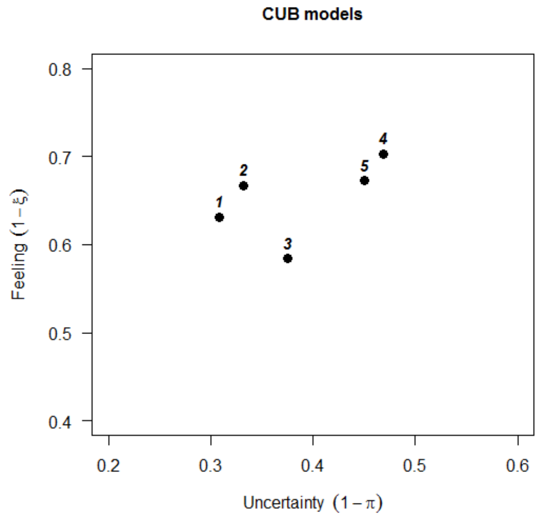

Figure 3: Satisfaction and opinion variables as coded in Table 9 with the increasing satisfaction/belief (feeling) and uncertainty when parameters tend to 1

Since the model does not fit well to the observed data (see *Diss* indexes in Table 9), we hypothesize there are groups of respondents that behave in a different way. We introduced covariates described in Table 3 and report the results in Table 10.

Table 10: Significant covariates for CUB models

Coding	*Variable name*	π	ξ	$2(\ell_{pq}-\ell_{00})$	*Df, p-value*
1	Preservation	Age	Purchase frequency	35.584	2, < .0001
2	Reseal-ability	-	Nationality		
		-	Purchase frequency		
		-	Reseal-able packaging	62.471	3, <.0001
3	Easy peel	-	Nationality		
		-	Attention to easy peel		
		-	Reseal-able packaging	63.335	3, <.0001
4	Preservatives	Reseal ability attention	Income	50.129	2, <.0001
5	Packaging	-	Nationality		
		-	Purchase frequency	37.641	2, <.0001

For each variable, the log-likelihood decreases and CUB model improves. Significant clusters gave a different grade of “satisfaction” or they have different opinions on how preservatives and packaging affect the food preservation.
The satisfaction about food preservation, for instance, is not homogeneous among respondents so we are considering “age” and “purchase frequency” as significant covariates for uncertainty and feeling (satisfaction) respectively and ultimately as

significant subgroups that are satisfied in different ways. The continuous variable “age” has been discretized and probability distributions have been estimated (Figure 4).

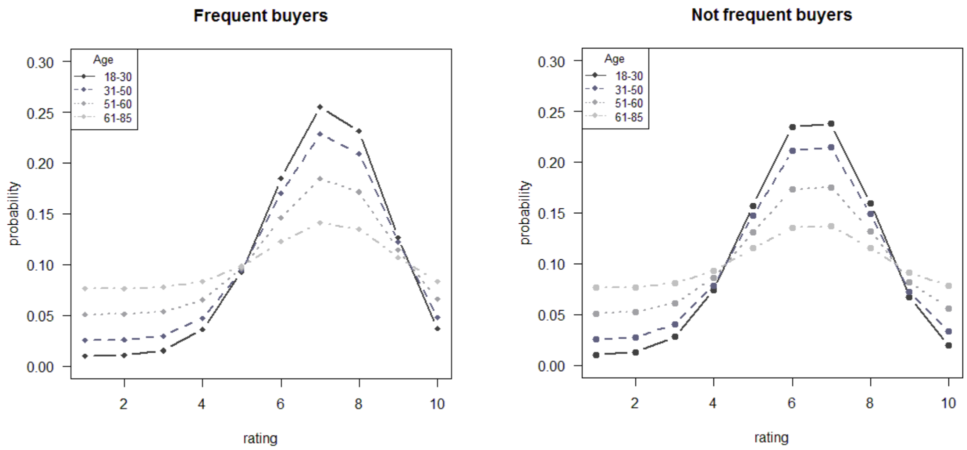

Figure 4: Estimated probability distributions of responses to the satisfaction variable “food preservation” for frequent buyers of packaged products (left panel) and not frequent buyers (right panel)

The older respondents have a flatter distribution with respect to the younger ones, the former have a great uncertainty, whereas frequent buyers are more satisfied that not frequent buyers with respect to packaged fresh foods.

4. Discussion and conclusions

CUB models have been developed with the aim to explain the psychological mechanism underlying the choice process (D’Elia, 2003; D’Elia and Piccolo, 2005). Several model extensions have been developed (Iannario, 2013) in order to take into account the multifaceted individual choice behaviour. Within the framework of preference evaluation CUB models are suited to many real cases (Piccolo and D’Elia, 2008; Corduas *et al.*, 2009; Cicia *et al.*, 2010; Iannario *et al.*, 2012), confirming CUB models as useful and theorem based (Iannario and Piccolo, 2014) statistical models.

Feeling and Uncertainty are supposed to be latent variables involved in the choice process of an item. The interpretation is very flexible with the “feeling” parameter explaining for the construct (satisfaction, preference or attention) the measurement scale is supposed to measure.

A real case study has been conducted by a survey on food packaging. In particular CUB models have been applied to specific questions: level of attention paid to specific characteristics at the grocery store and satisfaction level and opinions about some food packaging characteristics. Results showed that “price”, “discounted price”, “seasonality” and “provenance” have the highest attention level with different level of uncertainty. The variable “packaging” has received a medium-high grade of attention but the uncertainty was high indicating respondents pay very different level of attention towards “packaging” when they buy products at the supermarket. There were not significant covariates for “packaging” so the high uncertainty could indicate an attribute (packaging) respondents are not used to evaluate or to consider when they buy products at the supermarket. This could mean a low knowledge of the real utility/importance of packaging.

The introduction of covariates showed that some demographic characteristics are linked to specific variables. For instance, males pay more attention to brands than females or the seasonality and the provenance of the product are linked to “age” so that we see older respondents more interested in these characteristics than younger respondents.

A CUB model with shelter effect has been adopted for “advertisement”. With a shelter at c=1 the model fitting improved revealing that respondents tend to simplify the answer. Maybe the choice of what grade measures the right attention paid to products that are advertised is not simple.

CUB models for satisfaction variables showed a higher satisfaction for the preservation of the food and the reseal-ability of the packaging with respect to packaging with easy peel. Some covariates are involved in explaining, for instance, that the frequency of purchasing packaged food products is linked to the satisfaction for food preservation.

Concluding, the statistic models called CUB have proven to be very useful, flexible and constantly evolving.

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