Estimation of Gross Output of the Manufacturing Sector – A Longitudinal Approach

Sanping Chen[1] and Cindy Ubartas[2]

Abstract
Gross Output (GO) is the critical component for the "value-added" method to measure the Gross Domestic Product (GDP). Traditionally, both GO and GDP are calculated and published by statistical agencies without any quality measures. In this work, we establish a new longitudinal method for estimating Canada's manufacturing sector GO that produces variance estimates in a well-structured framework.

Key Words: Gross output, gross domestic product, longitudinal method, quality measure, manufacturing sector

1. Introduction

There are three prevailing methods for calculating the Gross Domestic Product (GDP) (Landefeld *et al*., 2008). The first is the value-added approach which defines the gross domestic output as

$$\text{GDP} = \text{gross output} - \text{intermediate inputs.}$$

Consequently, Gross Output (GO) is a critical economic concept in National Accounts. According to the definition in the *System of National Accounts 1993* published by the United Nations (p. 154), GO, nowadays often simply called output, "consists only of those goods or services that are produced within an establishment that become available for use outside that establishment." Roughly speaking, GO represents the total value of sales by producing enterprises in a reference period before subtracting the value of intermediate goods used up in production (gross sales less change in inventories). Given that National Accounts are typically sectorized, GO is usually also calculated for different sectors of the economy.

Together with GDP and other economic indicators, GO is estimated from multiple economic data, not necessarily of the same source, and its derivation is often not a straightforward and transparent process. This makes it usually difficult to assess the quality or accuracy of the final estimates. According to Landefeld *et al*. (2008),

> "Because the GDP estimates are based on administrative records and other nonsample data, confidence intervals and standard errors cannot be used to measure accuracy."

The situation has been the same with GO. Partly due to this lack of quality measures, sector-specific GO numbers are produced and used internally at Statistics Canada, but not publically disseminated.

[1] Business Survey Methods Division, Statistics Canada, Ottawa, Ontario K1A 0T6, Canada.
[2] Household Survey Methods Division, Statistics Canada, Ottawa, Ontario K1A 0T6, Canada.

An acute demand for rigorous quality measure of GO arises in our work in the field of energy statistics. A key measure of energy efficiency is the Energy Intensity Index at the detailed industrial sector level, roughly the amount of energy consumed for producing a monetary unit of GO. These indices would provide critical information for comparing and tracking energy efficiency across different industries as well as countries. With the growing concern over climate changes caused by human energy consumption, the need to provide reliable energy intensity indices at industrial sectors also increases. The lack of quality measures for GO estimate in each industry sector thus impedes the production of reliable sectorized energy intensity estimates.

2. Macro-estimates and the lack of quality measures

Within Statistics Canada, the key to calculating sectorized GO is the Monthly Survey of Manufacturing (MSM). The MSM is longitudinal in nature. Its monthly sample is refreshed rather infrequently, about once every five year. During the lifetime of the sample, all deaths are kept, and births are sampled proportionally to maintain the existing sample weights. In order to lessen response burden and to lower collection costs, the smallest units of the MSM population, approximately the bottom 5% based on the dollar value of sales of goods manufactured for each province, are excluded from being surveyed (the non-sampled or take-none component). The MSM publishes the values in Canadian dollars of sales of goods manufactured, inventories and orders. Its results are thus the primary inputs to Canada's key economic indicators, including GO.

The general formula for annual GO for an industrial sector can be formatted based on first principles as follows:

$$GO = \sum_{m=1}^{12} \frac{1}{I_m} \{Y_m^1 + (Y_m^2 - Y_{m-1}^2) + (Y_m^3 - Y_{m-1}^3)\}\ , \qquad (1)$$

where

$Y_m^1 =$ total sales for month m,
$Y_m^2 =$ total value of unfinished products for month m,
$Y_m^3 =$ total value of goods or work in process, estimated at end of month m,

and

$I_m =$ Industrial Product Price Index (IPPI) for the sector of interest for month m.

A technical note here is that month $m = 0$ refers to the last month of the previous calendar year.

The first three monthly numbers Y_m^1, Y_m^2, Y_m^3 are provided by the MSM, and the corresponding IPPI is calculated from other data sources. Therefore, for all sectors, GO can be calculated based on cumulated MSM monthly results throughout the reference year.

In addition, because there is a delay in value realization of the inventories, more sophisticated formulas are often used, which apply sector-specific delayed inflation deflators for Y_m^2 and Y_m^3 to reflect more timely “transaction prices”. This leads to slightly different but more accurate GO estimates.

Besides the price indices, the above formula for GO involves a total of 38 industrial sector measurements obtained from 13 monthly surveys. While price indices are usually regarded as fixed numbers, the other terms are all survey estimates based on statistical samples taken at different time points with various sampling errors. Thus, the overall quality of the final GO estimate is rather difficult to assess.

3. A time series approach based on macro-estimates

For our needs, price indices can be assumed to be deterministic without losing much in our variability assessment. The formula for GO written as a derived survey estimate based on 38 monthly estimates is then given by:

$$\widehat{GO} = \sum_{m=1}^{12} \frac{1}{I_m}\left\{\hat{Y}_m^1 + \left(\hat{Y}_m^2 - \hat{Y}_{m-1}^2\right) + \left(\hat{Y}_m^3 - \hat{Y}_{m-1}^3\right)\right\}. \quad (2)$$

These monthly survey estimates from MSM can be regarded as a multivariate time series

$$\left\{(\hat{Y}_m^1,\ \hat{Y}_m^2, \hat{Y}_m^3),\ \ m = 0,1,2,\dots,12\right\}.$$

Therefore, the variance of the estimated GO can be expressed as a straightforward linear combination of individual variance terms var($\hat{Y}_m^j$), and covariance (including auto-covariance) terms cov($\hat{Y}_m^j, \hat{Y}_{m+p}^k$), j,k=1,2,3. The number of these terms in the variance formula can be greatly reduced if we further impose a stationary assumption of the multivariate time series.

These variance and covariance terms can be estimated through time series models. In fact, most univariate variance and covariance terms can be readily extracted from existing time series diagnostics and seasonality analysis of individual MSM series. However, a key conceptual obstacle also arises: The quality measure of the estimated GO we are after is the variance due to sampling error (the uncertainty caused by using a monthly sample to represent the population of interest of the industry sector), whereas the residual terms in time series models that would provide the required variance and covariance estimates represent the combined component of both sampling error and the inherent time series variation term. It is technically very difficult to separate and estimate these two error terms. In addition, due to the unknown nature of the time series component, the combined error term may not necessarily be a conservative representation of the sampling error term. The time series approach was thus abandoned.

4. A longitudinal approach based on micro-data

4.1. General concept

Let us re-examine the calculation formula (2) of GO. It can be reformulated as

$$\begin{aligned}
\widehat{GO} &= \sum_{m=1}^{12} \frac{1}{I_m}\left\{\hat{Y}_m^1 + \left(\hat{Y}_m^2 - \hat{Y}_{m-1}^2\right) + \left(\hat{Y}_m^3 - \hat{Y}_{m-1}^3\right)\right\} \\
&= \sum_{m=1}^{12} \frac{1}{I_m}\left\{\hat{Y}_m^1 + \hat{Y}_m^2 + \hat{Y}_m^3\right\} - \sum_{m=0}^{11} \frac{1}{I_{m+1}}\left\{\hat{Y}_m^2 + \hat{Y}_m^3\right\} \\
&= \frac{1}{I_{12}}\left\{\hat{Y}_{12}^1 + \hat{Y}_{12}^2 + \hat{Y}_{12}^3\right\} + \sum_{m=1}^{11}\left\{\frac{1}{I_m}\hat{Y}_m^1 + \left(\frac{1}{I_m} - \frac{1}{I_{m+1}}\right)\left(\hat{Y}_m^2 + \hat{Y}_m^3\right)\right\}
\end{aligned}$$

$$-\frac{1}{I_1}\left\{\hat{Y}_0^2+\hat{Y}_0^3\right\}. \tag{3}$$

Therefore, let S_m be the MSM sample of month *m*. The monthly estimate is then

$$\hat{Y}_m^k=\sum_{i\epsilon S_m} w_{mi} y_{mi}^k,\ k=1,\ 2,\ 3, \tag{4}$$

where w_{mi} is the estimation weights at month *m* attached to unit *i*. Let us recall the longitudinal nature of MSM, particularly its practice of keeping deaths and sampling births proportionally to maintain the existing sample weights. We thus have almost always

$$w_{mi}=w_i,\ m=0,1,2,\cdots,12. \tag{5}$$

By plugging (4) into (3), and factoring out the above common weights, we obtain

$$\widehat{GO}=\sum_{i\epsilon S} w_i\times\left\{\frac{1}{I_m}\left(y_{12i}^1+y_{12i}^2+y_{12i}^3\right)+\sum_{m=1}^{11}\left[\frac{1}{I_m}y_{mi}^1+\left(\frac{1}{I_m}-\frac{1}{I_{m+1}}\right)\left(y_{mi}^2+y_{mi}^3\right)\right]-\frac{1}{I_1}\left(y_{0i}^2+y_{0i}^3\right)\right\}, \tag{6}$$

where

$$S=\bigcup_{m=0}^{12} S_m,$$

and

$$y_{mi}^k=0\ \text{ if }\ i\notin S_m\ .$$

In other words, by defining a new longitudinal variable

$$z_i=\frac{1}{I_m}\left(y_{12i}^1+y_{12i}^2+y_{12i}^3\right)+\sum_{m=1}^{11}\left[\frac{1}{I_m}y_{mi}^1+\left(\frac{1}{I_m}-\frac{1}{I_{m+1}}\right)\left(y_{mi}^2+y_{mi}^3\right)\right]$$
$$-\frac{1}{I_1}\left(y_{0i}^2+y_{0i}^3\right), \tag{7}$$

the calculation of (6) becomes a standard estimation problem of a population total $Z=\sum z_i$, with the joint sample *S*:

$$\widehat{GO}=\sum_{i\epsilon S} w_i\cdot z_i, \tag{8}$$

The standard formula can then be applied for calculating the variance of the estimated GO due to sampling for a stratified random sample (Cochran, 1977). This variance can be estimated using:

$$\widehat{var}\left(\widehat{GO}\right)=\sum_h N_h w_i\left(1-\frac{1}{w_i}\right)s_h^2, \tag{9}$$

where s_h^2 is the estimated population variance of *z*. Please note the finite-population correction factor $1-1/w_i$ in (9), where $1/w_i$ represents the sampling ratio f of each stratum.

If the original GO estimate uses the more sophisticated formula with time-delayed inflation deflators on inventory items, then the definition of the longitudinal variable z_i can be modified accordingly. Because month-to-month price changes in most sectors are usually very small, we will present our results with the basic formulas (6) and (7).

4.2. Initial treatment of non-sampled units

The new longitudinal approach described above meets our initial objective of finding a rigorous measure of sampling error for GO. However, before directly providing the GO estimates in (2) based on macro MSM estimates, we first need to account for the non-sampled (take-none) component of the MSM population.

Traditionally, the contribution of this portion of the population was accounted for by macro adjustment. During estimation, the exact fraction of the non-sampled portion in terms of sales with respect to the entire frame was calculated based on other (mostly administrative) sources for each domain. This fraction was then used to inflate the corresponding macro estimate to obtain the correct population total. This process is referred to as calibration (Deville and Särndal, 1992).

In theory, this adjustment factor is usually combined with the sampling weight at the micro level. However, with MSM, this factor is rather time-sensitive and varies considerably from month to month. Therefore, factoring this adjustment into the estimation weight w_{mi} would violate our basic assumption (5) of a constant estimation weight. To overcome this difficulty, we decided to use the adjustment factor to weight up the response value y_{mi}^{k} instead, thus preserving assumption (5). Clearly, this affects variance estimation since the adjustment factor is a random quantity that should be considered as such computation of the variance of the estimates.

In the process of our study, the MSM has introduced a new approach for the non-sampled portion of the survey population. The survey now mass-impute at micro-level all small units in the non-sampled portion of the survey frame based on administrative (mostly tax) data. Therefore the above treatment for estimating GO becomes unnecessary with the new MSM data, but would still be needed for historical data.

4.3. Accounting for variance due to imputation

The new MSM treatment of the non-sampled components also exemplifies another technical issue, namely how to account for the variance due to imputation. Because they contribute very little to the variance, the mass-imputed small units can be easily excluded from the variance calculation. On the other hand, the MSM, as all other business surveys, uses imputation for non-response. Ignoring the fact that many MSM values were imputed would lead to serious underestimation of the variance.

For instance, in several small industrial sectors, MSM represents a mini-census, with the sampling weight of all units being 1. This leads to zero sampling variance of the estimated GO by the variance formula (9), which is certainly not a realistic quality measure if a non-negligible part of the units have been imputed. It is obvious that a reasonable variance estimate would have to discount the information provided by imputed MSM values.

The longitudinal GO variable z_i in (7) is derived from a total of 38 original MSM variables. If we require all these 38 values to be true survey responses for z_i to be non-imputed, then we would complicate variance estimation by leaving out a great number of records that were only partially based on imputed MSM variables.

In the end, an upper bound for the variance accounting for imputation is obtained by adjusting the sampling ratio in the finite-population correction factor from $f = \frac{1}{w_i}$ to

$$f = \frac{r_i}{w_i}, \tag{10}$$

where r_i is the average response rate of the MSM variables used for that stratum. In other words, we regard r_i/w_i, not $1/w_i$, as the effective sampling ratio of each stratum. In actual application, we used the response rate of the main variable, i.e. sales of goods manufactured. Meanwhile, the stratum variance is estimated by the full longitudinal sample, because such an estimate would still be unbiased if imputation is unbiased.

5. Application of the method

We have applied the new longitudinal method for calculating both the GO and its variance for the eighteen most important industries for the Canadian Industry Program for Energy Conservation (CIPEC) over the years 2010-2012. The following is a summary of the results. For easy comparisons, we use the estimated coefficient of variation (CV) instead of the estimated variance or standard error.

5.1. Variance estimation and comparison with macro-estimates

Table 1: Summary of the CV of Longitudinal GO estimates and their relative difference from macro-estimates

CV and relative difference	Industries	
CV always < 10% Relative difference < 1%	Total manufacturing Chemicals Dairy* Electrical and Electronics Food and Beverage	Lime Plastic Pulp and Paper Transportation Wood Products
CV between 10% and 20% Relative difference < 1%	Aluminum* Brewery Cement Fertilizer	Petroleum Products Rubber Textiles*
CV around 25% Relative difference < 1%	Steel	
CV always < 10% Relative difference > 1%	Foundry	

* relative difference about 2% for one year.

Other than the small Foundry industry that underwent some NAICS transitions in these years, there is little difference between the longitudinal estimates and the macro estimates. It should be added that the small differences between the two methods do not show any fixed patterns from year to year. Other than the highly skewed steel industry with rather poor monthly survey response rates (~50%), the new longitudinal method resulted in mostly acceptable quality measures in terms of CV.

The summary Table 1 has already factored in the variance due to imputation as accounted for by the effective sampling ratio (10) in the finite-population correction. The following table provides examples of this adjustment, which is rather conservative, given the high percentages of reported monthly tax records used in imputing the survey data.

Table 2: Examples of CV changes after accounting for imputation

Industry	Unadjusted CV	Adjusted CV*
Total manufacturing	4%	5.5%
Brewery	0%	21.5%
Cement	0%	15.1%
Chemicals	0.8%	7.9%
Dairy	5%	7.2%
Electrical & electronics	3.3%	6.0%
Fertilizer	12.5%	20.3%
Lime	0%	8.1%
Petroleum products	0%	21.9%

* Based on new finite-population correction factor (10)

5.2 Validity of basic assumptions

In addition to the assumption that all business units in the monthly survey sample have had constant weights over the 13 months, our new longitudinal approach assumes tacitly that these units have had constant NAICS, and survey stratification classification as well.

In reality, all three assumptions hold only approximately. Indeed we have seen that in a small sector (Foundry), the violation of these assumptions may lead to significant changes in GO estimate. Nonetheless, in most cases, the overall validity of these assumptions turns out to be fairly good and robust.

Table 3: Validity of the assumption of constant weights

Year	% constant weights
2010	97.60%
2011	97.60%
2012	95.00%

Table 4: Validity of the assumption of constant NAICS

Year	# changes	% changes	% constant NAICS
2010	357	0.43%	99.57%
2011	311	0.39%	99.61%
2012	220	0.26%	99.74%

Table 5: Validity of the assumption of constant stratification

Year	# changes	% changes	% constant strata
2010	108	0.13	99.87%
2011	53	0.07	99.93%
2012	75	0.09	99.91%

From these tables, it is apparent that the great majority of the business units meet the three key assumptions. Further examination reveals that most violations occur with small- and medium-size units. While we are exploring a theoretic framework for additional variance components or biases caused by these minor violations, preliminary simulations indicate they cause little additional error in the GO estimates. Moreover, the great majority of these violations represent one-time changes in the period. This can be treated as a pseudo-death/birth by adding a new unit after such a change, and zero-filling the "post-death" and "pre-birth" values. With this modification, our longitudinal approach would still provide an approximately unbiased GO estimate, and a conservative variance estimate.

6. Summary and concluding remarks

Our results have demonstrated that, for the first time, the variance of GO in manufacturing sector can be estimated in a well-structured framework. In addition, for most sectors, GO estimates based on microdata are very close to that derived from macro MSM estimates. Meanwhile, the quality measures provided therein will facilitate the public release of GO, as well as their applications in other fields, with sector energy intensity/efficiency as a particular example.

The key assumptions for our method hold fairly well, and most violations can be accommodated within the new longitudinal methodology by defining pseudo-deaths/births.

As for future work, we are in the process of extending the longitudinal method to other industrial sectors, and are also exploring the possibility of developing variance estimates for intermediate inputs. The latter, if successful, would lead to quality measures for GDP estimates for some economic sectors for the first time.

References

Cochran, William G. *Sampling Techniques*, 3rd ed. New York: Wiley, 1977.

Deville, Jean-Claude, and Carl-Erik Särndal, "Calibration Estimators in Survey Sampling," Journal of the American Statistical Association 87(1992): 376-382.

Landefeld, J. Steven, Eugene P. Seskin, and Barbara M. Fraumeni. "Taking the Pulse of the Economy: Measuring GDP," *Journal of Economic Perspectives* 22(2008): 193–216.

Moyer, Brian C., Mark A. Planting, et al. "Improved Annual Industry Accounts for 1998–2003: Integrated Annual Input-Output Accounts and Gross-Domestic-Product-by-Industry Accounts," *Survey of Current Business*, June 2004, 21-57.

United Nations Department for Economic and Social Information and Policy Analysis, et al. *System of National Accounts 1993*, Brussels/Luxembourg, 1993.

Yuskavage, Robert E. and Yvon H. Pho, "Gross Domestic Product by Industry for 1987–2000: New Estimates on the North American Industry Classification System," *Survey of Current Business*, November 2004, 33-53.